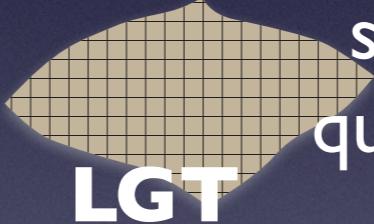
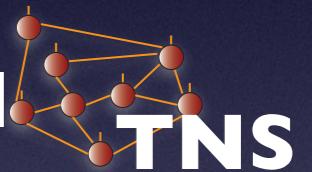


# TENSOR NETWORK STATES FOR LATTICE GAUGE THEORIES



about classical  
simulations of a  
quantum problem



Mari-Carmen Bañuls

with K. Cichy (Poznan), K. Jansen (DESY), H. Saito,  
J.I. Cirac (MPQ), S. Kühn (Perimeter)



Max-Planck-Institut

für Quantenoptik

(Garching b. München)



Würzburg, 2.8.2018

# TENSOR NETWORKS

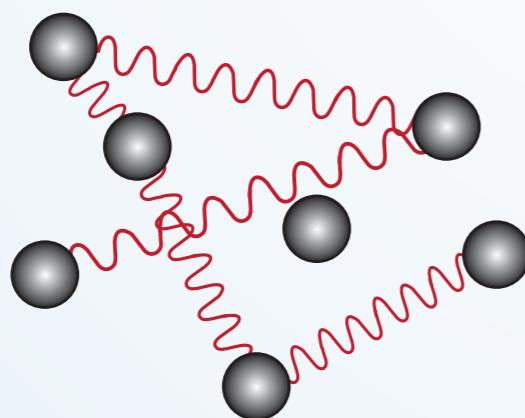
# WHAT ARE TNS?

- TNS = Tensor Network States

A general state of the N-body Hilbert space has exponentially many coefficients

$$|\Psi\rangle = \sum_{i_j} c_{i_1 \dots i_N} |i_1 \dots i_N\rangle$$

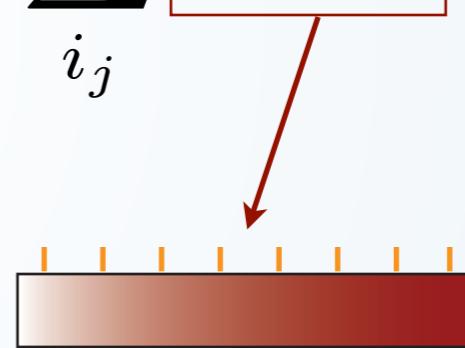
$N$



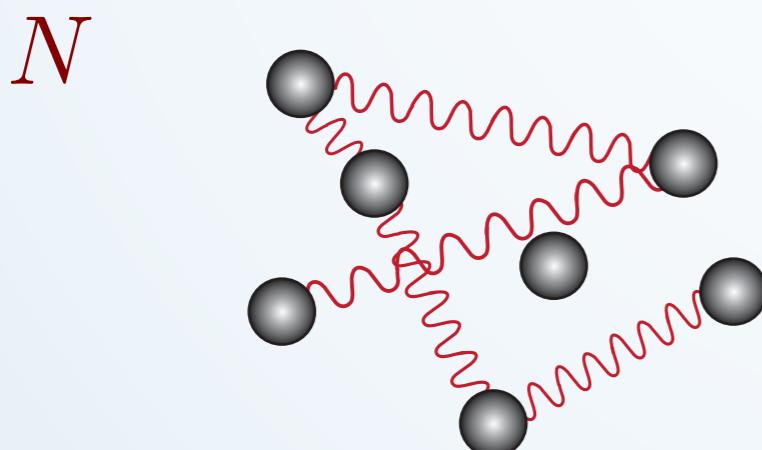
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N-legged tensor

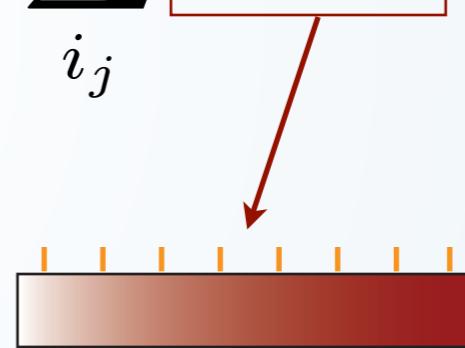


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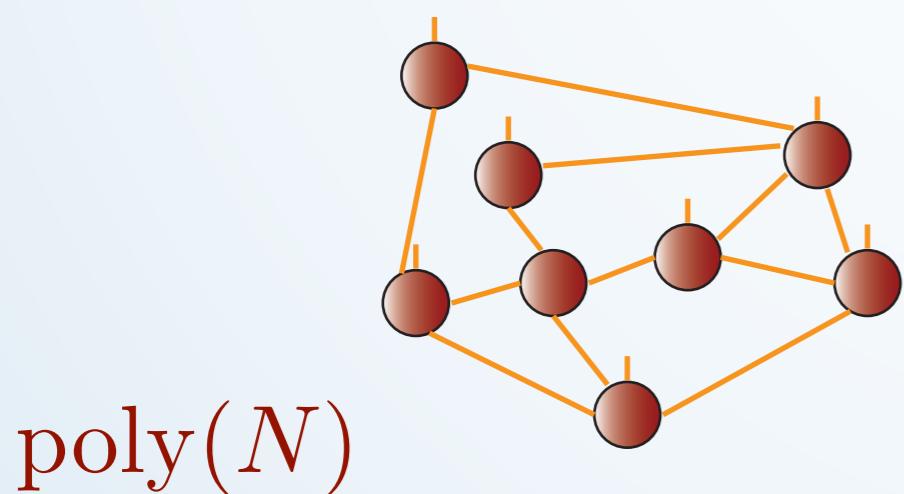
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N-legged tensor

ATNS has only a polynomial number of parameters

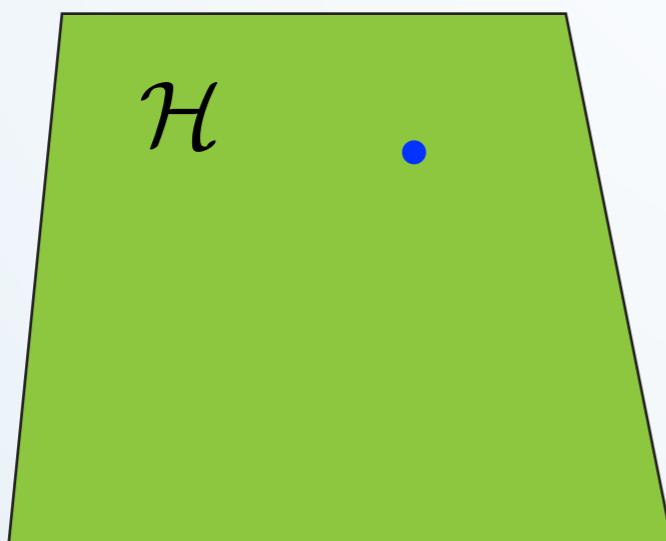
$d^N$



# WHY SHOULD TNS BE USEFUL?

States appearing in Nature are peculiar

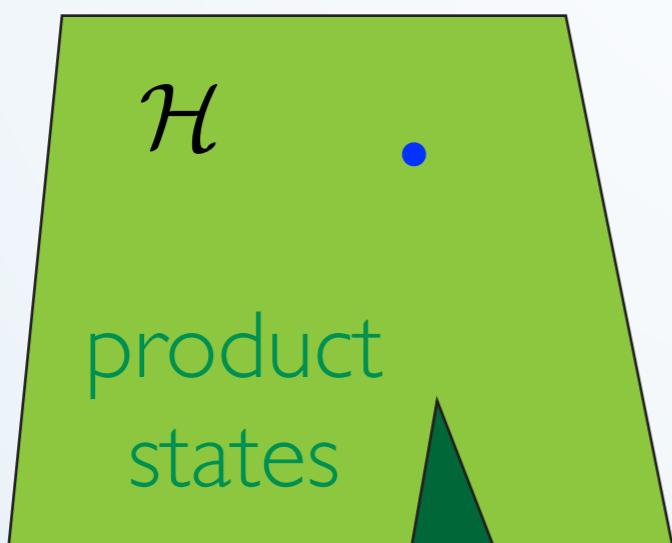
State at random from  
Hilbert space is not  
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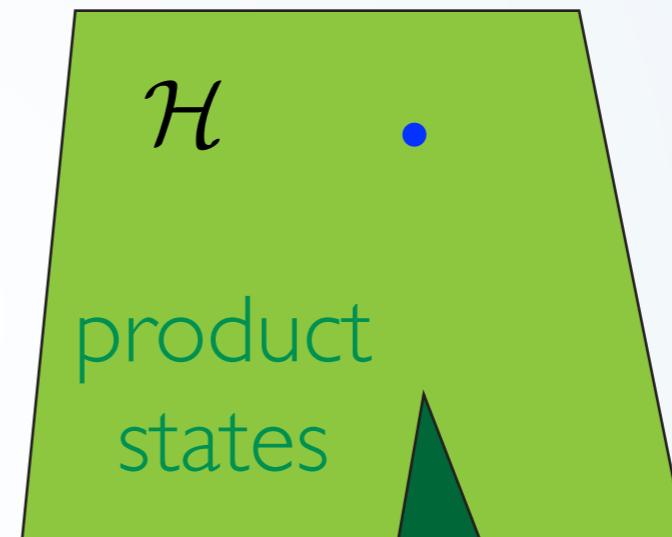
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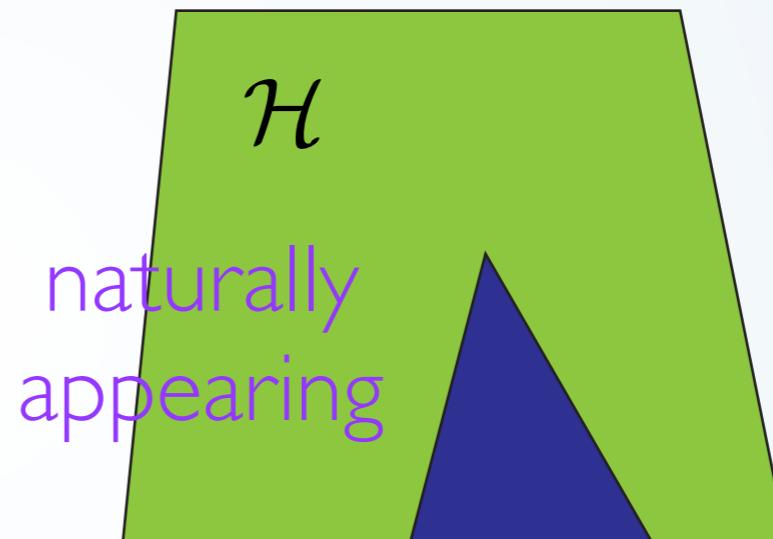
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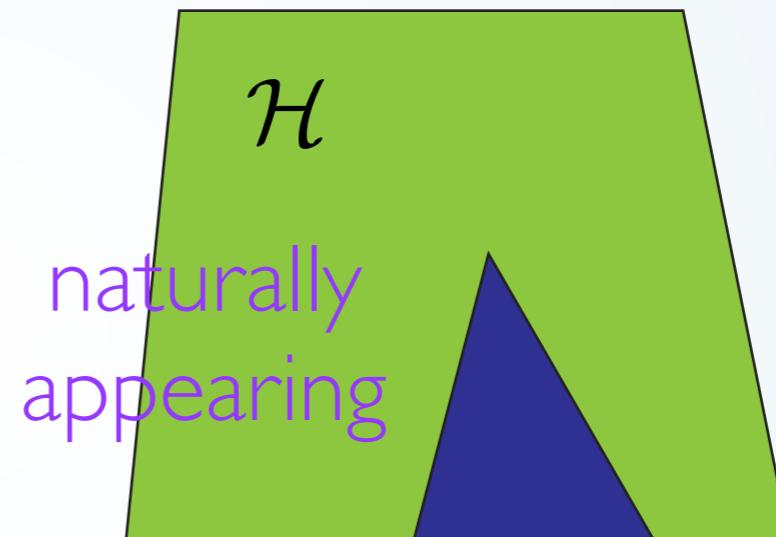


We look for the  
particular corner of the  
Hilbert space

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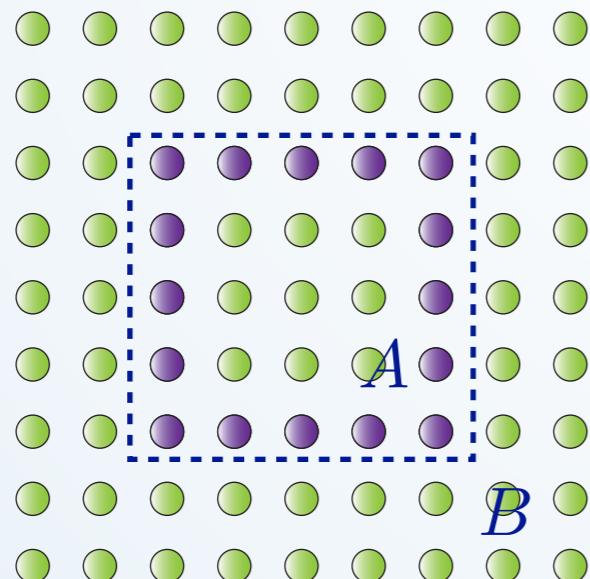
We look for states with  
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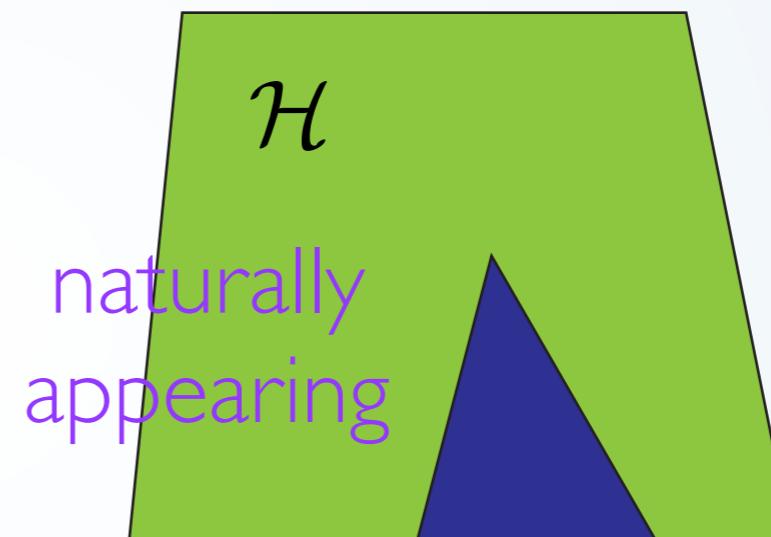
State at random from Hilbert space is not close to product

area law



Hastings 2007

Calabrese, Cardy 2004; Wolf 2006



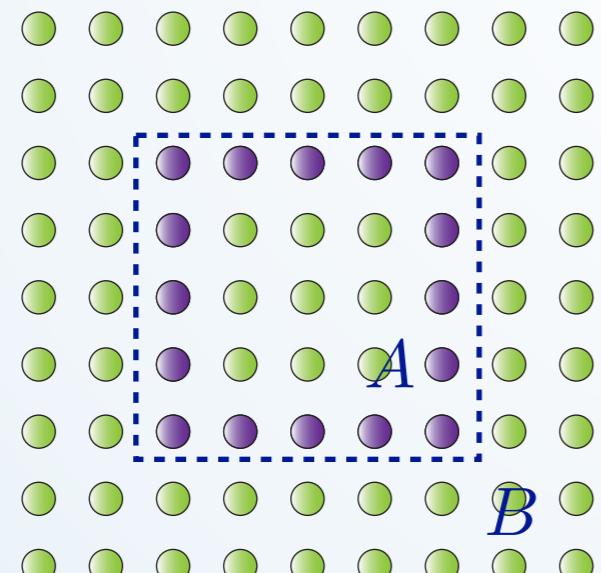
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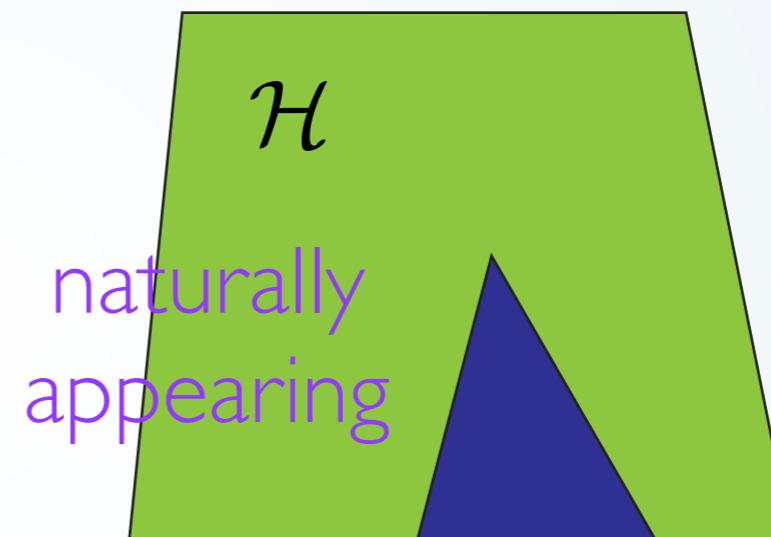
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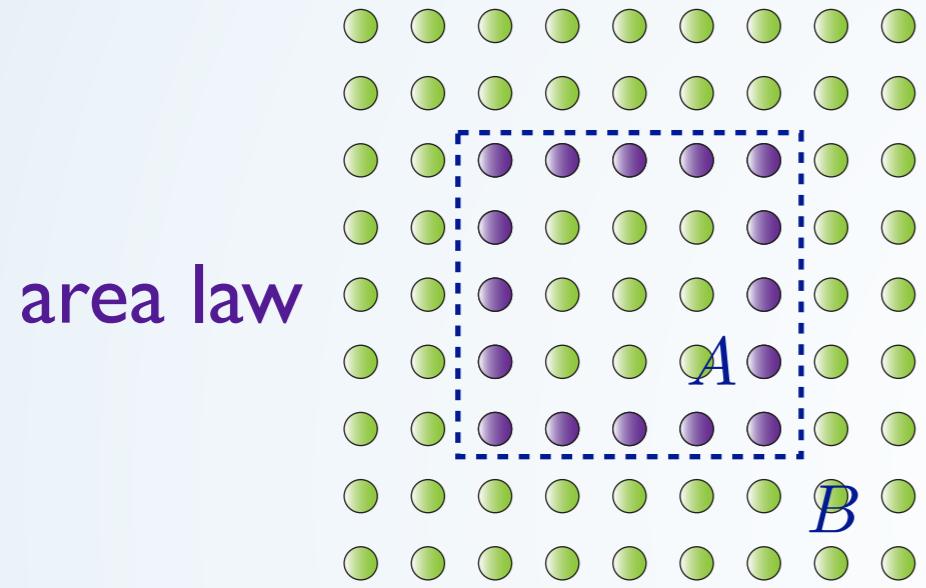
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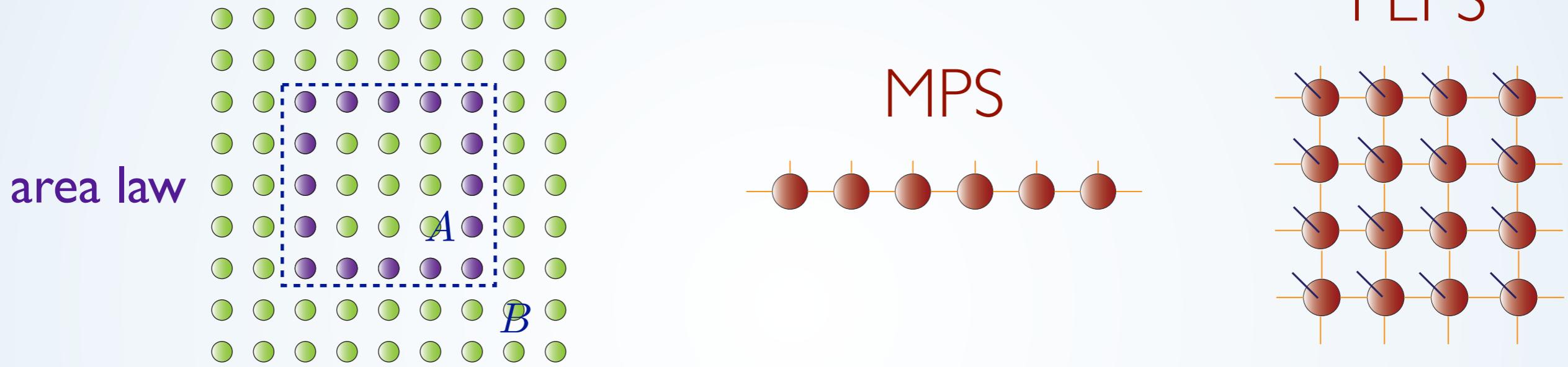
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TNS = entanglement based ansatz

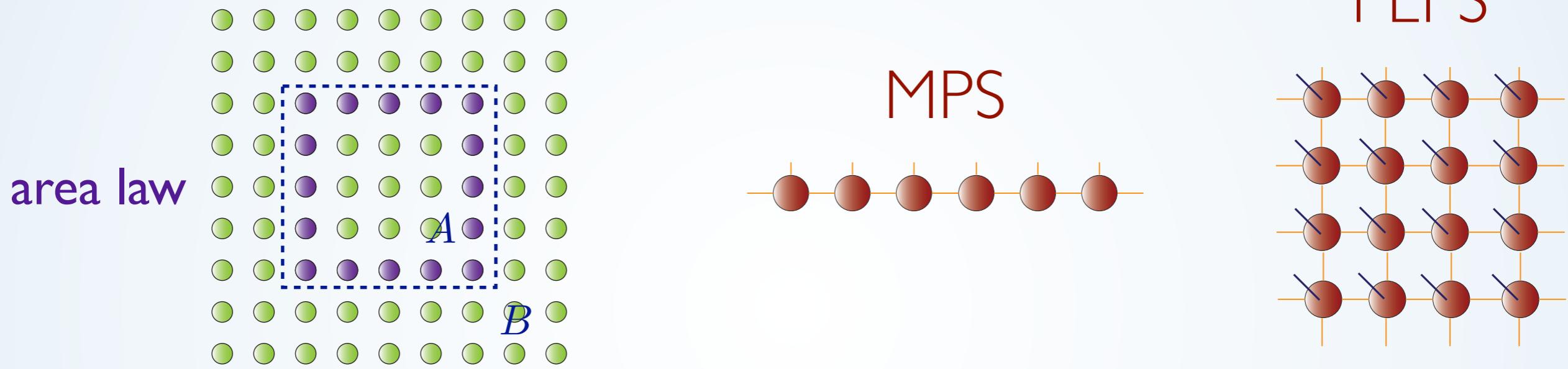
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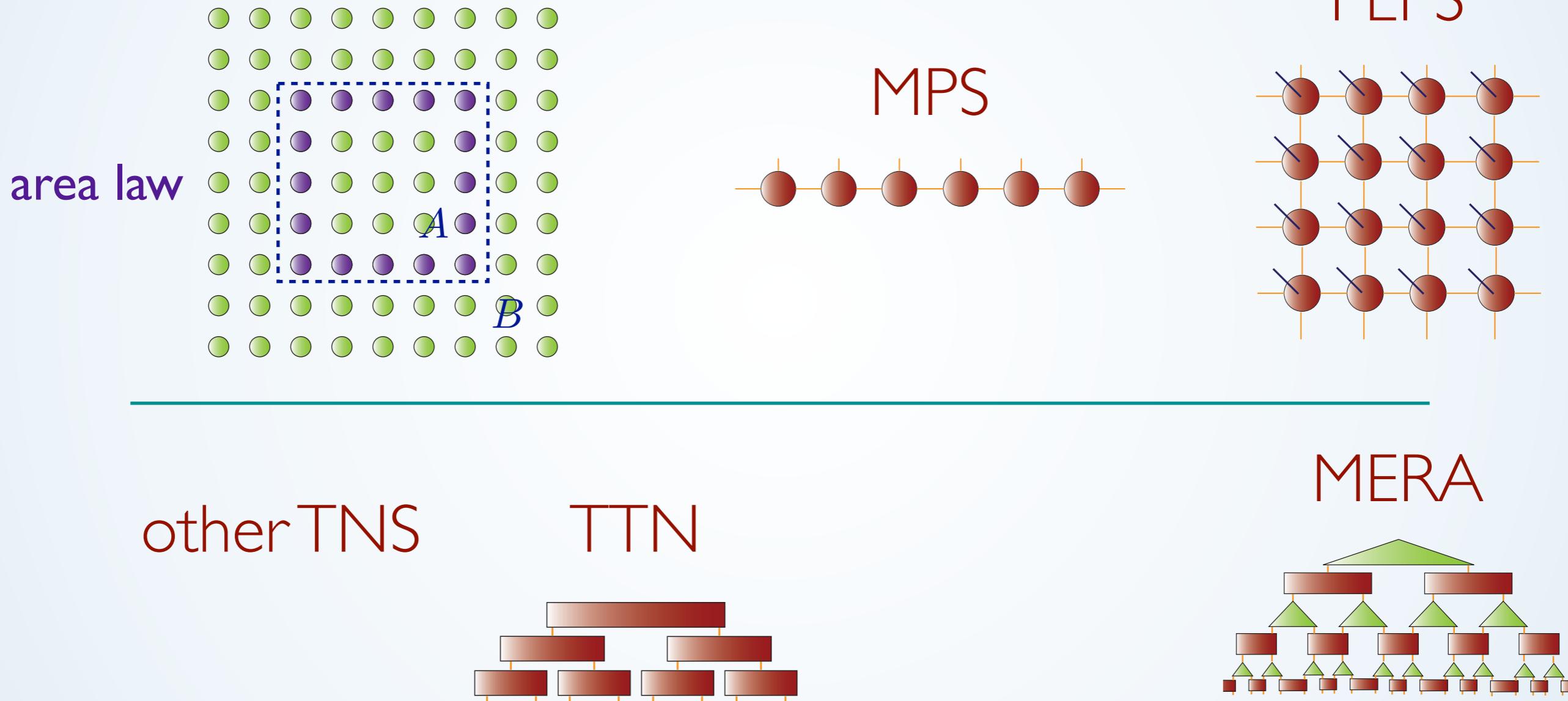


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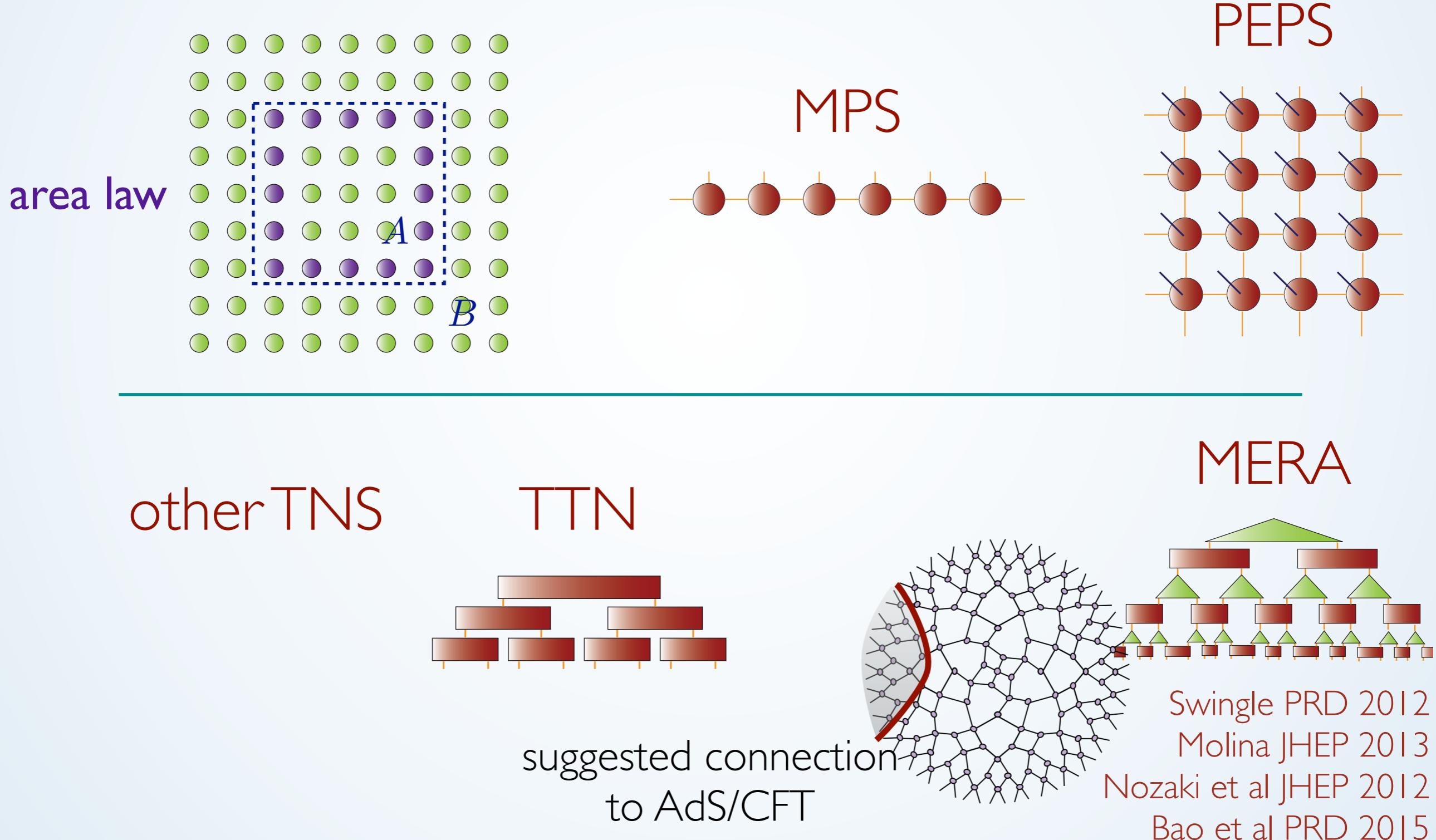


other TNS

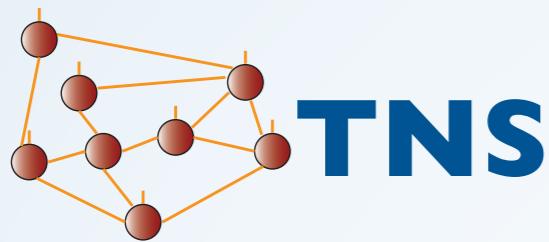
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# WHY FOR LGT?

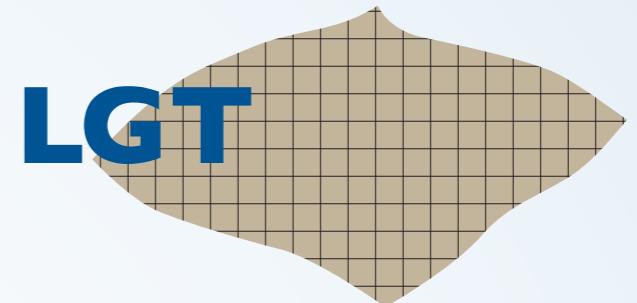


Non-perturbative for  
Hamiltonian systems

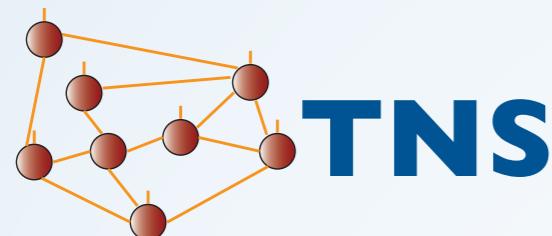
Extremely successful for  
1D systems (MPS)

Promising improvements  
for higher dimensions

ground states  
low-lying excitations  
thermal states  
time evolution



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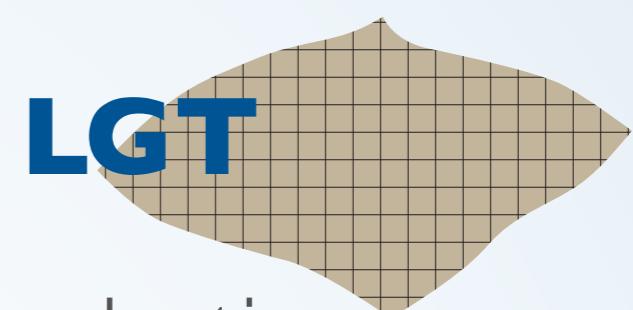
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Non-perturbative way of  
solving QFT (QCD)

Mostly path-integral  
formalism & MC

4D lattice

spectrum

finite T

$64^3 \times 96$

chemical potential  
time evolution

# USING TNS FOR LGT

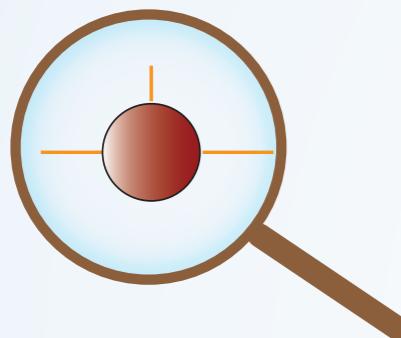
# USING TNS FOR QMB

a formal approach

no sign problem  
numerical algorithms

# USING TNS FOR QMB

a formal approach



classifying tensors  
constructing states

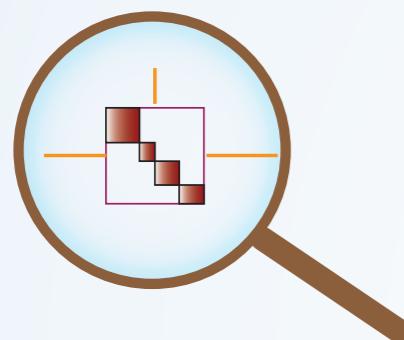
Chen et al PRB 2011  
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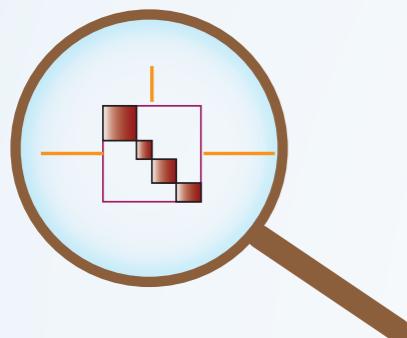
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great descriptive power: phases,  
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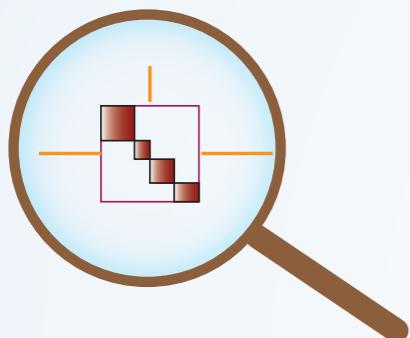
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TRG approaches

Nishino, JPSJ 1995  
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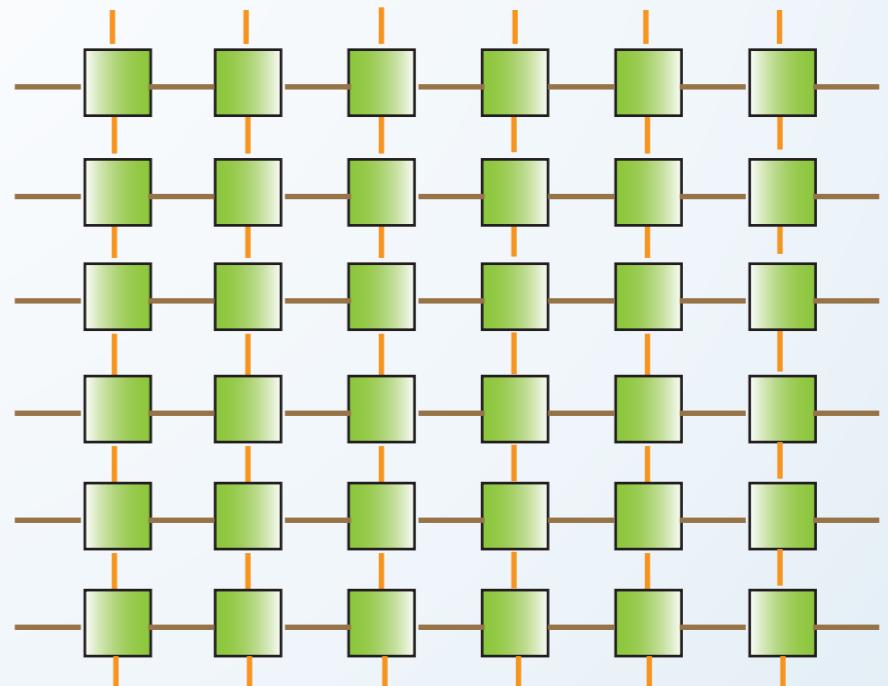
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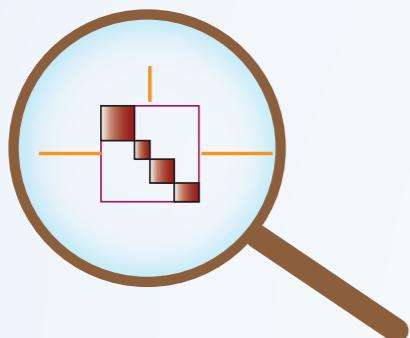
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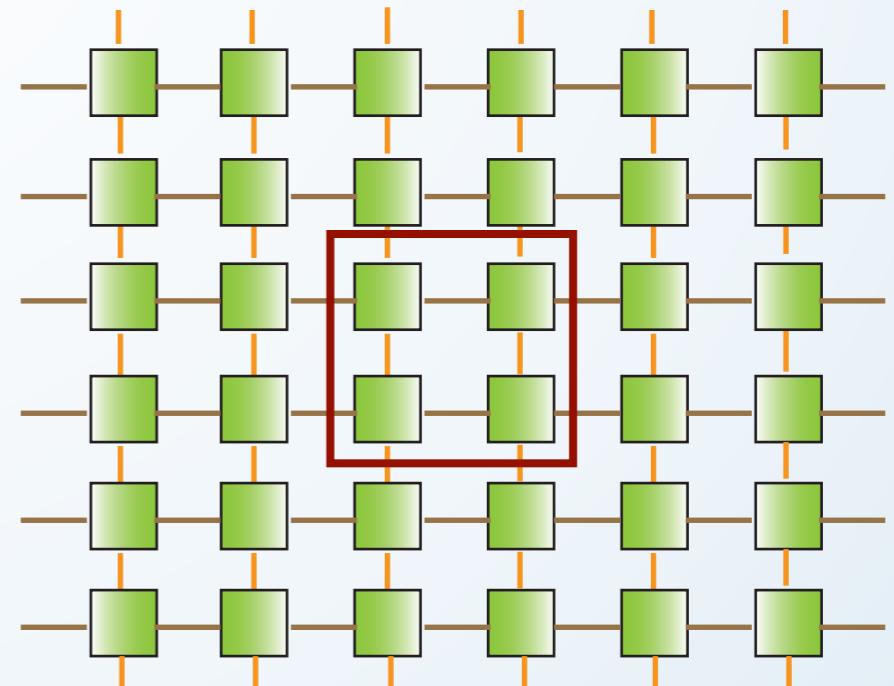
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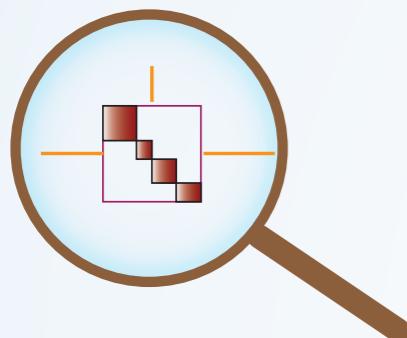
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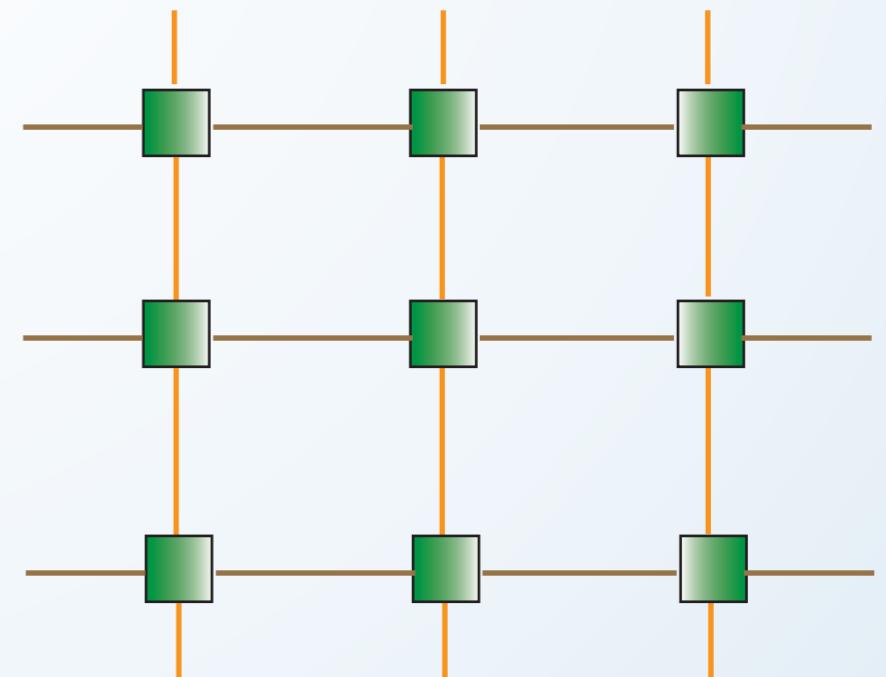
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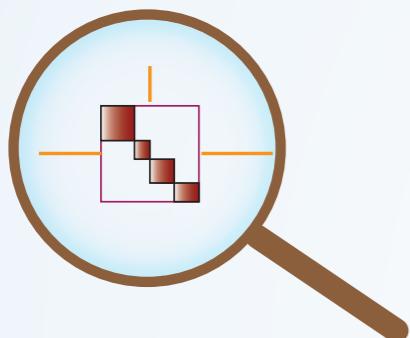
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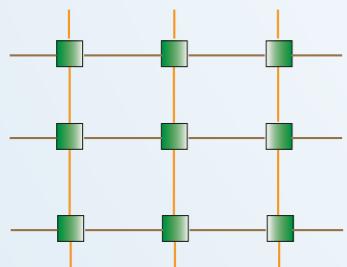


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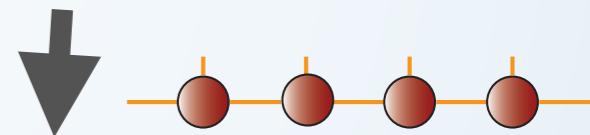
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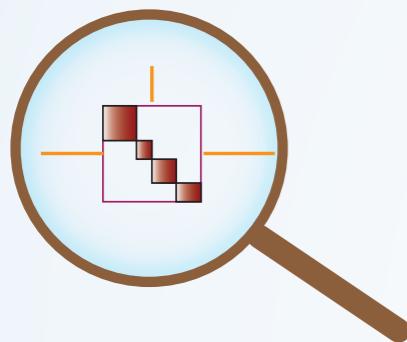
TNS as ansatz for the state

efficient algorithms for GS, low  
excited states, thermal, dynamics

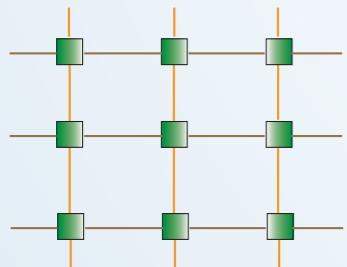
White PRL 1992; Schollwöck RMP 2011  
Vidal PRL 2003; Verstraete et al PRL 2004  
Verstraete et al Adv Phys 2008; Orús Ann Phys 2014

# USING TNS FOR LGT

a formal approach

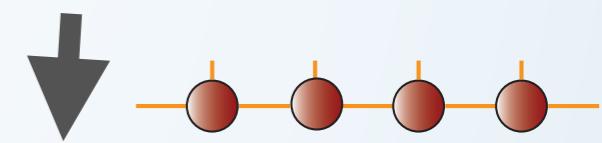


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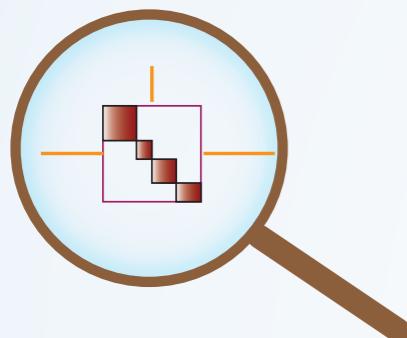
no sign problem  
numerical algorithms

TNS as ansatz for the state



# USING TNS FOR LGT

a formal approach



gauging the symmetry  
explicitly invariant states

general prescriptions,  $U(1)$ ,  $SU(2)$

Tagliacozzo et al PRX 2014  
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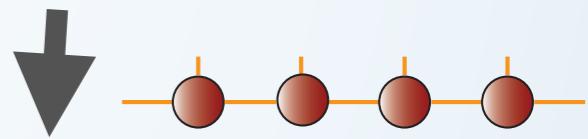
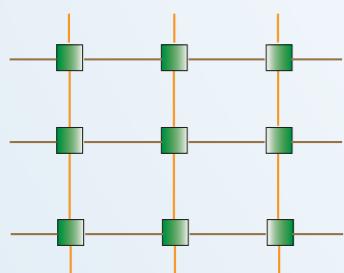
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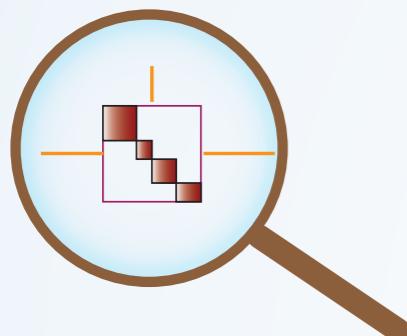
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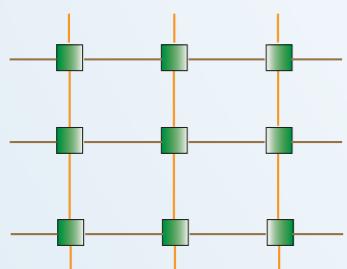
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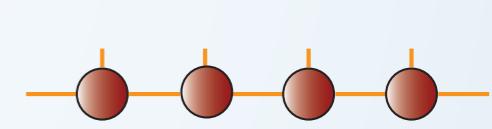
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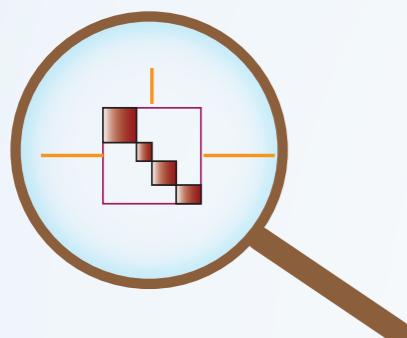
TRG approaches to classical  
and quantum models

Liu et al PRD 2013  
Shimizu, Kuramashi, PRD 2014,...  
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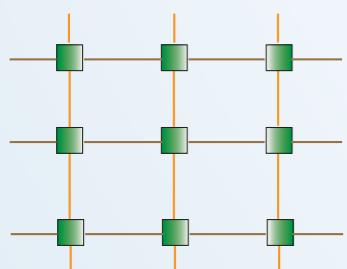


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next...

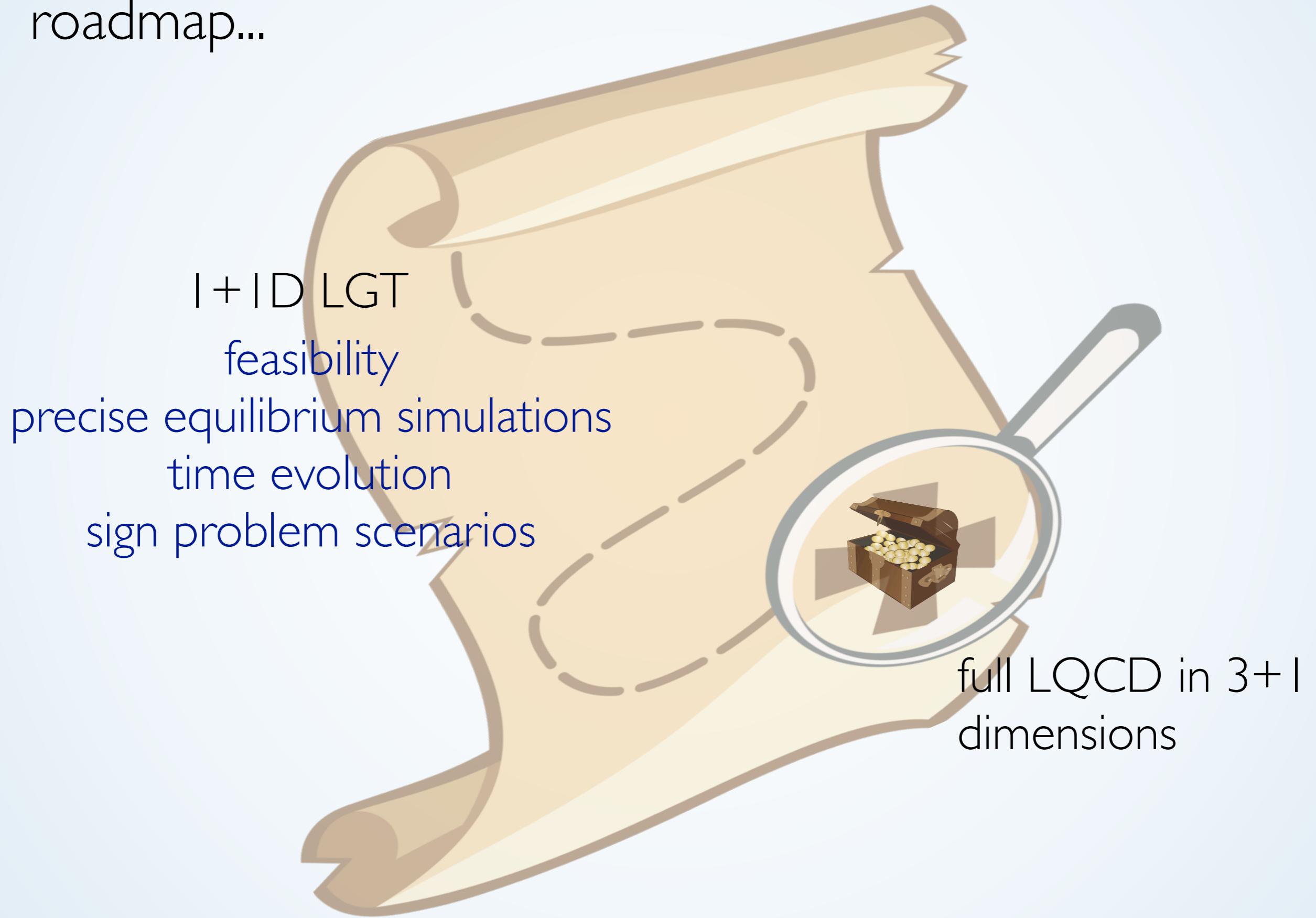
a possible LGT-TNS  
roadmap...



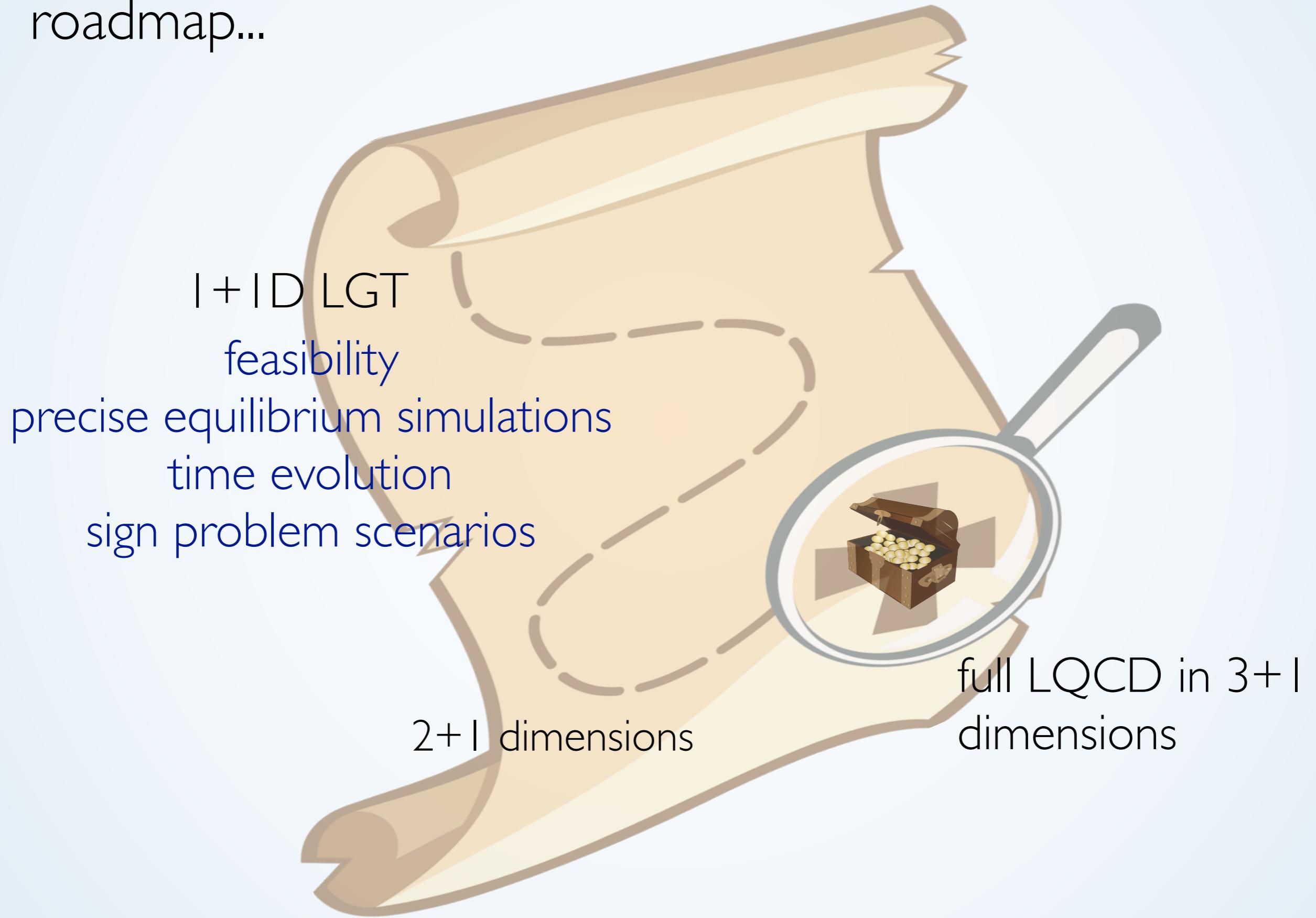
a wishful LGT-TNS  
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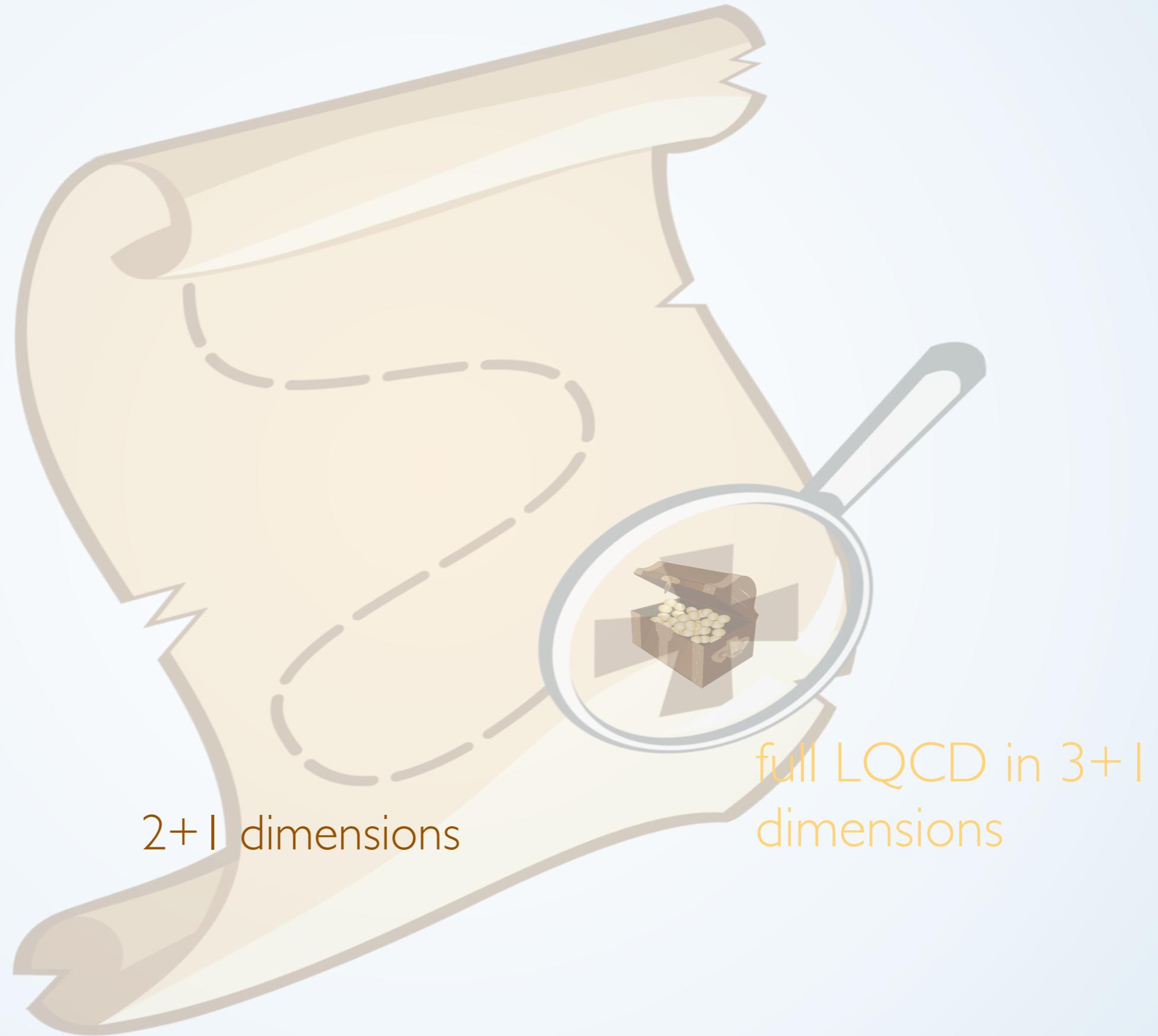
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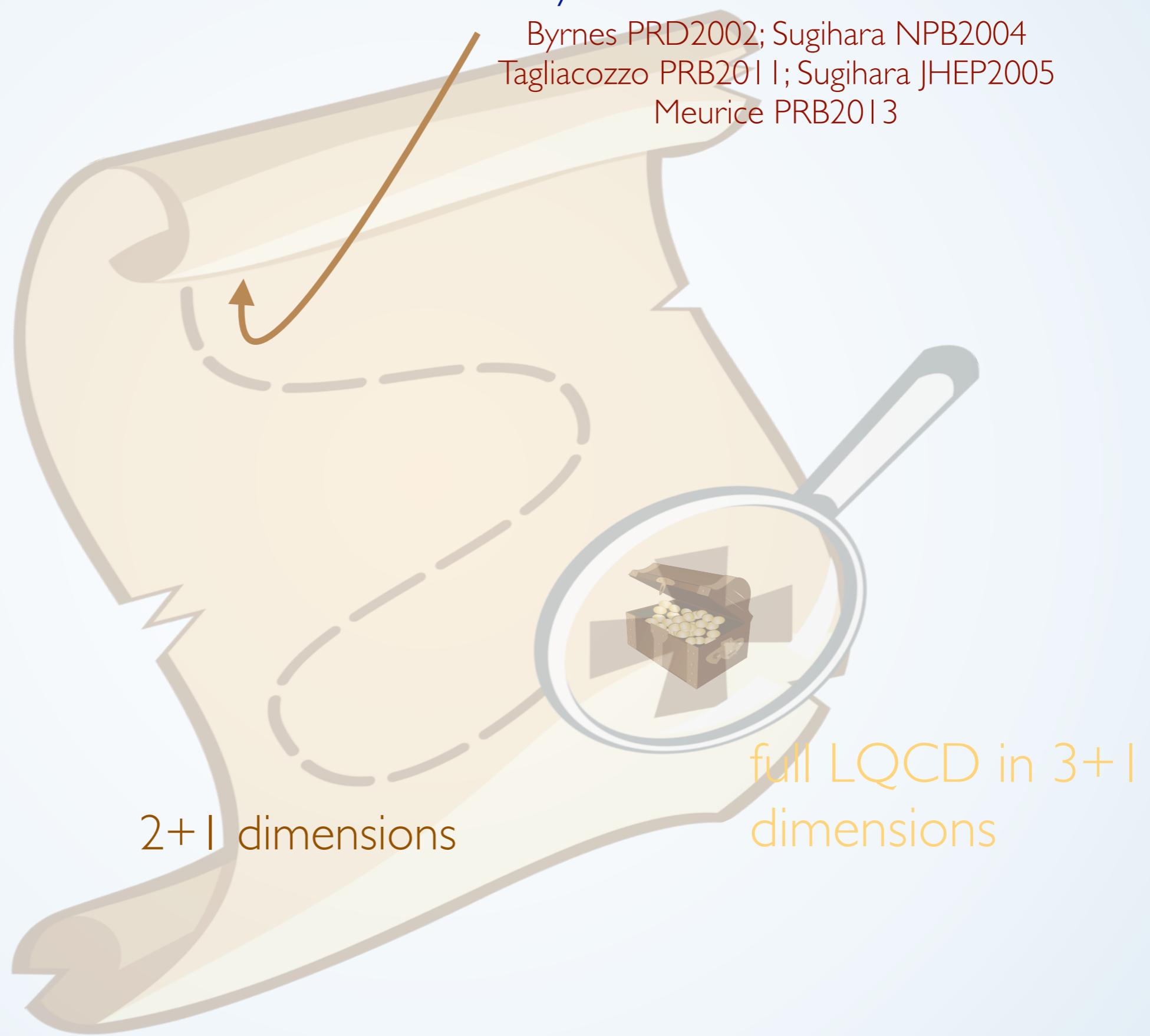
# a wishful LGT-TNS roadmap...



an ongoing LGT-TNS  
roadmap...



# an ongoing LGT-TNS roadmap...



# an ongoing LGT-TNS roadmap...

Schwinger model  
 $U(1)$  in 1D

early works with DMRG/TNS  
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Tagliacozzo PRB2011; Sugihara JHEP2005  
Meurice PRB2013

2+1 dimensions

full LQCD in 3+1  
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feasibility of QSim

MCB et al JHEP11(2013)158;

Rico et al PRL 2014; Buyens et al. PRL 2014;

S. Kühn et al., PRA 90, 042305 (2014);

MCB et al PRD 2015, Buyens et al. PRD 2016;

Pichler et al. PRX 2016;

review Dalmonte, Montangero, Cont. Phys. 2016

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S. Kuehn et al, PRL 118 (2017) 071601

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Non-Abelian in 1D  
string breaking dynamics  
S. Kühn et al., JHEP 07 (2015) 130;  
Silvi et al, Quantum 2017  
S. Kühn et al. PRX 2017

full LQCD in 3+1  
dimensions

2+1 dimensions

# an ongoing LGT-TNS roadmap...

Schwinger model  
 $U(1)$  in 1D  
precise equilibrium  
simulations,  
feasibility of QSim

MCB et al JHEP 11 (2013) 158;  
Rico et al PRL 2014; Buyens et al. PRL 2014;  
S. Kühn et al., PRA 90, 042305 (2014);  
MCB et al PRD 2015, Buyens et al. PRD 2016;  
Pichler et al. PRX 2016;  
review Dalmonte, Montangero, Cont. Phys. 2016

early works with DMRG/TNS  
Byrnes PRD2002; Sugihara NPB2004  
Tagliacozzo PRB2011; Sugihara JHEP2005  
Meurice PRB2013

finite density  
S. Kuehn et al, PRL118 (2017) 071601

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other 1+1 D  
models



2+1 dimensions

SOLVING LGT WITH TNS

# GENERAL STRATEGY

Hamiltonian formulation  
acting on a Hilbert space

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Common ingredients for quantum simulation

# SCHWINGER MODEL

continuum

$$H = \int dx \left[ -i\bar{\Psi}\gamma^1\partial_1\Psi + g\bar{\Psi}\gamma^1A_1\Psi + m\bar{\Psi}\Psi + \frac{1}{2}E^2 \right]$$

plus constraint: Gauss' Law

$$\partial_1 E = g\bar{\Psi}\gamma^0\Psi$$

# SCHWINGER MODEL

discretized

$$H = -\frac{i}{2a} \sum_n (\phi_n^\dagger e^{i\theta_n} \phi_{n+1} - \text{h.c.}) + m \sum_n (-1)^n \phi_n^\dagger \phi_n + \frac{ag^2}{2} \sum_n L_n^2$$

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$$L_n - L_{n-1} = \phi_n^\dagger \phi_n - \frac{1}{2} [1 - (-1)^n]$$

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$$| \dots s_e \ell s_o \ell s_e \ell s_o \dots \rangle$$

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work directly in the  
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SPECTRUM

# COMPUTING THE LOW ENERGY LEVELS

Efficient algorithms to find ground state and excitations

$$|E_0\rangle \simeq \text{---} \circlearrowleft \text{---} \circlearrowleft \text{---} \circlearrowleft \text{---} \circlearrowleft \text{---} \circlearrowleft \text{---}$$

variational, imaginary time...

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$$\sum_n e^{ikn} \text{---} \circlearrowleft \text{---} \circlearrowleft \text{---} \overset{(n)}{\text{---}} \text{---} \circlearrowleft \text{---} \circlearrowleft \text{---}$$

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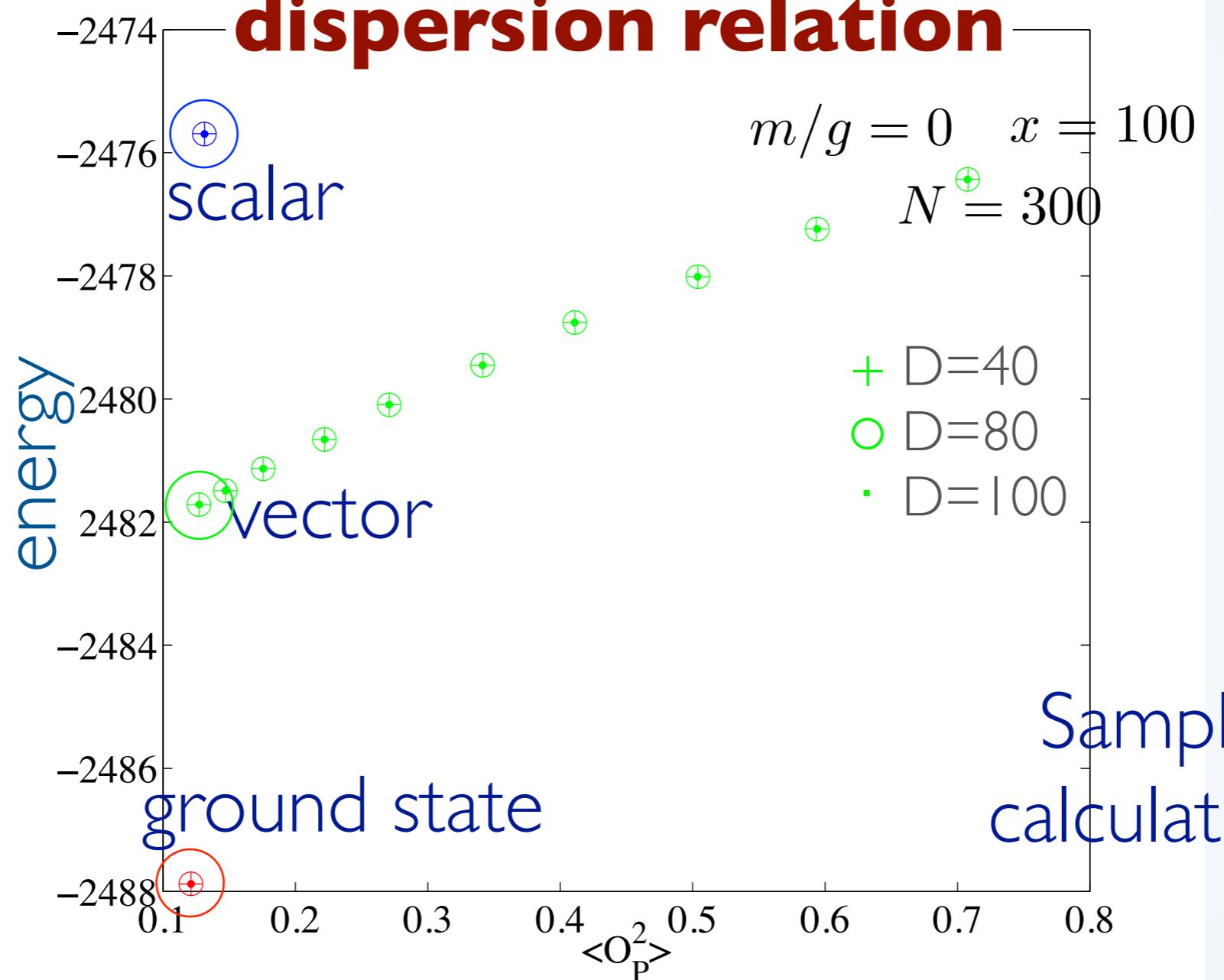
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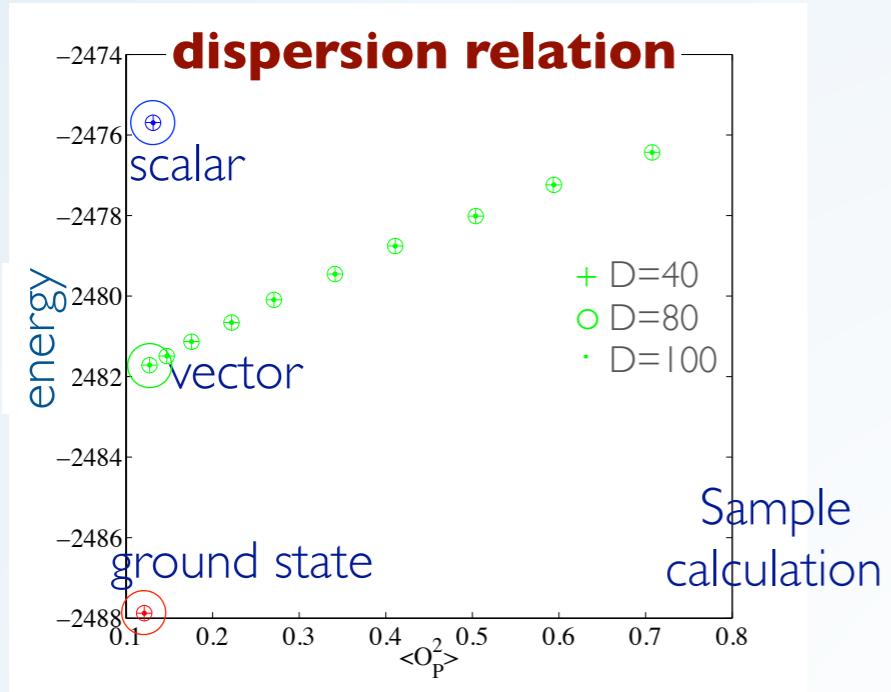
$$\sum_n e^{ikn} \text{---} \circlearrowleft \text{---} \circlearrowright \text{---}^{(n)}$$

Different strategies possible for LGT

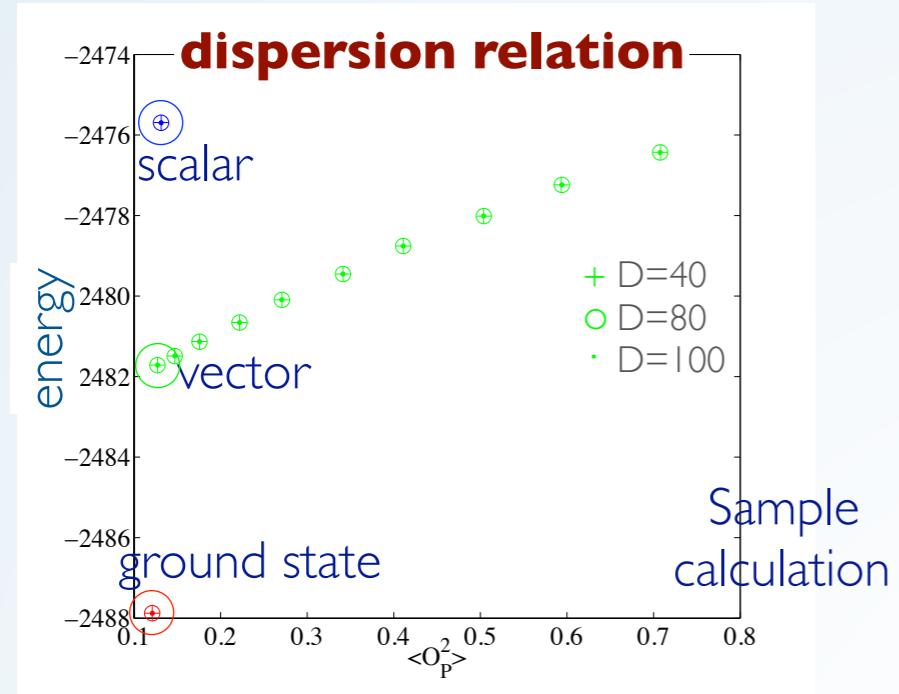
truncate the gauge dof, integrate out  
explicit symmetries in tensors

## dispersion relation

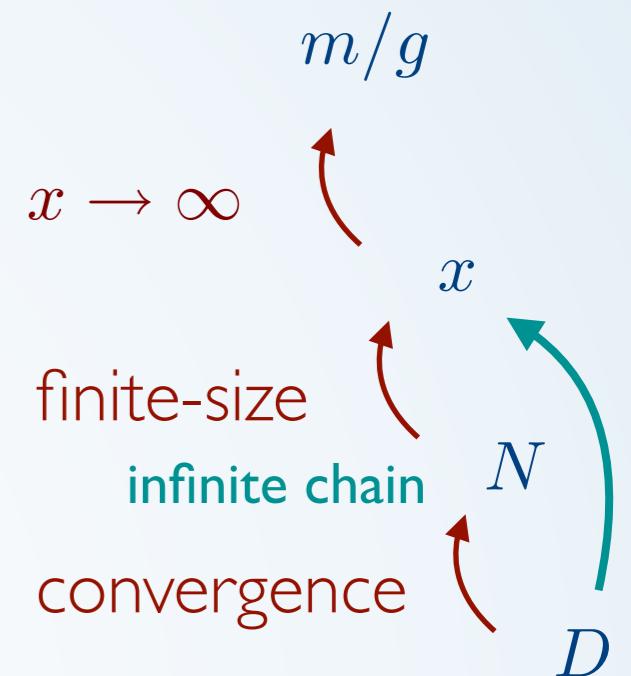




$m/g = 0 \quad x = 100$   
 $N = 300$

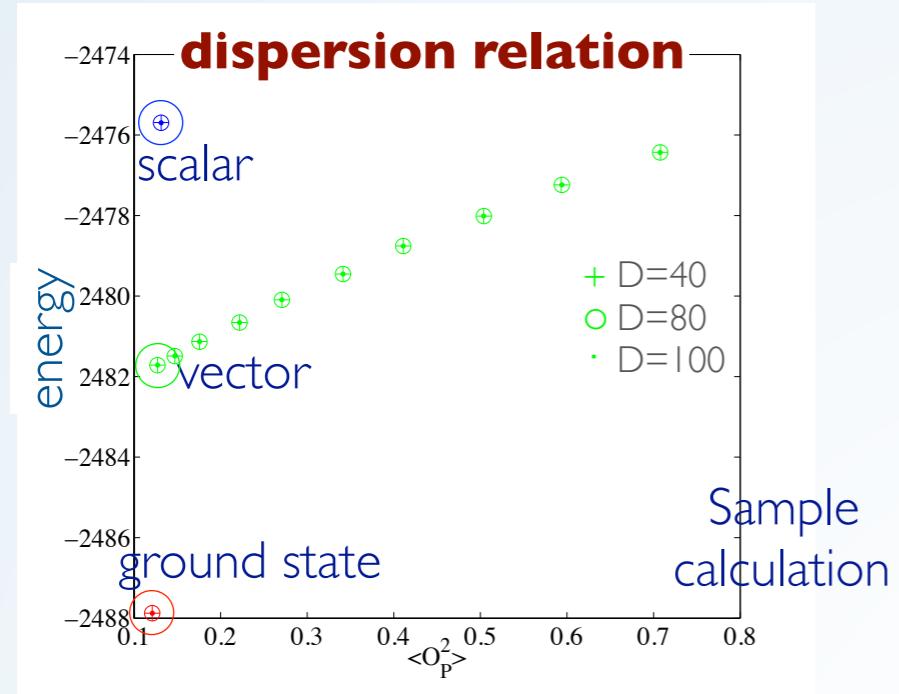


# after the continuum limit

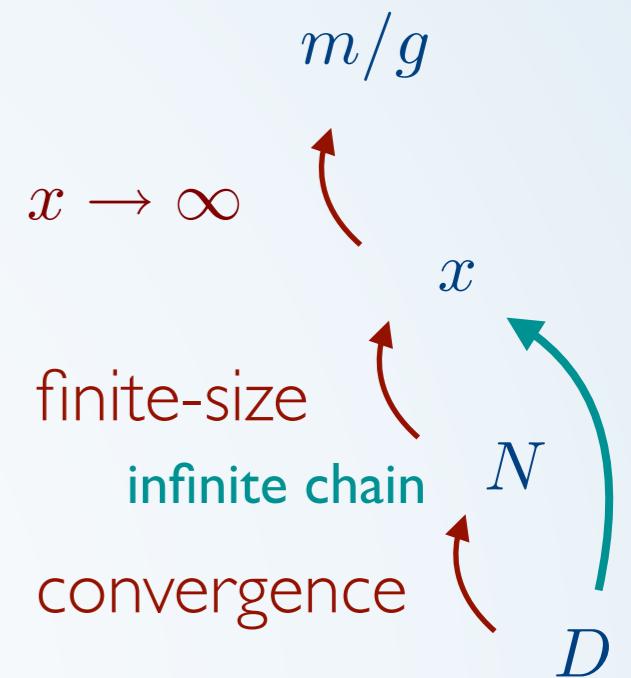


very precise for all masses

$m/g$	DMRG	MPS with OBC	SCE	MPS with OBC
0	0.5641859	0.56414(26)	1.128379	1.1283(10)
125	0.53950(7)	0.53946(20)	1.22(2)	1.221(2)
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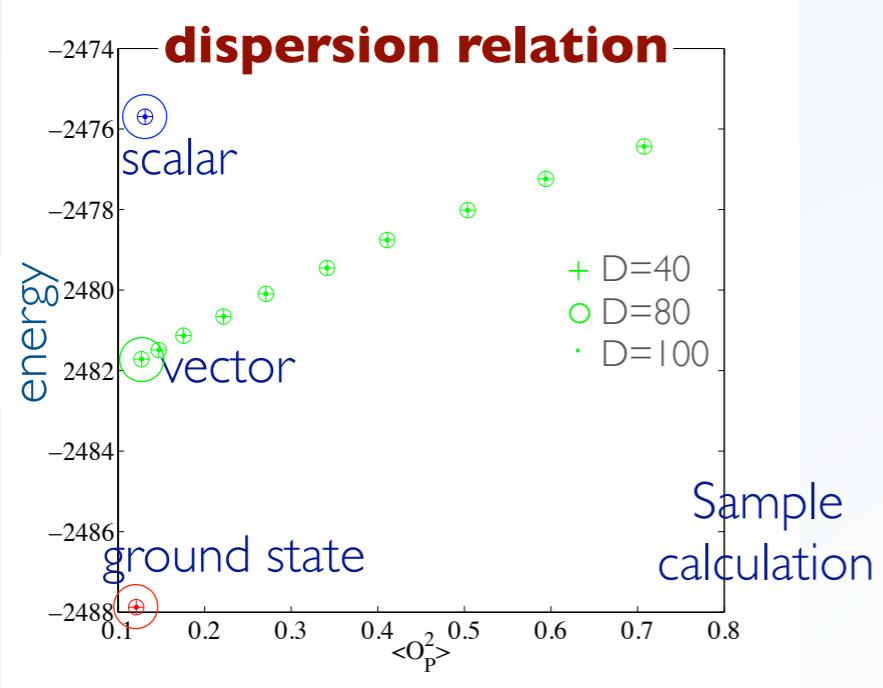
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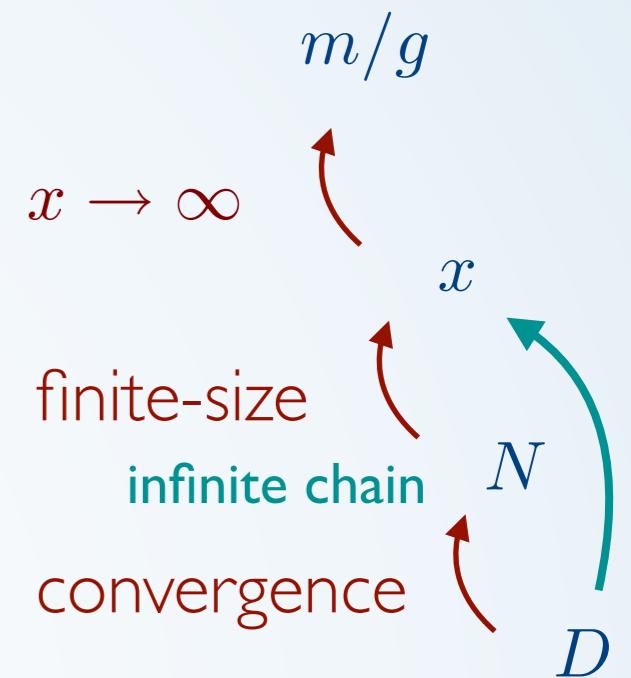
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also for SU(2) LGT  
MCB, Cichy, Cirac, Jansen, Kühn PRX 7, 041046 (2017)



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MCB, Cichy, Cirac, Jansen, Kühn PRX 7, 041046 (2017)

real time, too

Buyens et al. PRL 113 (2014) 091601, PRD96 (2017) 114501

MCB, Cichy, Cirac, Jansen JHEP 11 (2013) 158  
Buyens et al. PRL 113 (2014) 091601

ENTROPY

Entropy can be efficiently computed from MPS state

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gauge constraints not purely local  $\Rightarrow$  not all entropy physical

Casini et al 2014; Gosh et al JHEP 2015  
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$$S(\rho) = - \sum_j p_j \log_2(p_j) + \sum_j p_j S(\rho_j)$$

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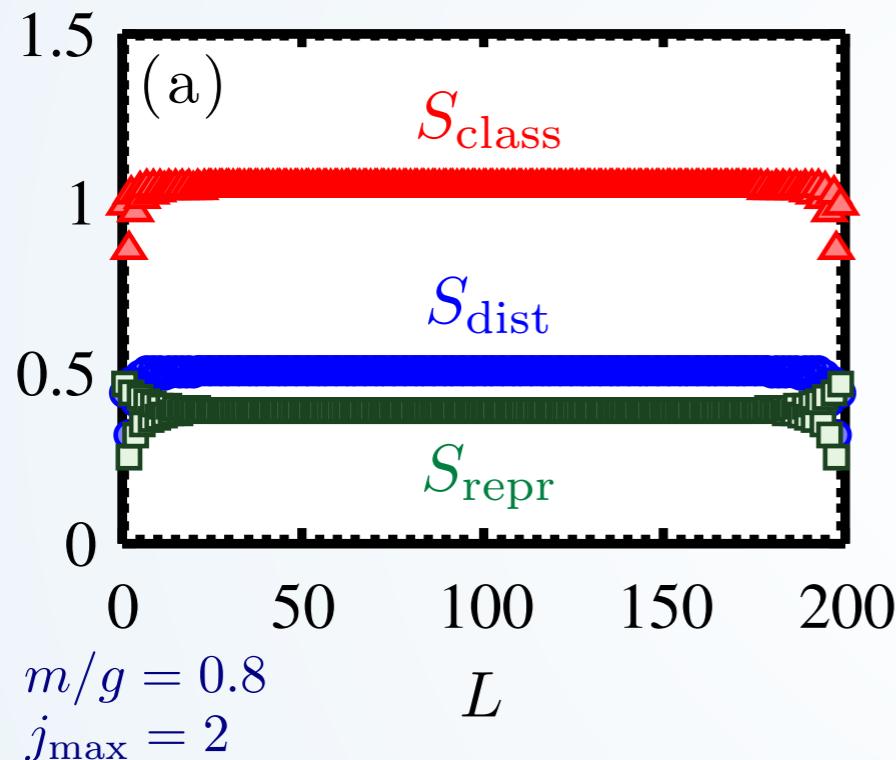
$S_{\text{class}}$        $S_{\text{repr}}$        $S_{\text{dist}}$

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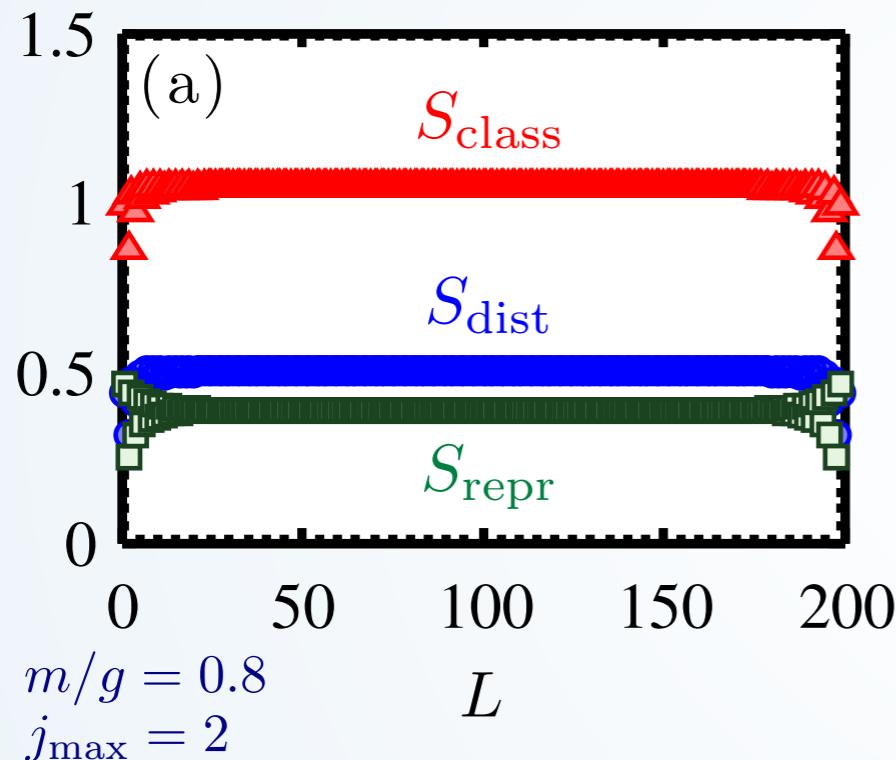
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PRX7, 041046 (2017)

divergence in the continuum limit

$$S \propto \frac{c}{6} \log_2 \frac{\xi}{a}$$

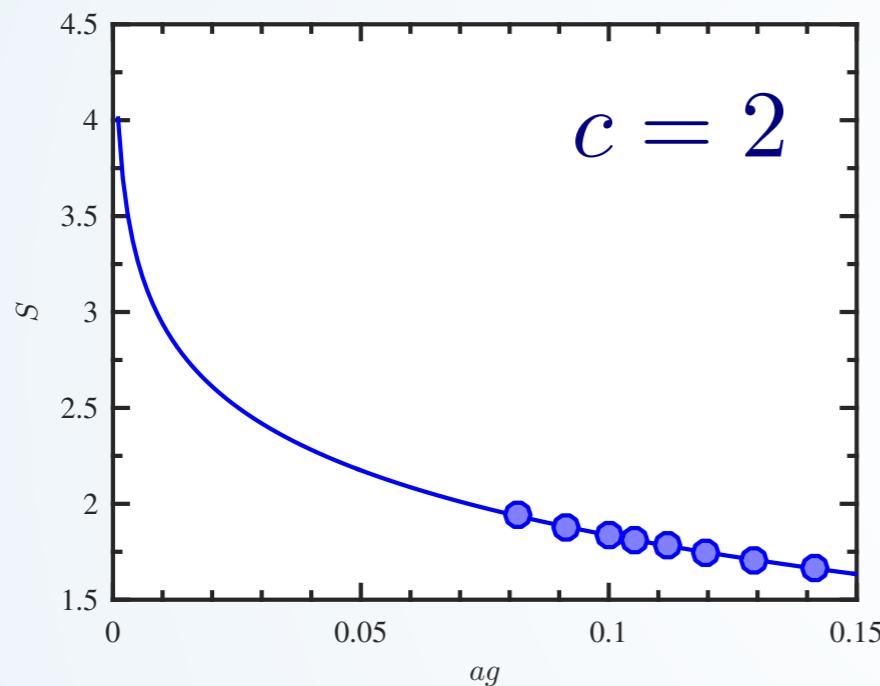
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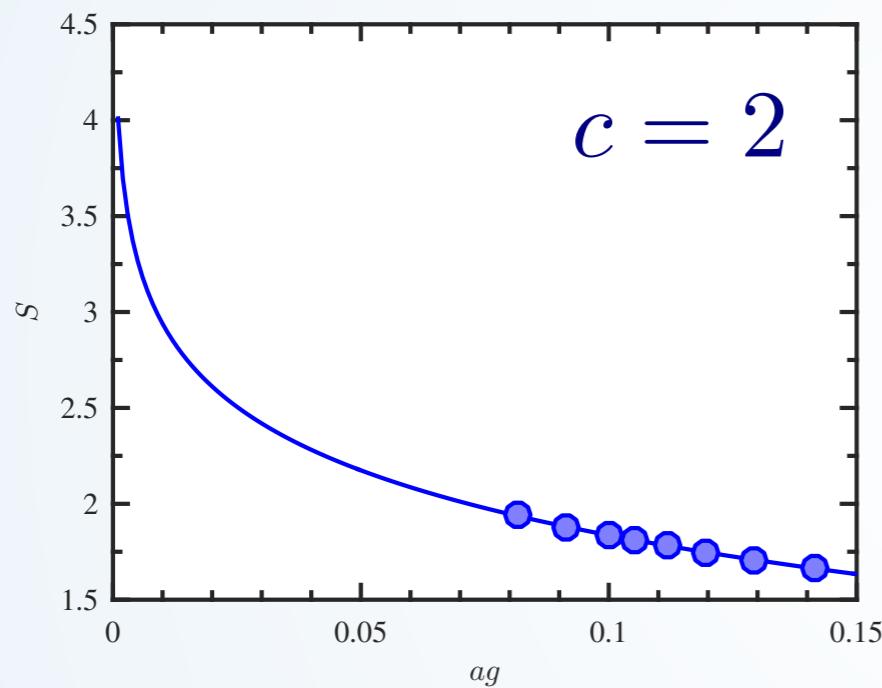
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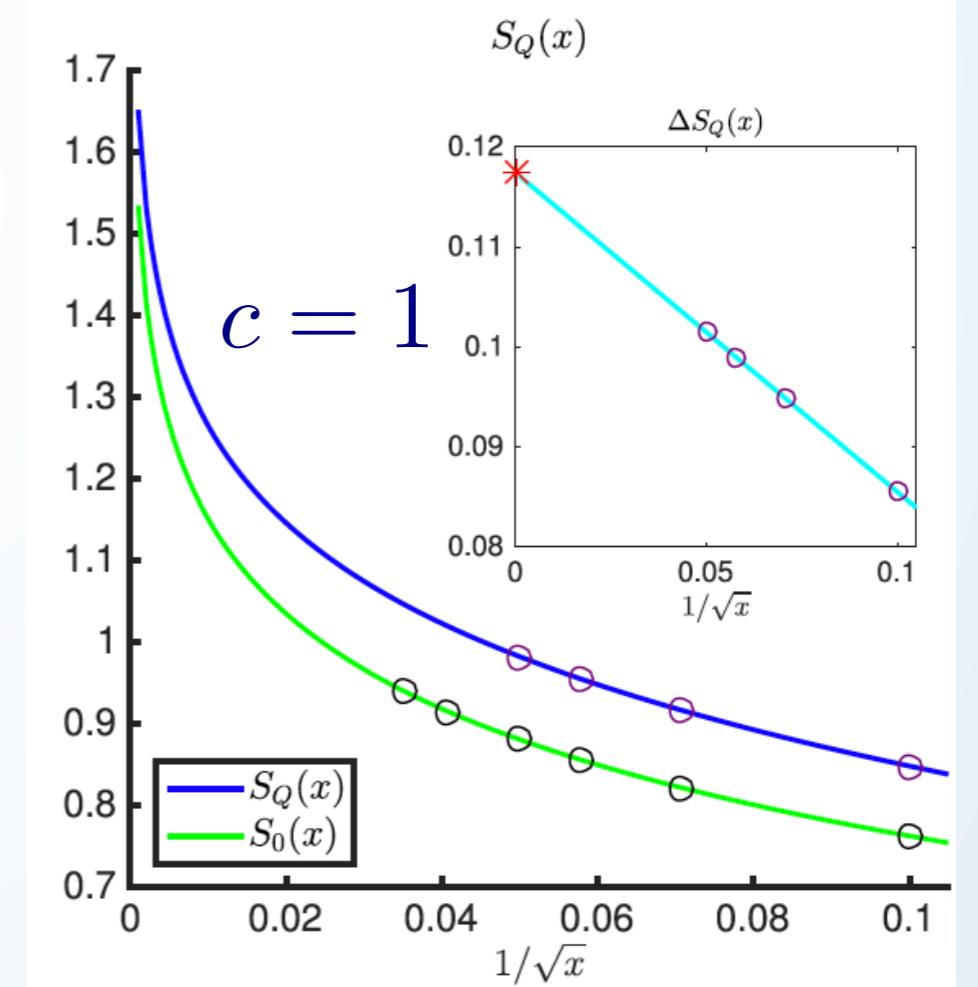
Soni,Trivedi JHEP 2016; van Acleyen et al PRL 2016

$SU(2)$



PRX7, 041046 (2017)

Schwinger



Buyens PRX6, 041040 (2016)

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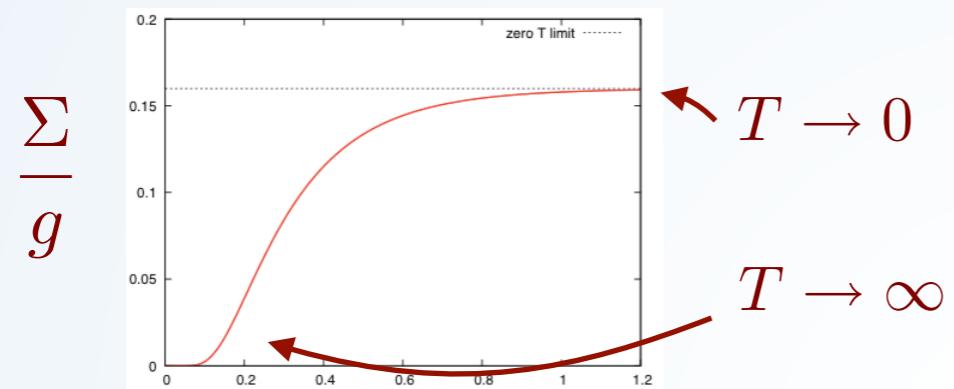
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THERMAL EQUILIBRIUM

# THERMAL PROPERTIES SCHWINGER

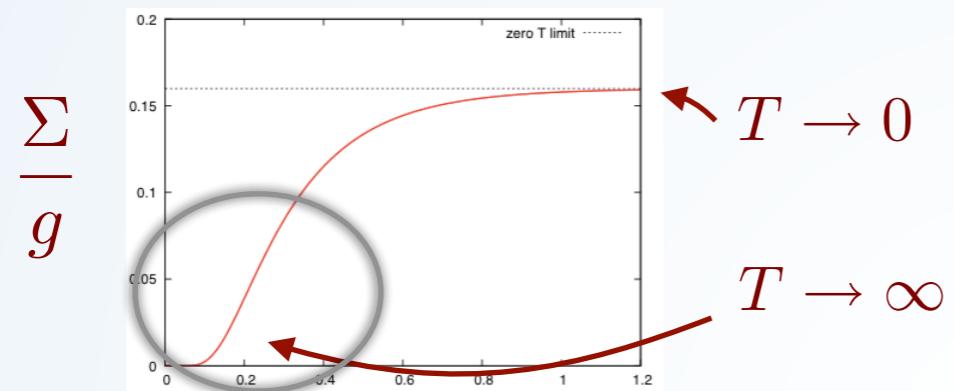
chiral condensate at finite T: analytical for  $m/g=0$



Sachs, Wipf 92

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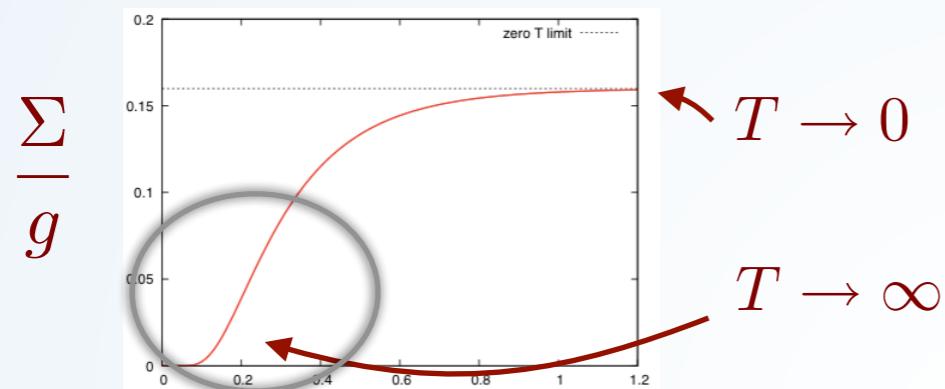


smooth  
restoration of  
chiral symmetry

Sachs, Wipf 92

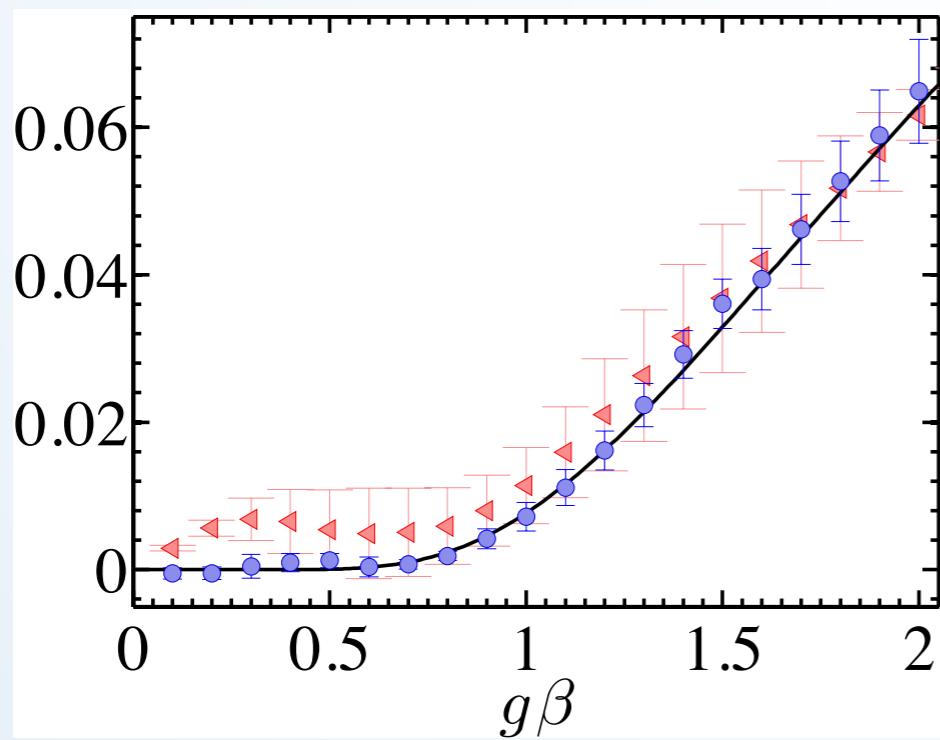
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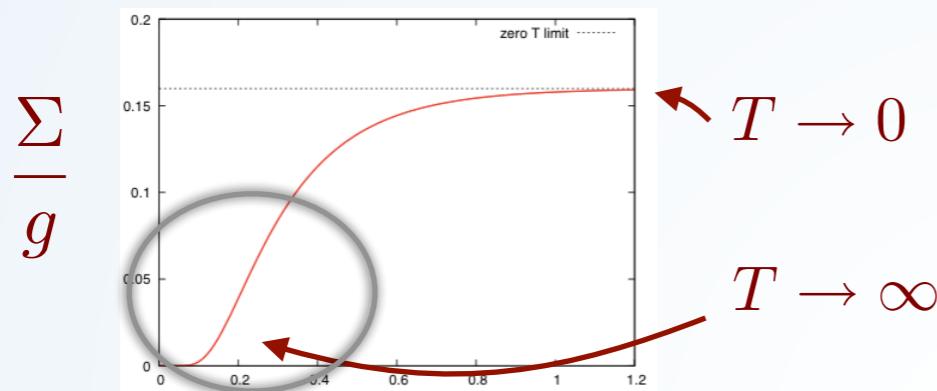
Sachs, Wipf 92



PRD 92, 034519 (2015); PRD 93, 094512 (2016)

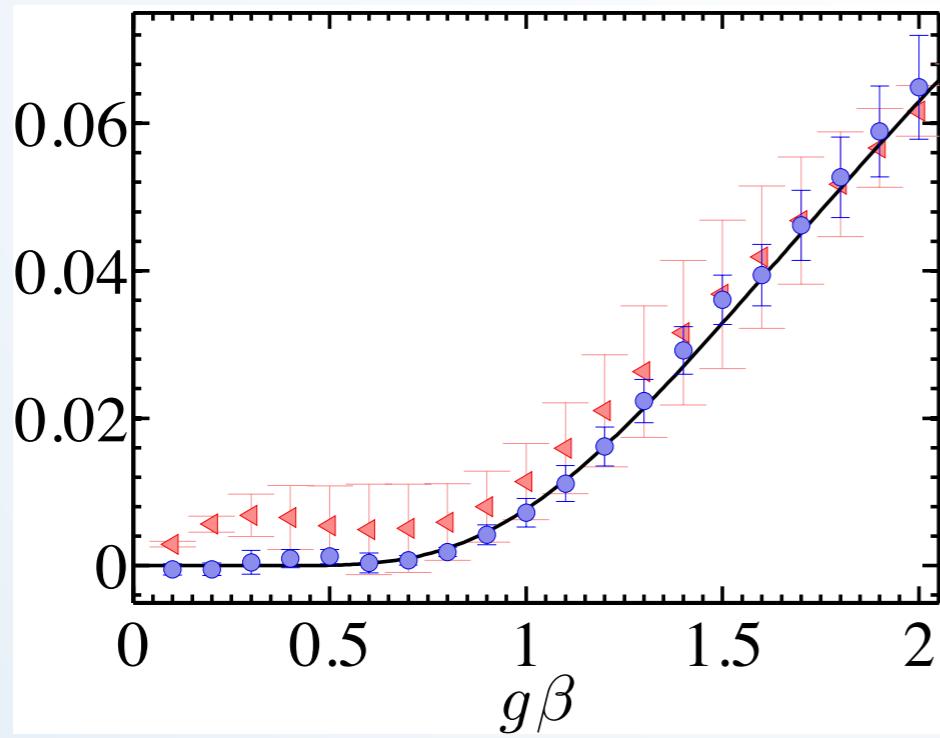
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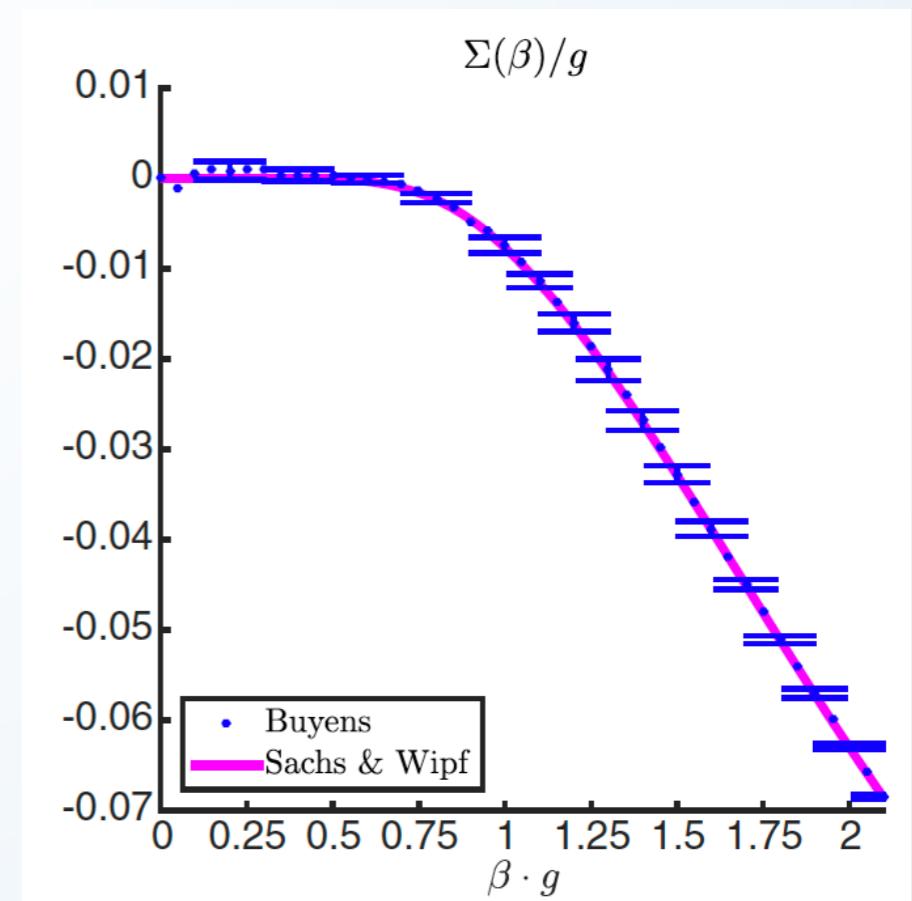


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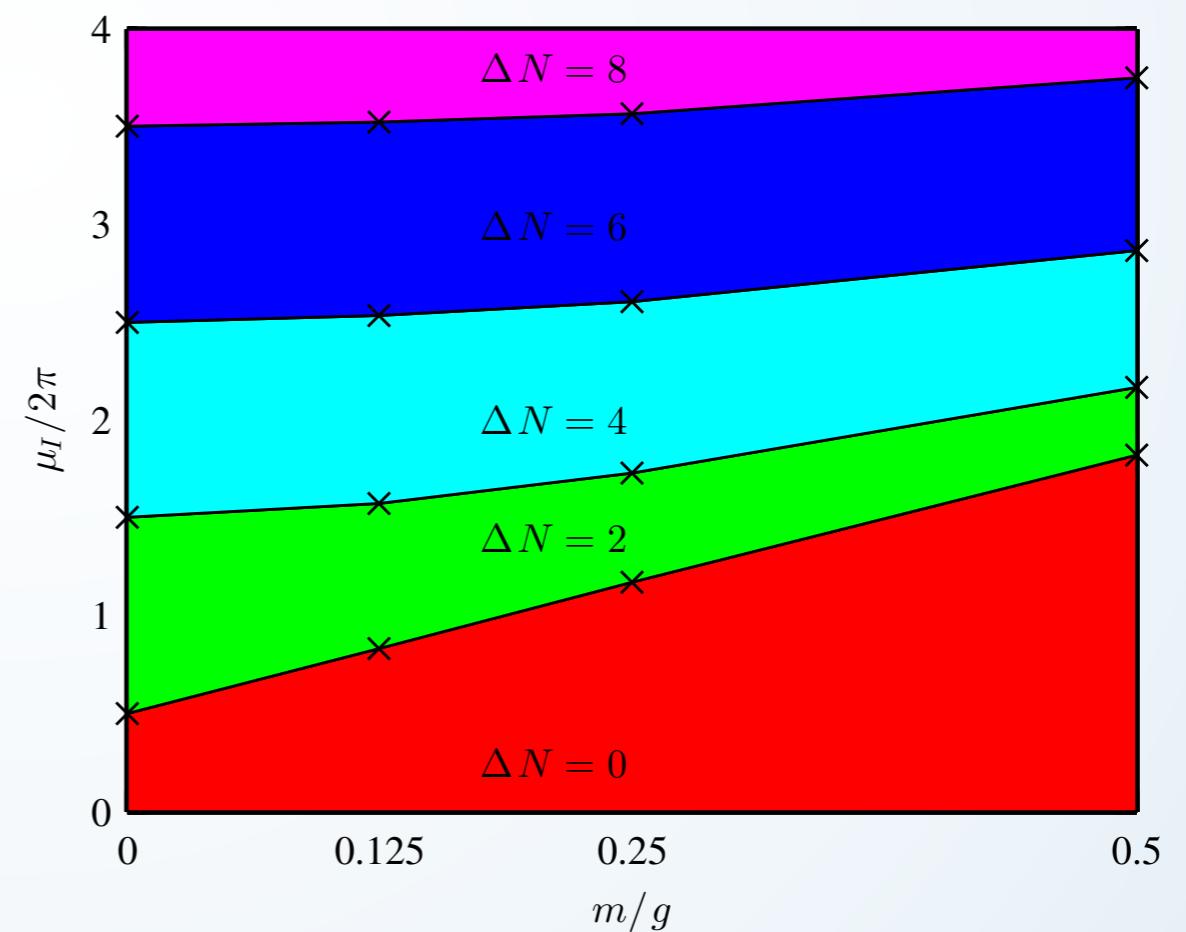
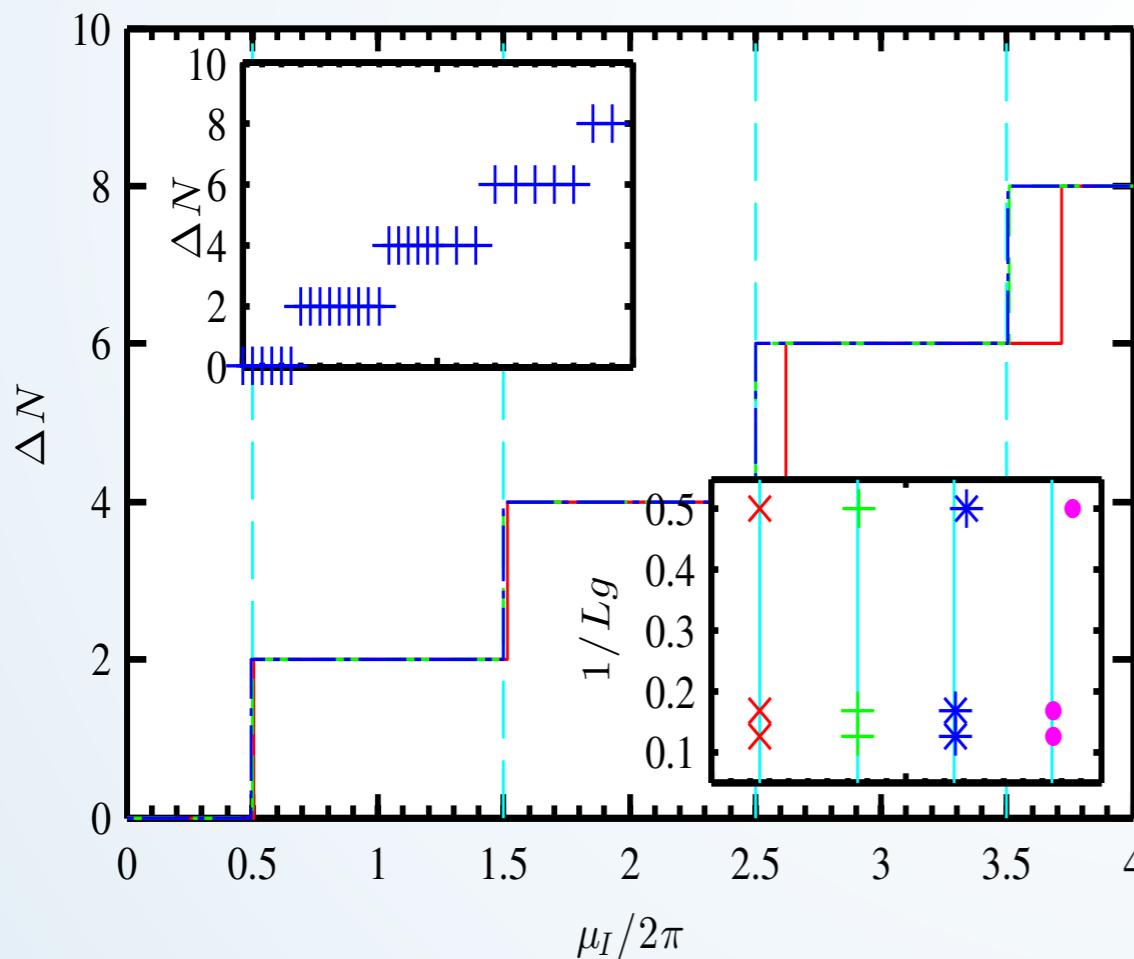


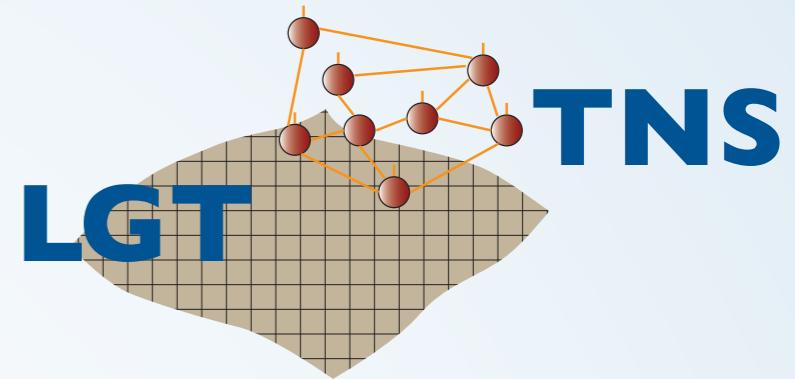
Buyens PRD 94, 085018 (2016)

# CHEMICAL POTENTIAL

# FINITE DENSITY WITH MPS

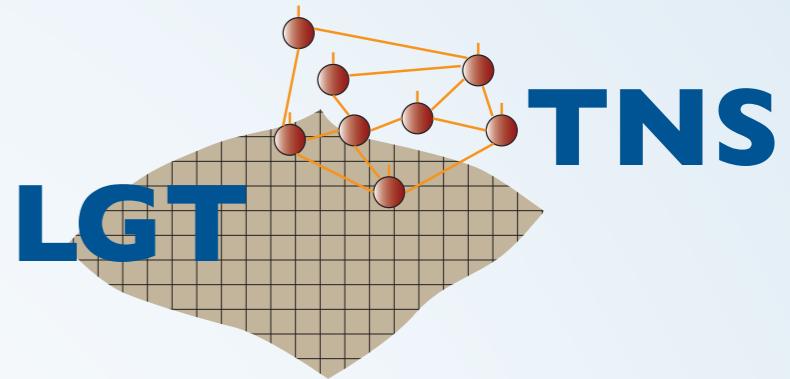
Several fermion flavors, different chemical potentials  
ground state density changes (first order PT)  
Montecarlo has sign problem





To conclude...

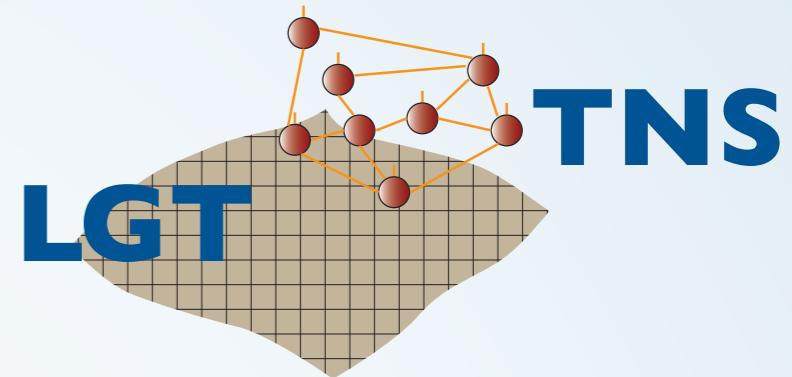
TNS = entanglement based ansatz



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TNS = entanglement based ansatz

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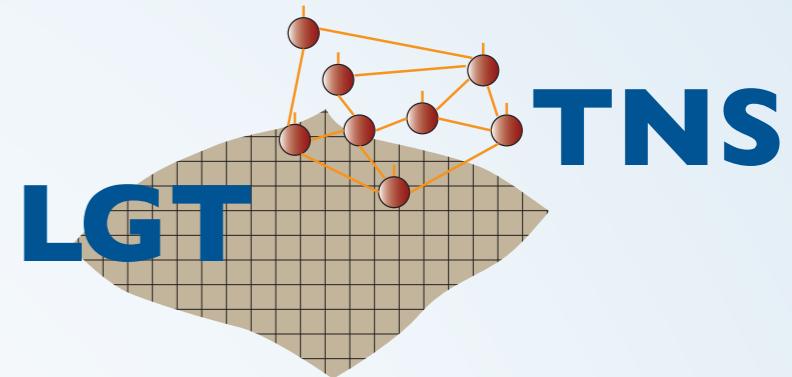
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spectrum, thermal equilibrium, finite density,  
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Abelian and non-Abelian models



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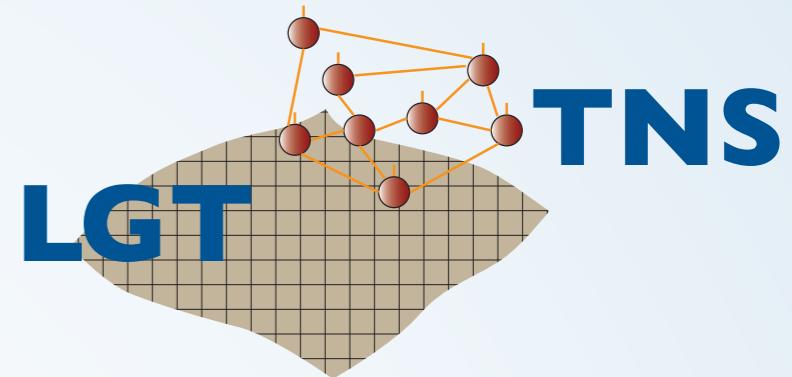
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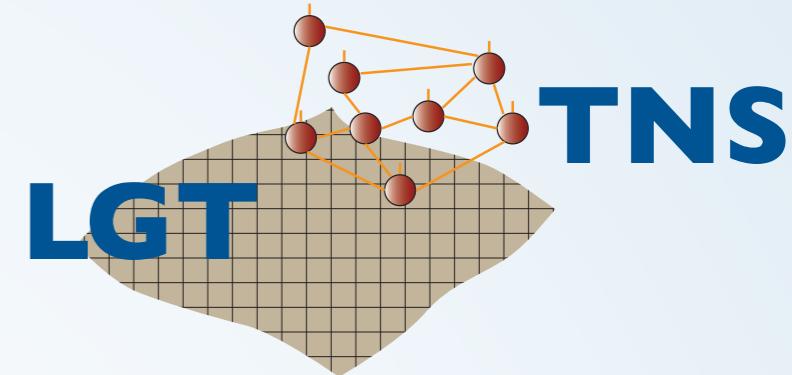
Related: proposals for quantum simulation of LGT with  
ultracold atoms

Zohar et al. PRL 2010, 2012 ,  
Tagliacozzo et al., Nat. Comm. 2013  
Banerjee et al., PRL 2012

Rico et al. PRL 2014  
Pichler et al, PRX 2016  
Zohar; Burrello, PRD 2015

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# THANKS



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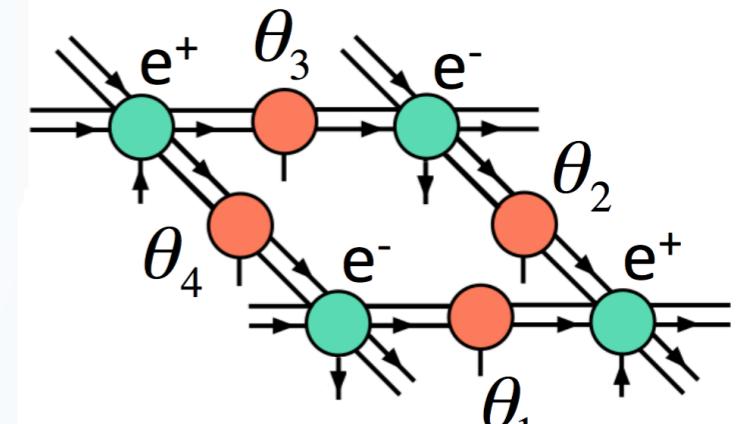
2D

# ACTIVE RESEARCH: PEPS FOR LGT

explicitly gauge invariant PEPS  
restricted ansatz calculations

Tagliacozzo et al PRX 2014  
Haegeman et al PRX 2014  
Zohar et al Ann Phys 2015  
arXiv:1807.01294

standard PEPS toolbox contains all ingredients  
for full variational computation  
computational cost, required D



Zapp, Orús PRD 2017

restriction of the ansatz may be better strategy

e.g. fully Gaussian PEPS

Zohar, Cirac PRD 2018