Gravity, Quantum Fields and Information

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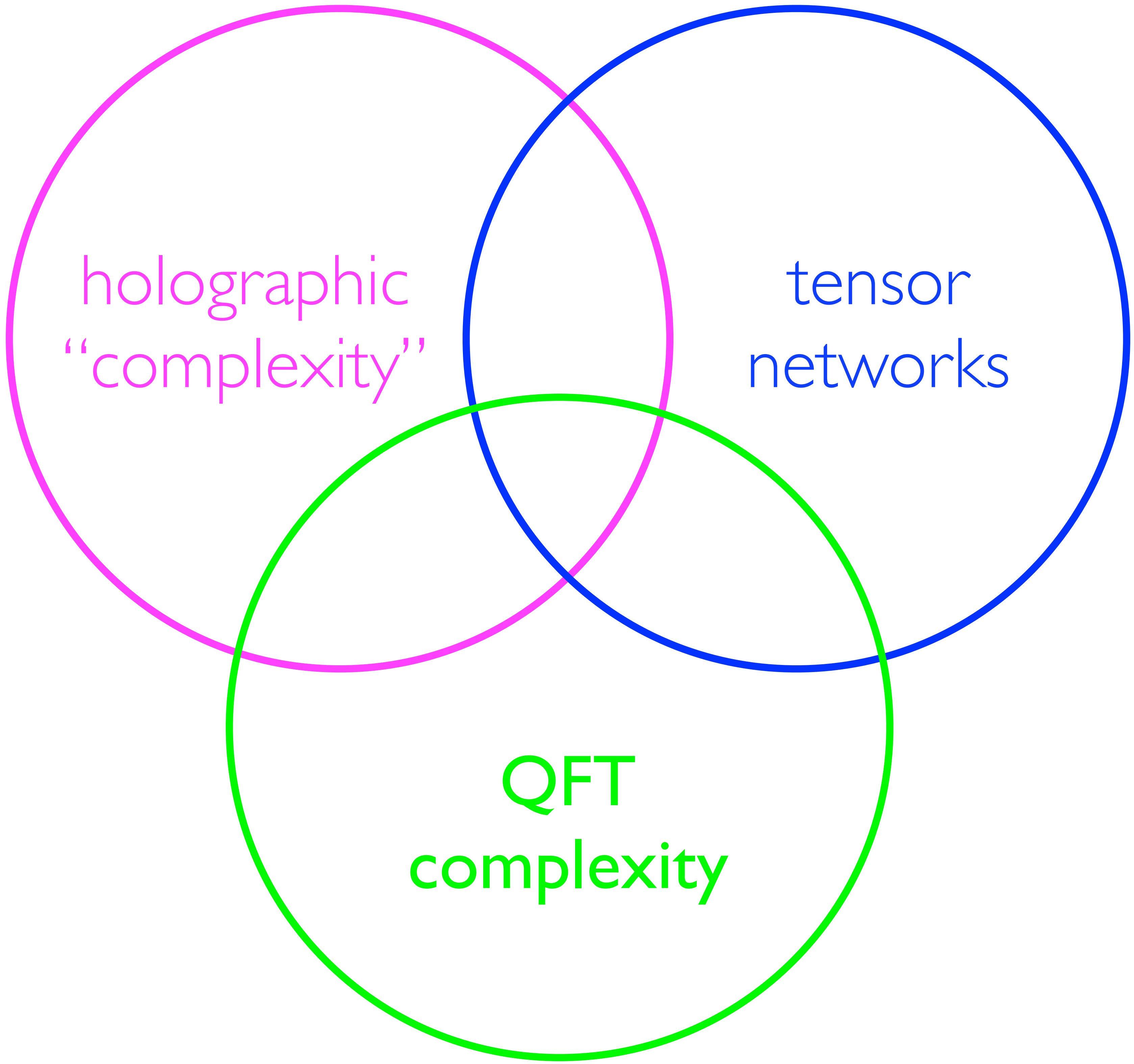
What this talk is about?



Part I: holographic “complexity”

quant-info ∩ cond-mat ⟶ hep-th

Entanglement / $\underset{B}{|\uparrow\rangle}\underset{\bar{B}}{|\downarrow\rangle} - \underset{B}{|\downarrow\rangle}\underset{\bar{B}}{|\uparrow\rangle}$ vs $|\uparrow\rangle|\downarrow\rangle$ / - key prop. of quantum-many bodies

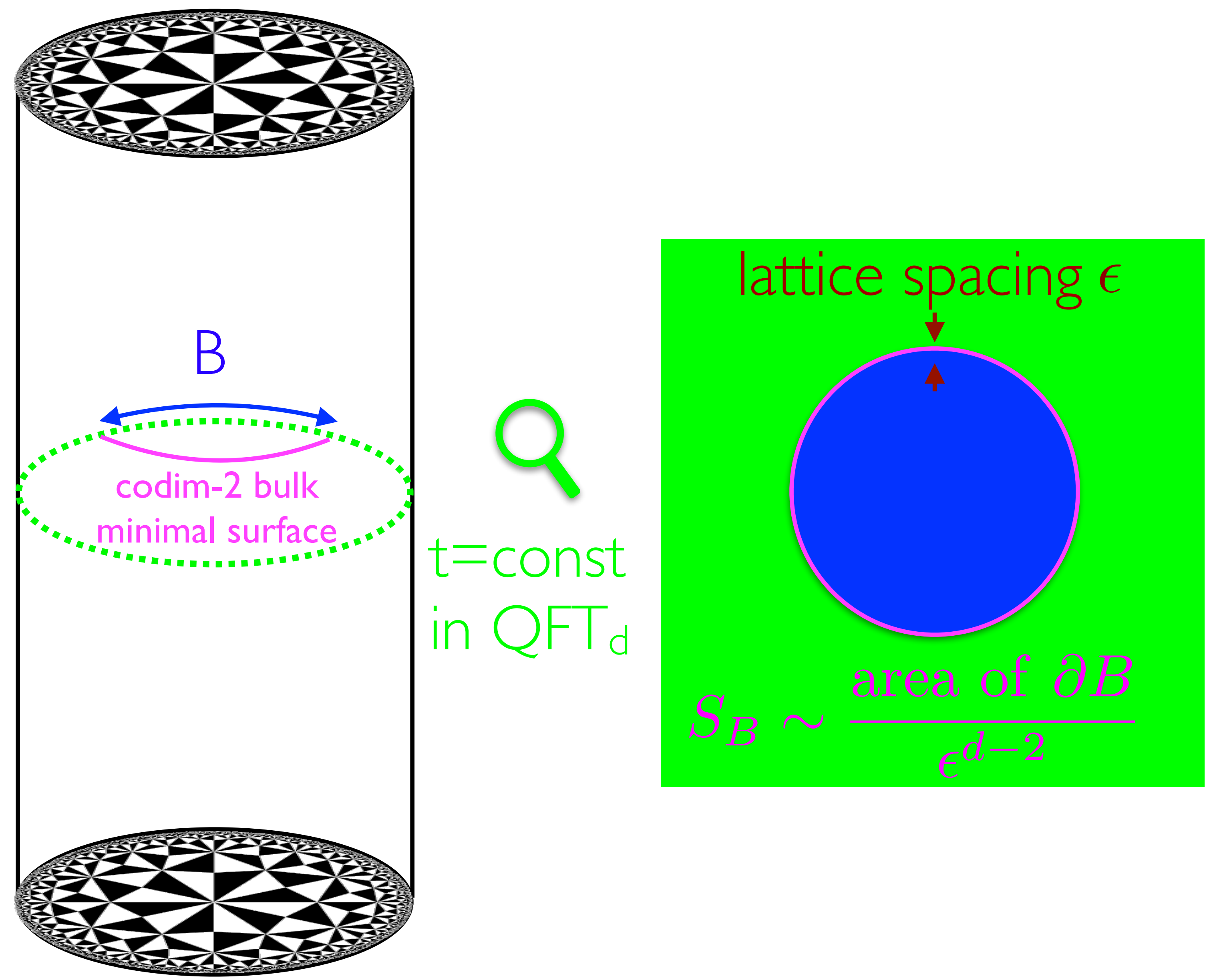
A powerful way to quantify it: entanglement entropy $S_B = -\mathrm{tr}\,(\rho_B \log \rho_B)$

see, e.g., **0808.3773** by Eisert, Cramer, Plenio

In SUGRA holography : $S_B = \dfrac{\text{bulk area}}{4\,G_N}$

hep-th/0603001 by Ryu & Takayanagi

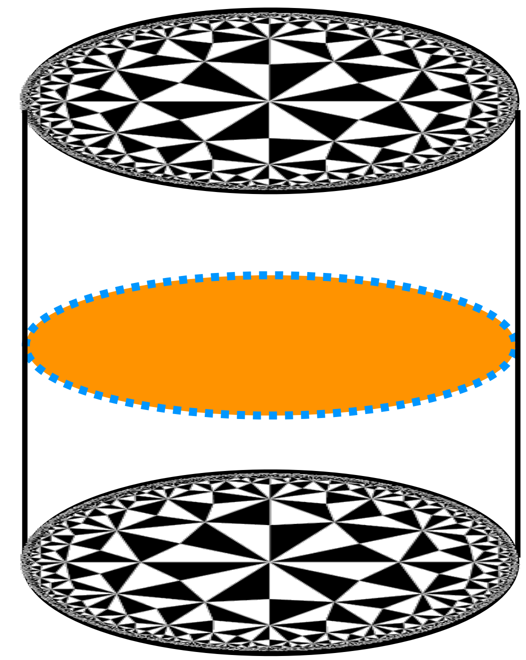
...



Volumes 1406.2678 by Stanford & Susskind

In the bulk, RT surfaces are just non-local objects defined by UV bdry condtion

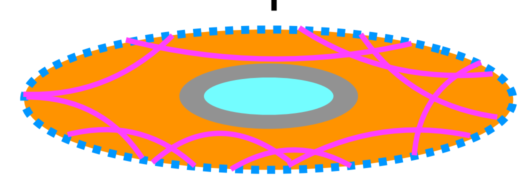
Are there other similarly defined objects? Yes:



bdry anchored
max. codim-1 vol
$\mathcal{C}_V \sim \mathrm{vol}$

hole-ography / kinematic space
1310.4204 / **1505.05515** by Czech et al. [1,2]

geometry from entanglement
1005.3035 by van Raamsdonk



Are codim-1 independent from RT: sometimes
see e.g. **1412.5175** by Freivogel et al.

"Entanglement (entropy a la RT) is not enough"
1411.0690 by Susskind

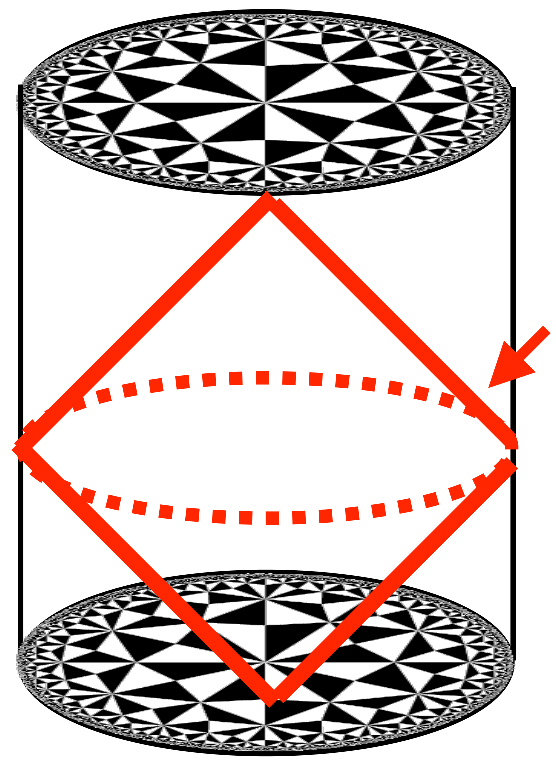
entwinement **1406.5859**, **1609.03991**, **1806.02871** by Balasubramanian et al. [1,2,3]

codim-1's as stand-alone objects

Wheeler-deWitt-patch actions

1509.07876, **1512.04993** by Brown et al.
1609.00207 by Poisson et al.
1612.05439 by Reynolds and Ross
1804.07410 by Chapman et al.
$\dots$

Another covariant codim-0 object:



$$\pi\,\mathcal{C}_A = \frac{1}{16\,\pi\,G_N}\int \mathrm{d}^{d+1}x\,(R - 2\,\Lambda)$$

$$+\dots$$

non-std variational problem (null bdries)

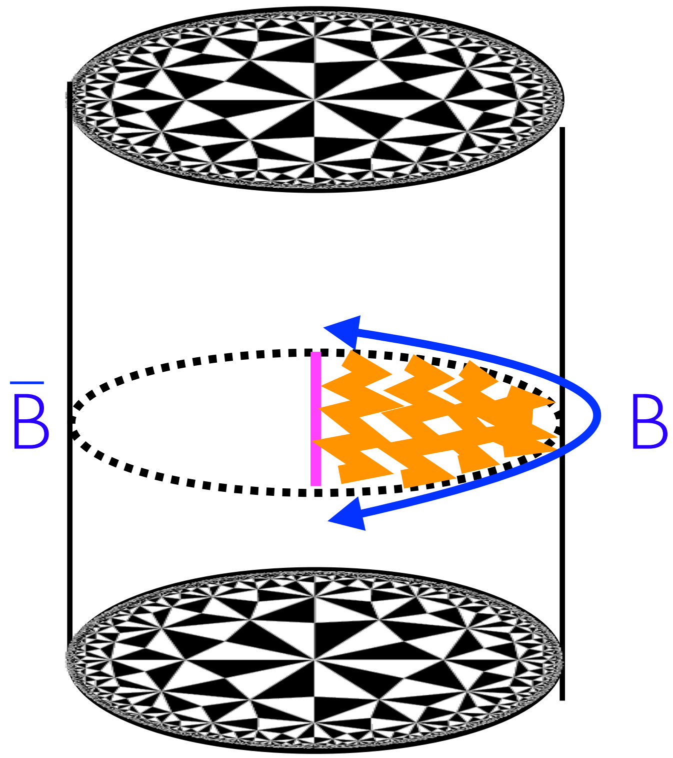
Despite appearance non-uniquely defined due to $\frac{1}{8\pi G_N}\int_{B'} d\lambda\, d^{d-1}\theta\,\sqrt{\gamma}\,\Theta\log(\ell_{ct}\Theta)$

1609.00207 by Poisson et al.
1804.07410 by Chapman et al.

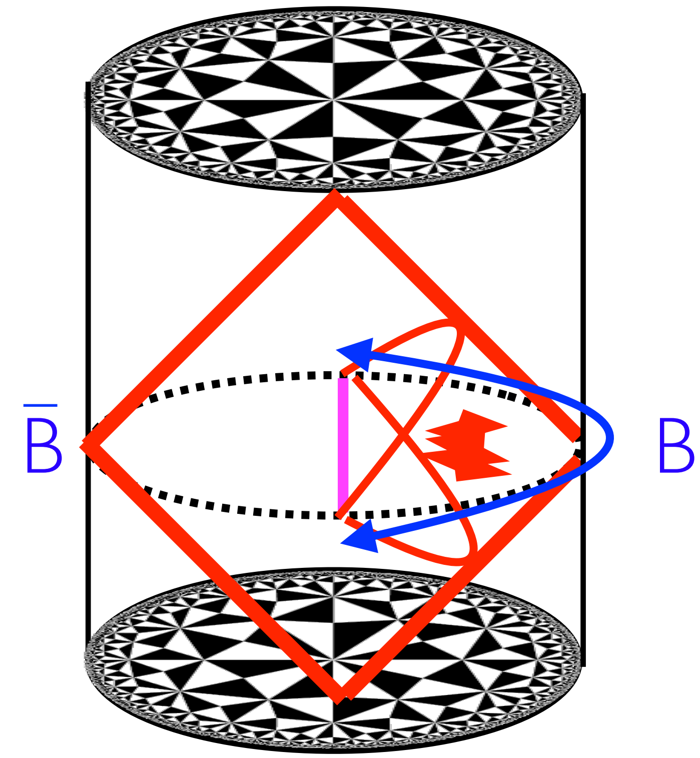
Subregions

1509.06614 by Alishahiha
1612.00433 by Carmi, Myers & Rath

$\mathcal{C}_V$:



$\mathcal{C}_A$:



Props: $\mathcal{C}_V[B] + \mathcal{C}_V[\bar{B}] \leq \mathcal{C}_V$ [always] $\mathcal{C}_A[B] + \mathcal{C}_A[\bar{B}] \geq \mathcal{C}_A$ [1 checked case]

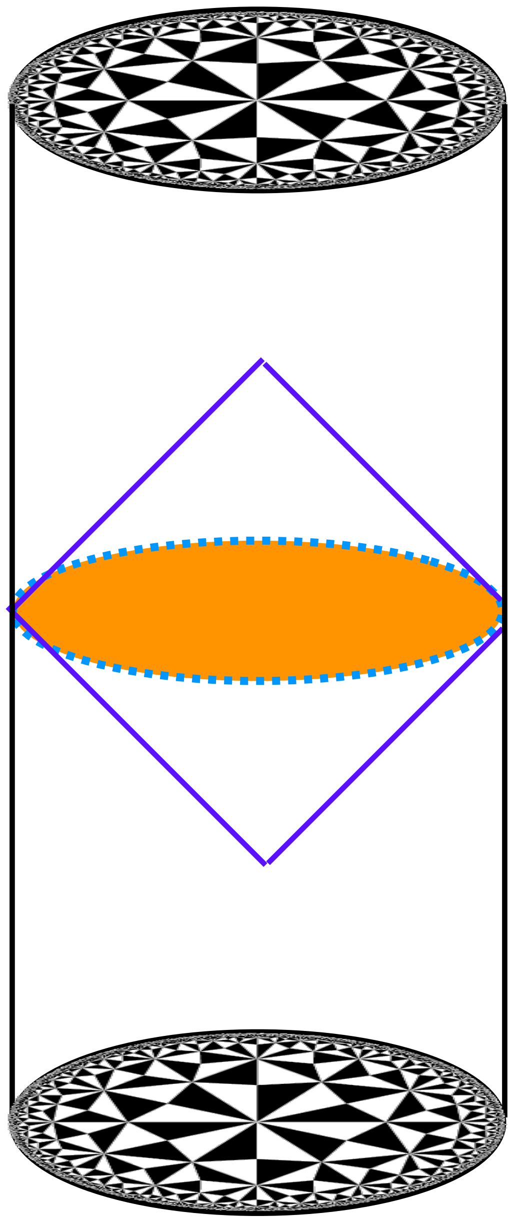
1804.01561 by Agón, Swingle & Headrick

For subregion volumes in AdS3 one can use hole-ography/kinematic space to express them in terms of entanglement entropy [more generally entwinement]

1707.01327, 1805.10298 by Abt et al.[1,2]

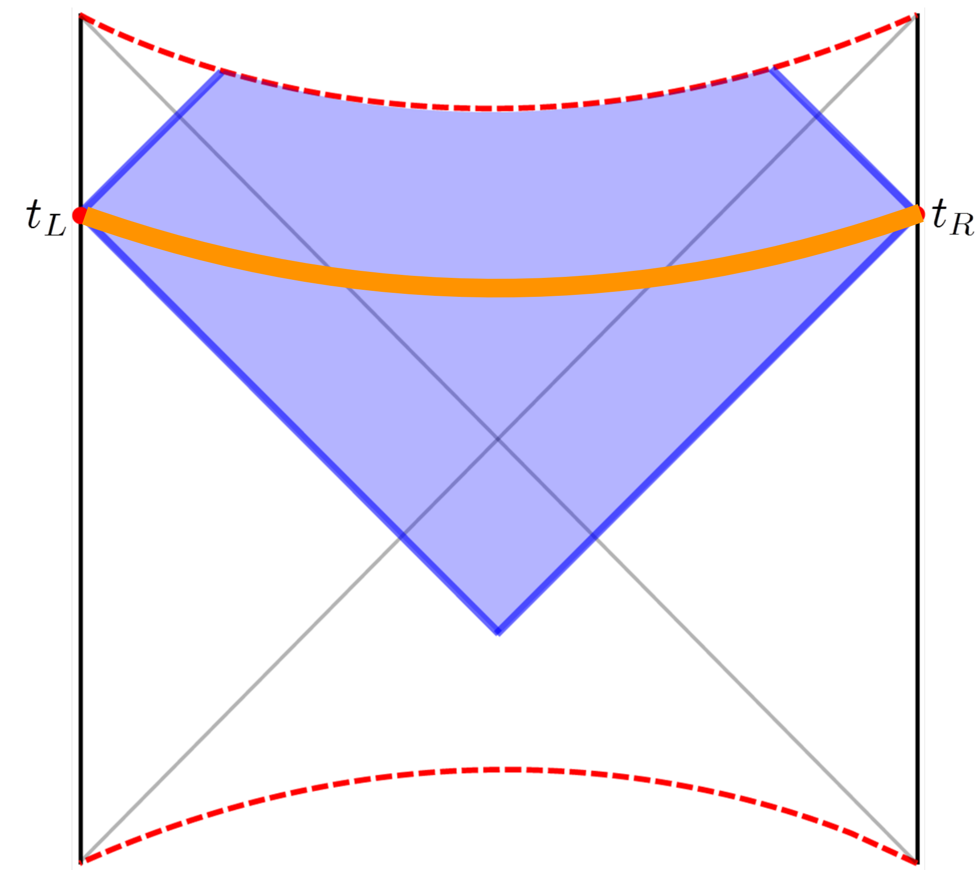
Some props of max volumes and WdW actions

aforementioned refs. + 1610.08063, 1709.10184 by Chapman et al.[1,2]
see also 1807.02186 by Couch et al.



$\mathcal{C}_V$ ~ Volume of codim-1 max volume bulk slice

$\mathcal{C}_A$ ~ Action in codim-0 bulk region with null bdries



I

$$\mathcal{C}_V[\mathrm{AdS}_{d+1}] \sim \frac{\text{vol occupied by hCFT}_\mathrm{d}}{\epsilon^{d-1}}$$

$$\mathcal{C}_A[\mathrm{AdS}_{d+1}] \sim \frac{\text{vol occupied by hCFT}_\mathrm{d}}{\epsilon^{d-1}} \left|\log \frac{l_{ct}}{L_{\mathrm{AdS}}}\right|$$

II

$$\mathcal{C}_{A/V}[\mathrm{AdS}-\mathrm{Schw}_{d+1}]\Big|_{t_L+t_R=0} - 2\,\mathcal{C}_{A/V}[\mathrm{AdS}_{d+1}] \sim S_\beta$$

III

$$\partial_{t_L+t_R}\mathcal{C}_{A/V}[\mathrm{AdS}-\mathrm{Schw}_{d+1}] \sim \mathrm{const}$$

Holographic complexity proposals

What $\mathcal{C}_V$ and $\mathcal{C}_A$ represent in dual hQFTs$_d$?

Unclear so far. No argument so far in terms of $Z_{\text{SUGRA}} = Z_{\text{hQFT}}$

We know they are distinct from RT (RT saturates in AdS-Schwarzschild + shadows)

For volumes arguments based on state overlaps

1507.07555 by Miyaji et al.
1806.10144 by Belin, Lewkowycz, Sárosi

Here we focus on complexity interpretation and links with tensor networks

Part II: tensor networks

Tensor networks see Banuls' talk and, e.g. , review **1407.6552** by Orús

For quantum-many bodies *Hilbert space* gets fast vast: *N*-qubits → dim($\mathcal{H}$)=2^N

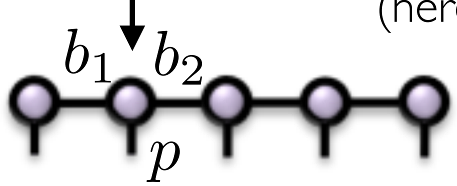
Tensor networks were born as **variational ansatzes** for the tiny corner of Hilbert space relevant for ground states of <u>local</u> Hamiltonians

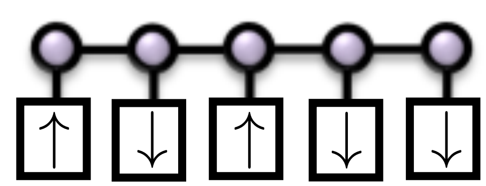
Key idea: GS of local Hamiltonian (~~criticality in 1+1~~) are locally entangled (area law)

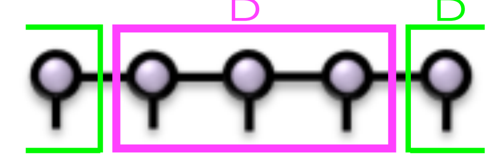
proven in 1+1 in **0705.2024** by Hastings; RT is one of many indications it is correct in higher d

In 1+1 one can then construct matrix product state variat. ansatz for GS:

matrix (tensor) $M_p^{b_1 b_2}$ ↗ bond indices (run over χ vals) ↘ physical index (here runs over 2 vals)

$|\mathrm{MPS}\rangle =$ 

$\langle\uparrow\downarrow\uparrow\downarrow\downarrow|\mathrm{MPS}\rangle =$ 



$\equiv$

$$|\mathrm{MPS}\rangle = \sum_{b_{eff}=1}^{\chi^2} c_{b_{eff}} |\psi_{b_{eff}}\rangle |\phi_{b_{eff}}\rangle$$

↓

$$S_B \leq \log \chi^2$$

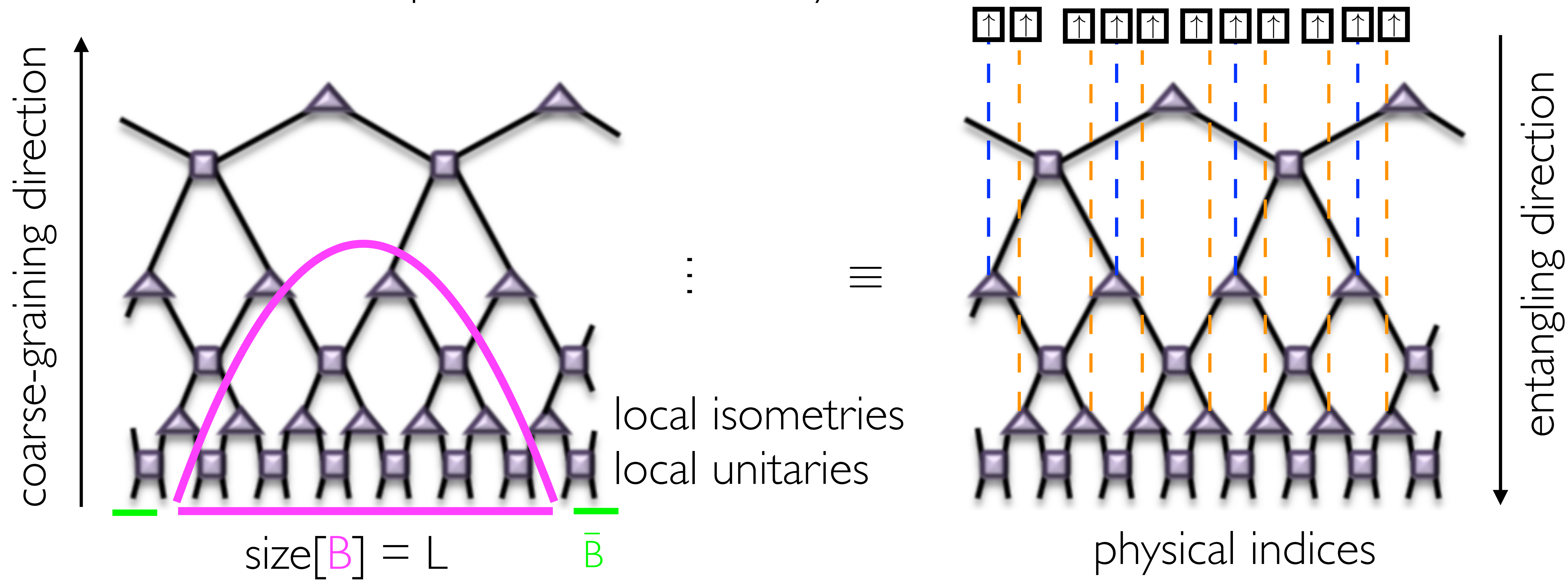
Key features: efficient contractibility and energy minimization $\langle \mathrm{MPS}|H|\mathrm{MPS}\rangle$

figs. adapted from 1407.6552 by Orús

Multiscale Entanglement Renormalization Ansatz

cond-mat/0512165 by Vidal

MERA is a TN that captures GS of critical systems in 1+1



Now we have $|\mathrm{MERA}\rangle = \sum_{b_{eff}=1}^{\chi^{\log L}} c_{b_{eff}} |\psi_{b_{eff}}\rangle |\phi_{b_{eff}}\rangle \longrightarrow S_B \leq \log \chi^{\log L} \sim \log L$ ✓

Why we care: symmetries of (some) H_2 or ***dS*₂** — holographic interpretation?

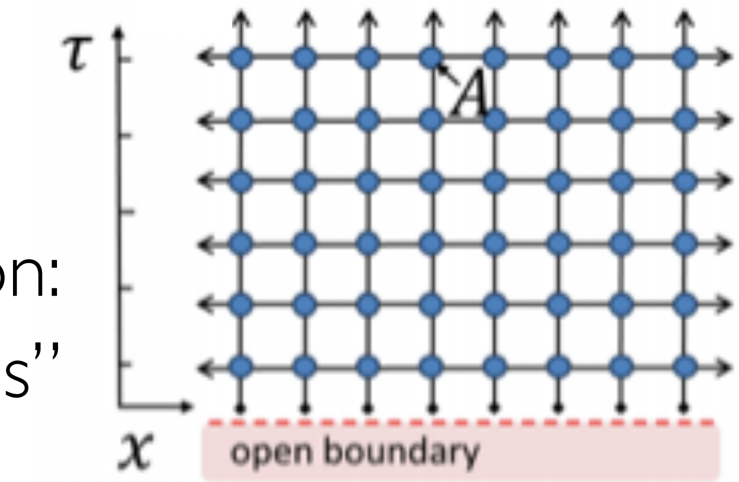
0905.1317 by Swingle
1512.01548 by Czech et al.

tensors in MERA gives a volume of $H_2 \longrightarrow \mathcal{C}_V$?

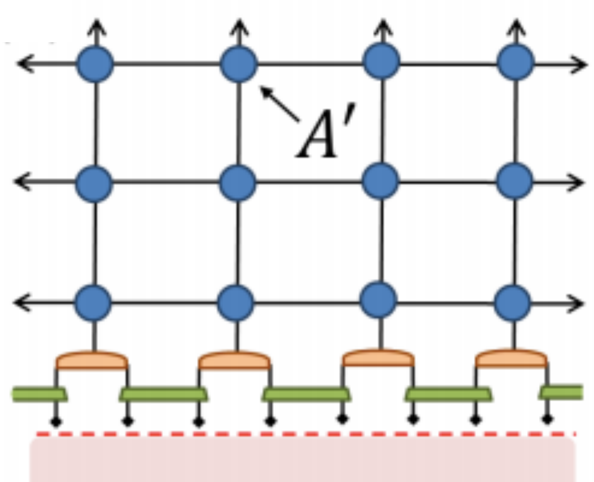
figs. adapted from 1407.6552 by Orús

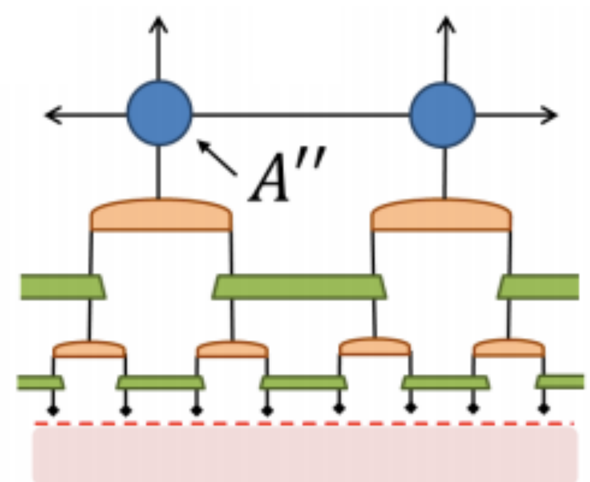
MERA from Tensor Network Renormalization

$\exp(-\tau\, H)$ acts as a projector to GS. TN representation:
"euclideons"



Such a TN can be coarse-grained, but open indices enforce a layer of MERA:
1412.0732 by Evenbly & Vidal

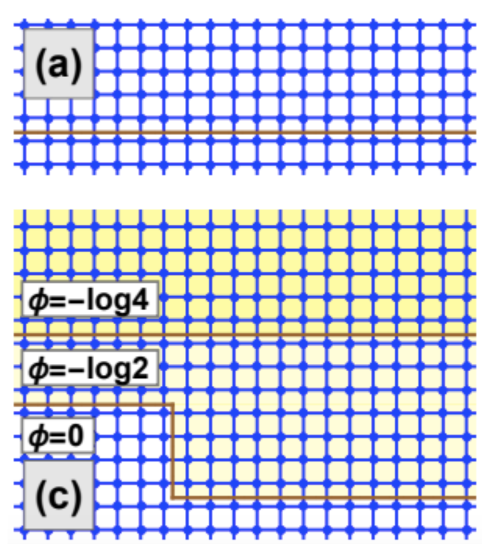




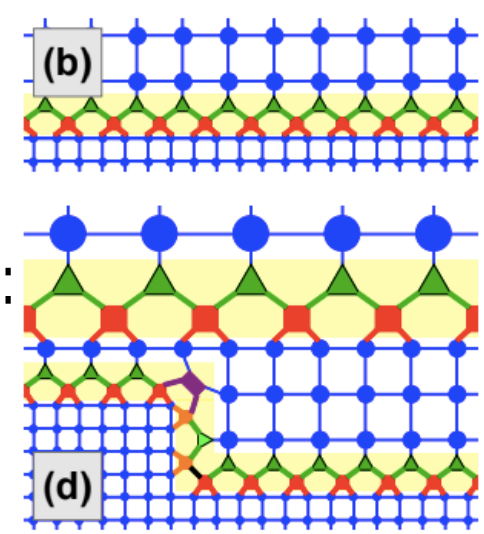
isometries

disentanglers

1502.05385 by Evenbly & Vidal



Finally, $d\tau^2 + dx^2$: $\longrightarrow e^{2\,\phi}\,(d\tau^2 + dx^2)$:



observation:

density of tensors $\sim e^{2\,\phi}$

MERA layer when $\partial\phi \neq 0$

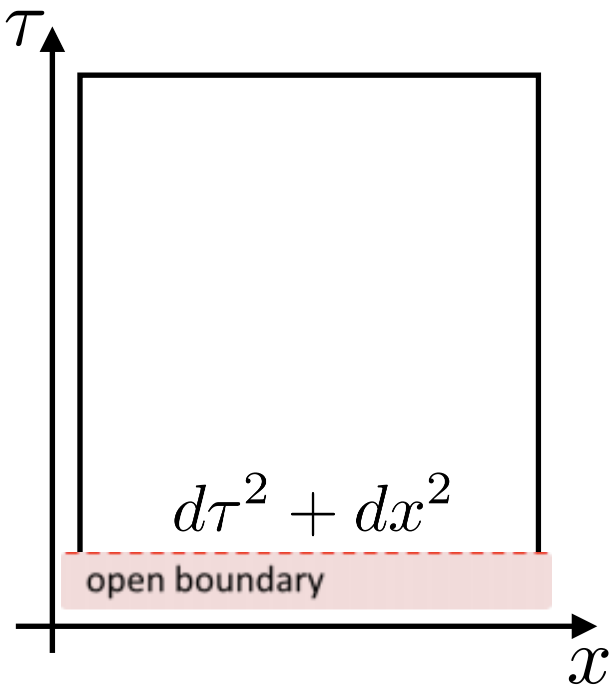
1706.00965 by Czech

figs. adapted from 1502.05385 by Evenbly & Vidal and 1706.00965 by Czech

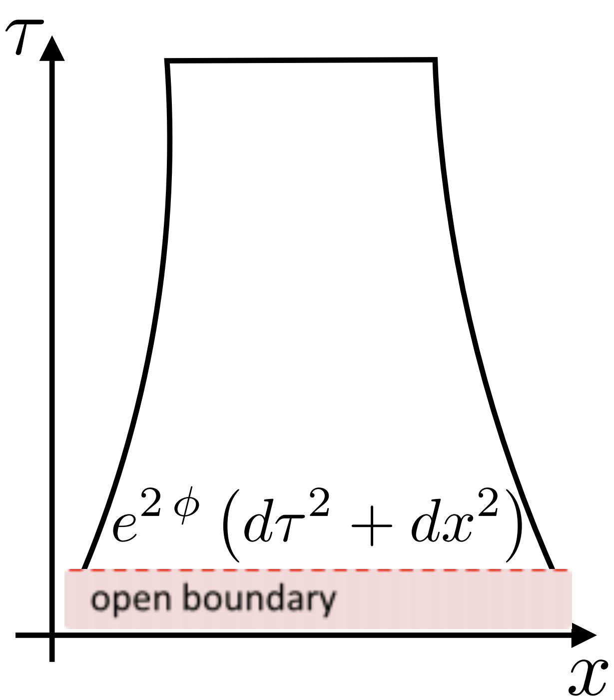
Path integral optimization

1703.00456, 1706.07056 by Caputa et al. , 1804.01999 by Bhattacharyya et al.

One can also consinder $\exp(-\tau H)$ in a QFT (here CFT$_2$) using path integrals:



$$\Psi[\varphi_0(x)] = \int D\varphi\, e^{-S_E[\varphi]}\, \delta\left(\varphi(\epsilon, x) - \varphi_0(x)\right)$$



$$\tilde{\Psi}[\varphi_0(x)] =$$

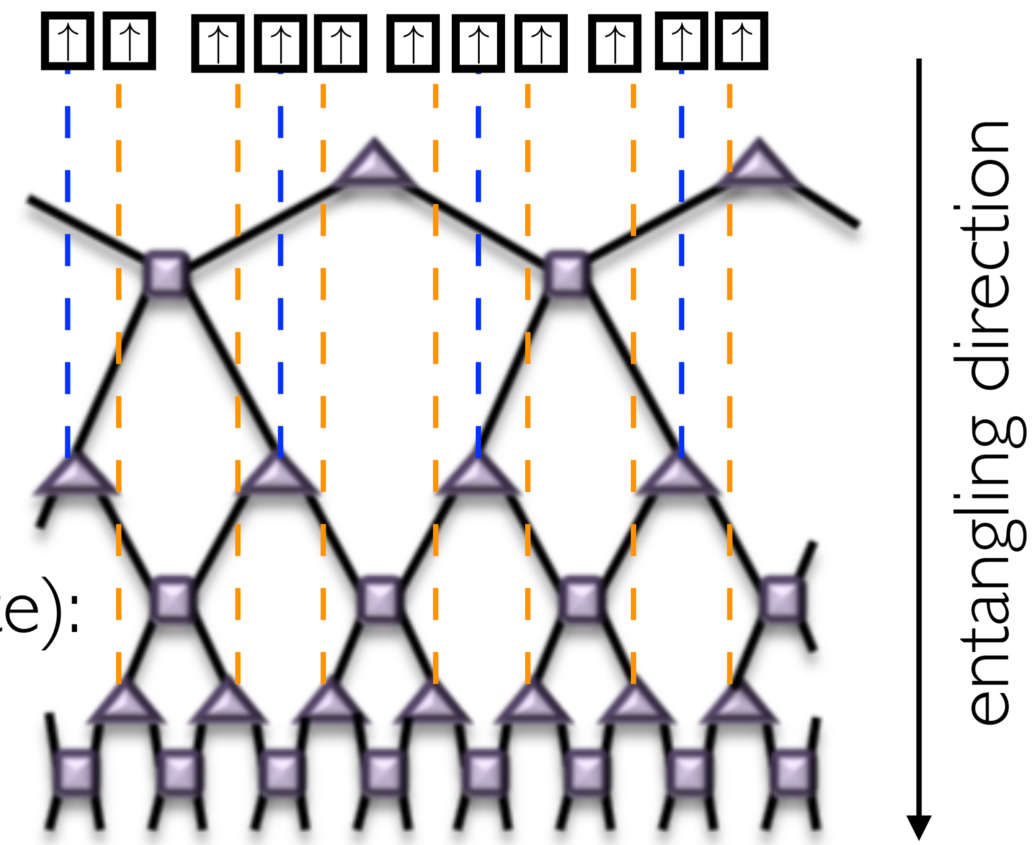
$$\exp\left\{\frac{c}{24\pi}\int_{-\infty}^{\infty} dx \int_{\epsilon}^{\infty} dz\, \left((\partial_x\phi)^2 + (\partial_\tau\phi)^2 + \mu\, e^{2\phi}\right)\right\} \times \Psi[\varphi_0(x)]$$

↓ minimization

optimal path integral defined on H_2 (c.f. MERA / TNR)

cMERA

1102.5524 by Haegeman et al.,
see also 1208.3469 by Nozaki et al., 1703.04798 by Hu & Vidal



One can view MERA as a unitary x (a product state):

Generalization to a free QFT:

$$\mathcal{P}e^{-i\int_{-\infty}^{0} du(L+K(u))}$$

GS of $H = \frac{1}{2}\int d^{d-1}x\,(\pi^2 + M^2\phi^2)$

univ. spatial scaling trafo

variationally determined disentangler

One can generalize it in the perturbation theory to LO

1806.02831, 1806.02835 by Cotler et al.

Part III: QFT complexity

$\mathcal{C}_{A/V}$ stands for complexity?

4) How to count gates and deal with UV divergences?

3) How to make sense now of the approximation?

Complexity $\mathcal{C}$: min. number of elem. unitary operations δU s.t. $|T\rangle \approx \overbrace{\delta U \ldots \delta U}^{\# \equiv \mathcal{C}} |\uparrow \ldots \uparrow\rangle$

2) What can now act as a set of elementary unitary operations (gates)?

1) What can be a simple reference state in continuum?

$<$ 2017: entanglement entropy in a QFT ✓ vs. complexity in a QFT ✗

5) We want an approach that is computable $\longrightarrow$ Gaussian States and free QFTs$_{d+1}$

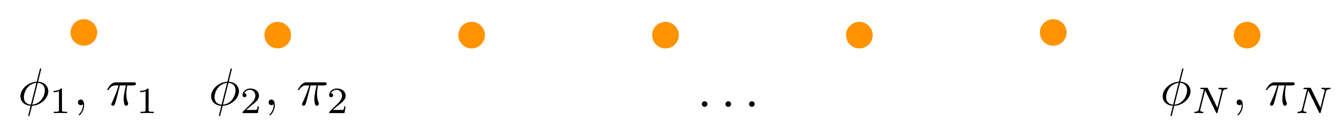
I. Vacuum

1707.08582 with Chapman, Marrochio & Pastawski, **1707.08570** by Jefferson & Myers and **1808.xxxxx** with Chapman, Eisert, Hackl, Jefferson, Marrochio & Myers
see also Jefferson's talk

Holography = strong coupling QFTs. We do free QFTs. Universality to the rescue?

Now target / reference state is GS of $\int \mathrm{d}^{d-1}x \left\{ \pi^2 + (\partial_x \phi)^2 + m_{1/2}^2 \phi^2 \right\}$

We put the theory on the lattice to UV regulate it



Gates: $\delta U = e^{i\phi_1 \pi_3 \, \delta s}$ etc $\longrightarrow$ $\mathrm{SP}(2N, \mathbb{R})$ group.

To calculate complexity, we will define a metric on* $\mathrm{SP}(2N, \mathbb{R})$ and calc. geodesics

Many choices, but soluble ones $\xrightarrow[\text{limit}]{\text{cont.}}$ $\mathcal{C} \sim \sqrt{\mathrm{vol} \int_{|k| \le \Lambda} \mathrm{d}^{d-1}k \left(\log \frac{m_1^2 + k^2}{m_2^2 + k^2} \right)^2}$ ✗

What compares ✓ with $\mathcal{C}_{V/A}$ is $\mathcal{C} \sim \mathrm{vol} \int_{|k| \le \Lambda} \mathrm{d}^{d-1}k \left| \log \frac{k}{m_2} \right|$

→ $|R\rangle$ GS of $\int \mathrm{d}^{d-1}x \left\{ \pi^2 + m_2^2 \phi^2 \right\}$

→ L^1 norm

II. Formation of TFD

1808.xxxxx with Chapman, Eisert, Hackl, Jefferson, Marrochio & Myers
see also Jefferson's talk

For the TFD state, we have additional gates such as $\delta U = e^{i\,\phi_1^L\,\phi_3^R}$

However, there are choices one can make such that

$$\mathcal{C}_{|TFD(t_L+t_R=0)\rangle} \sim \underbrace{\text{vol}\int_{k\leq\beta^{-1}} d^{d-1}k\,(\ldots)}_{S_\beta} + 2\times\underbrace{\text{vol}\int_{k\leq\Lambda} d^{d-1}k\,\left|\log\frac{k}{m_2}\right|}_{\mathcal{C}_{|0\rangle}}$$

As a result we get sth very similar to

I

$$\mathcal{C}_V[\text{AdS}_{d+1}] \sim \frac{\text{vol occupied by hCFT}_\text{d}}{\epsilon^{d-1}}$$

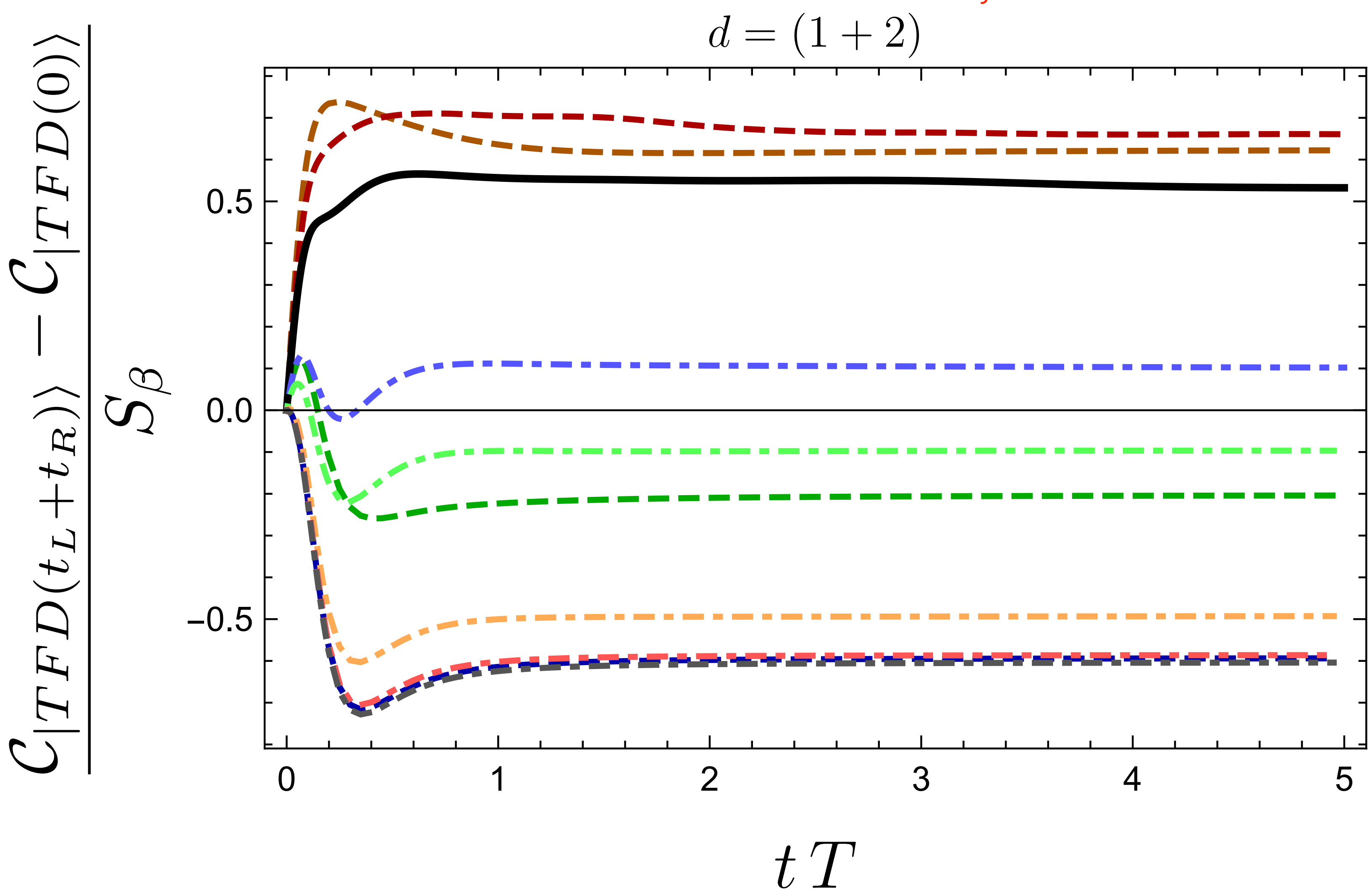
$$\mathcal{C}_A[\text{AdS}_{d+1}] \sim \frac{\text{vol occupied by hCFT}_\text{d}}{\epsilon^{d-1}}\left|\log\frac{l_{ct}}{L_{\text{AdS}}}\right|$$

II

$$\mathcal{C}_{A/V}[\text{AdS}-\text{Schw}_{d+1}]\Big|_{t_L+t_R=0} - 2\,\mathcal{C}_{A/V}[\text{AdS}_{d+1}] \sim S_\beta$$

III. Time-dependence of TFD

1808.xxxxx with Chapman, Eisert, Hackl, Jefferson, Marrochio & Myers
see also Jefferson's talk



Complexity saturates since it is a sum of oscillatory funcs (free QFT!) that dephase

Not surprisingly, this is in stark contrast with holography:

$$\partial_{t_L+t_R}\mathcal{C}_{A/V}[\mathrm{AdS}-\mathrm{Schw}_{d+1}] \sim \mathrm{const}$$

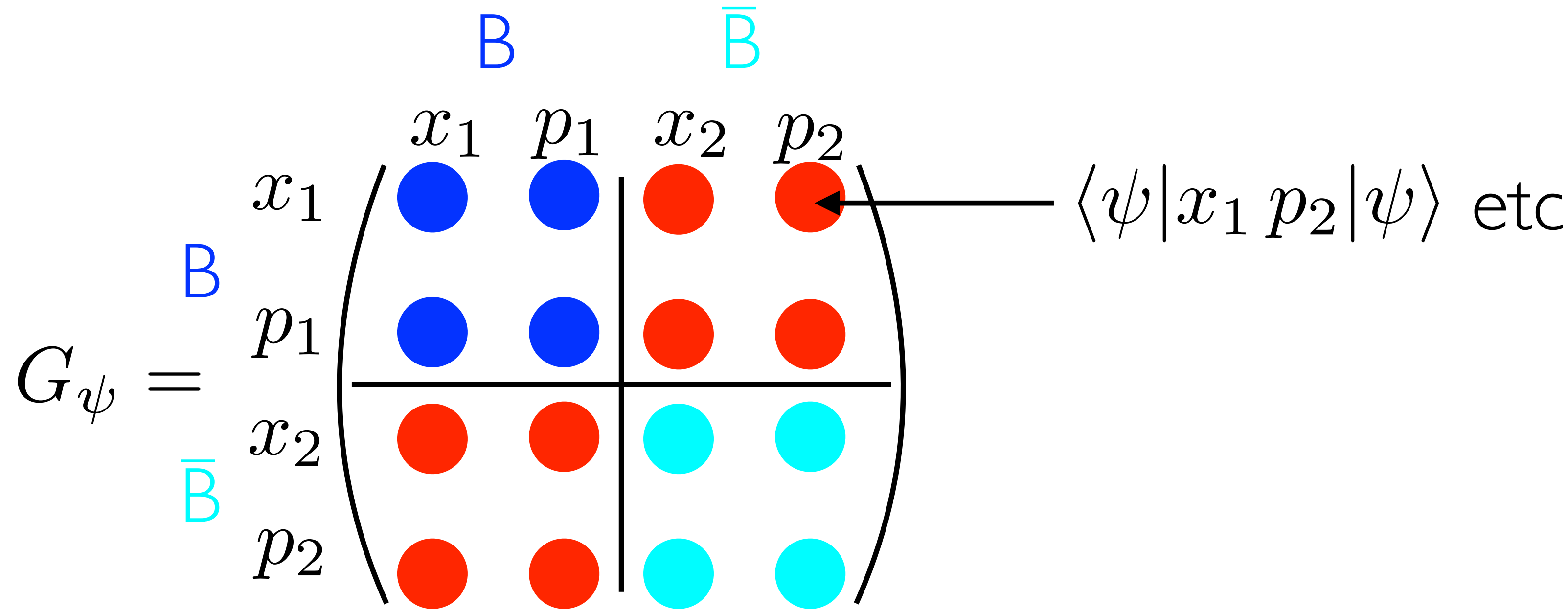
III

✗

Entanglement vs. complexity

1807.07075 with Camargo et al.
see also Jefferson's talk

Consider 2 harmonic oscillators in a pure Gaussian state:



Observations:

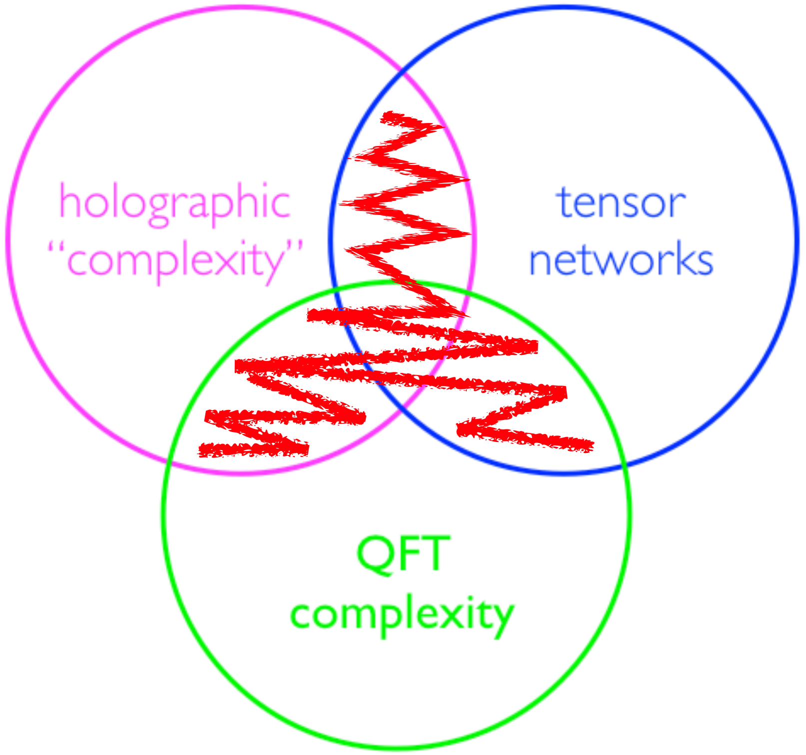
- Entanglement entropy$_B$ fixed by G_1 or G_2
- Entanglement entropy $\neq$ cross-correlations [well-known, but nice demo]
- Complexity sensitive to the whole G_ψ

Subregion complexity:

fix, e.g., $G_1 \longrightarrow$ consider $G_{\psi'}$ with the same $G_1 \longrightarrow C[\rho_B] := \min_{\psi'} C[|\psi'\rangle]$

Evidences for $C[\rho_B] + C[\rho_{\bar{B}}] \geq C[|\psi\rangle]$ (c.f. with $\mathcal{C}_V[B] + \mathcal{C}_V[\bar{B}] \leq \mathcal{C}_V$)

Towards convergence



Towards convergence