UNIVERSITY OF ICELAND

Stockholm University

Charged black holes and near AdS_2 holography

Lárus Thorlacius

based on arXiv:1808.xxxxx by
A. Brown, H. Gharibyan, H. Lin, A. Streicher, L. Susskind, LT, Y. Zhao

Gauge/Gravity Duality 2018

Motivation

Explore holographic complexity conjectures ("C = V" and "C=A") in a simple setting

— 1+1-dilaton gravity model

Teitelboim 1993; Jackiw 1995
Almheiri, Polchinski 2014; Jensen 2016; Engelsöy, Mertens, Verlinde 2016
Maldacena, Stanford, Yang 2016; Harlow, Jafferis 2018; …..

— broken conformal symmetry —> low-energy dynamics governed by Schwarzian effective action

— the same (broken) symmetry is realized in the SYK model
—> Schwarzian action captures important aspects of SYK dynamics

— SYK model has discrete field variables with q-local Hamiltonian
—> quantum complexity better defined than in continuum QFT

Jackiw-Teitelboim model

Action

$$S_{JT} = \frac{1}{2}\int d^2x\sqrt{-g}\,\varphi\left(R+\frac{2}{L^2}\right)+\int dy^0\sqrt{-\gamma_{00}}\,\varphi\left(K-\frac{1}{L}\right)$$
$$+\,\varphi_0\left(\frac{1}{2}\int d^2x\sqrt{-g}\,R+\int dy^0\sqrt{-\gamma_{00}}\,K\right)$$

topological term

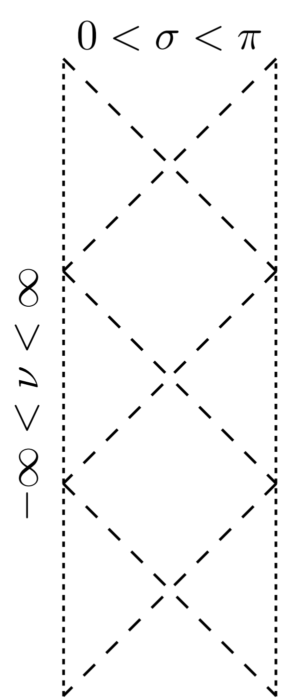
Field equations

AdS$_2$ geometry

$$0 = R+\frac{2}{L^2}$$
$$0 = \nabla_\alpha\nabla_\beta\,\varphi - g_{\alpha\beta}\left(\nabla^2\varphi-\frac{1}{L^2}\varphi\right)$$

Global coordinates on AdS$_2$

$$ds^2 = \frac{L^2}{\sin^2\sigma}\left(-\,d\nu^2+d\sigma^2\right)$$

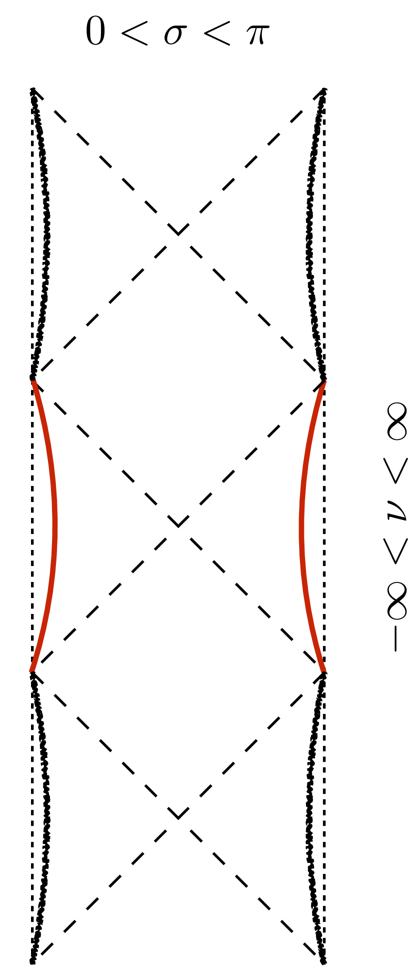


Dilaton field

$$\varphi(\nu,\sigma) = \varphi_H\,\frac{\cos\nu}{\sin\sigma}$$

Jackiw-Teitelboim black hole

$$ds^2 = \frac{L^2}{\sin^2\sigma}\left(-\,d\nu^2 + d\sigma^2\right) \qquad \varphi(\nu,\sigma) = \varphi_H\,\frac{\cos\nu}{\sin\sigma}$$



Boundary curve:

$$\varphi = \varphi_B \ll \varphi_0 \quad \Rightarrow \quad \sin\sigma = \frac{\varphi_H}{\varphi_B}\cos\nu$$

JT singularity:

$$\varphi + \varphi_0 = 0$$

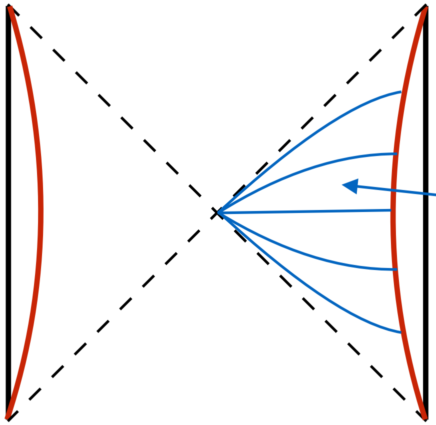
JT black hole in "Schwarzschild" coordinates

$$ds^2 = -\frac{r^2 - r_H^2}{L^2}\,dt^2 + \frac{L^2}{r^2 - r_H^2}\,dr^2 \qquad\qquad \varphi = \varphi_H\,\frac{r}{r_H}$$

AdS$_2$ scaling: $r \to \lambda r\,, \quad t \to \lambda^{-1} t$

Event horizon: $r = r_H$

BH temperature: $T = \dfrac{r_H}{2\pi L^2}$



Relation between global time and Schwarzschild time (at boundary):

$$\tan\left(\frac{\nu}{2} + \frac{\pi}{4}\right) \approx \exp\left(\frac{r_H\,t}{L^2}\right) \quad \text{at} \quad r \gg r_H$$

$$\nu = 0 \;\Leftrightarrow\; t = 0$$

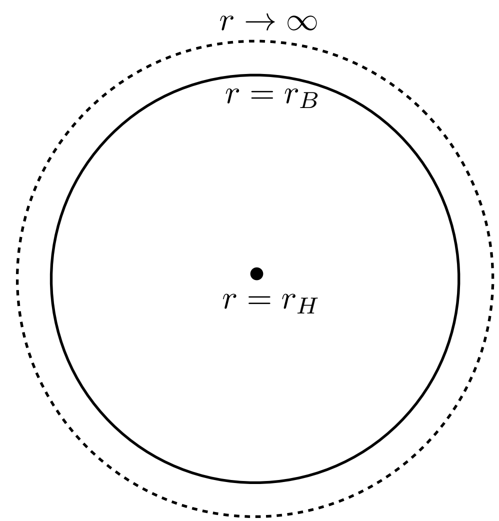
$$\nu \to \pm\frac{\pi}{2} \;\Leftrightarrow\; t \to \pm\infty$$

$$\frac{d\nu}{dt} \approx \frac{2r_H}{L^2}\,e^{-r_H t/L^2} \quad \text{as} \quad t \to \infty$$

JT black hole thermodynamics

$$ds^2 = \frac{r^2 - r_H^2}{L^2}\, d\tau^2 + \frac{L^2}{r^2 - r_H^2}\, dr^2$$

$$\varphi = \varphi_H\, \frac{r}{r_H}$$



On-shell Euclidean action:

$$S_E = \beta\, F = -S + \beta\, E$$

zero temperature entropy $S_0 = 2\pi\varphi_0$

BH entropy: $$S = 2\pi\varphi_0 + 4\pi^2 L^2 \frac{\varphi_B}{r_B} T$$

BH mass: $$E = 2\pi^2 L^2 \frac{\varphi_B}{r_B} T^2$$

Almheiri, Polchinski 2014
Maldacena, Stanford, Yang 2016
…..

$$\frac{dE}{dt} = T\, \frac{dS}{dt}$$

Holographic quantum complexity

Model BH scrambling dynamics by a quantum circuit with a total number of qubits of order S and a universal set of primitive gates. Hayden, Preskill 2006

The quantum complexity of a circuit state is the minimum number of primitive gates needed to obtain that state from a given reference state.

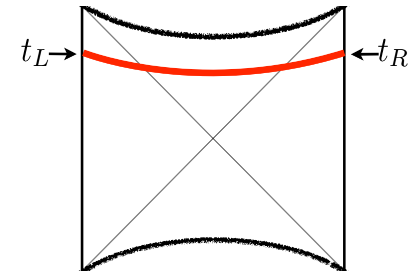
Assuming each qubit gets acted on by at most one primitive gate per cycle we expect $\frac{\Delta\mathcal{C}}{\Delta\tau} \sim S$

or, if each cycle takes of order one unit of Rindler time: $\boxed{\frac{d\mathcal{C}}{dt_S} \sim S\,T}$ $\tau_R = \frac{2\pi}{\beta}\, t_S$

Holographic complexity conjectures:

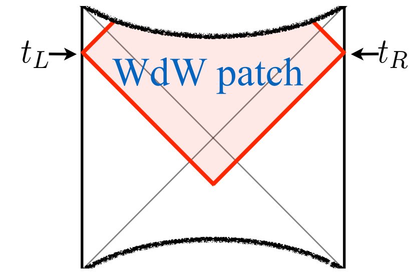
1) Complexity equals volume $\mathcal{C} \sim \frac{V}{G_N R_0}$

Susskind 2014

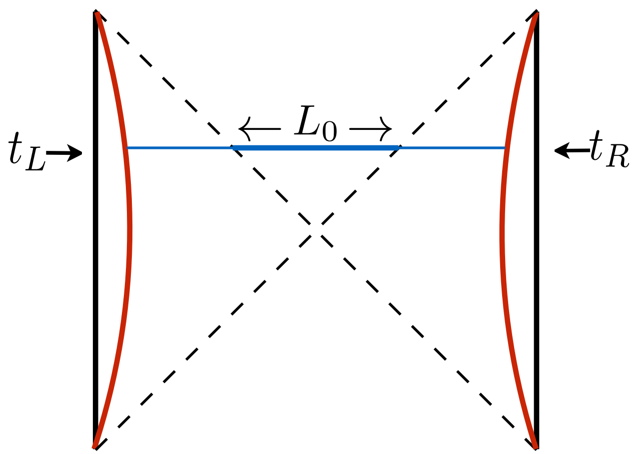


2) Complexity equals action $\mathcal{C} = \frac{\mathcal{A}}{\pi}$

Brown, Roberts, Susskind, Swingle, Zhao 2015



"$C = V$" for JT black hole



Consider geodesic connecting t_L and t_R on left and right boundaries and calculate the geodesic length L_0 inside BH.

"volume" of maximal slice: $V(t_L, t_R) \sim (G_N \varphi_0)\, L_0$

transverse area

Complexity: $\mathcal{C} \sim \dfrac{V}{G_N L} \sim \dfrac{\varphi_0\, L_0}{L}$

Calculation simplifies for $t_L = t_R = t$

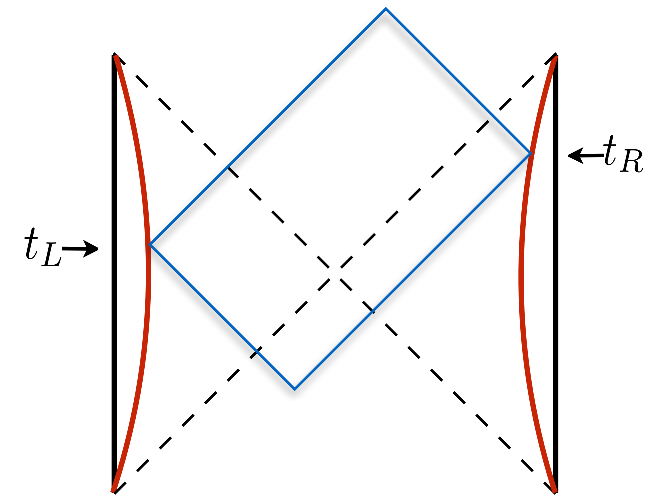
$$L_0 = L \int_{\frac{\pi}{2}-\nu}^{\frac{\pi}{2}+\nu} \frac{d\sigma}{\sin\sigma} = 2L\,\log\left[\tan\left(\frac{\pi}{4}+\frac{\nu}{2}\right)\right] \approx \frac{r_H\, t}{L} \quad \text{as} \quad t \to \infty$$

Now use $S = 2\pi\varphi_0 + O(T)$ and $T = \dfrac{r_H}{2\pi L^2}$

$\longrightarrow \quad \dfrac{d\mathcal{C}}{dt} \sim S\,T$ at late times (up to $O(T^2)$ terms)

"$C = A$" for JT black hole

$$\mathcal{A}_{JT} = \frac{1}{2}\int d^2x\sqrt{-g}\,\varphi\left(R+\frac{2}{L^2}\right)+\int dy^0\sqrt{-\gamma_{00}}\,\varphi\left(K-\frac{1}{L}\right)$$
$$+\varphi_0\left(\frac{1}{2}\int d^2x\sqrt{-g}\,R+\int dy^0\sqrt{-\gamma_{00}}\,K\right)$$



Euler characteristic

i) The topological term gives $2\pi\,\chi\big|_{WdW} = 2\pi i$ ← constant

ii) $R = -\dfrac{2}{L^2}$ on AdS_2 $\longrightarrow$ bulk JT term gives 0

iii) Careful evaluation of remaining boundary term gives $\dfrac{d\mathcal{A}}{dt} \to 0$ as $t \to \infty$

$\longrightarrow$ the action on the WdW patch does not grow at late times!

This does not mean that "$C = A$" fails but rather that we need to remember how the JT theory arises in the context of higher-dimensional charged BH's

3+1-dimensional charged BH

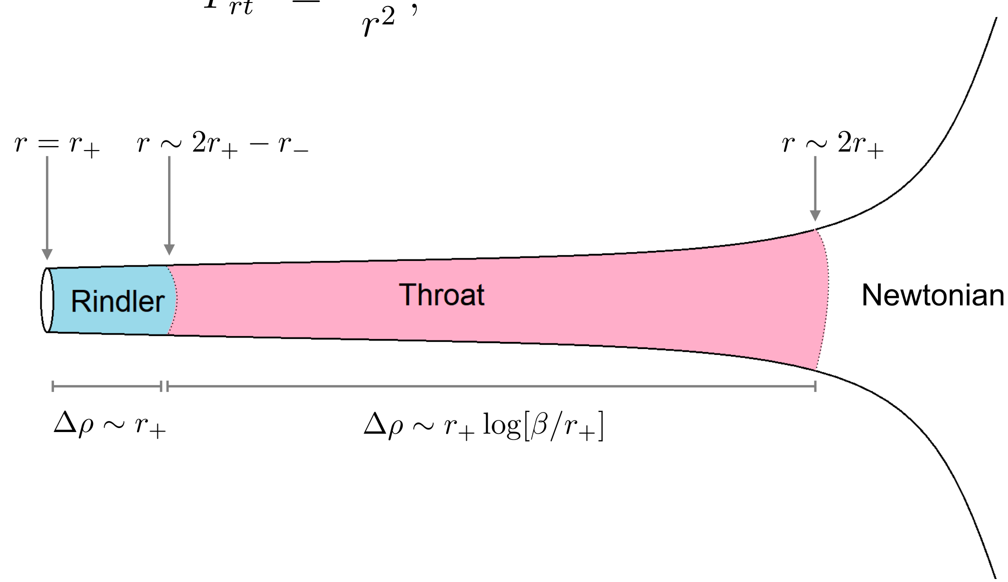
Our starting point is the 3+1-dimensional Einstein-Maxwell theory with action

$$\mathcal{S} = \frac{1}{16\pi}\int_{\mathcal{M}} d^4x\,\sqrt{-G}\,\left(\frac{1}{\ell^2}R - \,F_{\mu\nu}F^{\mu\nu}\right) + \frac{1}{8\pi\ell^2}\int_{\partial\mathcal{M}} d^3y\,\sqrt{-h}\,(K - K_0)\,,$$

where $\ell = \sqrt{G_N}$ is the 3+1-dimensional Planck length

Reissner-Nordström black hole with electric charge $Q > 0$ and mass $M \geq Q/\ell$

$$\begin{aligned} ds^2 &= -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2\,, \\ f(r) &= \left(1-\frac{r_+}{r}\right)\left(1-\frac{r_-}{r}\right)\,, \\ F_{rt} &= \frac{Q}{r^2}\,, \end{aligned}$$



JT model from spherical reduction

Navarro-Salas, Navarro 1999

Spherically symmetric ansatz:

$$ds^2 = \frac{1}{\sqrt{2\Phi}} g_{\alpha\beta}\, dx^\alpha dx^\beta + 2\ell^2 \Phi\, d\Omega^2$$

1+1 D metric

transverse area is a scalar field in 1+1 D

Inserting into original action gives 1+1 D action of an Einstein-Maxwell-Dilaton theory

$$\mathcal{S}_{2d} = \frac{1}{2}\int d^2x\sqrt{-g}\Big(\Phi R + \frac{1}{\ell^2}(2\Phi)^{-\frac{1}{2}} - \frac{\ell^2}{2}(2\Phi)^{\frac{3}{2}} F_{\alpha\beta}F^{\alpha\beta}\Big) + \int dy^0\sqrt{-\gamma_{00}}\Big(\Phi K - \frac{1}{\ell}(2\Phi)^{\frac{1}{4}}\Big)$$

The field equations of the 1+1-dimensional theory are,

$$\begin{aligned}
0 &= \nabla_\alpha\left(\Phi^{3/2}F^{\alpha\beta}\right), \\
0 &= R - \frac{1}{\ell^2}(2\Phi)^{-3/2} - \frac{3}{2}\ell^2(2\Phi)^{1/2}F^2, \\
0 &= \nabla_\alpha\nabla_\beta\Phi - g_{\alpha\beta}\Big(\nabla^2\Phi - \frac{1}{2\ell^2}(2\Phi)^{-1/2}\Big) + \ell^2(2\Phi)^{3/2}\Big(F_{\alpha\gamma}F_\beta{}^\gamma - \frac{1}{4}g_{\alpha\beta}F^2\Big)
\end{aligned}$$

The Maxwell equation determines the electromagnetic field strength in terms of the dilaton

$$F_{\alpha\beta} = \frac{Q}{\ell^2}(2\Phi)^{-3/2}\,\varepsilon_{\alpha\beta}$$

and this can be used to eliminate the gauge field from the remaining equations

Spherical reduction (p.2)

Remaining field equations for the metric and dilaton

$$\begin{aligned} 0 &= R - \frac{1}{\ell^2}(2\Phi)^{-3/2} + \frac{3Q^2}{\ell^2}(2\Phi)^{-5/2}\,, \\ 0 &= \nabla_\alpha \nabla_\beta \Phi - g_{\alpha\beta}\Big(\nabla^2\Phi - \frac{1}{2\ell^2}(2\Phi)^{-1/2} + \frac{Q^2}{2\ell^2}(2\Phi)^{-3/2}\Big) \end{aligned}$$

Now expand the dilaton around its value at the horizon of an extremal RN black hole: $\Phi = \frac{Q^2}{2} + \varphi$

and work order by order in φ/Q^2

$$\longrightarrow \qquad \begin{aligned} 0 &= R + \frac{2}{L^2}\,, \\ 0 &= \nabla_\alpha \nabla_\beta \varphi - g_{\alpha\beta}\Big(\nabla^2\varphi - \frac{1}{L^2}\,\varphi\Big) \end{aligned}$$

JT equations with $L = Q^{3/2}\,\ell$

Q: Can the JT action be obtained by integrating out the gauge field and considering the near-horizon limit?

A: Yes, but there is a twist.

Eliminating the gauge field from the 1+1 action, as it stands, leads to a dilaton gravity theory but one with a wrong-sign effective potential for the dilaton.

This kind of sign flip occurs any time a dynamical variable carrying kinetic energy is integrated out in favor of a potential energy term.

The problem is solved by adding an EM boundary term to the original action.

Electromagnetic boundary terms

Our 3+1 D action did not have any boundary terms for the Maxwell field and A_μ is kept fixed at the boundary.

In the Euclidean formalism this corresponds to a thermal ensemble at fixed chemical potential where the total electric charge of the system is allowed to fluctuate.

$$\mathcal{S}_E = \beta F\big|_\mu = -S + \beta M - \beta\mu Q$$

If we add the following boundary term to the action

$$\mathcal{S}_b^{\text{em}} = \frac{1}{4\pi}\int_{\partial\mathcal{M}} d^3y\,\sqrt{-h}\,\hat{n}_\mu\, F^{\mu\nu} A_\nu$$

then free variations of A_μ at the boundary are allowed and the corresponding thermal ensemble is that of fixed charge but varying chemical potential

$$\beta F\big|_Q = -S + \beta M$$

with $\quad S = \pi Q^2 + 4\pi^2 Q^3 \ell T \quad$ and $\quad M = \dfrac{Q}{\ell} + 2\pi^2 Q^3 \ell T^2$

Comparing expressions for JT black hole: $\quad S = 2\pi\varphi_0 + 4\pi^2 L^2 \dfrac{\varphi_B}{r_B} T \qquad M_{\text{2d}} = 2\pi^2 L^2 \dfrac{\varphi_B}{r_B} T^2$

$$\longrightarrow \qquad \varphi_0 = \frac{Q^2}{2} \qquad \text{and} \qquad \varphi = \frac{r}{\ell}$$

(1) JT model describes RN black holes at fixed Q

(2) Higher dimensional embedding provides a reference scale

Electromagnetic boundary terms (p.2)

If the electromagnetic boundary term is included in the 3+1D action, then the 1+1 D action will include its spherical reduction

$$\mathcal{S}^{\text{em}}_{b,2d} = \ell^2 \int dy^0 \sqrt{-\gamma_{00}}\,(2\Phi)^{\frac{3}{2}}\,\hat{n}_\alpha\,F^{\alpha\beta} A_\beta$$

Adding a boundary term involving the gauge field does not change its dynamical equations, i.e. the Maxwell equations are not affected, but the boundary term contributes to the effective dilaton potential that results from integrating out the gauge field

Write the boundary term as a 1+1-dimensional bulk term involving a total derivative,

$$\begin{aligned}\mathcal{S}^{\text{em}}_{b,2d} &= \ell^2 \int d^2x\sqrt{-g}\,\nabla_\alpha\big((2\Phi)^{\frac{3}{2}}\,F^{\alpha\beta} A_\beta\big)\\ &= \frac{\ell^2}{2}\int d^2x\sqrt{-g}(2\Phi)^{\frac{3}{2}}\,F^{\alpha\beta}F_{\alpha\beta}\,.\end{aligned}$$

This has the same form as the electromagnetic bulk but with a coefficient in front that is twice as large and of opposite sign

$$\longrightarrow \qquad \mathcal{S} = \frac{1}{2}\int d^2x\sqrt{-g}\Big(\Phi R + \frac{1}{\ell^2}(2\Phi)^{-\frac{1}{2}} - \frac{Q^2}{\ell^2}(2\Phi)^{-\frac{3}{2}}\Big)$$

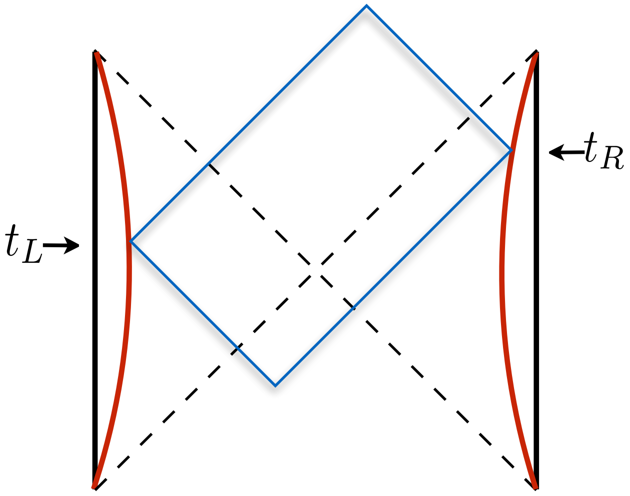
Now write $\Phi = \dfrac{Q^2}{2} + \varphi_0$ and work order by order in φ

JT theory

$$\longrightarrow \qquad \mathcal{S} = \frac{Q^2}{4}\int d^2x\sqrt{-g}\,R + \frac{1}{2}\int d^2x\sqrt{-g}\,\varphi\,\Big(R + \frac{2}{L^2}\Big) + \dots$$

"$C = A$" revisited

The improved WdW patch action for "$C = A$" calculation gives a finite growth rate at late times



$$\mathcal{A} = \mathcal{A}_{JT} + \frac{Q^2}{\ell^2}\int d^2x\sqrt{-g}\,(2\Phi)^{-3/2}$$

$$\approx \mathcal{A}_{JT} + \frac{1}{Q\ell^2}\int d^2x\sqrt{-g}$$

$$= -2Q^2\log(\cos\nu_R) - 2Q^2\log(\cos\nu_L) + \ldots$$

$$\longrightarrow \quad \frac{d\mathcal{A}}{dt_R} = 4\,S\,T + O(T^2) \quad \text{as} \quad t_R \to \infty$$

Conclusion

Both “C = V” and “C = A” give expected results for near-AdS_2 BH’s

— but not all actions are equal

Thank you!