A Tale of Tails

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Gauge / Gravity Duality
Würzburg
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work (mainly) with

Benjamin Withers

Igor Novak
introduction
Non equilibrium holography

Holography:

quantum gravity $\overset{=}={} \text{quantum field theory}$

Thermalization/non-equilibrium $\rightarrow$ BH formation (& evaporation)

**CFT**: late-time quantum physics of black holes

**AdS**: extract universal notions of non equilibrium physics from gravity
The moral of this tale

Let us consider stationary flows of strongly coupled liquids

Universal features of spatial structure of NESS are determined cleanly by

$$\frac{\eta}{s}$$

Signatures in thermoelectric probes of strongly coupled liquids
old tails of quenches

Nonequilibrium dynamics of holographic superfluid

[Bhaseen, Gauntlett, JS, Simons, Wiseman, PRL 110 (2013)]
Quench dynamics in holographic CFT

- Initial state: initial condition or result of operator deformation
- Quench is non-linear, but quickly gives way to linear QNM
- Can track n-pt functions, EE post quench
- Closed quantum system, but at late times can think of (large) BH as bath
**Example: Order Parameter Quench**

Holographic superconductor quench in Poincaré AdS

\[ J_{\psi}(t) = \delta e^{-\left(\frac{t}{\bar{\tau}}\right)^2} \]

conjugate to

\[ \langle \mathcal{O}(t, x) \rangle \]

- initial state
  - horizon
- boundary condition / quench
  - \( t = t_0 \)
  - \( t = t_f \)
- final state at effective temperature
  - static holographic superfluid
- non-linear non-equilibrium evolution
Universal Relaxation Dynamics

The dynamics of this quench give rise to three distinct regimes

I. Oscillation

II. Decay to finite gap

III. Decay to zero gap

FIG. 1. Schematic representation of the space-time coordinates. We show data for the time evolution of the real part of the scalar field $(t, z)\text{following a Gaussian quench at }t=0\text{ with }\delta=0.15\text{, from a superfluid black hole initial state as }t\rightarrow 1\text{ with }T_i/T_c=0.5\text{.}

The behaviour near the AdS boundary at $z=0$ is used to extract the dynamics of the superfluid order parameter $h_O(t)$ shown in Figs 2, 3.

For large quench strengths we exit the initial superfluid phase completely. In contrast, in region II it exhibits non-oscillatory exponential decay with Re($h_O$) = 0 towards $\Delta h = 0$. As we shall see later, this corresponds to the presence of a gapped "amplitude" mode and a gapless "phase" mode in the superfluid phase. However, in region I it exhibits exponentially damped oscillations with Re($h_O$) $\neq 0$ towards $\Delta h = 0$, so that for smaller quench strengths there is another regime of dynamics.

For the parameters used in Fig. 2, the transition from I to II occurs at a critical quench strength $\delta^*=0.14\text{, whilst the transition from II to III occurs at }\delta^* = 0.21\text{.}

The behaviour shown in Fig. 2 is reminiscent of the dynamical phase diagram for a BCS superconductor [16], despite the fact that the holographic superfluid is strongly coupled, and that the effects of thermal damping are incorporated. Indeed, the persistent oscillations of the integrable BCS Hamiltonian are replaced here by an under-damped approach towards $\Delta h = 0$, whilst the power-law damped BCS oscillations are replaced by an exponentially damped approach. The transition at $\delta^*$ provides a finite temperature and collision dominated analogue of the collisionless Landau damping transition [16].

Emergent Temperature Scale.

We can gain further insight by considering the phase diagram as a function of the final temperature, $T_f$, corresponding to the equilibrium temperature of the final state black hole. In Fig. 3(a) we plot $T_f$ versus $\Delta h$, showing that stronger quenches lead to greater final temperatures, consistent with the notion that the quench leads to heating. Using this relationship we may re-plot the data in Fig. 2 as a function of $T_f$; see Fig. 3(b). The data collapse on to the equilibrium phase diagram of the holographic superfluid [35], as indicated by the solid line. The transition from II to III is associated with increasing $T_f$ above $T_c$. However, Fig. 3(b) contains more information than the equilibrium phase diagram; there is an emergent dynamical temperature scale $T^*$, associated with $\delta^*$, where the dynamical phase transition occurs.
Universal Relaxation Dynamics

Universal explanation in terms of QNM: poles in correlation functions at real momentum and complex frequency

I. Oscillation

II. Pure Decay

III. Destruction of order (with pure decay)

Boundary between I & II is a dynamical phase transition. Also observed in e.g. BCS. [Barankov, Levitov, Spivak]
tails of steady states

Universal structure of non equilibrium steady states
[JS, Benjamin Withers, PRL 119 (2017), Igor Novak, JS, Benjamin Withers (2018)]
Nonequilibrium Steady States

NESS: out of equilibrium, but independent of time

**Example I:** current-driven steady states (apply E field)

- out-of-equilibrium fluctuation theorem [JS, Andrew Green]

**Example II:** heat-driven steady states (local thermal quench)  
[J. Bhaveen, Doyon, A. Lucas, K. Schalm;…]

- Characterisation of steady state region
- full out-of-equilibrium fluctuation relations
Fluid NESS

Consider a stationary flow of a strongly coupled liquid over an obstacle

\[ J_g(x) = \delta e^{-\left(x/\bar{x}\right)^2} \]

Compressible flow non-linearly disturbed by the obstacle

Left equilibrium

\[ \mathbf{v}_L \]

\[ \theta \]

Right equilibrium

\[ \mathbf{v}_R \]
Perspective: ’Spatial Quench’ [Figueras & Wiseman (2013)]

Holographic dual:

Turn a ‘standard’ quench by 90 degrees

Duals are stationary black holes with non-Killing horizon
Tails of NESS

How is asymptotic equilibrium reached (e.g. energy density)?

$$\epsilon(t, x) = \epsilon_{L,R} + e^{ik \cdot x} \delta \epsilon$$

Seek time independent mode on background with finite velocity

SCM:

$$\omega \in \mathbb{R}, \quad k \in \mathbb{C}$$
Spatial Collective Modes (SCM)

Depending on sign of \( \text{Im } k \) can have diverging mode. Regularity:

\[ \text{Im } k > 0 \implies \text{right mode} \]
\[ \text{Im } k < 0 \implies \text{left mode} \]

Approach of expectation value of some operator at position \( x \):

\[
\delta \langle \Phi(t, x) \rangle = \int_{-\infty}^{\infty} F(x') (-i) \theta(x - x') \langle [\Phi(t, x), \Phi(t', x')] \rangle
\]

This object has SCM poles in upper-half complex momentum plane

[see also: Amado, Hoyos, Landsteiner, Montero]

“The QNM of breaking spatial translation symmetry”
Aside: relation to QNM

But aren’t these just the usual quasinormal modes?

Let’s look at BTZ for simplicity

$$\omega = \pm q - 4\pi iT \left( \frac{\Delta}{2} + n \right)$$

Boosting SCM into the fluid rest frame $\rightarrow \omega = -\gamma kv, \; q = \gamma k$

$$k = \frac{4\pi iT}{\gamma(v \pm 1)} \left( \frac{\Delta}{2} + n \right)$$

SCM is boosted QNM and then analytically continued to complex k. This also applies in higher dimensions

But there is no such argument in a nonrelativistic fluid!
Example: hydro

Such modes can be constructed in hydro

\[ \epsilon(t, \mathbf{x}) = \epsilon_{L,R} + e^{i \mathbf{k} \cdot \mathbf{x}} \delta \epsilon \]

Seek time independent mode on background with finite velocity

<table>
<thead>
<tr>
<th>2 longitudinal</th>
<th>[ k = -iT \frac{s}{d-1} \eta + \frac{1}{2} \zeta \sqrt{1 - \frac{c_s^2}{c_s^2}(v \pm c_s)} + O(k)^2 ]</th>
</tr>
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<tbody>
<tr>
<td>1 transverse</td>
<td>[ k = -iT \frac{s}{\eta} v + O(k)^2 ] (angles suppressed)</td>
</tr>
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(in charged case also have a mode analogous to charge diffusion)
Beyond hydro

SCM of charged planar AdS\textsubscript{4} black hole

We get ‘hydro SCM’ and, additionally, more rapidly decaying non-hydro ones

Pole motion leads to interesting hydro-to-nonhydro phase transitions of NESS!
Connection to non-linear NESS

Can construct dual geometry to full non-linear NESS: nonKilling black holes in AdS$_4$ using ‘generalized harmonic’ method (deTurck)

In all cases the strongly non-linear behaviour around the obstacle transitions rapidly to fall-offs predicted by SCM analysis
A fully analytical example

In AdS$_3$ can construct black Janus, a finite-temperature defect solution [Bak, Gutperle, Janik]

\[ k_{\pm} = \pm i4\pi T(1 + n) \]

Contains scalar field with

\[ \langle O_\phi \rangle = \pm 2 \frac{\sqrt{m}(2\pi T)^2}{1 + m} \text{csch}^2(2\pi T x) \]

Matches precisely SCM spectrum of BTZ black hole

Fun corollary: can understand emergence of branch cut in $T \to 0$ limit which gives power law spatial decay
Accessing $\eta/s$

Decay length of transverse mode depends cleanly on $\eta/s$:

$$k = -iT \frac{s}{\eta} \cos \theta + O(k^2)$$

Setup NESS in strongly coupled material, use spatial structure to measure $\eta/s$ (See also [Falkovich & Levitov, Crossno et al.] and poster on Poiseuille flow at this conference)

Decay lengths at in real life (graphene near charge neutrality):

Estimate using Geim group parameters:

$$|\text{Im}k|^{-1} = 0.7 \mu m$$

at room temperature for normal incidence
Conclusions

A ubiquitous notion throughout holography: collective phenomena manifest as damped poles on complex w/k plane

**Time dependent case (QNM):** hydro and beyond + transitions between them
→ nonequilibrium phase transitions, critical phenomena

**Stationary case (SCM):** hydrostatic and beyond + transitions between them
→ excellent numerical and analytical evidence of relevance of SCM
→ nonequilibrium phase transitions, critical phenomena
→ stationary manifestation of shear viscosity

**Nonlinear:** both time dependent and NESS case under full control at large N, thanks to holography → new nonKilling horizons
Outlook

**Broadly speaking:** A key task to my mind is to understand QNM/SCM beyond holography

microscopic approach? resurgent gradient expansion [Heller & Spalinski, Withers]? 

**More focused:**
Large space of NESS to be explored (spatial oscillations, relation to black funnels & droplets) nonrelativistic case (SCM independently defined)

How far can we push this idea in graphene or other strongly correlated materials exhibiting hydro behavior? Relevant to experiment?