A Tale of Tails

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Gauge / Gravity Duality
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work (mainly) with

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introduction

Non equilibrium holography

Holography:

quantum gravity $=$ quantum field theory

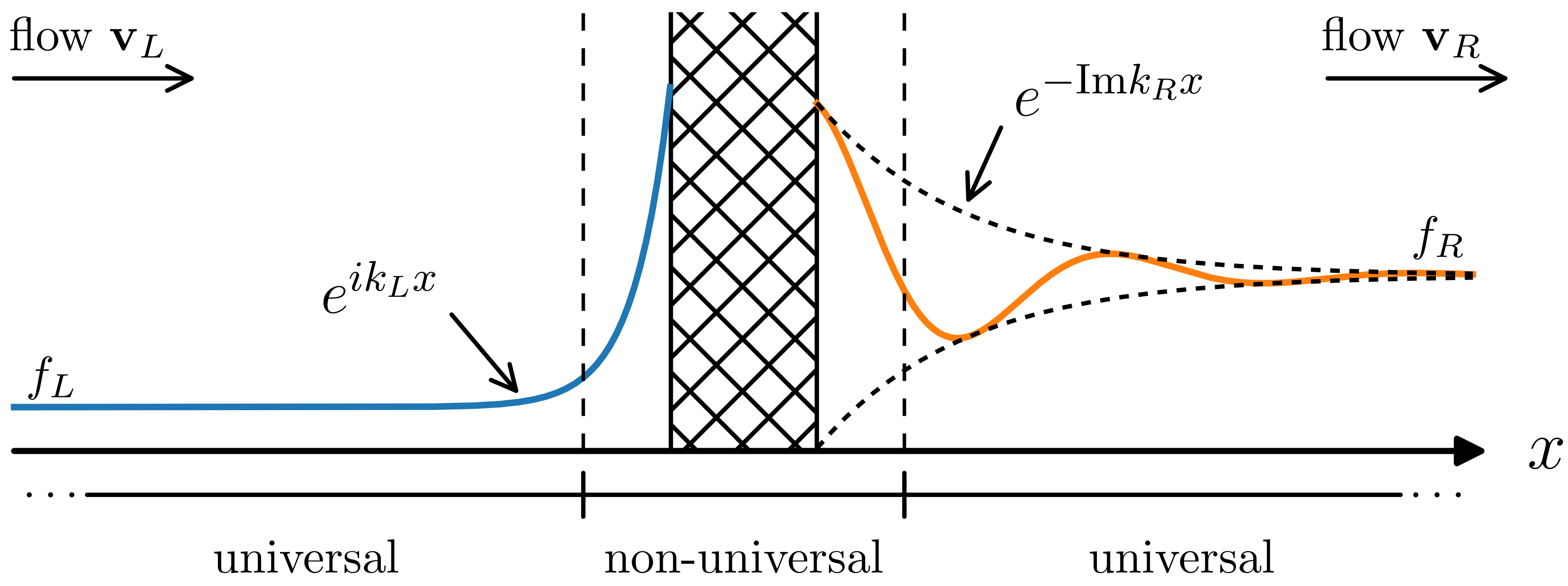
Thermalization/non-equilibrium ➛ BH formation (& evaporation)

CFT: late-time quantum physics of black holes

AdS: extract universal notions of non equilibrium physics from gravity

The moral of this tale

Let us consider stationary flows of strongly coupled liquids



Universal features of spatial structure of NESS are determined cleanly by

$$\eta/s$$

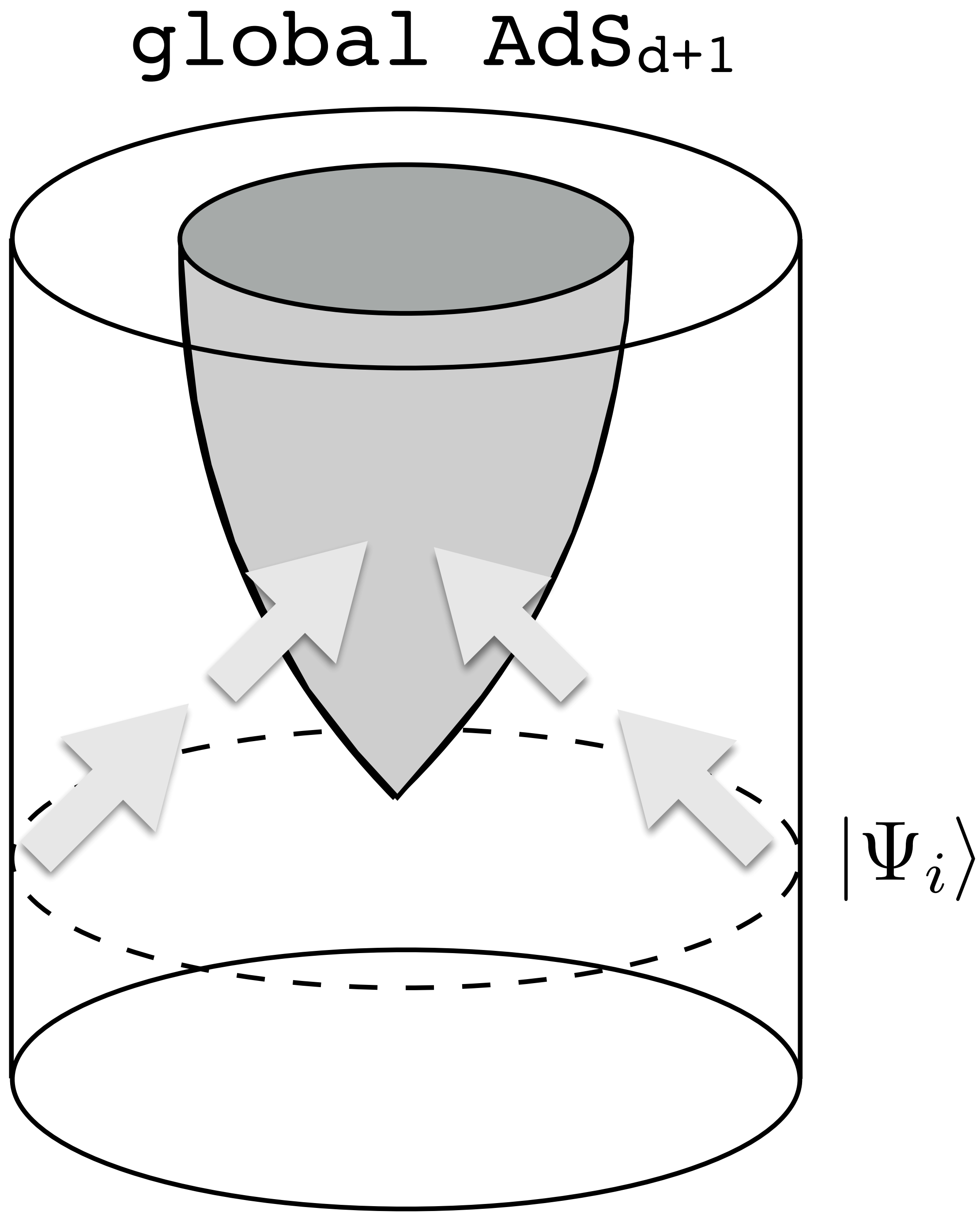
Signatures in thermoelectric probes of strongly coupled liquids

old tails of quenches

Nonequilibrium dynamics of holographic superfluid

[Bhaseen, Gauntlett, JS, Simons, Wiseman, PRL 110 (2013)]

Quench dynamics in holographic CFT



- Initial state: initial condition or result of operator deformation
- Quench is non-linear, but quickly gives way to linear QNM
- Can track n-pt functions, EE post quench
- Closed quantum system, but at late times can think of (large) BH as bath

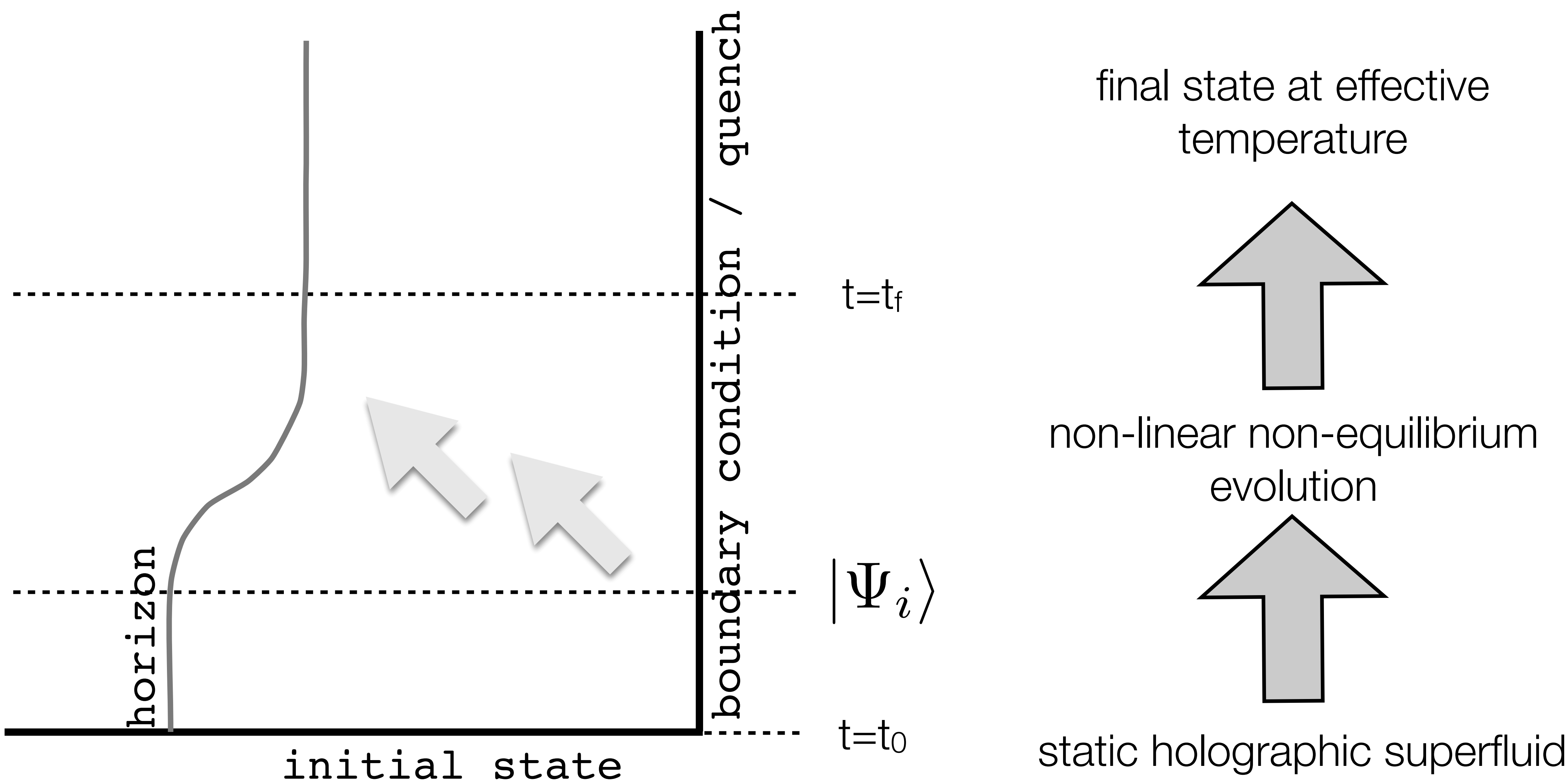
Example: Order Parameter Quench

Holographic superconductor quench in Poincaré AdS

conjugate to

$\langle \mathcal{O}(t,x) \rangle$

$$J_\psi(t) = \delta e^{-(t/\bar{\tau})^2}$$



Universal Relaxation Dynamics

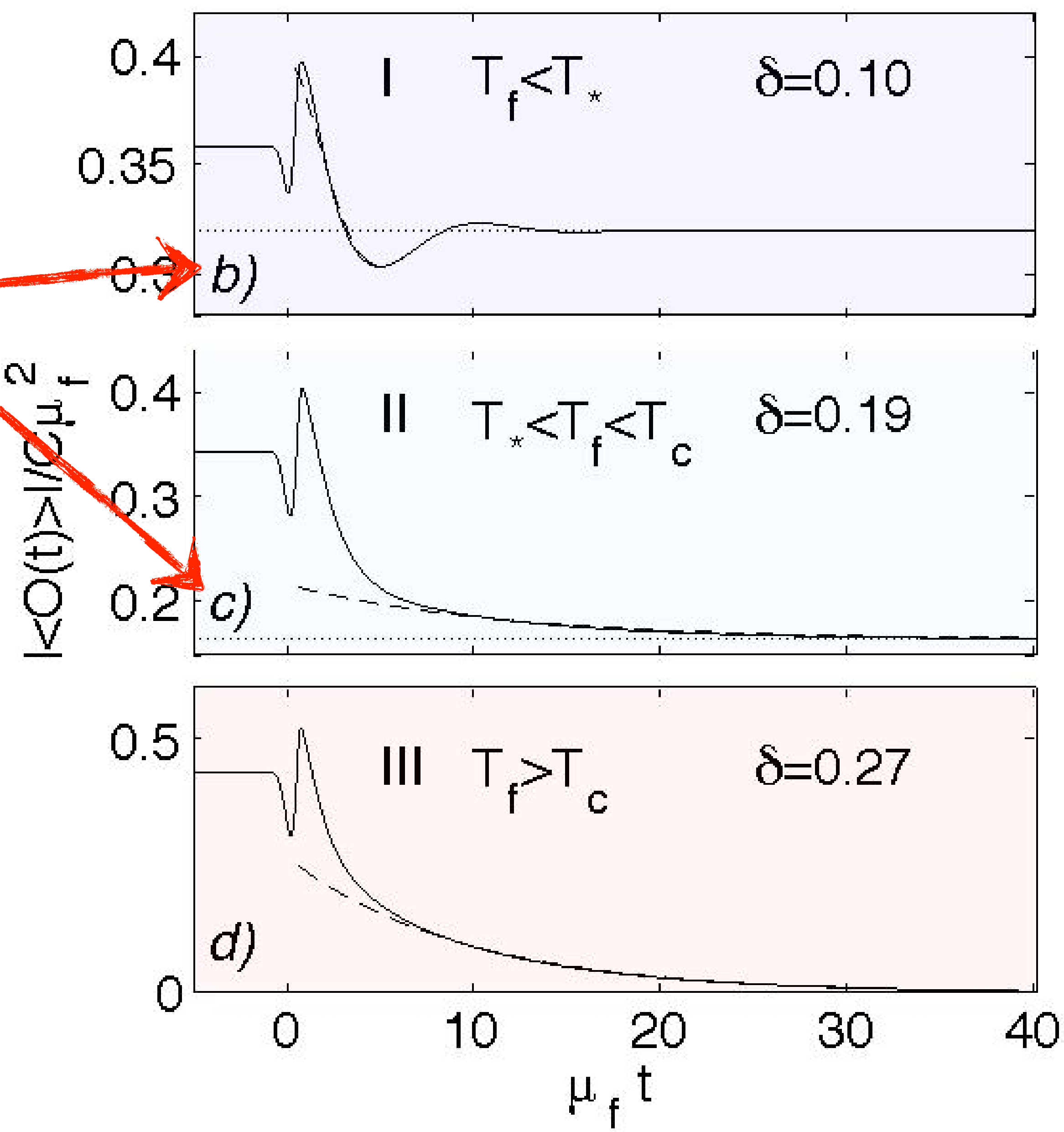
The dynamics of this quench give rise to three distinct regimes

I. Oscillation

dynamical phase transition

II.Decay to finite gap

III.Decay to zero gap



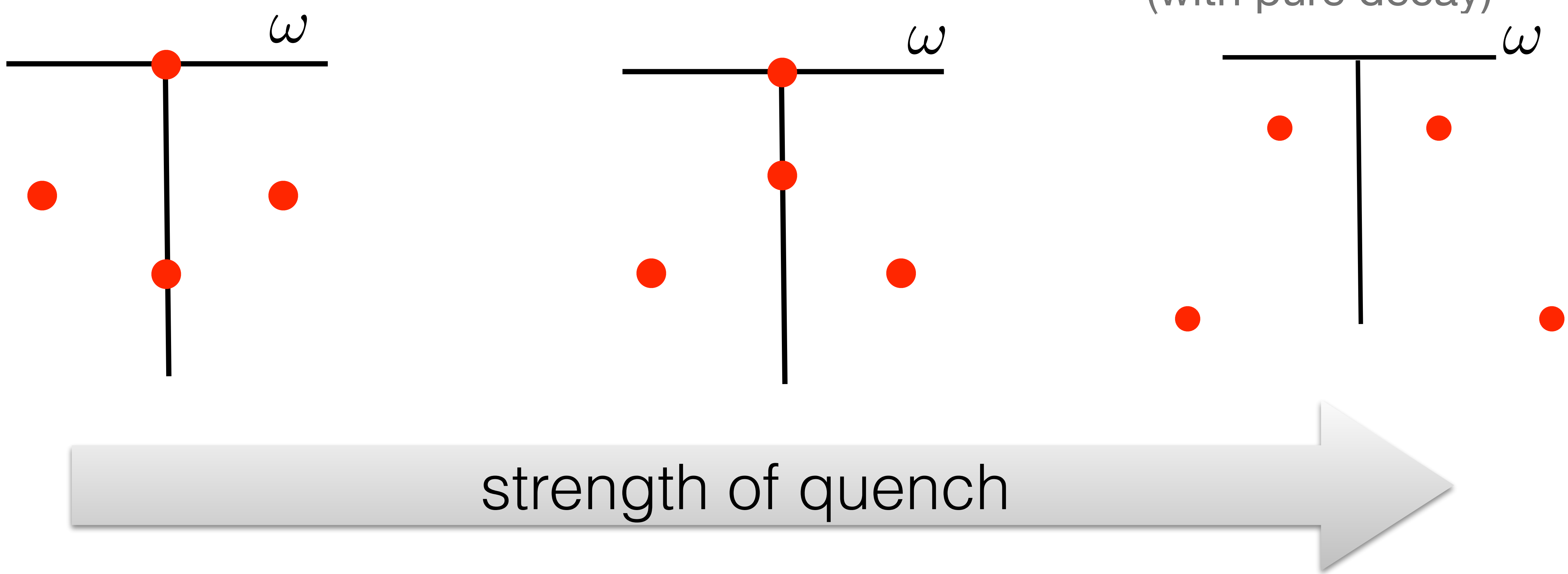
Universal Relaxation Dynamics

Universal explaination in terms of QNM: poles in correlation functions at real momentum and complex frequency

I. Oscillation

II. Pure Decay

III. Destruction of order (with pure decay)



Boundary between I & II is a dynamical phase transition. Also observed in e.g. BCS . [Barankov, Levitov, Spivak]

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tails of steady states

Universal structure of non equilibrium steady states

[JS, Benjamin Withers, PRL 119 (2017), Igor Novak, JS, Benjamin Withers (2018)]

Nonequilibrium Steady States

NESS: out of equilibrium, but independent of time

Example I: current-driven steady states (apply E field)

- Non-linear conductivity [A. Karch, A. O'Bannon; A. Karch, S. Sondhi]

- out-of-equilibrium fluctuation theorem [JS, Andrew Green]

Example II: heat-driven steady states (local thermal quench)
[J. Bhaseen, Doyon, A. Lucas, K. Schalm;…]

- Characterisation of steady state region

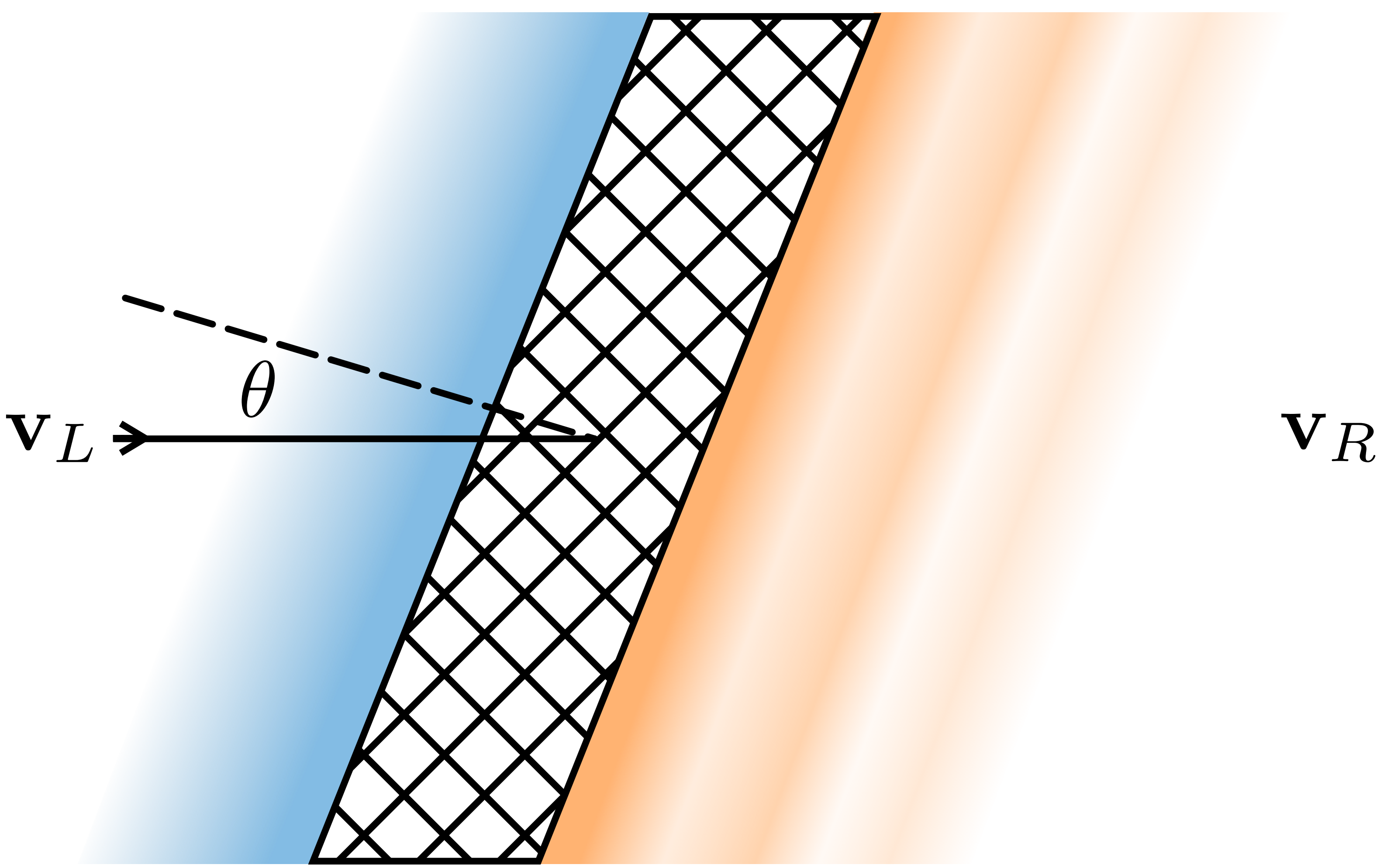
- full out-of-equilibrium fluctuation relations

Fluid NESS

Consider a stationary flow of a strongly coupled liquid over an obstacle

$$J_g(x) = \delta e^{-(x/\bar{x})^2}$$

Left equilibrium



Right equilibrium

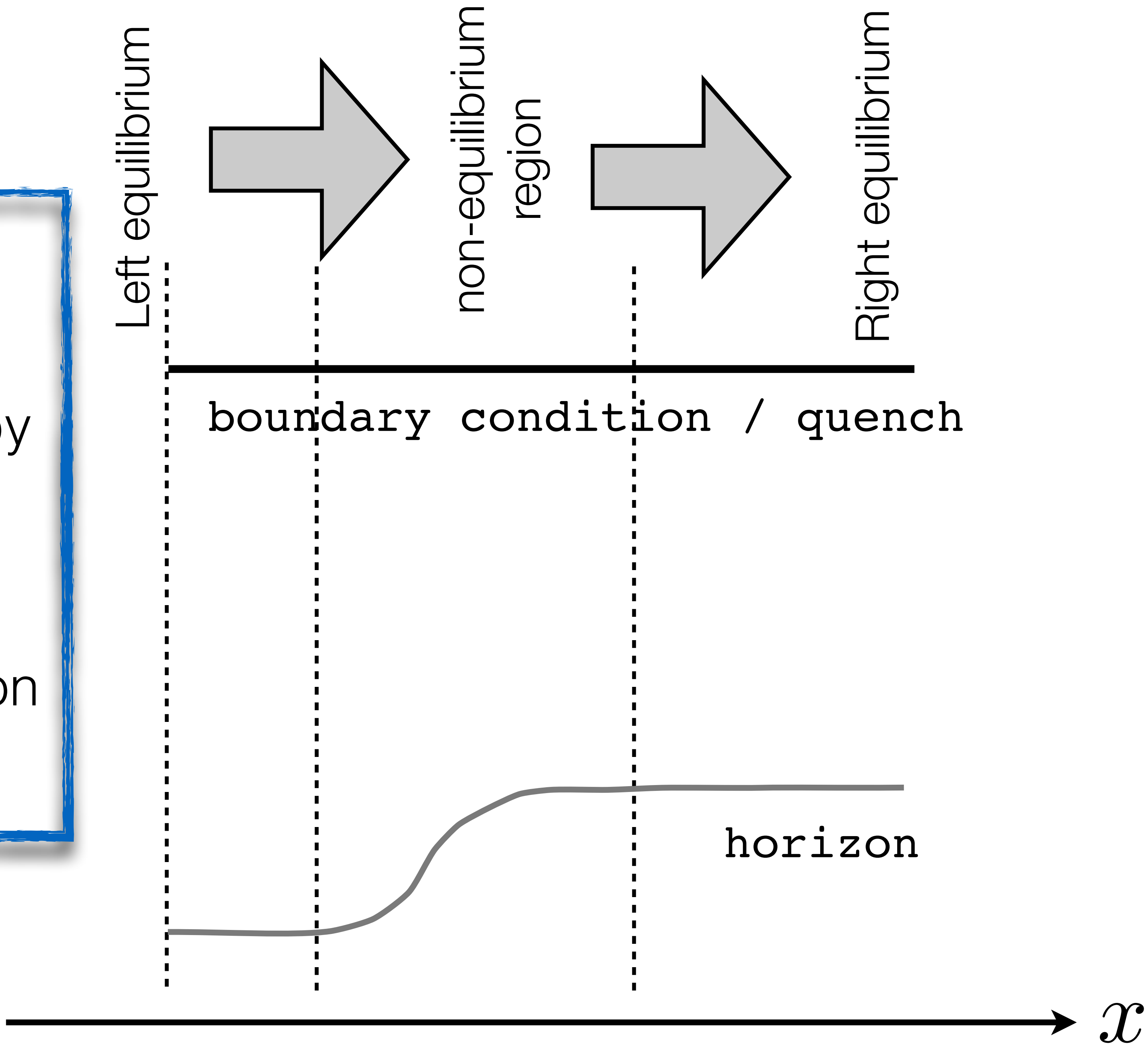
Compresssible flow non-linearly disturbed by the obstacle

Perspective: ’Spatial Quench’ [Figueras & Wiseman (2013)]

Holographic dual:

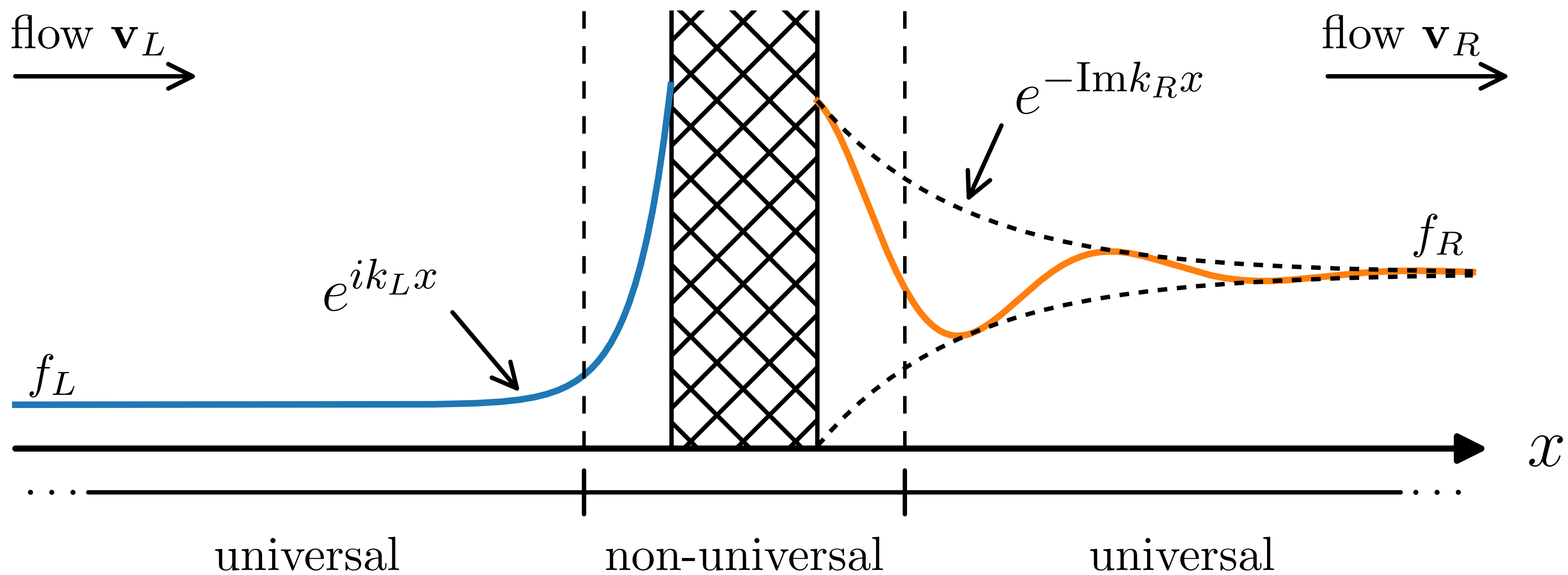
Turn a ‘standard’ quench by 90 degrees

Duals are stationary black holes with nonKilling horizon



(have compactified spatial direction)

Tails of NESS



How is asymptotic equilibrium reached (e.g. energy density)?

$$\epsilon(t, \mathbf{x}) = \epsilon_{\mathrm{L,R}} + e^{i\mathbf{k}\cdot\mathbf{x}}\delta\epsilon$$

Seek time independent mode on background with finite velocity

SCM: $(\omega \in \mathbb{R})$ $k \in \mathbb{C}$

Spatial Collective Modes (SCM)

Depending on sign of Im k can have diverging mode. Regularity:

$$\mathrm{Im}\, k > 0 \implies \text{right mode}$$
$$\mathrm{Im}\, k < 0 \implies \text{left mode}$$

Approach of expectation value of some operator at position x:

$$\delta \langle \Phi(t,x) \rangle = \int_{-\infty}^{\infty} F(x') \underbrace{(-i)\theta(x-x')\langle[\Phi(t,x),\Phi(t',x')]\rangle}$$

‘decay to right correlator’ $G^{[\searrow]}(x-x')$

This object has SCM poles in upper-half complex momentum plane

[see also: Amado, Hoyos, Landsteiner, Montero]

“The QNM of breaking spatial translation symmetry”

Aside: relation to QNM

But aren't these just the usual quasinormal modes?

Let's look at BTZ for simplicity

$$\omega = \pm q - 4\pi i T \left(\frac{\Delta}{2} + n \right)$$

Boosting SCM into the fluid rest frame → $\omega = -\gamma k v \, , \;\; q = \gamma k$

$$k = \frac{4\pi i T}{\gamma(v \pm 1)} \left(\frac{\Delta}{2} + n \right)$$

SCM is boosted QNM and then analytically continued to complex k.
This also applies in higher dimensions

But there is **no** such argument in a nonrelativistic fluid!

Example: hydro

Such modes can be constructed in hydro

$$\epsilon(t, \mathbf{x}) = \epsilon_{\mathrm{L,R}} + e^{i\mathbf{k}\cdot\mathbf{x}}\delta\epsilon$$

Seek time independent mode on background with finite velocity

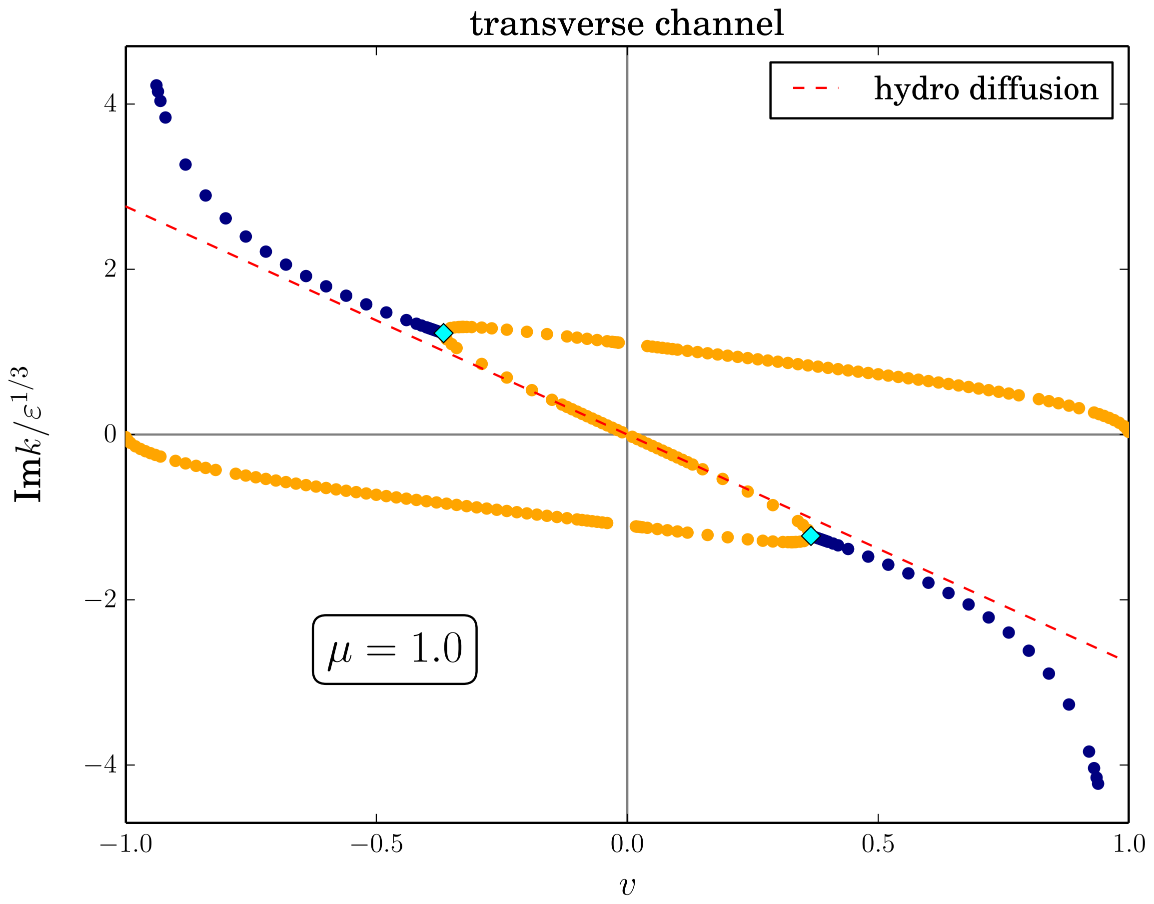
2 longitudinal $$k = -iT\frac{s}{\frac{d-2}{d-1}\eta + \frac{1}{2}\zeta}\sqrt{1-c_s^2}(v \pm c_s) + O(k)^2$$

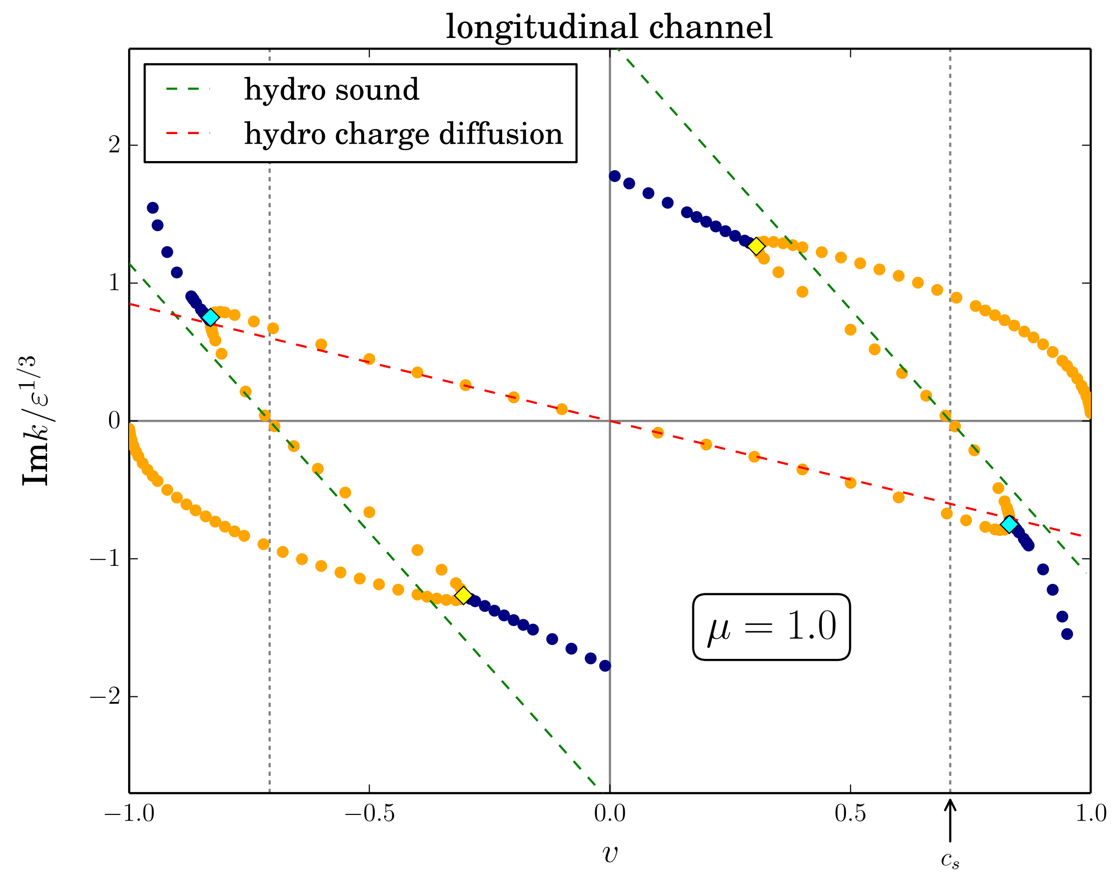
1 transverse $$k = -iT\frac{s}{\eta}v + O(k)^2$$ (angles suppressed)

(in charged case also have a mode analogous to charge diffusion)

Beyond hydro

SCM of charged planar AdS_4 black hole



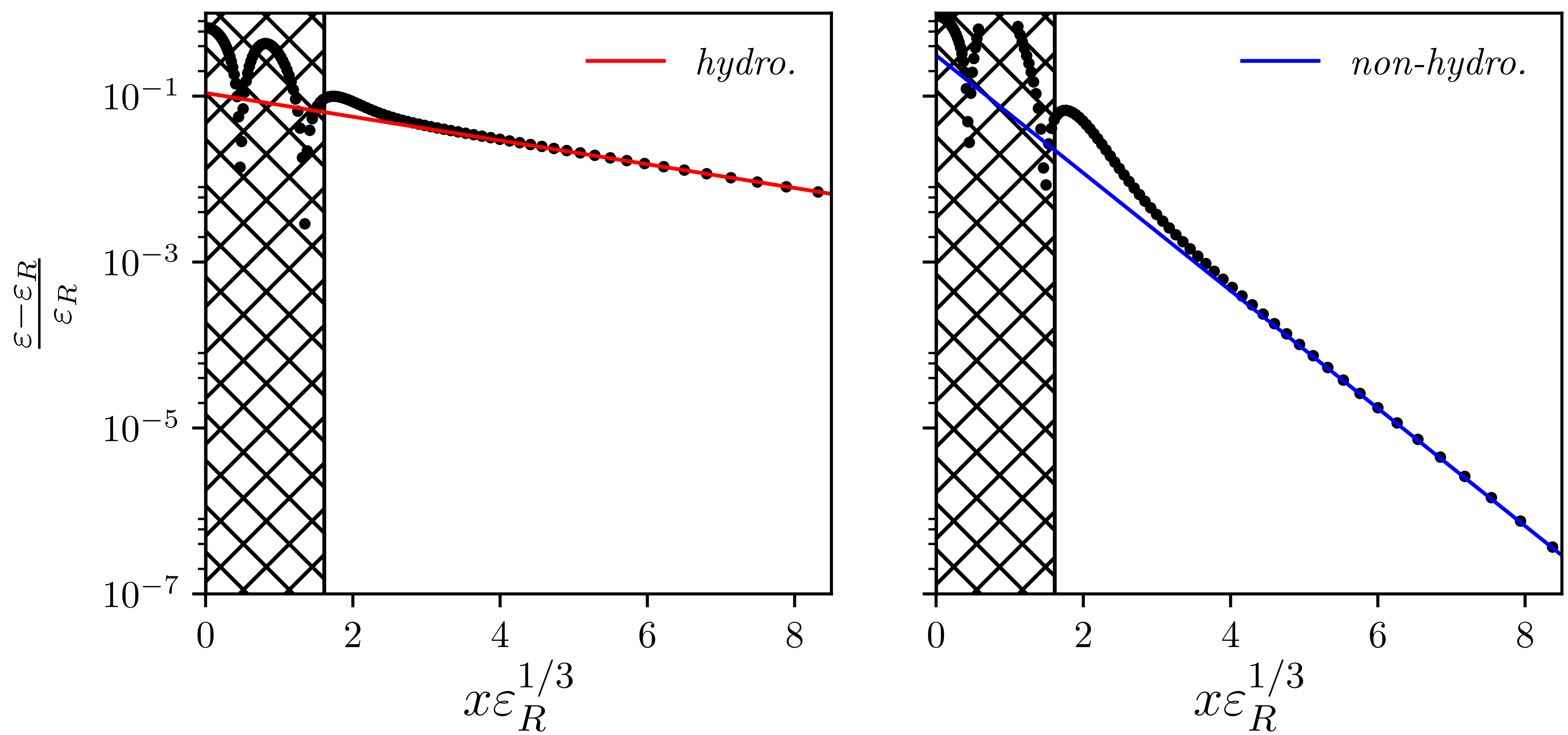


We get 'hydro SCM' and, additionally, more rapidly decaying non-hydro ones

Pole motion leads to interesting hydro-to-nonhydro phase transitions of NESS!

Connection to non-linear NESS

Can construct dual geometry to full non-linear NESS: nonKilling black holes in AdS_4 using ‘generalized harmonic’ method (deTurck)

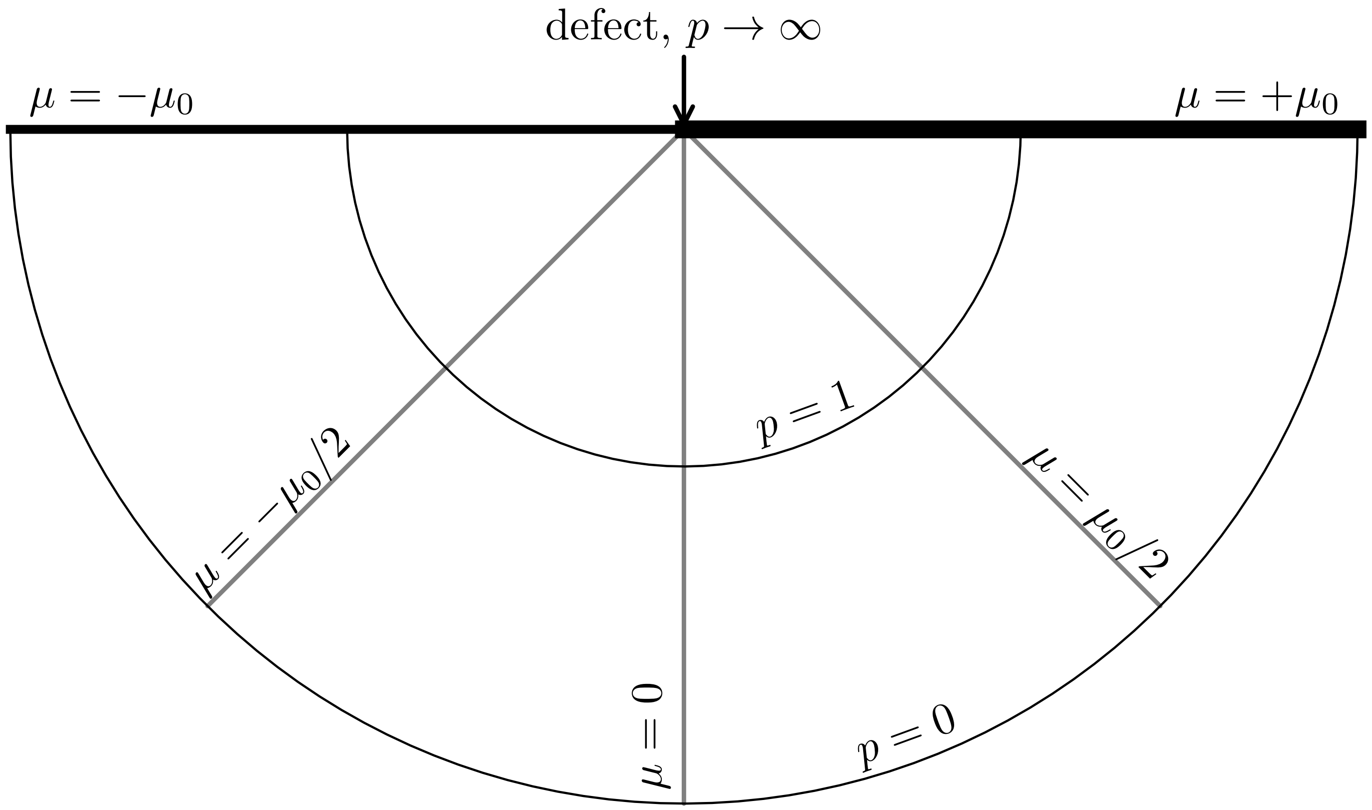


In all cases the strongly non-linear behaviour around the obstacle transitions rapidly to fall-offs predicted by SCM analysis

A fully analytical example

In AdS_3 can construct black Janus, a finite-temperature defect solution

[Bak, Gutperle, Janik]



Contains scalar field with

$$\langle O_\phi \rangle = \mp 2 \frac{\sqrt{\mathsf{m}}(2\pi T)^2}{1+\mathsf{m}} \operatorname{csch}^2(2\pi T x)$$

Inverse ↓ Laplace

$$k_n^{\pm} = \pm i 4\pi T(1+n)$$

Matches precisely SCM spectrum of BTZ black hole

Fun corollary: can understand emergence of branch cut in $T \to 0$ limit

which gives power law spatial decay

Accessing η/s

Decay length of transverse mode depends cleanly on η/s

$$k = -iT\frac{s}{\eta}\cos\theta + \mathcal{O}(k^2)$$

Setup NESS in strongly coupled material, use spatial structure to measure η/s (See also [Falkovich & Levitov, Crossno et al.] and poster on Poiseuille flow at this conference)

Decay lengths at in real life (graphene near charge neutrality):

Estimate using Geim group parameters:

$$|\mathrm{Im}k|^{-1} = 0.7\mu m$$

at room temperature for normal incidence

Conclusions

A ubiquitous notion throughout holography: collective phenomena manifest as damped poles on complex w/k plane

Time dependent case (QNM): hydro and beyond + transitions between them
→ nonequilibrium phase transitions, critical phenomena

Stationary case (SCM): hydrostatic and beyond + transitions between them
→ excellent numerical and analytical evidence of relevance of SCM
→ nonequilibrium phase transitions, critical phenomena
→ stationary manifestation of shear viscosity

Nonlinear: both time dependent and NESS case under full control at large N, thanks to holography → new nonKilling horizons

Outlook

Broadly speaking: a key task to my mind is to understand QNM/SCM beyond holography

microscopic approach?
resurgent gradient expansion [Heller & Spalinski, Withers]?
….

More focused:
Large space of NESS to be explored
(spatial oscillations, relation to black funnels & droplets)
nonrelativistic case (SCM independently defined)

How far can we push this idea in graphene or other strongly correlated materials exhibiting hydro behavior? Relevant to experiment?