Holographic viscoelastic hydrodynamics

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$\Longrightarrow$ Relativistic hydrodynamics:

- ideal hydrodynamics,

  $T^{\mu\nu} \equiv T^{\mu\nu}_{eq} = \epsilon\; u^\mu u^\nu + P(\epsilon)\; \Delta^{\mu\nu}\,, \qquad u^\mu u_\mu = -1\,, \qquad \Delta^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$

  $\{\epsilon, P\}$ — energy density and pressure of the fluid, u^μ — local fluid 4-velocity;

- Navier—Stokes hydrodynamics,

$$T^{\mu\nu} = T^{\mu\nu}_{eq} - \eta(\epsilon)\; \sigma^{\mu\nu} - \zeta(\epsilon)\; \Delta^{\mu\nu}\; (\nabla \cdot u)$$

  $\{\eta, \zeta\}$ — shear and bulk viscosities; $\sigma^{\mu\nu} = \mathcal{O}(\nabla^\mu u^\nu)$

- all-orders,

$$T^{\mu\nu} \equiv T^{\mu\nu}_{eq} + \Pi^{\mu\nu}\left(\nabla u, \{(\nabla u)^2, \nabla^2 u\}, \cdots\right)$$

$\Longrightarrow$ We will be interested in $n \to \infty$ order in the hydrodynamic expansion, *i.e.*, focusing on terms $(\nabla u)^n$ or more generally

$$(\nabla^{k_1} u)^{p_1} (\nabla^{k_2} u)^{p_2} \cdots (\nabla^{k_m} u)^{p_m}$$

with $k_1 p_1 + k_2 p_2 + \cdots k_m p_m = n$

$\Longrightarrow$ Too many indices, and too many different ways to describe flows....

We take the following steps to simplify index structure of the observables:

- we focus on the entropy density s production rate,

$$\frac{d}{dt}\ \ln(s) = \frac{1}{T}\,\mathcal{S}\left(\nabla u, \{(\nabla u)^2, \nabla^2 u\}, \cdots\right)$$

$$\mathcal{S} = \left[(\nabla\cdot u)^2\ \frac{\zeta}{s} + \frac{2\eta}{s}\ \sigma_{\mu\nu}\sigma^{\mu\nu}\right] + \cdots$$

- and a specific flow, *i.e.*, the homogeneous and isotropic expansion:

$$u^\mu = (1,0,0,0)\ , \qquad \nabla_\mu u^\mu = 3\frac{\dot{a}}{a} = 3H = \text{const}$$

This flow can be alternatively though as a co-moving frame expansion of the fluid in de Sitter Universe

$$ds^2 = -dt^2 + a^2(t)\ d\boldsymbol{x}^2\,, \qquad a(t) = e^{Ht}$$

Notice that for such a flow

$$\sigma^{\mu\nu} \equiv 0$$

$\Longrightarrow$ The full co-moving entropy production is due to conformal symmetry breaking:

$$\mathcal{L} = \mathcal{L}_{CFT} + \lambda_{4-\Delta}\mathcal{O}_\Delta$$

where Δ is a dimension of the CFT breaking operator,

$$\frac{d}{dt}\ \ln(a^3 s) \propto \frac{H^2}{T}\left(\frac{\lambda_{4-\Delta}}{T^{4-\Delta}}\right)^2\ \Omega_\Delta^2$$

$$\Omega_\Delta = \Omega_\Delta\left(\nabla u, \{(\nabla u)^2, \nabla^2 u\}, \cdots\right) = \Omega_\Delta(\frac{H}{T})$$

$\Longrightarrow$ for some models of holographic QGP fluids we can explicitly compute

$$\Omega_\Delta = \sum_{n=0}^{\infty} c_n\ \left(\frac{H}{T}\right)^n$$

and find

$$\frac{c_{n+1}}{c_n}\ \propto\ (n+4-\Delta) \qquad \Longrightarrow \qquad c_n \propto \Gamma(n+4-\Delta) \sim n!$$

$\Longrightarrow$ Thus:

- hydrodynamic expansion for fluids has zero radius of convergence
- the series in the derivative expansion can be Borel-resummed
- the poles in the Borel transform identify that the physical reason for the asymptotic character of the hydrodynamics are the

  non-hydrodynamic

  excitation in fluids (black brane QNMs in the dual holographic picture)

$\Longrightarrow$ this is an old story [**Michal Heller+Romuald Janik+**..., 2013]

$\Longrightarrow$ Now, an even older story [**Alex Buchel+Jim Sethna**, 1996]:

$\Longrightarrow$ Recall the Hooke's Law:

$$F = k\ x$$

where k is a spring constant

- Of course, if can not be a full story:

$$F = k\ x + k_2\ x^2 + k_3\ x^3 + \cdots$$

where k_i are non-linear elastic coefficients

$\Longrightarrow$ We argued that in brittle materials (those that can develop cracks under the stress), the Hooke's Law is the first term in otherwise asymptotic series, *i.e.*,

Elastic theory has zero radius of convergence

$\Longrightarrow$ Specifically,

- consider the fully non-linear in external pressure P expression for the bulk modulus K of a solid:

$$\frac{1}{K(P)} = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T = c_0 + c_1 \; P + c_2 \; P^2 + \cdots$$

- c_0 represents the Hooke's Law and $c_i \, , i \geq 1$ are higher-order coefficients
- as $n \to \infty$, for 2D elastic materials at temperature T, the crack surface tension α, Yong's modulus Y and the Poisson's ratio σ,

$$\frac{c_{n+1}}{c_n} \; \longrightarrow \; -n^{1/2} \; \left(\frac{\pi T(1-\sigma^2)}{8Y\alpha^2}\right)^{1/2}$$

or

$$c_n \; \propto \; \Gamma(\frac{n+1}{2}) \; \sim \; (\frac{n}{2})!$$

$\Longrightarrow$ Elastic theory and hydrodynamics are **similar**:

- both have a well-defined effective description, akin to derivative expansion in EFT;
- both expansions are asymptotic series (gradient expansion in fluids, powers of strain expansion in solids)
- both have 'non-perturbative' effects responsible for zero radius of convergence of effective description

$\Longrightarrow$ Elastic theory and hydrodynamics are **different**:

- non-perturbative effects in hydrodynamics: non-hydro modes in plasma
- non-perturbative effects in theory of elasticity: cracks

$\Longrightarrow$ BUT solids and fluids are rather different:

- there is no shear in fluids; as a result the transverse long-wave length fluctuations are non-propagating, *i.e.*, purely dissipative:

  $$\omega = -iD\, q^2$$

  where D is the diffusive constant, $TD = \frac{\eta}{s}$

- on the contrary, in solids we have transverse sound waves:

  $$\omega = c_\perp q\,, \qquad c_\perp^2 = \frac{\mu}{\epsilon + P}$$

  where μ is the shear elastic modulus

$\Longrightarrow$ In this talk

$$\text{solids+fluids} = \text{viscoelastic materials}$$

- Embed viscoelastic materials in holography
- Have a control parameter that interpolates from

  more solid like—to—more fluid like
- study all-derivative viscoelastic hydrodynamics
- signature of holographic cracks?

$\Longrightarrow$ The holographic model (think in microcanonical ensemble — we are interested in dynamics)

- start with the holographic superconductor

$$S = \frac{1}{16\pi G_N} \int_{\mathcal{M}_5} d^5x \sqrt{-g} \left[R + 12 - \frac{1}{2}(\partial\phi)^2 - \frac{1}{4}F^2 + \frac{\Delta(\Delta-4)}{2}\phi^2 \right]$$

  as usual, for a fixed charge density Q, below some critical energy density ϵ below which ϕ condenses

- add a 'lattice' (J.Gauntlett + others)

$$\left[\cdots - \frac{1}{2}\phi^2 \sum_{i=1}^{3} \left\{ \lambda_1 (\partial\psi_i)^2 + \lambda_2 \left((\partial\psi_i)^2 \right)^2 \right\} \right]$$

  where $\lambda_i > 0$ are coupling constants; we will be turning on the non-normalizable component for ψ_i as

$$\psi_i = k\ \delta_i^j\ x_j\,, \qquad \text{where} \qquad \{i,j\} = 1 \ldots 3 \qquad \text{and} \qquad k = \text{const}$$

- where is the lattice?
  - for simplicity, set $\lambda_1 = 1$ and $\lambda_2 = 0$;

$$\{\phi, \psi_i\} \qquad \Longrightarrow \text{ field redefinition } \Longrightarrow \qquad \Phi_i \equiv \frac{\phi}{\sqrt{2}} e^{i\sqrt{2}\psi_i}$$

results in a standard kinetic term for 3 complex fields Φ_i:

$$-\delta^{ij} \partial\Phi_i \partial\Phi_j^\star$$

and identifies ψ_i as axions:

$$\psi_i \sim \psi_i + \pi\sqrt{2}$$

  - since we are turning on $\psi_i = k\ \delta_i^j\ x_j$, the (boundary) spatial coordinates x_j must be periodically identified:

$$x_j \sim x_j + \frac{\pi\sqrt{2}}{k}$$

since we have a lattice, it will not be a surprise that we have nonzero elastic modulus;

■ turns out, elastic modulus in the model exists robustly for any set of $\{\lambda_1, \lambda_2\}$;

■ elastic modulus exists independently whether or not the non-normalizable component of ϕ is turned on:

— in the former case transverse phonons are gapped

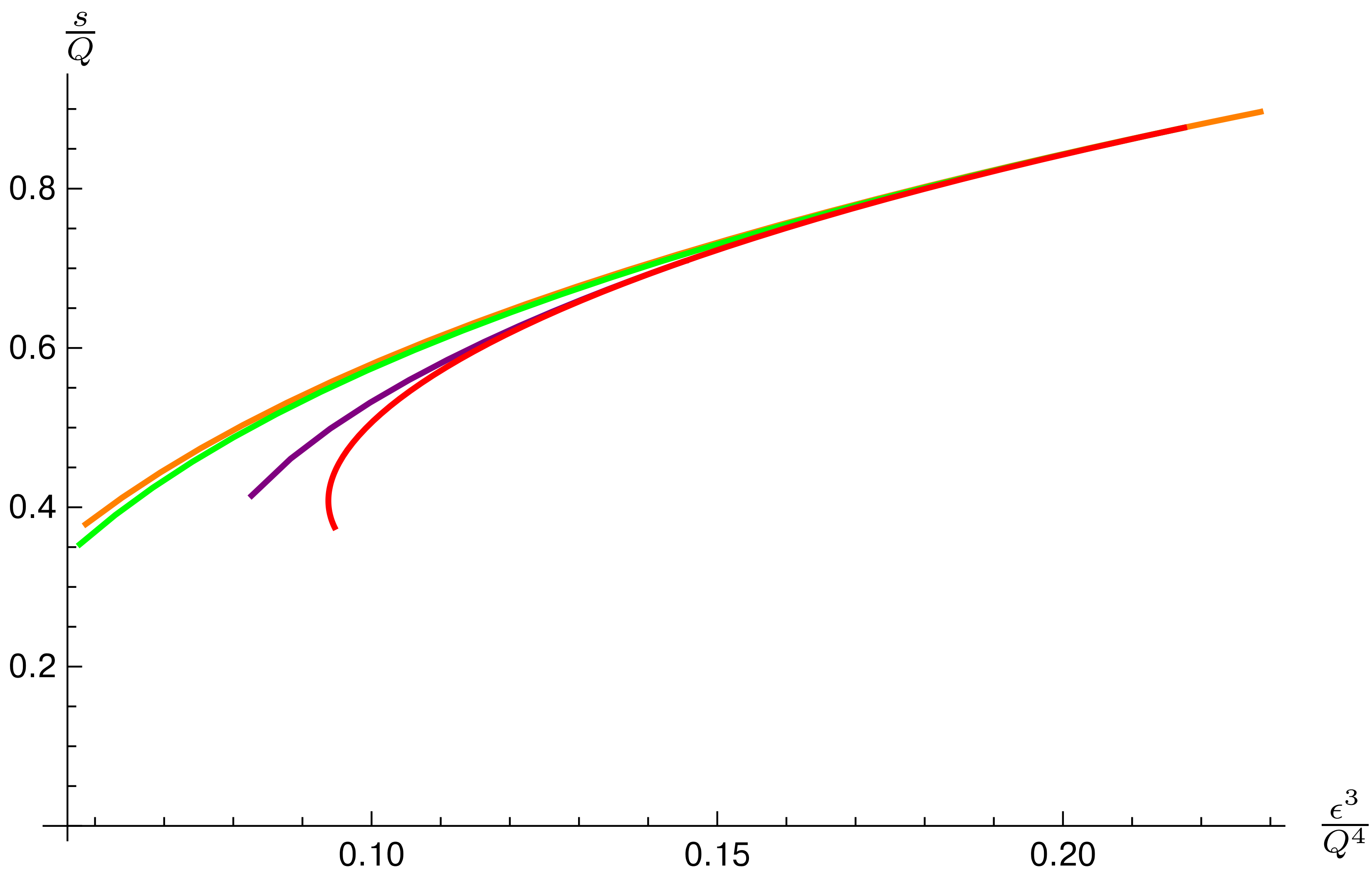
— in the latter case transverse phonons are gapless, with expected dispersion relation dictated by the shear elastic modulus

- enhance the 'lattice' effects in the model

$$-\frac{1}{4}F^2 \qquad \Longrightarrow \qquad -\frac{1}{4}(1+\gamma\phi^2)F^2\,, \qquad \gamma > 0$$

$\Longrightarrow$ I will now highlight the computational results in the model introduced

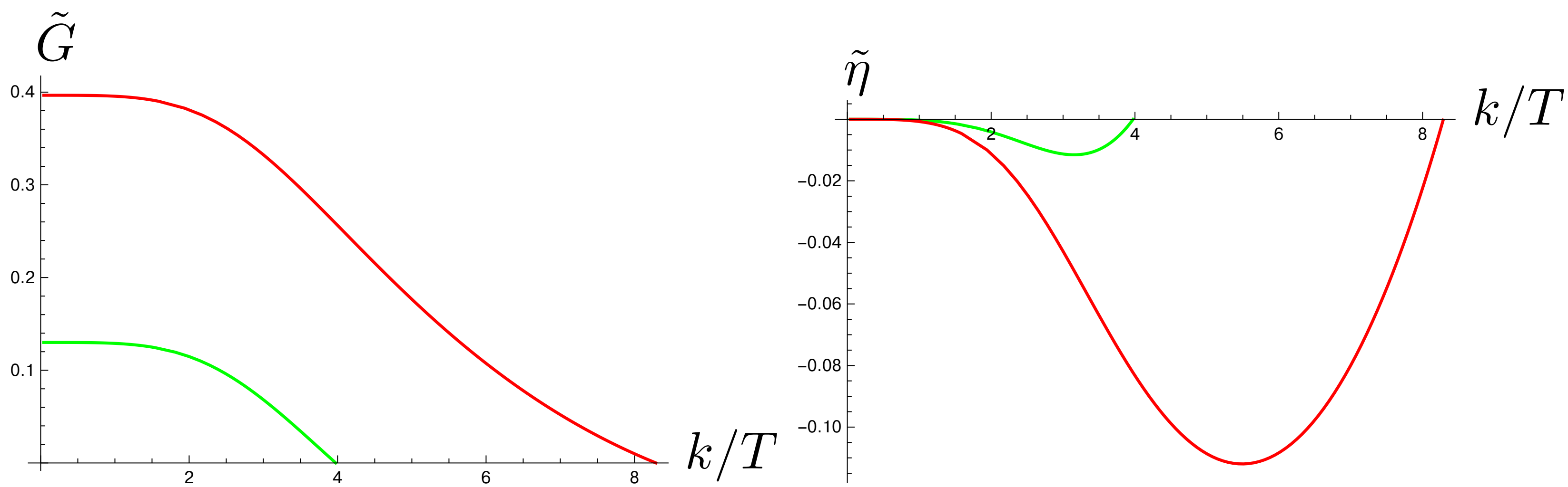
$\Longrightarrow$ Thermodynamics (energy density ϵ, charge density Q, entropy density s):



red: RN black hole; **orange**: broken phase at $k = 0$;

green: broken phase at $\frac{k}{\epsilon^{1/4}} = 1$; **purple**: broken phase at $\frac{k}{\epsilon^{1/4}} = 10$

$\Longrightarrow$ Elastic shear modulus $G \propto k^4 \tilde{G}$ and the shear viscosity $4\pi\eta/\mathcal{S} = 1 + \tilde{\eta}$ in the model:



The reduced shear elastic modulus $\tilde{G} = 16\pi G_N G/k^4$ (left panel) and the reduced shear viscosity $\tilde{\eta} = (4\pi\eta/\mathcal{S} - 1)$ (righ panel) as a functions of k/T for select values of $\frac{T}{\mu} = \{\frac{1}{12}, \frac{1}{6}\}$, {red,green} curves, at the criticality.

$\Longrightarrow$ To study large-order hydrodynamics of our holographic viscoelastic model we focus on a divergent series for Ω_Δ:

$$\Omega_\Delta = \sum_{n=0}^{\infty} c_n g^n$$

- construct a Borel transform

$$\Omega_\Delta^{(B)}(\xi) = \sum_{n=0}^{\infty} \frac{c_n}{n!} \xi^n$$

- Borel resummation is performed as

$$\Omega_\Delta^{(R)} = \int_{\mathcal{C}} d\xi \, e^{-\xi} \, \Omega_\Delta^{(B)}(\xi \, g) \equiv \frac{1}{g} \int_{\mathcal{C}} d\xi \, e^{-\xi/g} \, \Omega_\Delta^{(B)}(\xi)$$

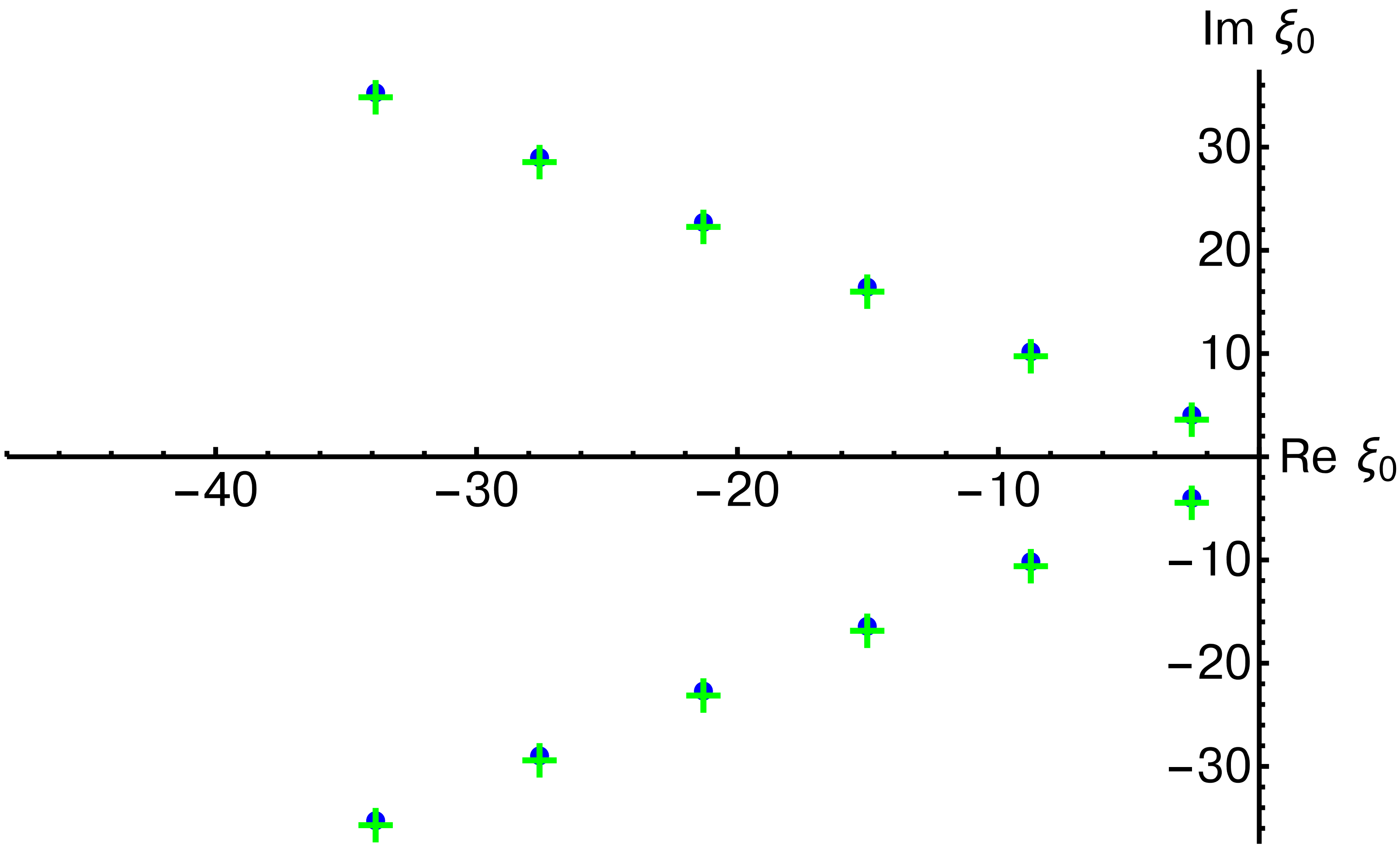
  where the contour $\mathcal{C}$ connects 0 and ∞.

- Ambiguities in $\Omega_\Delta^{(R)}$ come from the poles in $\Omega_\Delta^{(B)}(\xi)$:

$$\delta\Omega_\Delta^{(R)} \sim e^{-\xi_0/g}\,, \qquad \text{once} \qquad \frac{1}{\Omega_\Delta^{(B)}(\xi_0)} = 0$$

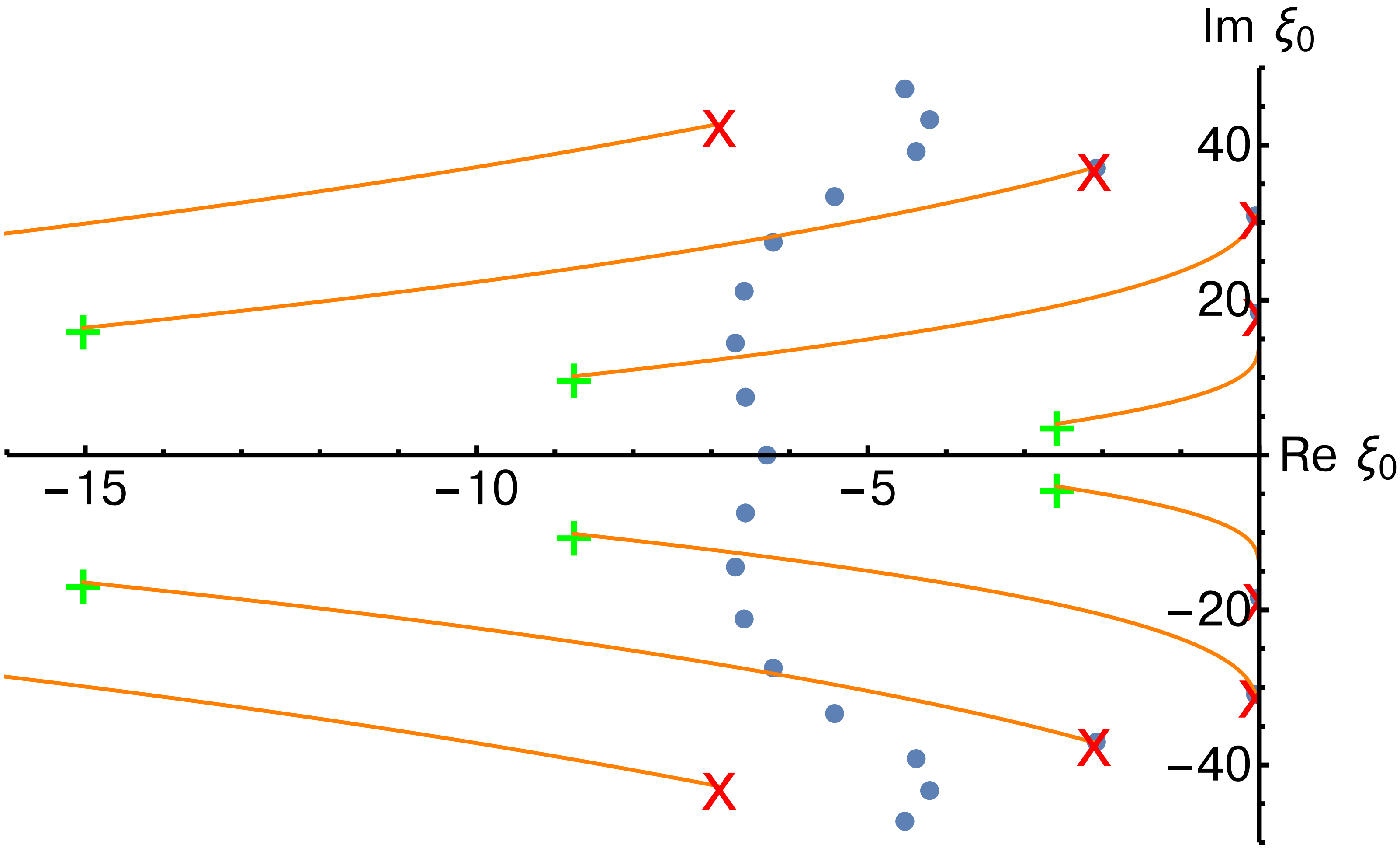
$\Longrightarrow$ For small g, poles in $\Omega_\Delta^{(B)}(\xi)$ generate essential singularity in $\Omega_\Delta^{(R)}$, responsible for the asymptotic character of Ω_Δ

$\Longrightarrow k = 0$ case (fluid)



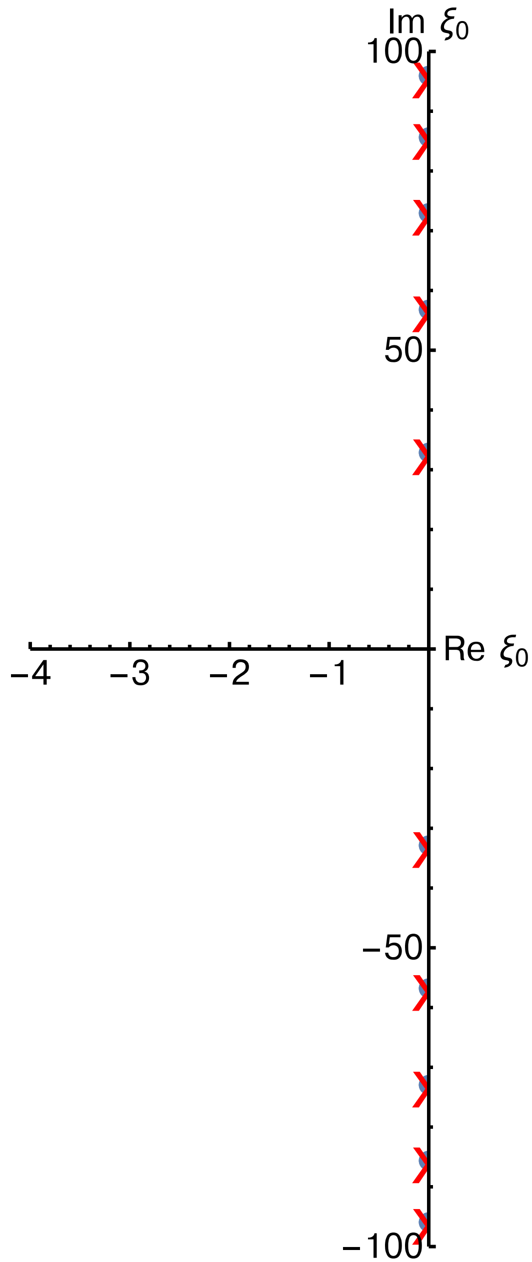
- blue filled circles: poles of the (Pade approximation of the) Borel transform of $\Omega_{\Delta=2}$
- green crosses: Starinets-Nunez QNMs

$\Longrightarrow \frac{k}{T} = 100$ case (viscoelastic)



- red crosses: QNMs in the model at $\frac{k}{T} = 100$
- orange lines: spectral flows of QNMs from $\frac{k}{T} = 0$ to $\frac{k}{T} = 100$

$\Longrightarrow \frac{k}{T} = 1000$ case (solid)



$\Longrightarrow$ **I did not have time to discuss:**

- G with explicit symmetry breaking
- elastic bulk modulus $\mathcal{K}$
- gapped-vs.-gapless phonons
- general Δ results
- how large orders of the hydrodynamics know about spontaneous symmetry breaking
- how and why G depends on the charge density
- critical exponents of G and $\mathcal{K}$ for spontaneous symmetry breaking

$\Longrightarrow$ **Open questions:**

- what are limitations of Pade approximation of Borel transform?
- where are 'cracks' in the model?
- or it is not a brittle solid?
- is there a physics in the wall-of-Borel-poles?
- can we study boost-invariant expansion of the viscoelastic model?