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Holography and thermalization in optical pump-probe spectroscopy

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Phys.Rev. D97 (2018) no.8, 086005 and JHEP 1807 (2018) 065

Conference Gauge/Gravity Duality 2018

Würzburg, 2 August 2018

Quantum non-equilibrium physics of strongly interacting many body systems

Experimental progress:

- ultrarelativistic heavy ion collisions
- cold atom systems
- “ultrafast” techniques in condensed matter physics

Theoretical challenge: dense many body entanglement

→ use holography (already successful near equilibrium)

Pump-probe spectroscopy

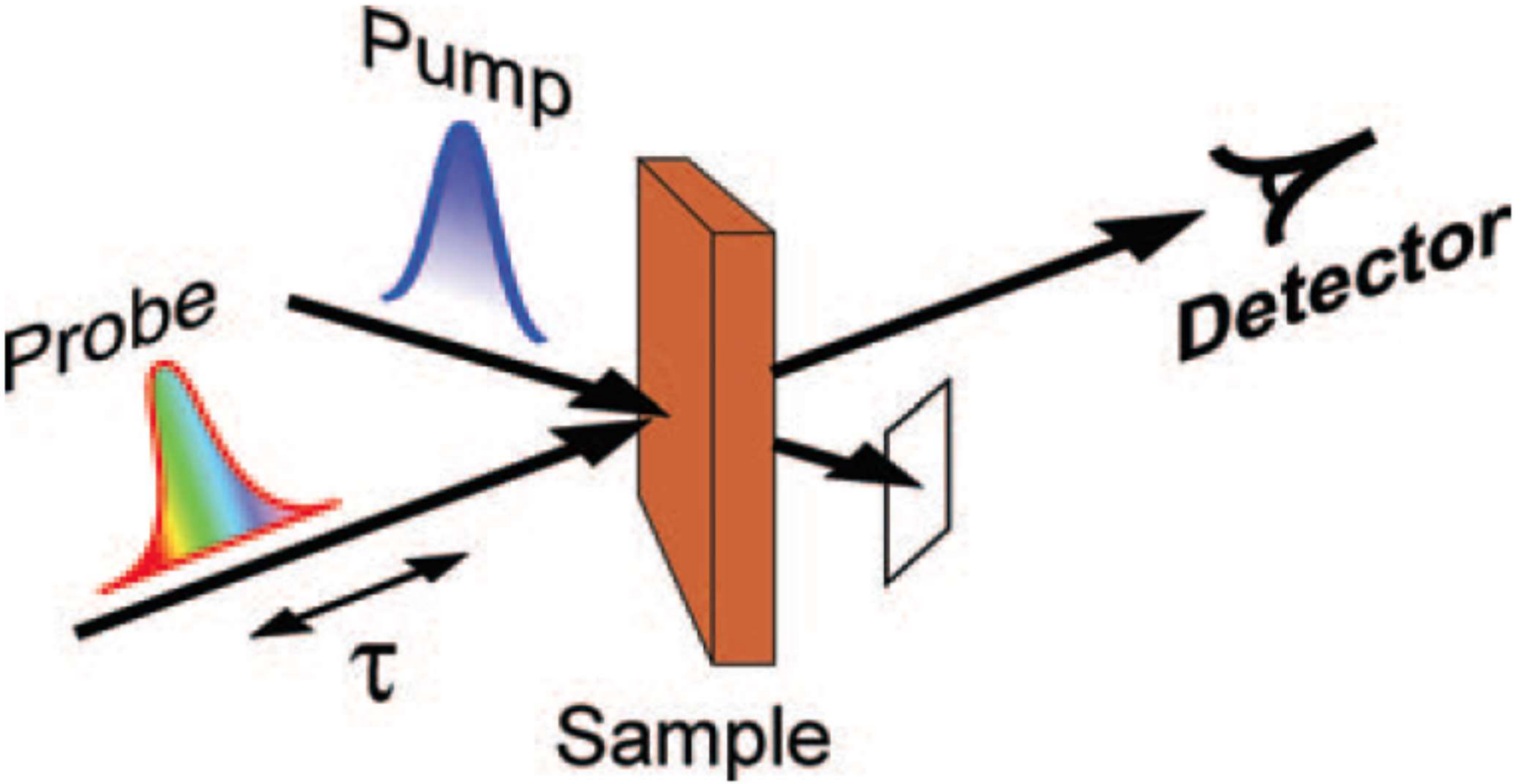


Figure from [G. Cerullo , C. Manzoni , L. Lüer , D. Polli , Photocem. Photobiol. Sci. 2007 , 6 , 135]

Electric field pump at zero density

Consider zero-density quantum critical system (CFT).

Start with vacuum or thermal state.

Pump it out of equilibrium by laser pulse (homogeneous pulse of transversal electric field).

Holographically dual to Vaidya geometry (lightlike shell collapsing into black hole). [Horowitz, Iqbal, Santos 2013]

1-point functions and retarded 2-point functions assume thermal equilibrium values immediately after energy injection ends.

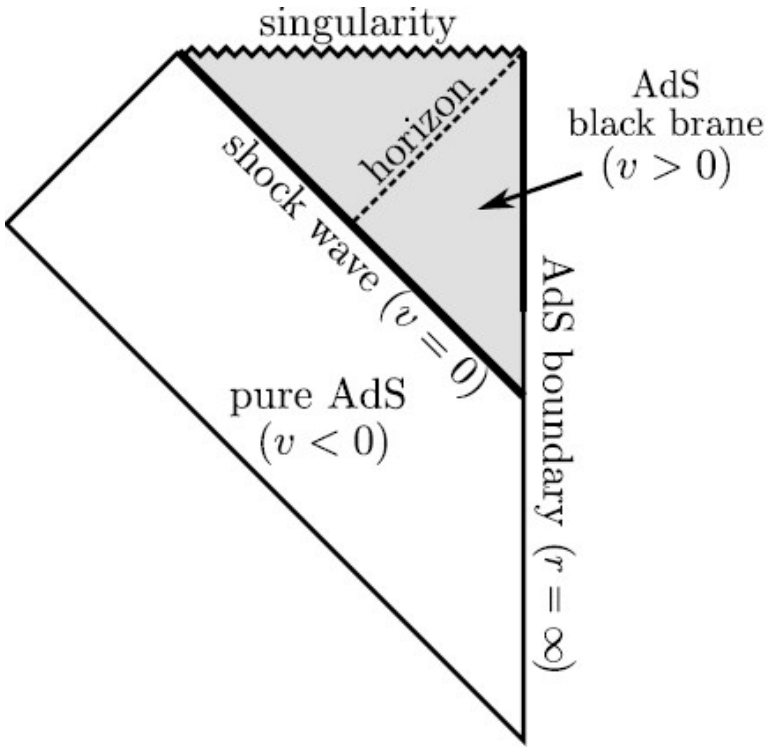
[Bhattacharyya, Minwalla 2009]

Instantaneous thermalization

<u>Bulk perspective</u>:

Instantaneous thermalization in field theory dual to Vaidya follows from bulk causality.

[Bhattacharyya, Minwalla 2009]



<u>Field theory perspective</u>:

Quantum thermalization: local observer quickly loses track of information. Cf. ETH.

Nonlocal observables do not thermalize instantaneously.

Pump-probe at nonzero density

Model 2+1D strange metal found in high-T_c superconductor: turn on chemical potential.

Apply pump laser pulse: short, spatially homogeneous transversal electric field pulse.

Probe time-evolution of optical conductivity.

(Mimic mild breaking of translation invariance by momentum relaxation via linear axions.)

[Andrade, Withers 2013]

Related models:
[Vegh 2013]
[Blake, Tong, Vegh 2013]
[Donos, Gauntlett 2013]

Main result

Thermalization of the optical conductivity continues to be instantaneous, except when the pump pulse contains a static electric field component.

The latter would set the charges in uniform motion and induce nonzero momentum density, delaying thermalization by (half of) the final state equilibrium momentum relaxation time (which is long for mildly broken translation invariance).

Mechanism for instantaneous thermalization: deviation of conductivity from thermal equilibrium takes the form $Ae^{-t/\tau}$.
In the absence of a static electric field component, the amplitude A vanishes.

Holographic setup: equilibrium

Einstein-Maxwell with linear axion scalar fields

$$S = \frac{1}{2\kappa_4^2}\int d^4x\,\sqrt{-g}\Big[R - 2\Lambda - \frac{1}{2}\sum_{I=1}^{2}(\partial\phi_I)^2 - \frac{1}{4}F^2\Big]$$

Equilibrium solution (RN strange metal w/ momentum dissipation)

$$ds^2 = \frac{1}{z^2}\Big(-f dt^2 + \frac{dz^2}{f} + dx^2 + dy^2\Big)$$

$$f = 1 - \frac{1}{2}k^2z^2 - mz^3 + \frac{1}{4}\rho^2 z^4$$

$$A = (\rho z - \mu)dt$$

$$\phi_1 = kx\,, \qquad \phi_2 = ky$$

[Andrade, Withers 2013]

Holographic setup: electric pump

Study effect of $E_x(t)$ via ansatz:

$$ds^2 = -F_z(z,v)dv^2 - \frac{2}{z^2}dvdz + 2F_x(z,v)dxdv$$
$$+\Sigma(z,v)^2(e^{-B(z,v)}dx^2 + e^{B(z,v)}dy^2)$$

$$A = (E_x(v)x + a_v(z,v))dv + a_x(z,v)dx$$

$$\phi_1 = kx + \Phi(z,v)\,, \quad \phi_2 = ky$$

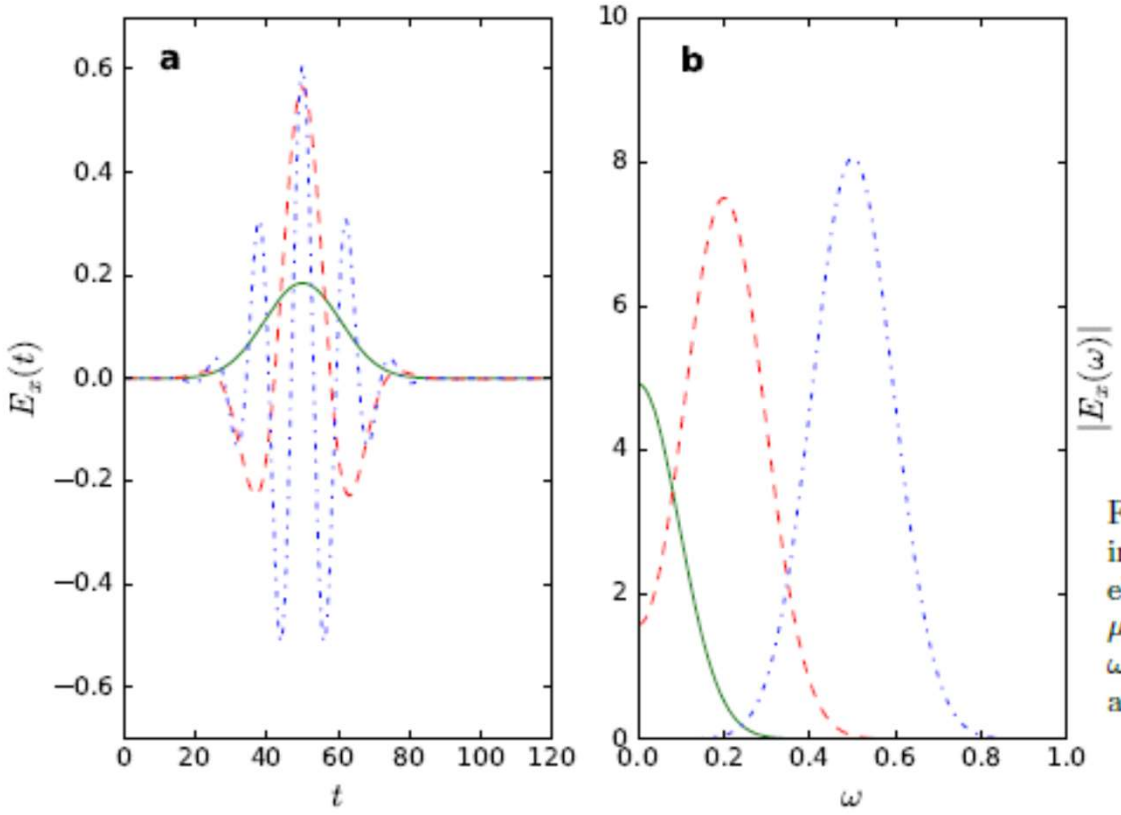
[Withers 2016]

Consider electric field profiles

$$E_x(t) = A\cos(\omega_P t)e^{-\frac{(t-t_0)^2}{(\Delta t)^2}}\frac{1-\tanh\frac{t-t_0-3\Delta t}{\delta}}{2}$$

Electric field profiles

$$E_x(t) = A\cos(\omega_P t)e^{-\frac{(t-t_0)^2}{(\Delta t)^2}}\frac{1-\tanh\frac{t-t_0-3\Delta t}{\delta}}{2}$$



$$\omega_P = (0, 0.2, 0.5)$$

$$\Delta t = 15$$

$$t_0 = 50$$

$$\delta = 0.01$$

$$t_{\text{end}} \equiv t_0 + 3\Delta t = 95$$

FIG. 1. Pump electric field profiles in the boundary both in the time- and frequency domain (left- and right panels) measured in units of the initial chemical potential $\mu = 1$. These are characterized by mean frequencies $\omega_P = (0, 0.2, 0.5)$ corresponding with the solid, dashed and dot-dashed lines, respectively.

Zero vs nonzero density

At zero charge density without momentum dissipation: Vaidya.

[Horowitz, Iqbal, Santos 2013]

Zero charge density with momentum dissipation: Vaidya-like.

[Bardoux, Caldarelli, Charmousis 2012]

Nonzero charge density: numerical GR required. [Withers 2016]

Physical mechanism: electric field → charges carry momentum
→ energy momentum tensor → additional bulk metric components.

One-point functions exhibit QNM decay

Expectation values of boundary operators: values of regularized bulk fields at AdS boundary, e.g.

$$\langle T_{tx} \rangle = 3F'_x \,,$$
$$p_x - p_y = \langle (T_{xx} - T_{yy}) \rangle = 12 B'_r$$

[Withers 2016]

For vanishing pump frequency:

$$\langle T_{tx} \rangle \propto e^{-\omega_i t}, \quad \langle (T_{xx} - T_{yy}) \rangle \propto e^{-2\omega_i t}, \quad \langle \mathcal{O} \rangle \propto e^{-\omega_i t}, \quad \langle J_x \rangle \propto e^{-\omega_i t}$$

with $\omega_* = -i\omega_i$ the leading vector QNM frequency, which is purely imaginary. Mild breaking of translation invariance → slow decay.

Expanding around late-time black brane solution, tensor mode B is sourced by square of vector mode F_x, explaining double decay rate.

QMN amplitude decreases with increasing pump frequency

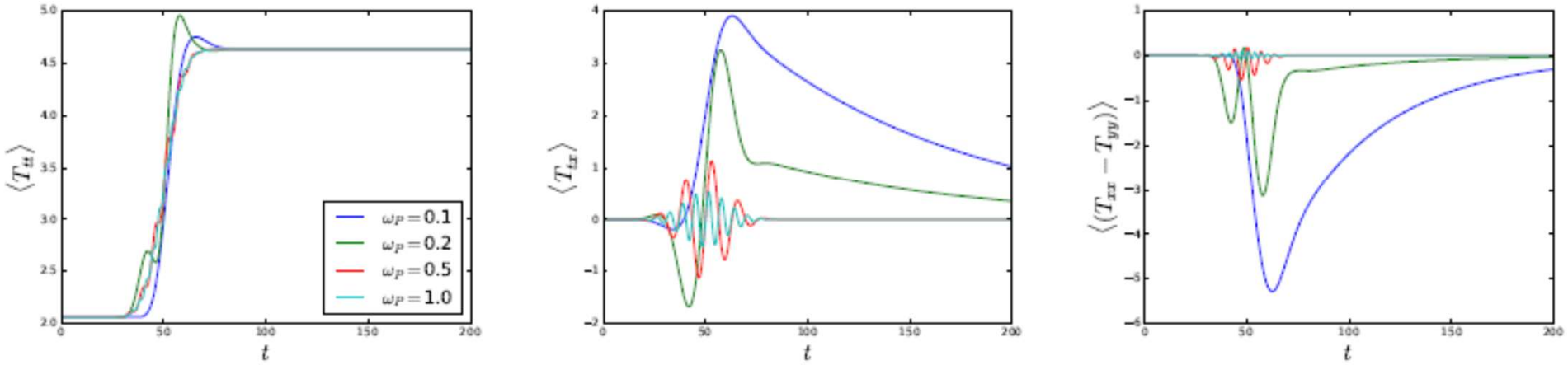


Figure 6: Plots of expectation values of one point functions for different pump frequencies $\omega_P = (0.1, 0.2, 0.5, 1.0)$. Again $\mu_I = 1, T_I = 0.2, T_F = 0.3, k = 0.2$. The plots show the exponential QNM approach to equilibrium, with decreasing amplitudes for increasing ω_P.

Bulk solution for large pump frequency

Assume $E_x(t) = \cos(\omega t)\Omega(t)$ with $\Omega(t)$ a smooth function of compact support that varies more slowly than $\cos(\omega t)$.

Ansatz: $F_x \sim 1/\omega; \qquad B, \Phi, a_x \sim 1/\omega^2$

To leading order in $1/\omega$: Vaidya solution

$$ds^2 = \frac{1}{z^2}\Big[-\left(1 - \frac{1}{2}k^2 z^2 - M(v)z^3 + \frac{1}{4}\rho^2 z^4\right)dv^2 - 2dvdz + dx^2 + dy^2\Big]$$

with $M(v) = m + \frac{1}{2}\int_{-\infty}^{v} dv' E_x(v')^2$

First subleading correction $F_x = \frac{1}{3}\rho z \int_{-\infty}^{v} dv'\, E_x(v')$ is $\mathcal{O}(1/\omega)$

during the pulse, but is suppressed much more strongly after the pulse.

QNM amplitudes: linear response estimate

Evaluate effect of E_x on expectation value of operator $\chi(t)$:

$$\langle \chi(t) \rangle = \int_{-\infty}^{t} dt' G_R^{\chi, J_x}(t, t') A_x(t') \,,$$

with $G_R^{\chi, J_x}(t, t') = -i\theta(t - t') \int d^2x' \langle [\chi(t, x), J_x(t', x')] \rangle \,.$

At late times: QNM expansion $G_R^{\chi, J_x}(t, t') = \theta(t - t') \sum_n g_n e^{-i\omega_n (t - t')}$

Long after A_x turns off: $\langle \chi(t) \rangle = \sum_n g_n e^{-i\omega_n t} \int_{-\infty}^{t} dt' A_x(t') e^{i\omega_n t'}$

Integrate by parts: $\langle \chi(t) \rangle = -i \sum_n \frac{g_n}{\omega_n} e^{-i\omega_n t} \int_{-\infty}^{\infty} dt' E_x(t') e^{i\omega_n t'}$

QNM amplitude set by Fourier transform of E_x at QNM frequency.

Linear response captures QNM amplitudes

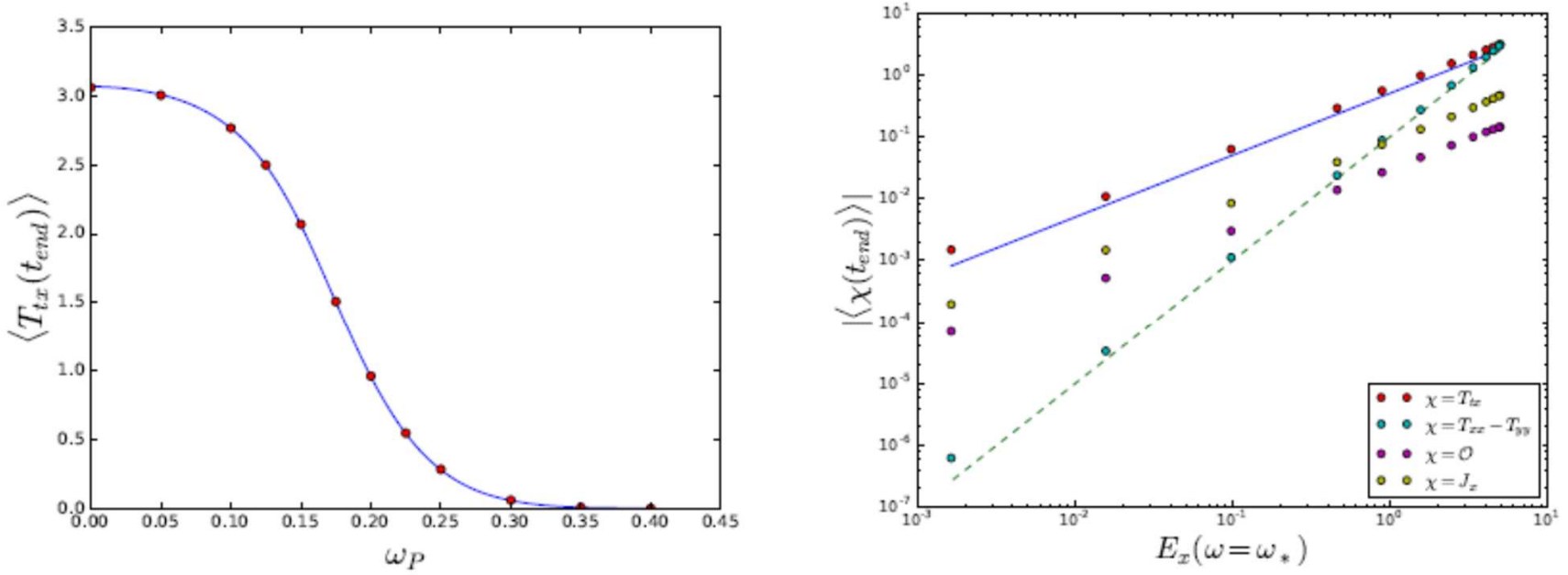


Figure 9: Left: The red dots are data points obtained from the full numerical solution. The blue curve is a fit to the functional form (4.27) with a single fitting parameter (the overall scale). The root-mean-square error of the fit is $RMSE \approx 0.0036$. Right: Different one point functions evaluated at $t = t_{end}$ plotted as functions of the Fourier transformed electric field (4.27). The blue solid line corresponds to a linear relation, while the green dashed line corresponds to a quadratic relation.

Probe conductivity

Differential conductivity $\sigma_{ij}(t,t')$ defined through:

$$\delta\langle J_i(t)\rangle = \int_{t_i}^{t} dt' \sigma_{ij}(t,t')\delta E_j(t') \qquad (t_i = t_{\mathrm{end}})$$

Computed by choosing probe electric field $\delta E_j(t) = \varepsilon\delta(t-\bar{t})\delta_{jx}$:

$$\sigma_{xx}(t,\bar{t}) = \delta\langle J_x(t)\rangle/\varepsilon$$

Define frequency space conductivity:

$$\sigma(\omega,t) = \int_{t}^{t_m} dt' e^{i\omega(t'-t)}\sigma_{xx}(t',t)$$

Probe optical conductivity: results

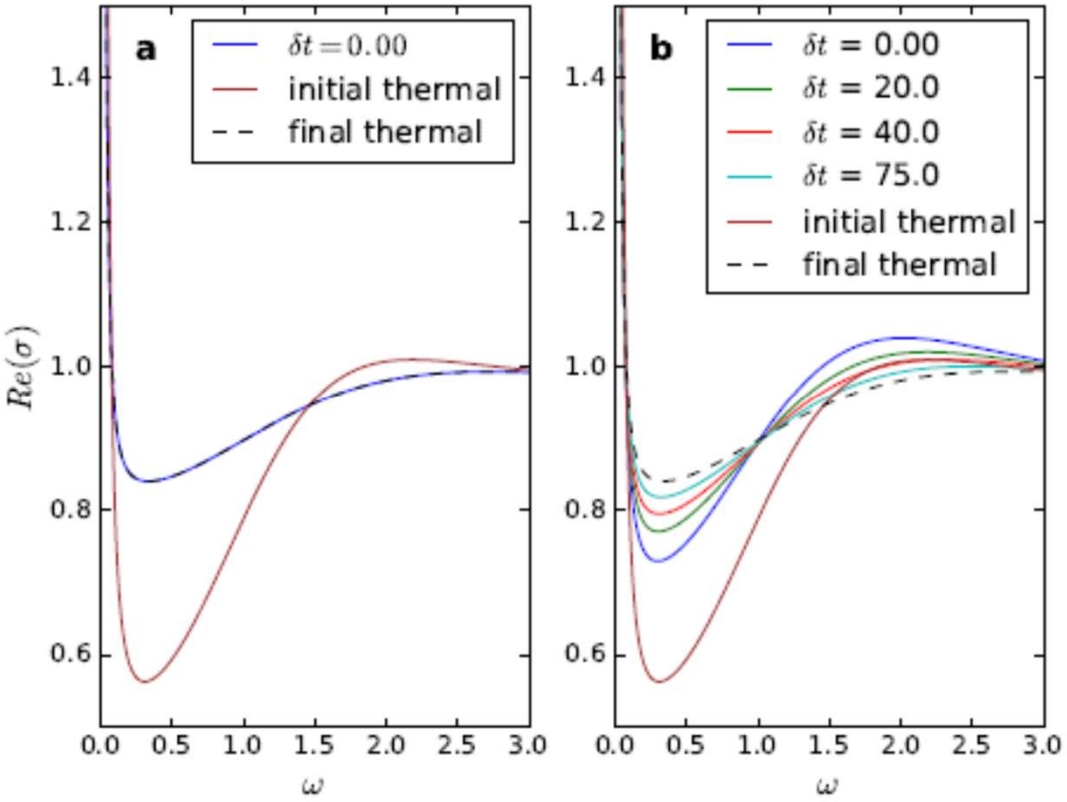


FIG. 2. Representative examples of the real part of (probe) conductivities as functions of ω. (a) The real part of the optical conductivity for the initial equilibrium state (solid brown) and for the excited state right after the pump pulse with $\omega_P = 0.5$ has ended (solid blue). For this pulse, the conductivity at all times after the system has been excited essentially coincides with the equilibrium result at the final temperature T_F (dashed black). (b) The time-dependence of the same system with the only difference that it is now excited by a pump electrical field with vanishing ω_P. The time $\delta t = t - t_{\rm end}$, measured after the termination of the pump pulse, and the parameters $T_I = 0.2$, $k = 0.2$ and $T_F = 0.3$ are all expressed in units where the initial chemical potential $\mu = 1$.

Pump pulse without zero frequency component

Pump pulse with zero frequency component

(More precisely, what matters is whether the Fourier transform of the pulse has support at the lowest QNM frequency ω_*, which is purely imaginary.)

Time-dependence of DC conductivity

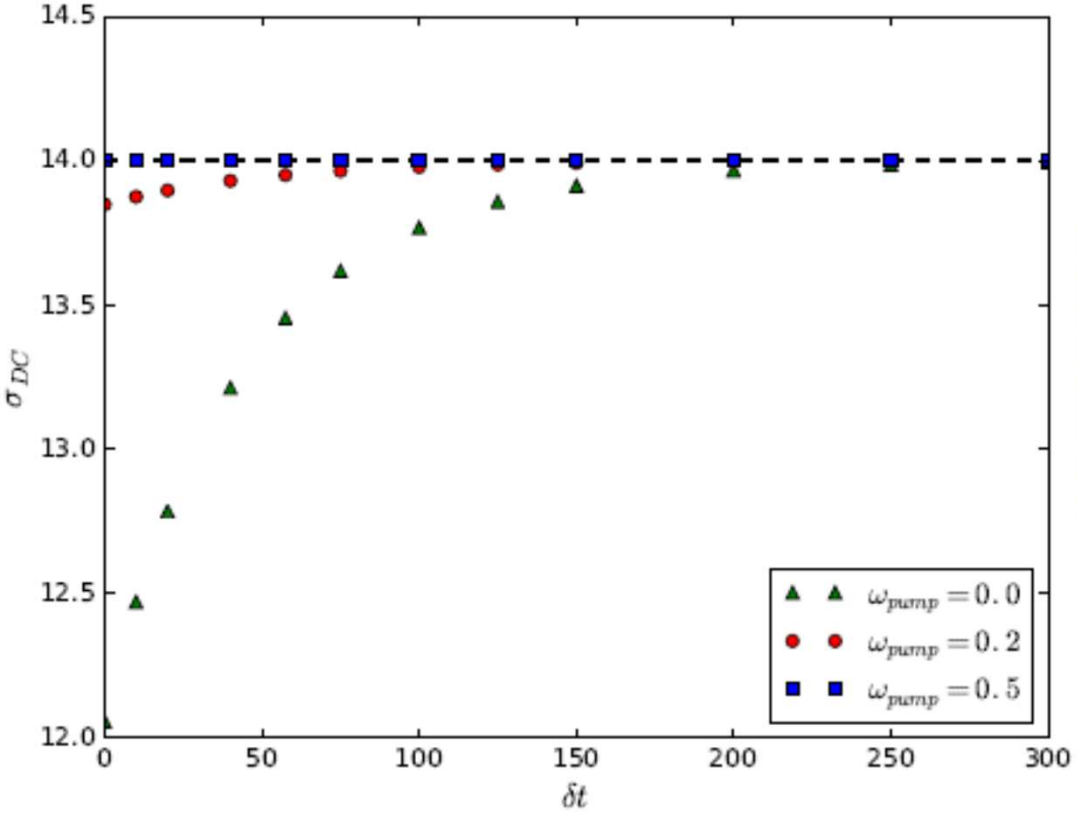


FIG. 3. The time-dependence of the DC conductivities for the same parameters as in Fig. 2: The green triangles, the red circles and blue boxes correspond to the pulses of Fig. 1 with $\omega_p = 0, 0.2$ and 0.5, respectively. The dashed line corresponds to the equilibrium DC conductivity at temperature T_F. The time scale is half the momentum relaxation time, while the magnitude is associated with the zero frequency components of the pulse.

If the DC conductivity does not assume its late-time equilibrium value instantaneously, then it relaxes exponentially towards it.

Decay rate of DC conductivity

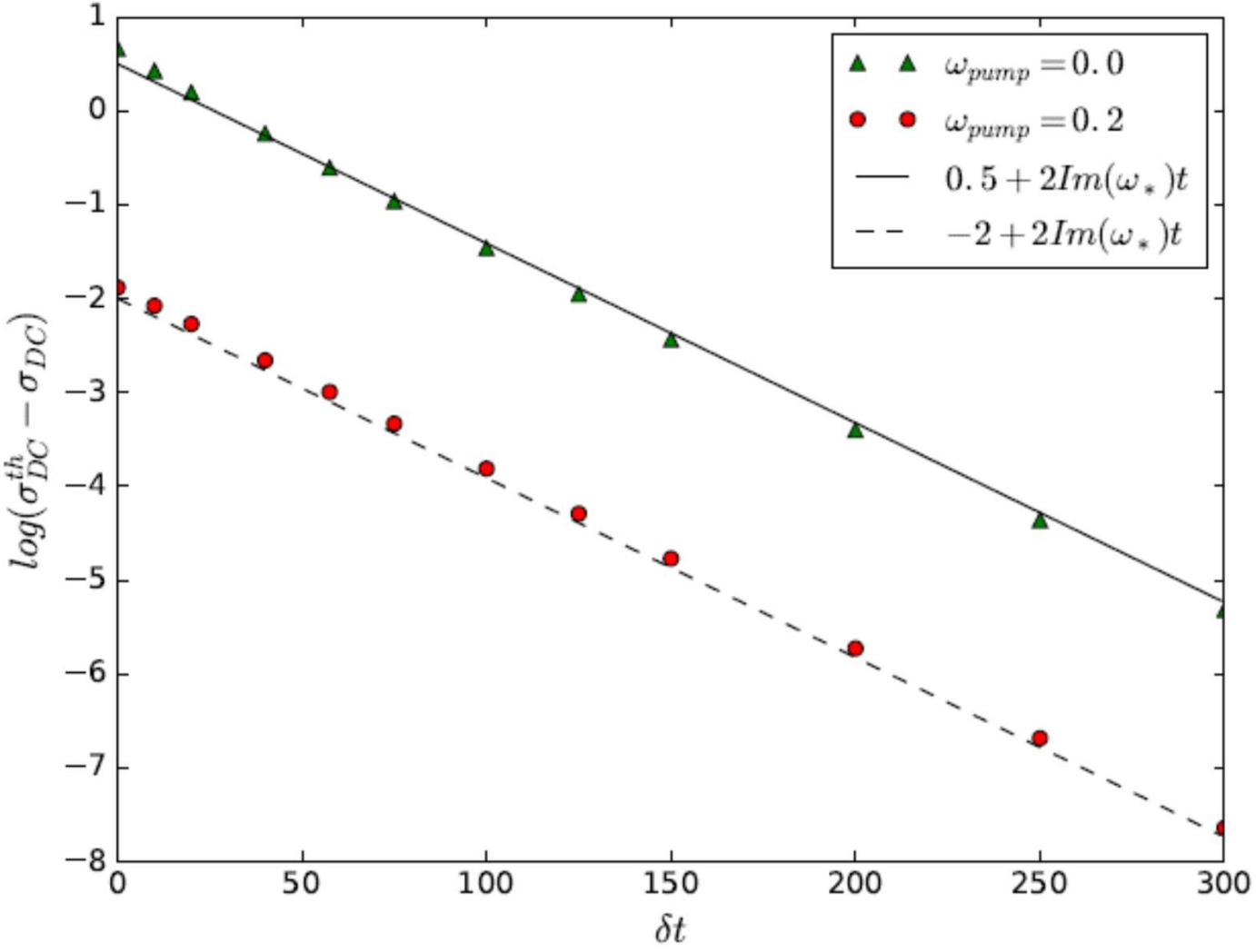


FIG. S1. The DC conductivity decays towards equilibrium with a rate consistent with $-2\mathrm{Im}(\omega_*)$.

Thermalization via momentum relaxation

How can thermalization be governed by (half) a momentum relaxation time?

Static electric field component accelerates finite density system, which carries nonzero momentum after the pulse. Momentum relaxation is required to reach equilibrium.

Bulk interpretation: in linearized approximation, decay towards late-time BH governed by lowest QNM frequency ω_*. Holds remarkably well in fully nonlinear regime. [cf. Withers 2016]

Metric components induced by momentum appear squared in computation of optical conductivity → conductivity relaxes twice as fast.

Holographic thermalization in the lab?

Pump-probe experiments appear to be promising testing grounds for holography far from equilibrium. Realization in the lab is not straightforward, though.

High frequency components in the pump pulse should be avoided to make sure one excites only the strongly entangled excitations of the strange metal.

Characteristic timescale of electronic relaxation: femtoseconds. To be sensitive to anomalously short thermalization times, the pump pulse should be turned off very fast and the probe pulse needs very good time resolution.

Important to test to what extent holographic predictions hold for finite N, real-life systems.

Summary

Using holography, we model experiments in which a 2+1D strange metal is pumped by a laser pulse into a highly excited state, after which the time evolution of the optical conductivity is probed.

We consider a finite-density state with mildly broken translation invariance and excite it by oscillating electric field pulses. At zero density, the optical conductivity would assume its thermalized value immediately after the pumping has ended.

At finite density, pulses with significant DC components give rise to slow exponential relaxation, governed by a vector quasinormal mode. In contrast, for high-frequency pulses the amplitude of the quasinormal mode is strongly suppressed, so that the optical conductivity assumes its thermalized value effectively instantaneously.