Holography for $\mathcal{N} = 1$ SQFT and the uplift of GPPZ solution

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Introduction

- The best understood holographic duality is that between $\mathcal{N} = 4$ SYM and $AdS_5 \times S^5$.
- Since the early days of AdS/CFT, a significant effort was devoted to obtain holographic dualities involving non-conformal theories with reduced supersymmetry.
- Here we will address this problem systematically for $\mathcal{N} = 1$ deformations of $\mathcal{N} = 4$ SYM ...
- ... and then analyze the case of $N = 1^*$ SYM.

References

The first part of the talk is based on

- Stanislav Schmidt, KS,
  $\mathcal{N}=1$ deformations of $\mathcal{N}=4$ SYM and SUGRA potentials,
  18xx.xxxxx

The last part is based on

- M. Petrini, H. Samtleben, S. Schmidt and KS,
  The 10d Uplift of the GPPZ Solution,
  JHEP07(2018)026, arXiv:1805.01919 [hep-th] and on-going.

  Related work appeared in

- N. Bobev, F. F. Gautason, B. E. Niehoff and J. van Muiden,
  Uplifting GPPZ: A Ten-dimensional Dual of $\mathcal{N}=1^*$,
  arXiv:1805.03623 [hep-th].

References

- Earlier relevant work:

  Andianopoli, Ferrara, Zaffaroni, Lledo, ... (1998-1999)

  Girardello, Petrini, Porrati, Zaffaroni (1998-1999)

  Pilch, Warner, ... (1998-2000)

  Townsend, KS (1999)-(2007), Freedman, Nuñez, Schnabl, KS (2003)

  ....
- Consistent truncation of IIB on S^5

  ... [Lee, Strickland-Constable, Waldram (2014)] ... [Baguet, Hohm, Samtleben (2015)]

Outline

Deforming holographic dualities

- Recall that 1/2 BPS operators $\mathcal{O}_\Delta$ of holographic SCFTs correspond to bulk supergravity fields ϕ
- We can use these operators to deform the SCFT to a SQFT:

$$L_{CFT} \to L_{QFT} = L_{CFT} + \phi_0 \mathcal{O}_\Delta$$

- The bulk description is in terms of a domain-wall spacetime,

$$\begin{aligned} ds^2 &= dr^2 + e^{2A(r)} dx^i dx^i \\ \phi &= \phi(r) \end{aligned}$$

where $A(r) \to r, \phi(r) \to e^{(d-\Delta)r}\phi_0$ as $r \to \infty$.

- The same type of spacetime but with $A(r) \to r, \phi(r) \to e^{-\Delta r}\phi_\Delta$ as $r \to \infty$ describes the dual to a QFT with a condensate, where now $\langle O_\Delta \rangle \sim \phi_\Delta$.

Fake supergravity

- Assuming $\phi(r)$ is invertible, there is a first order formulation:

$$\dot{A}(r) = -\frac{1}{d-1}W(\phi), \qquad \dot{\phi} = \partial_\phi W(\phi)$$

where $W(\phi) = -(d-1)\dot{A}(r(\phi))$ is the fake superpotential.

- The potential $V(\phi)$ is then given in terms of $W(\phi)$ by

$$V(\phi) = -\frac{1}{2}\left(\frac{d}{d-1}W^2 - (\partial_\phi W)^2\right)$$

- Theories with such potential enjoy a positive energy theorem. [Boucher (1984)] [Townsend (1984)]
- This then implies that the domain-wall solution is perturbatively (and often non-perturbatively) stable. [KS, Townsend (1999)(2006)], [Freedman, Nuñez, Schnabl, KS (2003)] ......

Obtaining the potential from QFT

- In holographic RG flows, the beta function given by

$$\beta(r) = \frac{1}{d-1}\frac{\partial \ln W(\phi)}{\partial \phi}$$

  where r is associated with the energy scale of the QFT.

- Suppose now we know $\beta(r)$ exactly.

- Then the field equation and the holographic beta function can be integrated to give:

$$W(r) = -\frac{1}{(d-1)^2 \int^r dr \beta(r)^2}, \quad \phi(r) = \pm(d-1)\int^r dr \beta(r) W(r)$$

Obtaining the potential

- Inverting $r = r(\phi)$ we obtain $W(\phi) = W(r(\phi))$ and from here we find the potential $V(\phi)$,

$$V(\phi) = -\frac{1}{2}\left(\frac{d}{d-1}W^2 - (\partial_\phi W)^2\right)$$

- **Summary:**

Given the beta function we can obtain the potential.

So ... when do we know the beta function exactly?

$N=1$ supersymmetric non-renormalization theorems

- In supersymmetric theories, the superpotential does not renormalize.
- Suppose we modify the superpotential

$$W \to W + \phi\hat{O}$$

where $\hat{O}$ is a chiral superfield. Then the beta function for the coupling ϕ is

$$\beta_\phi = \frac{d\phi(\mu)}{d\ln\mu} = (-d + (\Delta + \gamma_O))\phi(\mu)$$

where Δ is the classical dimension of the F-term of $\hat{O}$ and γ_O is its anomalous dimension.

Beta function

- If $\hat{O}$ has protected dimension, then we can readily integrate the beta function,

$$\frac{d\phi(\mu)}{d\ln\mu} = (-d+\Delta)\varphi(\mu) \qquad \Rightarrow \qquad \phi(\mu) = \phi_0 \exp(-(d-\Delta)\ln\mu)$$

- In holography, $\ln\mu \leftrightarrow r$, and the beta function is

$$\beta(r) = -(d-\Delta)\phi_0 \exp(-(d-\Delta)r)$$

Bulk potential

- Inserting in our previous formulae we obtain the fake superpotential

$$W(\phi)=-\frac{-(d-1)}{2}\left(\cosh\left(\frac{2\phi}{\alpha}\right)+1\right), \qquad \alpha^2=\frac{2(d-1)}{d-\Delta}$$

- ... and the potential,

$$V(\phi)=\frac{d-1}{4\kappa^2}\cosh^2\left(\frac{\phi}{\alpha}\right)\left[(d-2\Delta)\cosh\left(\frac{2\phi}{\alpha}\right)-(3d-2\Delta)\right]$$

Explicit realization

- Consider $\mathcal{N} = 4$ $SU(N)$ SYM. Its 1/2 BPS operators have no anomalous dimension.
- $\mathcal{N} = 4$ SYM may be expressed in an $\mathcal{N} = 1$ language and some of the 1/2 BPS operators are F-terms from the perspective of the $N = 1$ theory.
- Deform $\mathcal{N} = 4$ SYM using any of these operators.

$\mathcal{N} = 4$ in $\mathcal{N} = 1$ language

- The field content of $\mathcal{N} = 4$ SYM in terms of $\mathcal{N} = 1$ multiplets consist of three chiral superfields Z_i and one vector multiplet V.
- The $SU(4)_R$ R-symmetry is correspondingly broken to $SU(3) \times U(1)_R$.
- The action is given by

$$S_{\mathcal{N}=4} = \int d^2\theta d^2\bar{\theta}\, K(e^V Z, \bar{Z}) + \left(\int d^2\theta \;\left(f(Z) W^\alpha W_\alpha + \mathcal{W}(Z)\right) + c.c. \right)$$

with

$$K(Z, \bar{Z}) \sim \mathrm{tr}\, \bar{Z} Z, \quad f(Z) \sim \tau, \quad \mathcal{W}(Z) \sim \epsilon^{ijk} Z_i [Z_j, Z_k]$$

Holographic $\mathcal{N} = 1$ deformations of $\mathcal{N} = 4$ SYM

- To be able to study $\mathcal{N} = 1$ deformations holographically the deformations must involve 1/2 BPS operators of $\mathcal{N} = 4$ SYM.

  We need to identify the (parts of the 1/2 BPS) operators that deform:

➢ the superpotential $\mathcal{W}(Z)$

➢ the gauge kinetic term $f(Z)$

➢ D-term deformations

- In $\mathcal{N} = 1$ SQFT, the bottom components of chiral superfields may condense.

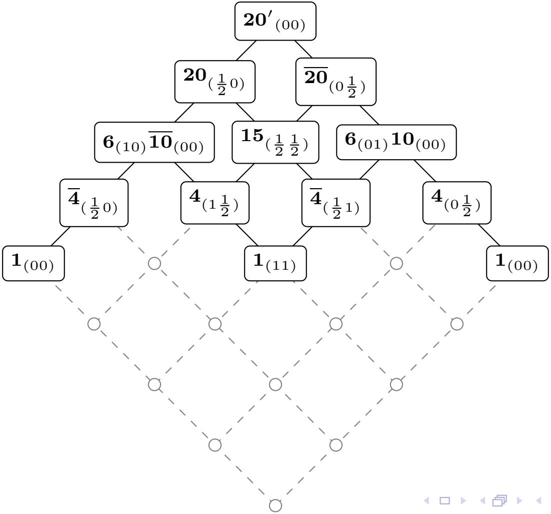
➢ To study those holographically we also need to identify where these components sit in the 1/2 BPS operator of $\mathcal{N} = 4$ SYM.

- We have classified all cases.

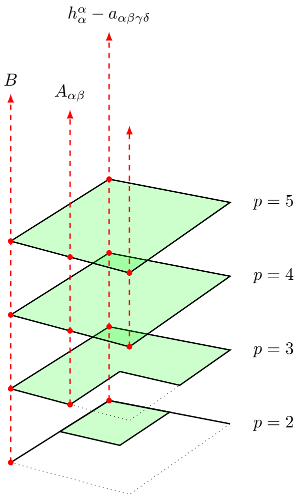
1/2 BPS operators of $\mathcal{N} = 4$ SYM

- Superconformal primaries are given by $\mathrm{Tr}\,\phi^{\{i_1} \ldots \phi^{i_p\}}$.
- The rest of the 1/2 BPS operators are obtained as super-conformal descendants.
- $p = 2$ is the energy-momentum multiplet and it corresponds to the gauged supergravity multiplet.
- $p > 2$ correspond to KK modes on S^5.

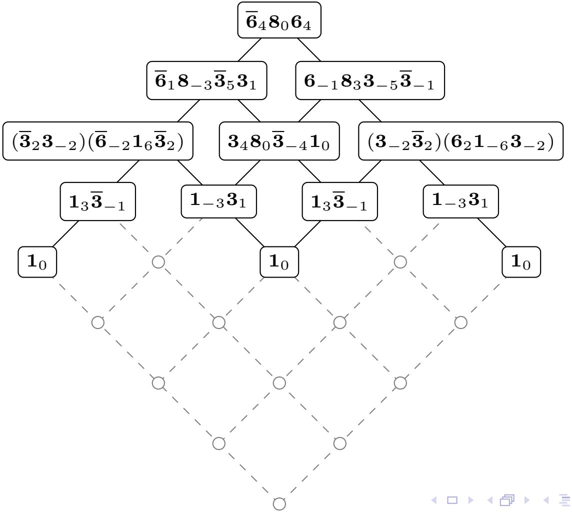
The $p = 2$ multiplet: $\mathrm{Tr}\,\phi^{\{i_1}\phi^{i_2\}}$



Correspondence with bulk fields

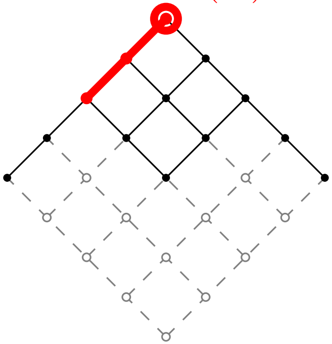


The $p = 2$ multiplet: $SU(4)_R \to SU(3) \times U(1)_R$



The $p = 2$ multiplet: $\mathcal{N} = 1$ decomposition

$$Z^2 = \overline{\mathbf{6}}_{4(00)}$$



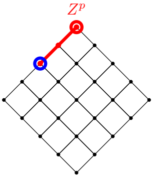
GPPZ flow

- A particular case of this deformation yields $N = 1^*$ SYM.
- In this case $\Delta = 3$ and the evaluating the formulae for the (super)potential we had earlier we find:

$$W = -\frac{3}{4}\left[\cosh\left(\frac{2m}{\sqrt{3}}\right) + 1\right]$$
$$V = -\frac{3}{8}\left[\cosh^2\left(\frac{2m}{\sqrt{3}}\right) + 4\cosh\left(\frac{2m}{\sqrt{3}}\right) + 3\right].$$

which are precisely the superpotential and potential worked out by GPPZ.

General p

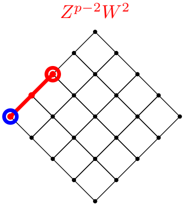


➢ These originate from KK modes of the $A_{\alpha\beta}$ field on S^5.

➠ There must exist massive single scalar consistent truncations of IIB SUGRA with the potential given earlier.

Gauge kinetic function

- Gauge kinetic function



where $W^2 = W^\alpha W_\alpha$.

- If we deform $\mathcal{N} = 4$ SYM by the F-component of this operator, then we deform $f(Z) \to \tau + Z^{p-2}$.

D-term deformations

- They appear for $p \geq 4$ and correspond to $\mathcal{N} = 1$ higher derivative terms.
- They are characterized by harmonic functions: $\partial^i \partial_i g = 0$
- For example,

$$\delta S = \int d^2\theta d^2\bar{\theta} \left(\overline{W}^2 Z_i Z_j \partial^i \partial^j g(Z_i, \bar{Z}^j) + \text{c.c.} \right)$$

Condensates

- $\mathcal{N}=4$ SYM does not have vacua that spontaneously break $\mathcal{N}=4 \rightarrow \mathcal{N}=1$.
- We can however first deform $\mathcal{N}=4$ SYM to $\mathcal{N}=1$ SYM, as discussed earlier. In the $N=1^*$ case,

$$\langle W^2 \rangle \sim m^3 e^{2\pi i \tau / N}$$

- In the large 't Hooft limit, $e^{2\pi i \tau/N} \rightarrow 1$. Then

$$\frac{d\langle W^2 \rangle}{d \log m} = 3\langle W^2 \rangle$$

- This is the analogue of the beta function for condensates.

Fake superpotential

- Using our earlier analysis, now for holographic RG flows corresponding to vevs, we find that the fake superpotential is

$$W_\sigma = -\frac{3}{4}\left(\cosh(2\sigma) + 1\right)$$

- Combined with the fake superpotential W_m that corresponds to the deformation (found earlier), we finally get

$$W = -\frac{3}{4}\left[\cosh\left(\frac{2m}{\sqrt{3}}\right) + \cosh(2\sigma)\right]$$

and from here we can get the potential.

- This agrees exactly with the GPPZ result.

GPPZ solution [Girardello, Petrini, Porrati, Zaffaroni (1999)]

$$m(y) = \frac{\sqrt{3}}{2} \log \left[\frac{1 + e^{-(y-C_1)}}{1 - e^{-(y-C_1)}} \right],$$

$$\sigma(y) = \frac{1}{2} \log \left[\frac{1 + e^{-3(y-C_2)}}{1 - e^{-3(y-C_2)}} \right],$$

$$A(y) = y - \log\cosh \frac{m(y)}{\sqrt{3}} - \frac{1}{3} \log\cosh \sigma(y) .$$

- m and σ are the norms of the complex sources that couple to the deforming operator and the gaugino bilinear.
- There is a naked curvature singularity.
- Solution physically acceptable if $C_2 \leq C_1$ (according to Gubser criterion).
- Proposed to be dual to a confining vacuum of $N = 1^*$.

Remarks

- The QFT sources are complex. We need to generalize the solution to include the phases of the sources.
- In QFT $\langle W^2 \rangle \sim m^3$, where m is the deformation parameter.
- It gauge/gravity duality we expect such relations to come from regularity, but there is no choice of the integration constants for which the solution is regular.
- There is also a computation that suggests that the spectrum contains a massless state [DeWolfe, Freedman (2000)]. Is this solution dual to a Coulomb vacuum?
- A proposal for the 10d dual of $N = 1^*$ was made by [Polchinski, Strassler (2000)]. What is the relation to GPPZ?

The uplift of the GPPZ solution

- We generalized the GPPZ solution to complex m and σ and uplfited it to 10d.
- The phases of m and σ are accounted by a combination of a rotation in S^5, which geometrizes the R-symmetry, and a rotation of $U(1) \in SL(2)_{IIB}$ (bonus $U(1)$ [Gunaydin, Marcus (1985)] [Intriligator (1998)]).
- The 10d metric and axion-dilaton agree exactly with those in [Pilch, Warner (2000)].
- All p-form are turned on and the entire solution is given in terms of elementary functions.

Singularity

- The solution is still singular but the singularity is milder in 10d than in 5d [in agreement with [Pilch, Warner (2000)]].
- There is either a ring singularity in internal space (if $C_2 < C_1$) or both a radial singularity and an angular singularity (if $C_1 = C_2$).
- Intriguingly, there are conformal frames where the singularity is even milder: there is a singularity only at one point on S^5 ($C_2 < C_1$) or only in the radial direction ($C_1 = C_2$).

Near boundary expansion

- We computed the near-boundary expansion of all fields to sub-leading order, and found agreement with [Freedman-Minahan (2000)].
- Boundary conditions (non-normalizable modes) are the same as Polchinski-Strassler.
- Sub-leading terms are different from those in [Polchinski, Strassler (2000)]: the Polchinski-Strassler configurartion does not solve the IIB equations of motion.

Is the solution supported by branes?

- We have explicitly checked that the IIB equations of motion hold at generic points.
- Checking for delta function sources in the field equations is more subtle.
- To diagnose possible delta function sources one may integrate the field equations against test functions.

  The results so far are as follows:
- There are no 5- and 7-branes localized away from the position of the 5d singularity.
- There may still be branes localized at the position of the 5d singularity.

Conclusions/Outlook

- We classified $\mathcal{N} = 1$ deformations of $\mathcal{N} = 4$ SYM.
- We discussed how to obtain the supergravity potential for single-scalar sectors directly from the QFT.

- It would be interesting to extend this discussion in other cases where we understand the QFT dynamics.
- The results suggests the existence of single-scalar truncations of massive modes.
- Extend to other dimensions. (The potential for $d = 3, \Delta = 1, 2$ appeared before in the literature.)

Conclusions/Outlook

- We uplifted the GPPZ solution to ten dimensionss.
- We should compute further observables to decide what this solution represents:
- Use the method Kaluza-Klein holography [KS, Taylor (2006)] to extract the expectation values of higher dimension chiral primaries.
- Compare with QFT expectations.