Transport from gravitational Chern-Simons: Theme and Variation

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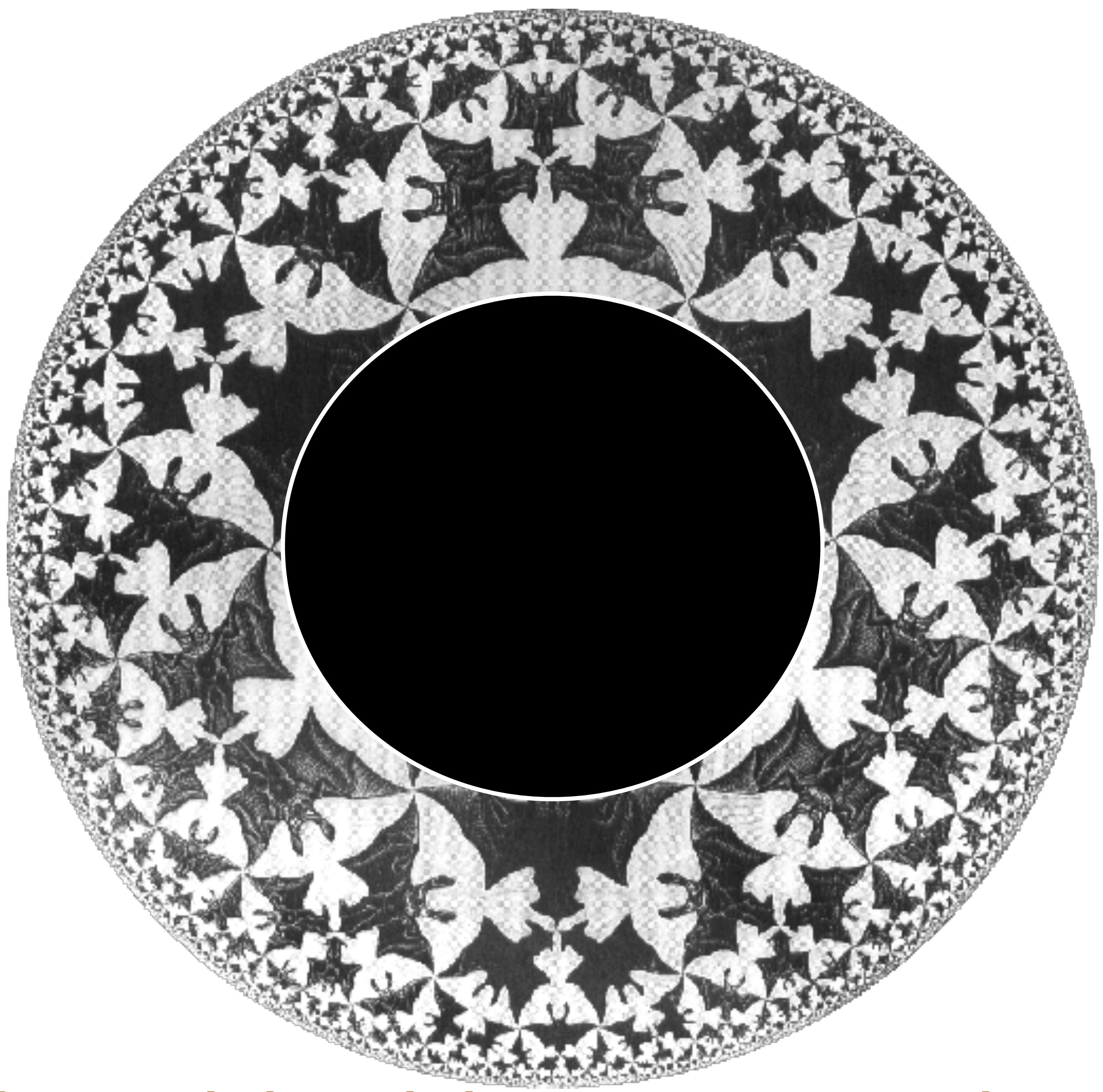
Outline

- Anomalous Transport
- 3 Examples
  - Translation breaking
  - Quenching the CME
  - Odd viscosity
- Outlook

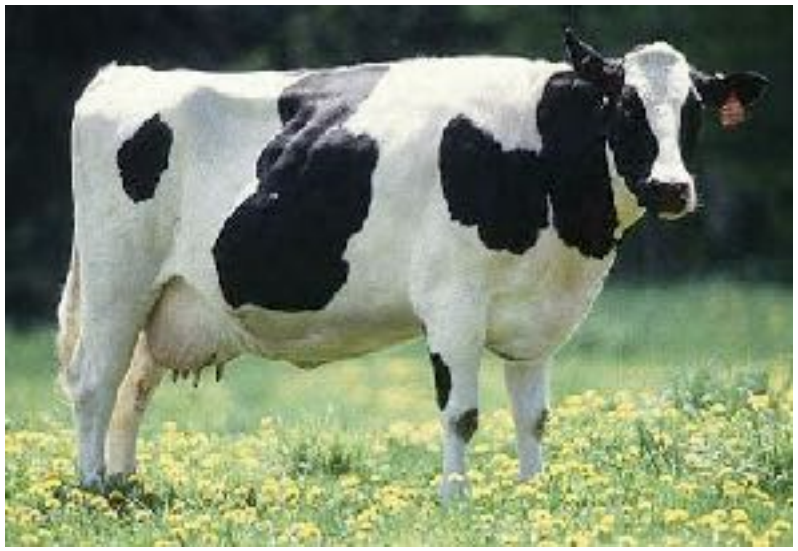
Holography:

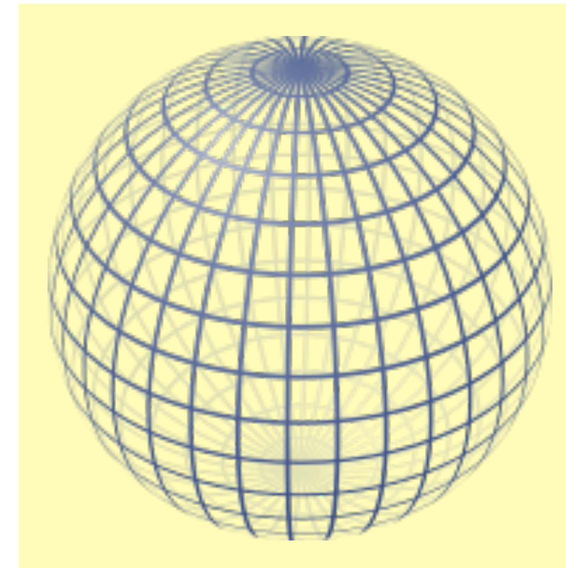
$$ds^2 = r^2(-f(r)dt^2 + d\vec{x}^2) + \frac{dr^2}{f(r)r^2}$$

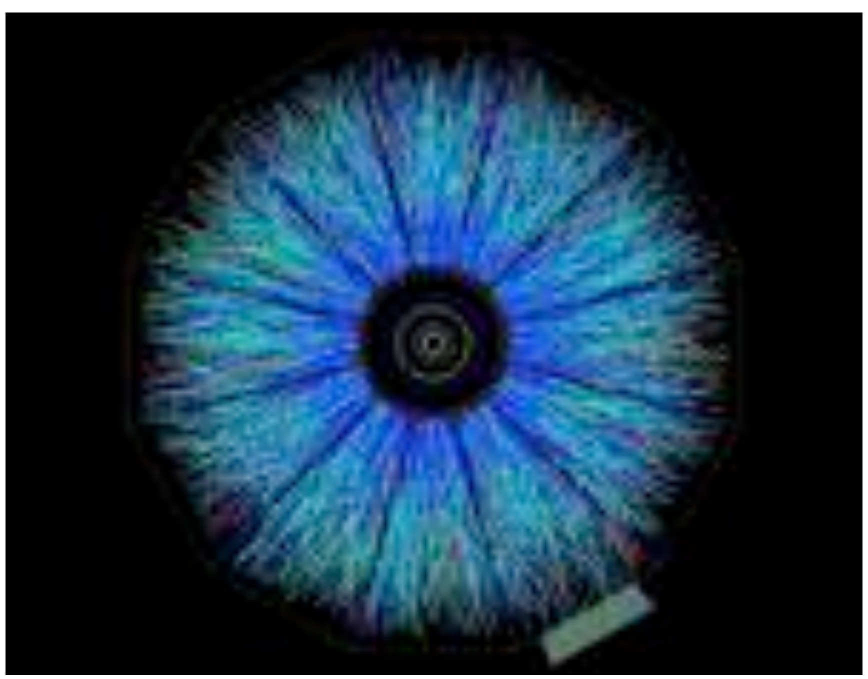


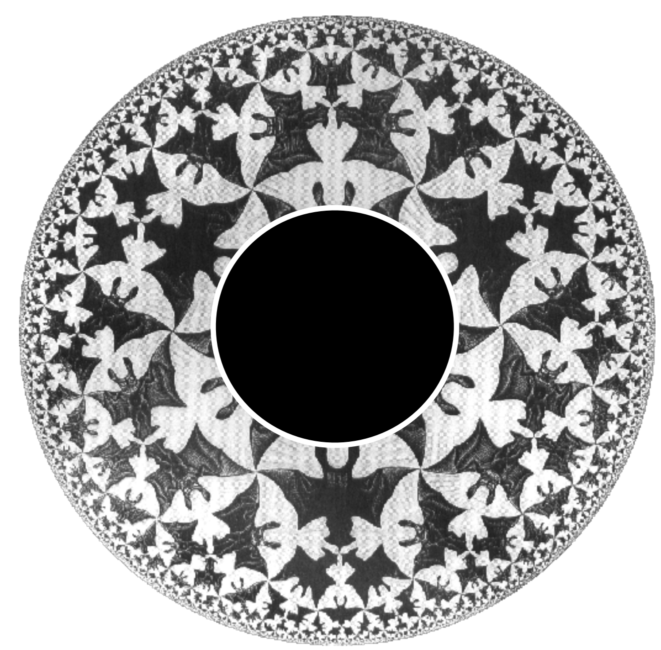
[Maldacena] [Witten] [Gubser, Klebanov, Polyakov]

AdS = spherical (hyperbolic) cow of sQGP



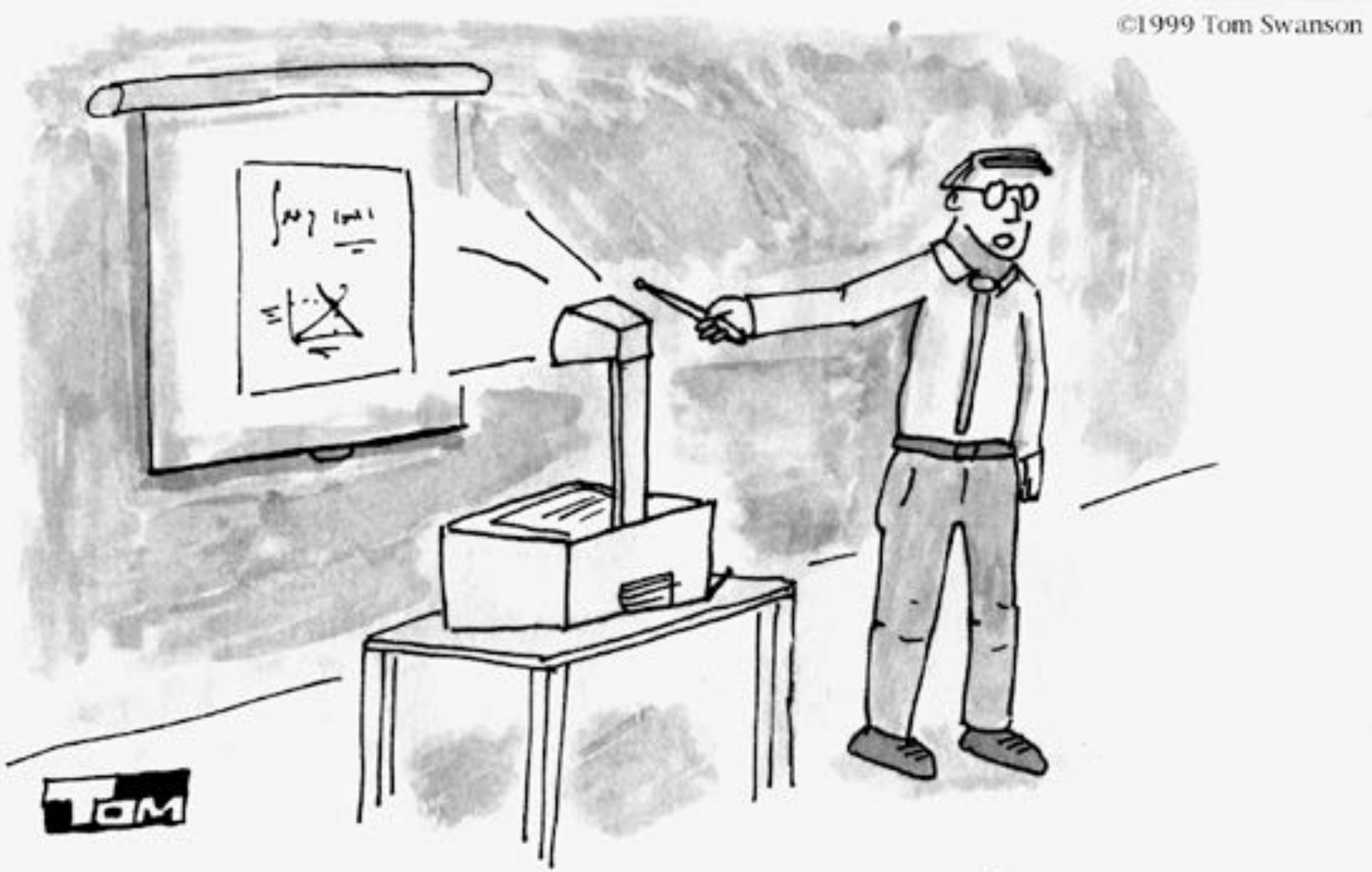






$$\frac{\eta}{s} = \frac{1}{4\pi}$$

[Policastro, Son, Starinets]
[Kovtun, Son, Starinets]



ACTUALLY, THAT ASSUMPTION ISN'T REALLY NECESSARY. WE CAN SEE HERE THAT THE POINT-COW APPROXIMATION WORKS EQUALLY WELL.

Anomalous Transport

$$S_{\mathrm{CS}} = \int (\frac{\kappa}{3} A \wedge F \wedge F + \lambda\, A \wedge \mathrm{tr}(R \wedge R)$$

$$\delta S_{\mathrm{CS}} = \int_{\partial} \theta \left(\frac{\kappa}{3} F \wedge F + \lambda\, \mathrm{tr}(R \wedge R) \right)$$

Chiral anomaly

"Gravitational"
Chiral anomaly

Anomalous Transport

$$\vec{J} = 8\kappa\mu\vec{B} + \left(4\kappa\mu^2 + 32\pi^2\lambda T^2\right) 2\vec{\omega}$$

$$\vec{J}_\epsilon = \left(4\kappa\mu^2 + 32\pi^2\lambda T^2\right) \vec{B} + \left(\frac{4}{3}\kappa\mu^3 + 32\pi^2\lambda\mu T^2\right) 2\vec{\omega}$$

- Chiral Magnetic Effect
- Chiral Vortical Effect
- Chemical Potential “κ“ chiral anomaly
- Temperature “λ“, grav. Anomaly

[Banerjee, Bhattacharya, Bhattacharyya, Dutta, Loganayagam, Surowka]
[Erdmenger, Haack, Kaminski, Yarom], [Megias, Melgar, K.L., Pena-Benitez]

Gravitational anomaly

- Objection: 4-th order in derivatives, too high

$$D_\mu J^\mu = \frac{1}{768\pi^2}\epsilon^{\mu\nu\rho\lambda} R^\alpha{}_{\beta\mu\nu} R^\beta{}_{\alpha\rho\lambda}$$

- BUT: AdS has 5-th dimension

$$\int d^5x A \wedge \mathrm{tr}\,(R\wedge R) = \int d^5x A \wedge \mathrm{tr}\left(R^{(4)}\wedge R^{(4)} + A\wedge D(K\wedge DK)\right)$$

- Is there even in flat space
- Vanishes at the boundary
- Picks up a contribution from the horizon
- Suggest "low energy anomaly"
- even in flat space, 2 space time derivatives

$$K_{ij} \approx \frac{\partial}{\partial r} g_{ij}$$

$$\partial_\mu J^\mu = D(K\wedge DK)$$

- BUT: K is covariant tensor w.r.t. to 4-dim spacetime

Quantum Currents from 5D

- At the black hole horizon

$$ds^2 = r^2 f(r)(-dt^2 + 2\vec{A}d\vec{x}dt) + r^2 d\vec{x}^2 + \frac{dr^2}{r^2 f(r)}$$

$$D(K \wedge DK) \propto f'(r_h)^2 \frac{\partial \vec{A}}{\partial t}(\vec{\nabla} \times \vec{A})$$

- Following Hawking $D(K \wedge DK) \propto 64\pi^2 T^2 \vec{E}_g \cdot \vec{B}_g$
- Current $J^\mu = \sqrt{-g}F^{\mu r} - \frac{\lambda}{2\pi G}\epsilon^{\mu\nu\rho\lambda}K_\nu^\sigma D_\rho K_{\lambda\sigma}$
- Luttinger $\vec{E}_g = -\frac{\vec{\nabla}T}{T}$ $\vec{B}_g = \nabla \times \vec{v}$

[POSTER -> C. Copetti]

$$\vec{J} = 64\pi^2 \lambda T^2 \vec{\omega}$$

Chiral Vortical Effect

Comments

- Black hole horizon
- **Without AdS**
- Holographic RG flow
- Without Holography: couple to black hole
- BH does RG flow for you
- Boundary condition: no outgoing current at horizon

[Stone, Kim]
[Jensen, Loganayagam, Yarom] (Cone)
Also [Wilczek, Robinson]

- Global anomaly: [Golkar, Sethi], [Glorioso, Liu]

Translation breaking

- In Holography "linear axion" background (massless scalar)

$$S = \int d^5x\sqrt{-g}\left(|\partial\phi|^2\right) \quad , \quad \phi \approx kx + \cdots$$

- Background breaks translations eoms are homogeneous
- Graviton has mass

$$T^{0i} = T^{i0}$$

- Charge (Momentum) = Current (Energy-current)

1. Intuition: Momentum density, broken symmetry
2. Intuition: Energy current is dissipationless

[Copetti, Fernandez-Pendas, K.L., Megias]

Translation breaking

- Can extrinsic curvature term be seen in UV?

$$ds^2 = -fr^2dt^2 + \frac{dr^2}{r^2 f} + r^2 d\vec{x}^2 \qquad f = 1 + \frac{k^2}{r^2} - (1 - \frac{k^2}{4})\frac{r_H^4}{r^4}$$

Unusual power!

- Extrinsic curvature as additional variable

$$\delta S_{on-shell} = \int_\partial \sqrt{-g}(t^{\mu\nu}\delta g_{\mu\nu} + u^{\mu\nu}\delta K_{\mu\nu})$$

- Energy momentum tensor (Ward identity)

$$\Theta^{\mu\nu} = t^{\mu\nu} + u^{\mu\lambda}K^{\nu}_{\lambda}$$

- New term is due to gravitational Chern-Simons term

Symmetry breaking - II

- CME and CVE without new term

$$J^i = \left(4\kappa\mu^2 + 32\pi^2\lambda T^2\right) 2\Omega^i$$
$$T^{0i} = \left(4\kappa\mu^2 + 32\pi^2\lambda T^2 - 4\lambda k^2\right) B^i$$

- Impossible in unitary theory
- CVE = 2 CME for energy current by Kubo formulas

- Including the new term

$$T^{0i} = \left(4\kappa\mu^2 + 32\pi^2\lambda T^2\right) B^i$$

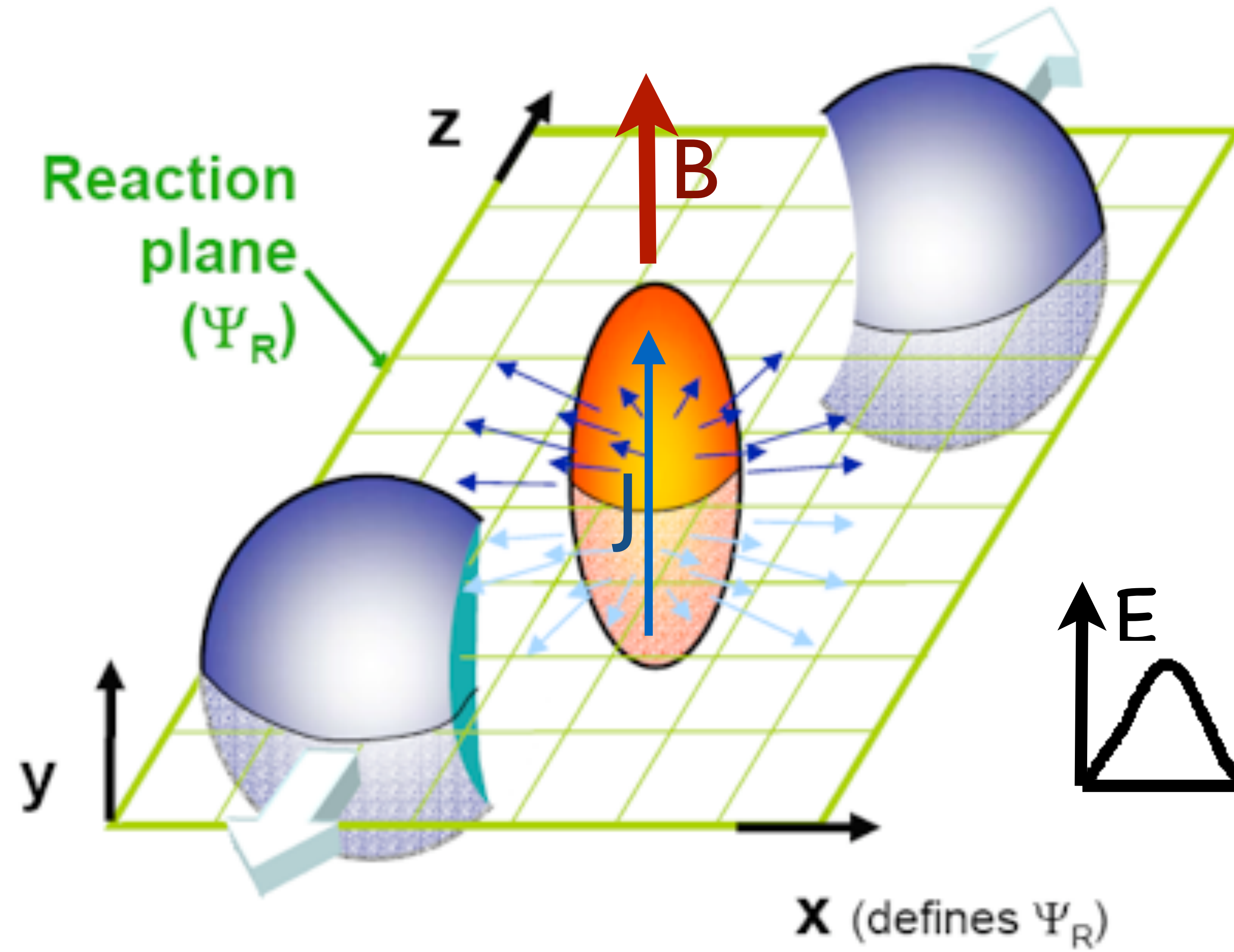
- All is well!
- Energy current being dissipation wins!

[Copetti, Fernandez-Pendas, K.L., Megias]

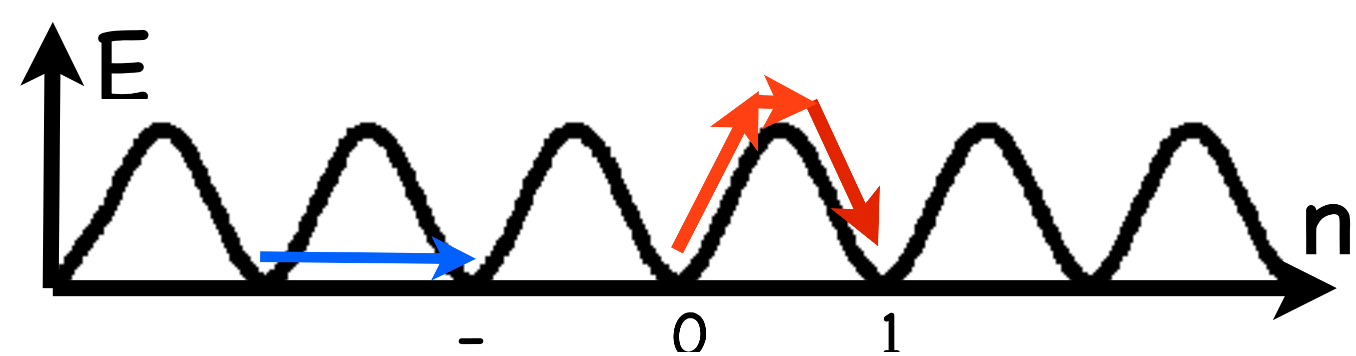
CME

- Aug. 2008: *"The Chiral Magnetic Effect"* [Fukushima, Kharzeev, Warringa]

$$\vec{J} = \frac{\mu_5}{2\pi^2}\vec{B}$$



- Topology of QCD fields
- CME induces charge separation
- First observable of anomalous transport



Quenching the CME

[E. Lopez, G. Milans del Bosch, K.L.]

- CME and CVE depend on equilibrium quantities T, μ

- Natural question: anomaly induced transport **far** from equilibrium physics

- Possible importance for Heavy Ion Collisions (magnetic field has already decayed in hydrodynamic regime)

- Holography allows both: study fast time evolution, quenches and anomalous transport

- Study CME via gravitational Chern-Simons term

What to look for

- "Minimal" setup: inject energy
- Equilibrium: energy — temperature $T_0 \longrightarrow T$
- CME in energy-momentum tensor
- First near equilibrium = hydro

$$T_{\mu\nu} = (\epsilon + p) u_\mu u_\nu + p\eta_{\mu\nu} + \hat{\sigma}_B (u_\mu B_\nu + u_\nu B_\mu) \, ,$$
$$J_\mu = \rho u_\mu + \sigma_B B_\mu \, ,$$
$$J_\mu^X = \rho_X u_\mu + \sigma_{B,X} B_\mu$$

- Landau frame

$$\hat{\sigma}_B = 0$$
$$\sigma_B = 24\alpha\mu - \frac{\rho}{\epsilon + p} \left(12\alpha\mu^2 + 32\lambda\pi^2 T^2\right)$$
$$\sigma_{B,X} = -\frac{\rho_X}{\epsilon + p} \left(12\alpha\mu^2 + 32\lambda\pi^2 T^2\right)$$

What to look for

- Energy current = Momentum density

$$T_{0i} = T_{i0}$$

- Momentum density = conserved charge

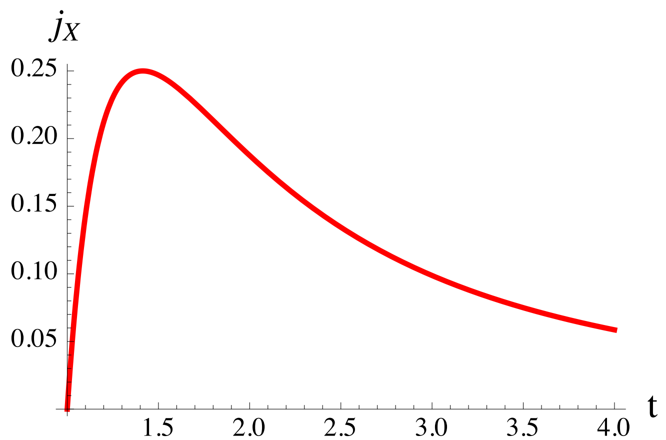
$$32\lambda\pi^2 T_0^2 \vec{B} = (\epsilon + p)\vec{v}$$

- Monitor response in tracer U(1) current

$$\vec{J}_X = 32\frac{\rho_X}{\epsilon + p}(T_0^2 - T^2)\pi^2\lambda\vec{B}$$

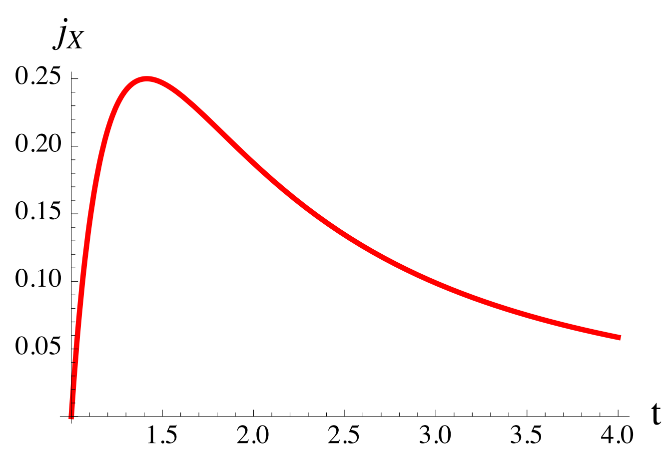
- Removing constants: benchmark near equilibrium curve

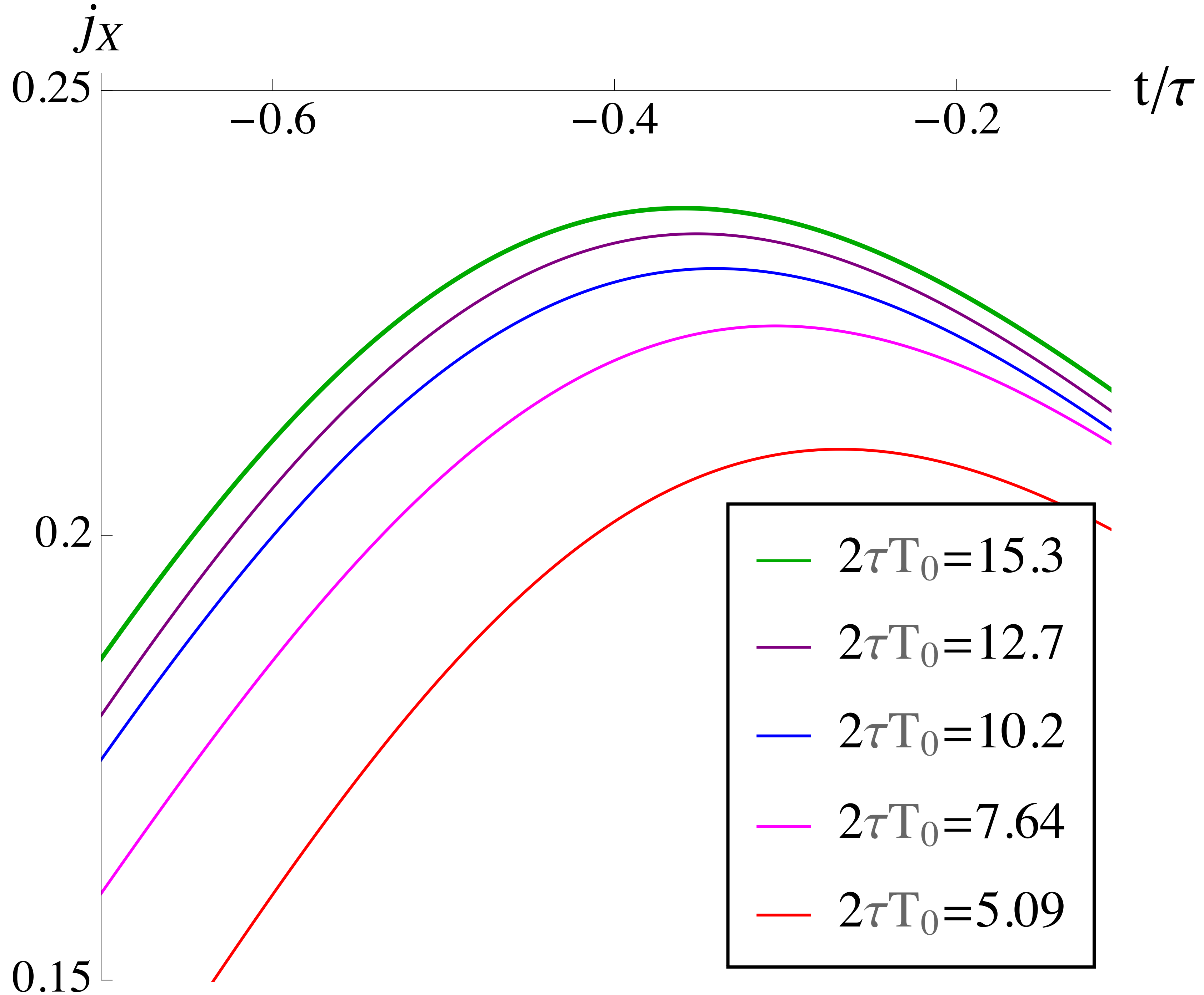
$$j_X = \frac{T^2/T_0^2 - 1}{T^4/T_0^4}$$



Holographic quench

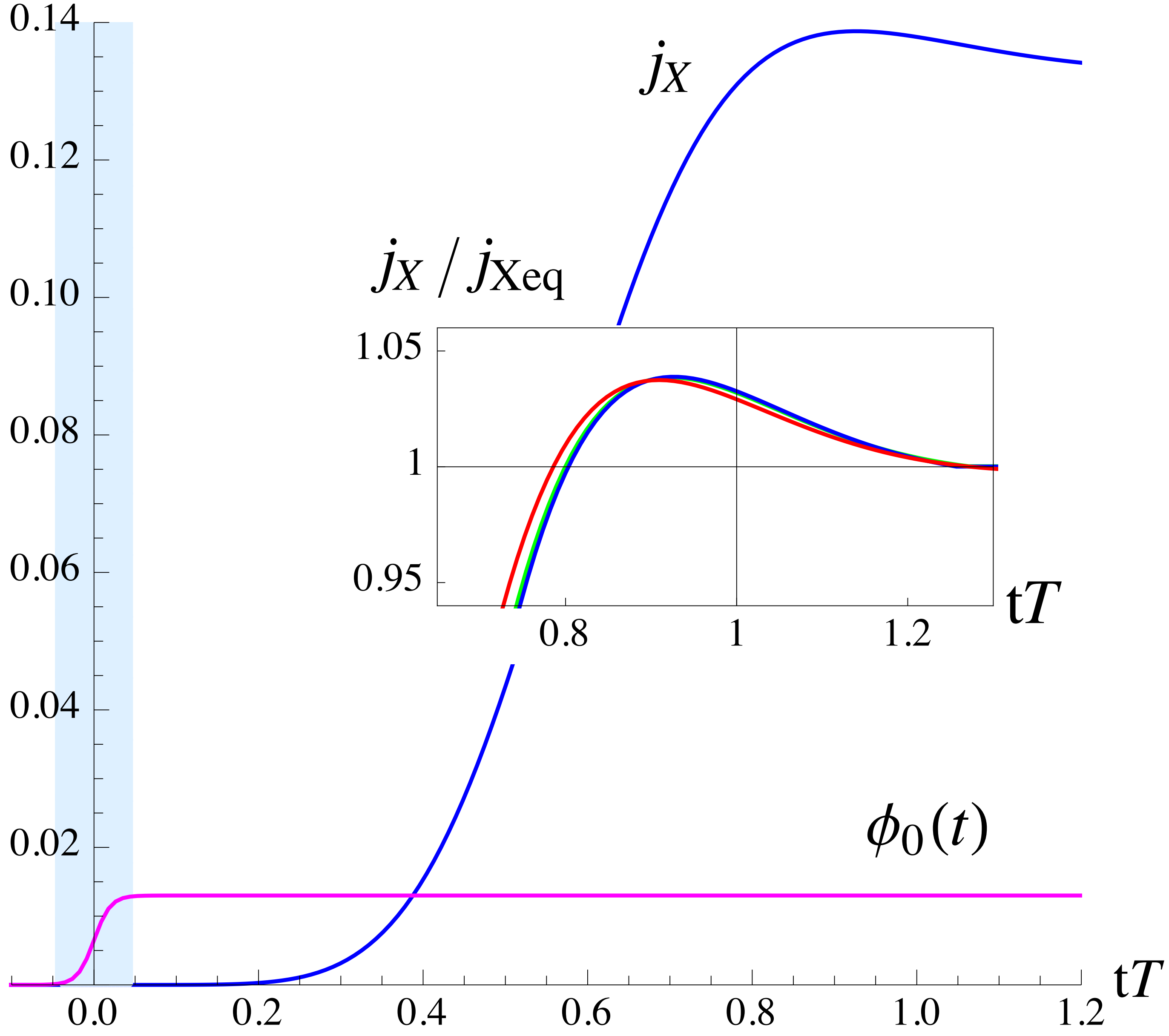
- Very slow quenches





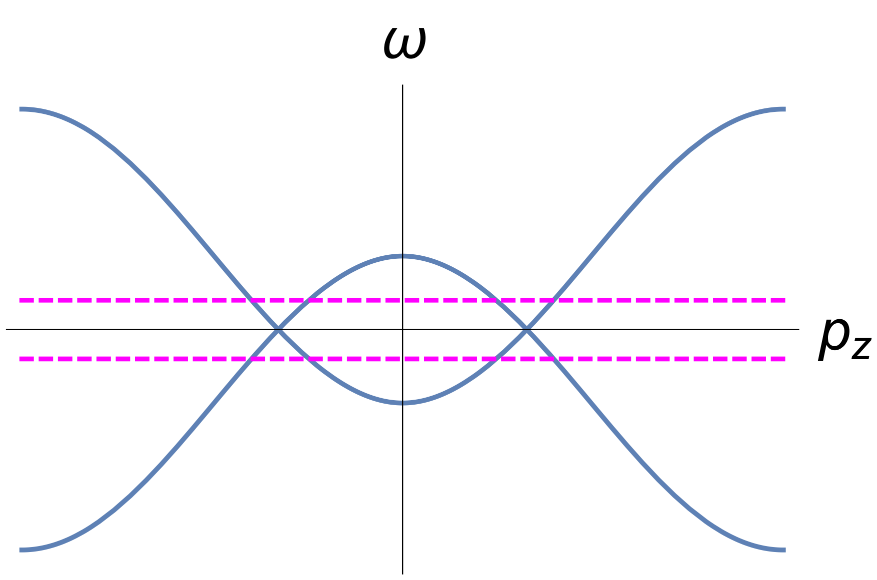
Holographic quench

- Fast quenches $2\tau T < 1$

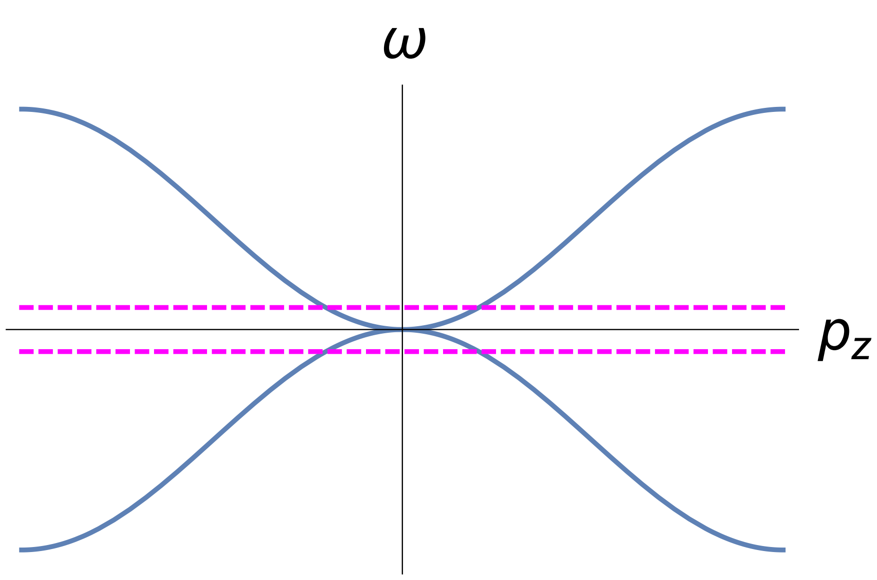


Holography for Weyl Semi-Metals

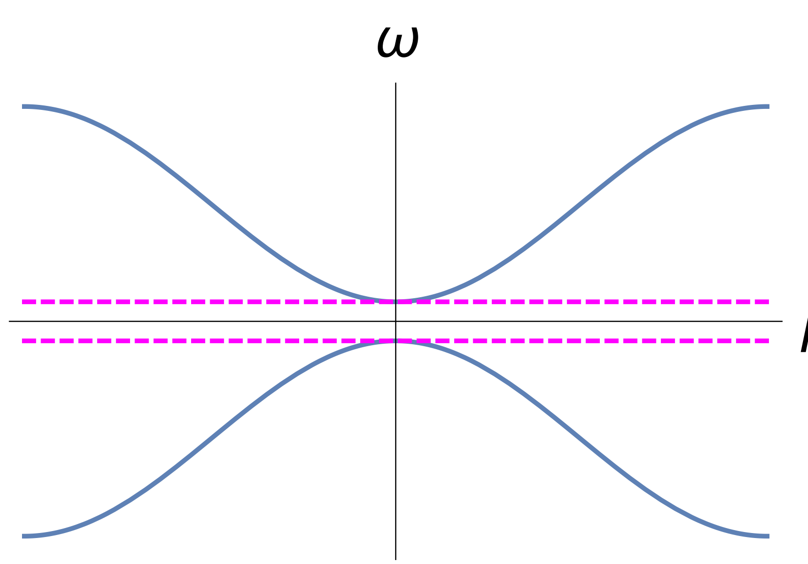
- Holographic model for quantum phase transition [K.L., Liu, Sun]



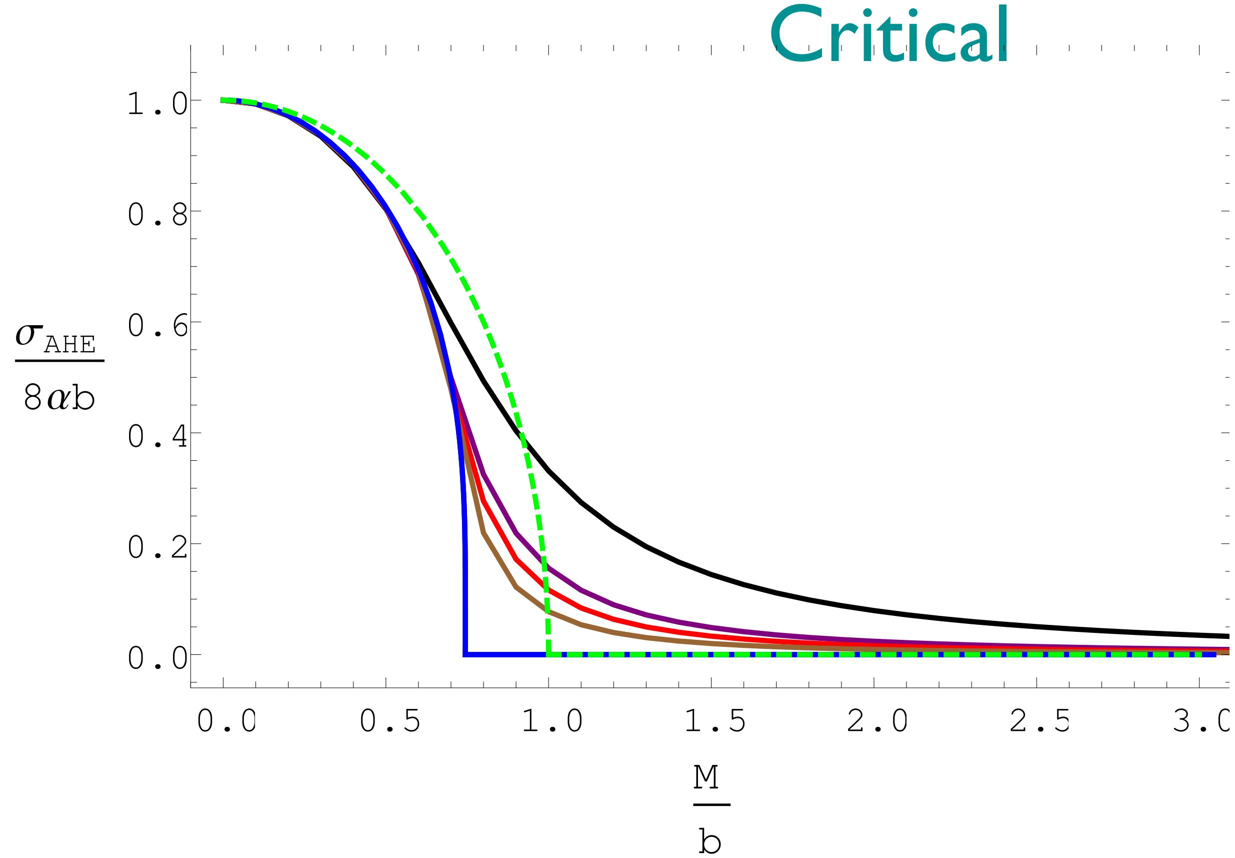
WSM



Critical



Trivial



$$\sigma_{xy} = 8\alpha A_z^5(0)$$

[POSTER -> J. Fernandez-Pendas]

Odd viscosity

- Hall viscosity in 2D Quantum Hall states [Avron, Seiler, Zograf]
- Time reversal breaking necessary
- 2D : invariant ε tensor
- 3D: need some anisotropy

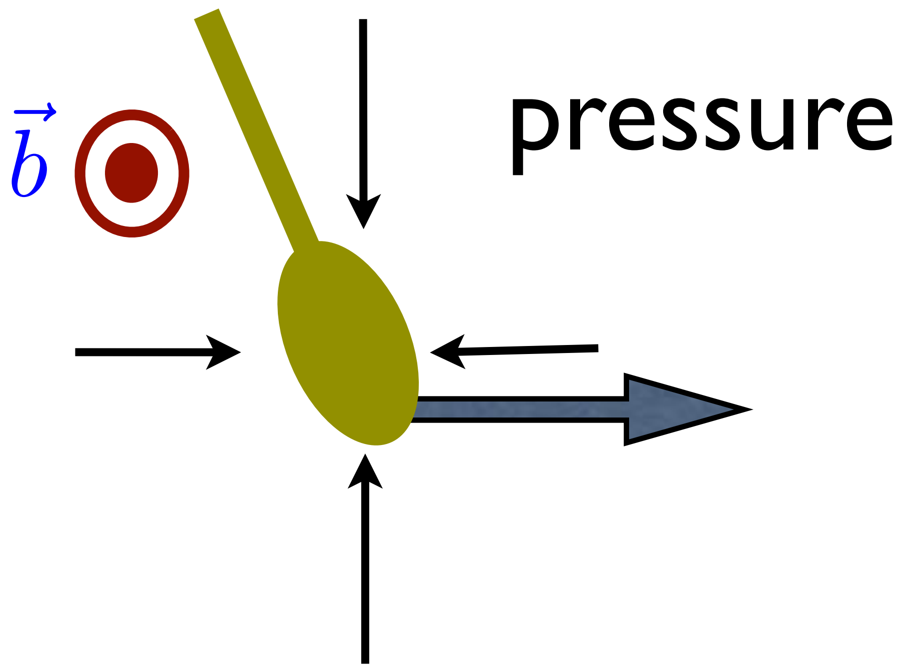
$$\tau_{xy} = \eta_\perp V_{xy} - \eta_\perp^H (V_{xx} - V_{yy})$$
$$\tau_{xz} = \eta_{||} V_{xz} + \eta_{||}^H V_{yz}$$
$$\tau_{yz} = \eta_{||} V_{yz} - \eta_{||}^H V_{xz}$$

[Landau, Lifshytz Vol. 10]

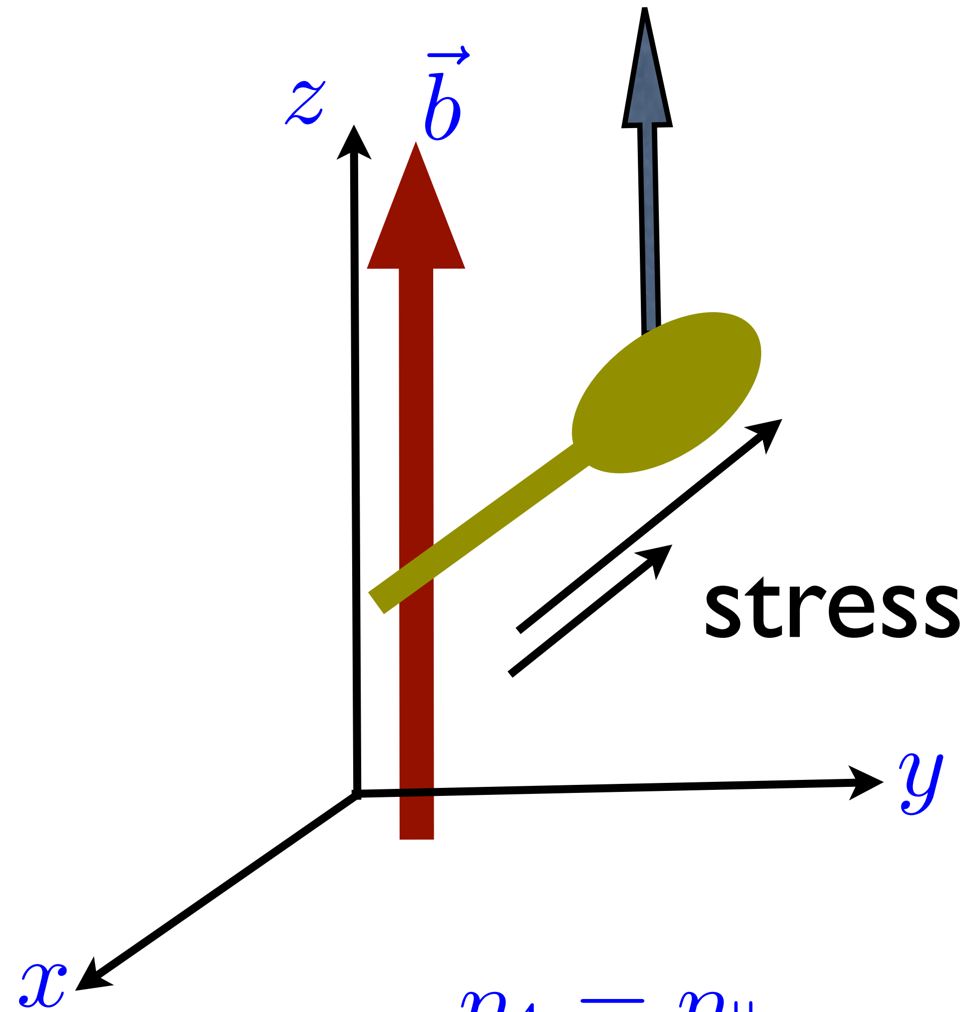
$$V_{ij} = \frac{1}{2}\left(\partial_i v_j + \partial_j v_i\right)$$

- In total: 3 shear, 2 "bulk" and 2 odd viscosities

Odd viscosity



$\eta_3 = \eta_\perp$

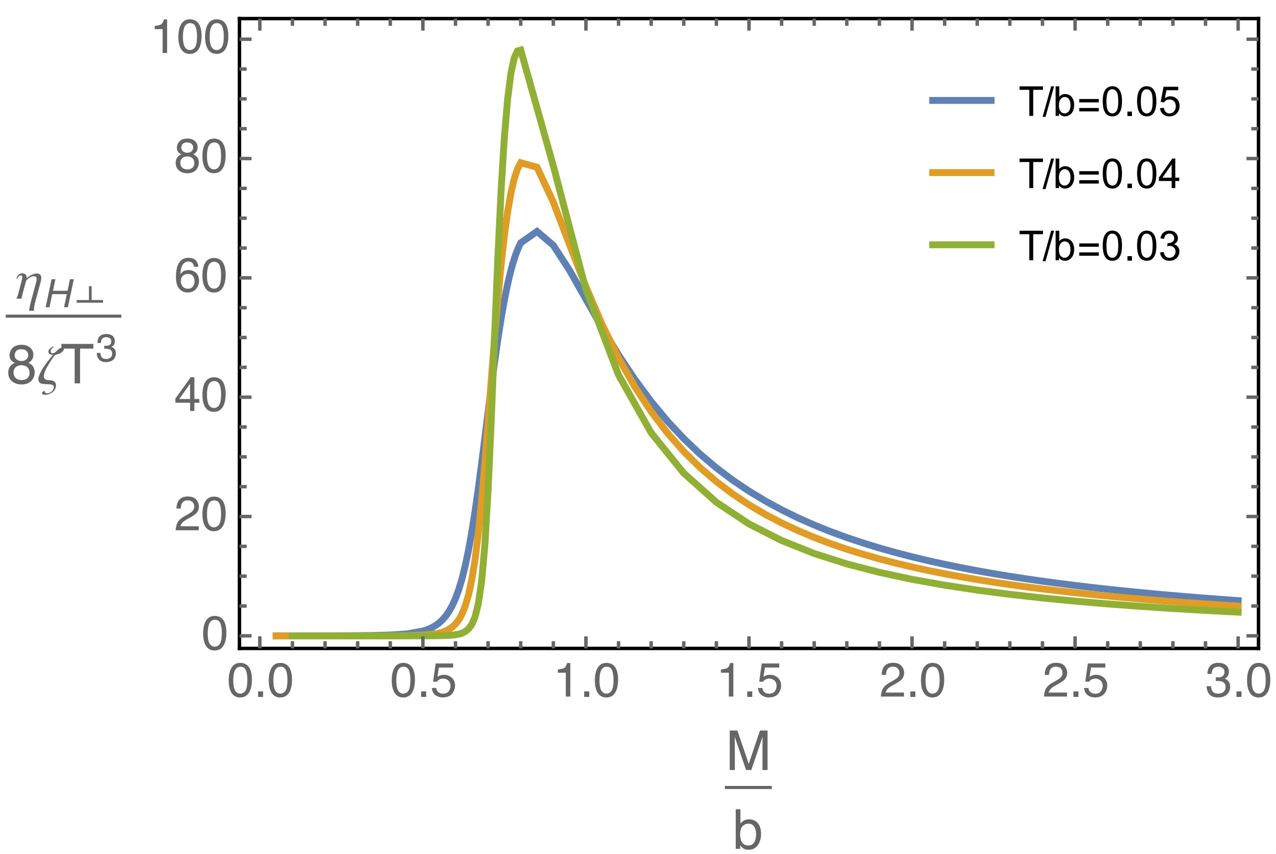


$\eta_4 = \eta_\parallel$

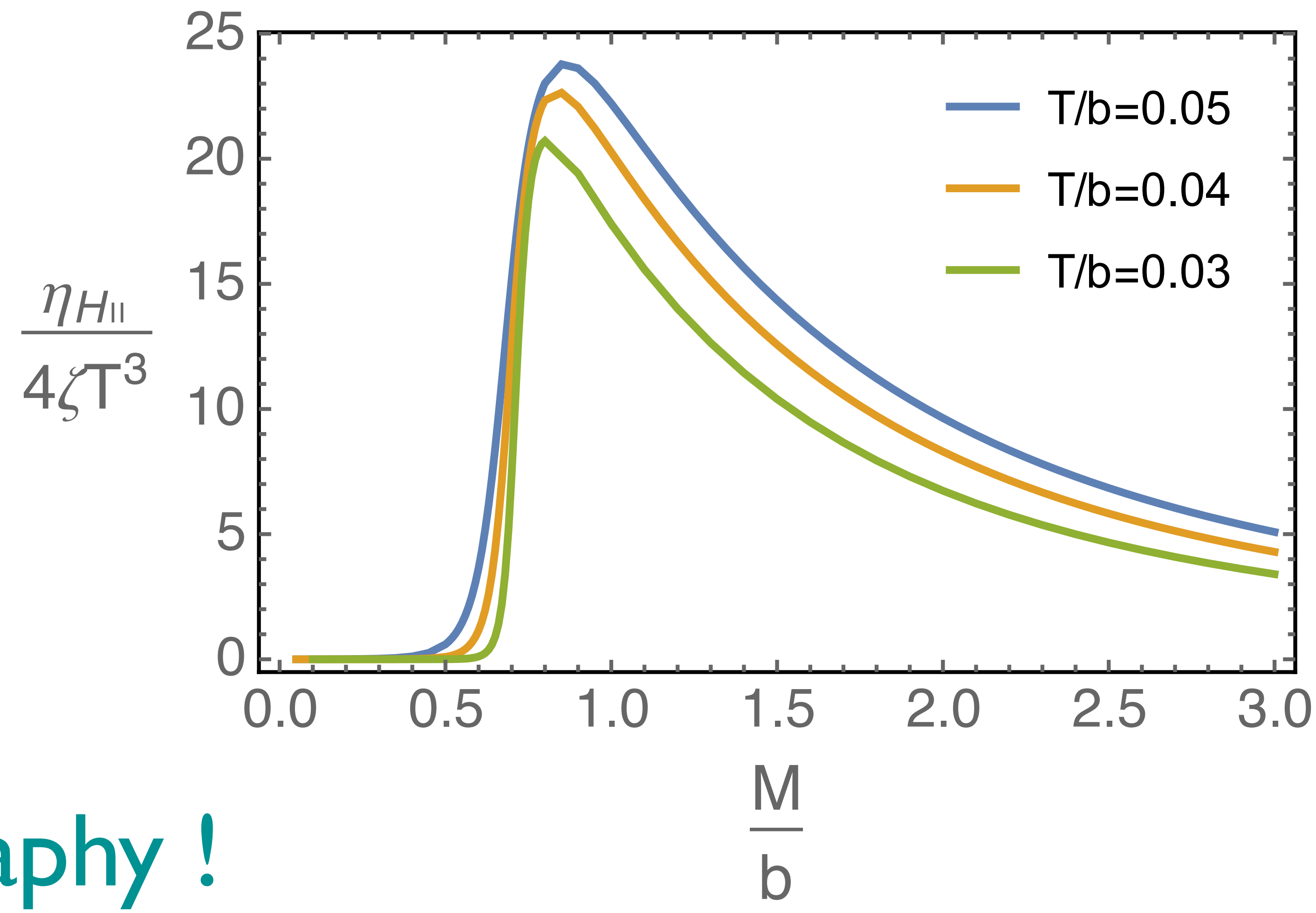
Odd viscosity

- Odd viscosities by adding gravitational anomaly term
- Probe IR region of geometry: Low T

transverse



parallel



- Prediction from Holography !
- Again: gravitational Anomaly at first order !

Summary

- Holography is efficient discovery tool for transport
- Fate of anomalous transport under symmetry breaking
- New questions:
  - What is the extrinsic curvature in field theory?
  - Anomalous transport far from equilibrium (QGP)
  - Odd viscosity and grav. anomaly?