Towards a Holographic Dictionary for SYK model

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in progress

SYK Model

- This is a quantum mechanical model of N real fermions which are all connected to each other by a random coupling. The Hamiltonian is

$$H = (i)^{\frac{q}{2}} \sum_{1 \le i_1 < i_2 < \cdots < i_q \le N} j_{i_1 i_2 \cdots i_q} \chi_{i_1} \chi_{i_2} \cdots \chi_{i_q} ; \qquad \{\chi_i, \chi_j\} = \delta_{ij}$$

- The couplings are random with a Gaussian distribution with width J

$$< j_{i_1 i_2 \cdots i_q}^2 > = \frac{J^2 (q-1)!}{N^{q-1}}$$

- This model is of interest since this displays quantum chaos and thermalization.
- When N is large, one can treat this using replicas, Sachdev and Kitaev showed that one can replace the quenched average by an annealed average.

- Averaging over the couplings gives rise to the action

$$S = \frac{1}{2}\int dt \sum_{i=1}^{N} \chi_i \partial_t \chi_i - \frac{J^2 N^{q-1}}{2q}\int dt_1 \int dt_2 (\sum_{i=1}^{N} \chi_i(t_1)\chi_i(t_2))^q$$

- We can now express the path integral in terms of bilocal collective field. *(Jevicki, Suzuki and Yoon)*

$$\Psi(t_1, t_2) \equiv \frac{1}{N}\sum_{i=1}^{N} \chi_i(t_1)\chi_i(t_2)$$

- The path integral is now

$$\int \mathcal{D}\Psi(t_1, t_2)\; e^{-S_c[\Psi]}$$

- Where the collective action includes the jacobian for transformation from the original variables to the new bilocal fields

$$S_{\rm col}[\Psi] \;=\; \frac{N}{2}\int dt\,\Big[\partial_t\Psi(t,t')\Big]_{t'=t} + \frac{N}{2}\,{\rm Tr}\log\Psi \;-\; \frac{J^2N}{2q}\int dt_1dt_2\,\Psi^q(t_1,t_2)$$

- The equations of motion are the large N Dyson-Schwinger equations

$$\partial_{t_1}\Psi(t_1,t_2) + \delta(t_1-t_2) - J^2\int dt_3[\Psi(t_3,t_1)]^{q-1}\Psi(t_3,t_2) = 0$$

- At strong coupling – which is the IR of the theory – the first term can be neglected, and there is an emergent reparametrization symmetry. $t \to f(t)$
- In this limit saddle point solution is (at zero temperature)

$$\Psi_0(t_1,t_2) \;=\; \frac{b}{|t_{12}|^{\frac{2}{q}}}{\rm sgn}(t_{12}) \qquad b^q \;=\; \frac{\tan(\frac{\pi}{q})}{J^2\pi}\left(\frac{1}{2}-\frac{1}{q}\right)$$

The Strong Coupling Spectrum

- Expand the bilocal action around the large N saddle point

$$\Psi(t_1, t_2) = \Psi_0(t_1, t_2) + \sqrt{\frac{2}{N}}\, \eta(t_1, t_2)$$

$$\eta(t_1, t_2) \equiv \Phi(t, z) = \sum_{\nu,\omega} \tilde{\Phi}_{\nu,\omega} u_{\nu,\omega}(t, z)$$

$$u_{\nu,\omega}(t, z) = \mathrm{sgn}(z)\, e^{i\omega t}\, Z_\nu(|\omega z|)$$

- Where $Z_\nu(x)$ denotes a complete orthonormal set of combinations of Bessel functions (*Kitaev*, *Polchinski and Rosenhaus)*

$$Z_\nu(x) = J_\nu(x) + \xi_\nu\, J_{-\nu}(x)\,, \qquad \xi_\nu = \frac{\tan(\pi\nu/2) + 1}{\tan(\pi\nu/2) - 1}$$

- The order is real discrete $\nu = 3/2 + 2n$ or purely imaginary continuous $\nu = ir$

- These are simultaneous eigenfunctions for the kernel for quadratic fluctuations and the SL(2,R) Casimir – which is a *Lorentzian* AdS_2 laplacian
-
$$\left[z^2(-\partial_t^2+\partial_z^2)+\frac{1}{4}\right] e^{-i\omega t} z^{1/2} Z_\nu(\omega z) = \nu^2\; e^{-i\omega t} z^{1/2} Z_\nu(\omega z)$$
-
- The orthonormality and completeness relations are

$$\int_0^\infty \frac{dx}{x}\, Z_\nu^*(x)\, Z_{\nu'}(x) \;=\; N_\nu\, \delta(\nu-\nu') \qquad N_\nu \;=\; \begin{cases} (2\nu)^{-1} & \text{for } \nu = 3/2+2n \\ 2\nu^{-1}\sin\pi\nu & \text{for } \nu = ir\,, \end{cases}$$

$$\int \frac{d\nu}{N_\nu}\, Z_\nu^*(|x|)\, Z_\nu(|x'|) \;=\; x\,\delta(x-x')\,.$$

- The integral here is a shorthand for a sum over discrete modes and an integral over imaginary values.
- This combination of Bessel functions is forced on us by the requirement that the SL(2,R) generators commute with the kernel. (*Polchinski and Rosenhaus*)

- This leads to the quadratic action

$$S^{(2)} \sim \int d\nu \int d\omega \tilde{\Phi}^{\star}_{\nu,\omega}[\tilde{\kappa}(\nu) - 1]\tilde{\Phi}_{\nu,\omega}$$

where

$$\tilde{\kappa}(\nu) = -\frac{1}{(q-1)} \frac{\Gamma(\frac{1}{2}+\frac{1}{q})\Gamma(\frac{1}{q})}{\Gamma(\frac{3}{2}-\frac{1}{q})\Gamma(1-\frac{1}{q})} \frac{\Gamma(\frac{5}{4}-\frac{1}{q}+\frac{\nu}{2})\Gamma(\frac{5}{4}-\frac{1}{q}-\frac{\nu}{2})}{\Gamma(\frac{1}{4}+\frac{1}{q}+\frac{\nu}{2})\Gamma(\frac{1}{4}+\frac{1}{q}-\frac{\nu}{2})}$$

- The spectrum is therefore given by the solutions of the equation

$$\tilde{\kappa}(\nu) = 1 \qquad \nu = p_m$$

- There are always an infinite number of solutions.
- For any q $p_m = 3/2$ is always a solution. This is a zero mode at strong coupling coming from reparametrization symmetry broken by the saddle point.

- The object $\Phi(t,z)$ appears to be as a field in 1+1 dimensions.
- However the field action in real space is non-polynomial in derivatives.

$$S^{(2)} = \int dtdz\{(z^{1/2}\eta(t,z)\left[\tilde{\kappa}(\sqrt{\mathcal{D}_B}) - 1\right](z^{1/2}\eta(t,z)\}$$

$$\mathcal{D}_B \equiv z^2(-\partial_t^2 + \partial_z^2) + \frac{1}{4}$$

- In fact the form of the propagator looks like a sum of contributions from an infinite number of fields in AdS
- The conformal dimensions of the corresponding operators are given by

$$h_m = \frac{1}{2} + p_m$$

The Bilocal Propagator

- The 4 point function of the fermions is the two point function of the bilocal fluctuations.
- The expression for this is

$$\mathcal{G}(t,z;t',z') \sim |zz'|^{1/2} \int d\omega e^{-i\omega(t-t')} \int \frac{d\nu}{N_\nu} \frac{Z_\nu(\omega z) Z_\nu(\omega z')}{\tilde{\kappa}(\nu) - 1}$$

- Performing the integral over ν the propagator can be expressed as a sum over poles

$$\mathcal{G}(t,z;t',z') \sim -\frac{1}{J}|zz'|^{\frac{1}{2}} \sum_m \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{-i\omega(t-t')} \frac{Z_{-p_m}(|\omega|z^>) J_{p_m}(|\omega|z^<)}{N_{p_m}} R_{p_m}$$

- $z^> (z^<)$ denotes greater (smaller) of z and z'.
- R_{p_m} is the residue of the pole at $\nu = p_m$
- *Actually there is a double pole at 3/2. In the above expression we have implicitly used a regulator to shift this pole – this anticipates a proper treatment of this mode.*

- This expression has a divergent contribution from the mode $\nu = \frac{3}{2}$ since

$$Z_\nu(x) = J_\nu(x) + \xi_\nu J_{-\nu}(x), \qquad \xi_\nu = \frac{\tan(\pi\nu/2)+1}{\tan(\pi\nu/2)-1} \qquad \boxed{\xi_{-3/2} = \infty}$$

- At finite coupling, however, this pole is shifted since the reparametrization invariance is explicitly broken (*Maldacena and Stanford*)

$$p_0 = 3/2 - \alpha\frac{\omega}{J}$$

- Using this shift the contribution from this mode can be calculated

$$\frac{12\alpha_0 J}{\pi\alpha_K}\int\frac{d\omega}{\omega^2}\,e^{i\omega(t'_+-t_+)}\left[\frac{\sin(\omega t_-)}{\omega t_-}-\cos(\omega t_-)\right]\left[\frac{\sin(\omega t'_-)}{\omega t'_-}-\cos(\omega t'_-)\right]$$

$$t_\pm \equiv \frac{t_1 \pm t_2}{2}, \qquad t'_\pm \equiv \frac{t_3 \pm t_4}{2}$$

- The answer is proportional to J - hence called an enhanced propagator
- This is the quantity which is reproduced from the Schwarzian action.

AdS Interpretation

- At strong coupling, it is natural to expect that this model is dual to some theory in two dimensional AdS_2
- *Maldacena, Stanford and Yan*g; *Engelsoy, Mertens and Verlinde* argued that the gravity sector should be Jackiw-Teitelboim type of dilaton-gravity in 1+1 dimensions coupled to some matter (*Alihemri and Polchinski*)

$$S = \frac{1}{16\pi G}\int \mathrm{d}^2x\sqrt{-g}\Big(\Phi^2 R - V(\Phi)\Big) + S_{\mathrm{matter}}$$

- Classical solutions have a AdS_2 metric, e.g. $ds^2 = \frac{1}{z^2}[-dt^2 + dz^2]$
- And a dilaton $\Phi^2 = 1 + \frac{a}{z}$
- A nonzero a breaks the conformal symmetry. Agrees with SYK where this breaking is in the UV (finite z). Naturally, $a \sim 1/J$

- There is also a black hole solution

$$
\begin{aligned}
ds^2 &= -\frac{4M^2 du dv}{\sinh^2 M(u-v)} \\
\Phi^2 &= 1 + aM \coth M(u-v)
\end{aligned}
$$

- This is supposed to be dual to the SYK model at finite temperature

$$T = \frac{M}{\pi}$$

- The gravity sector of the theory is naturally thought as coming from the mode of the SYK model which is a zero mode at infinite coupling.
- In fact, the action – which comes entirely from boundary terms – is a Schwarzian action. This is quite universal and appears generically near extremal black holes.

(Nayak, Shukla, Soni, Trivedi & Vishal; Gaikwad, Joshi, Mandal & Wadia)

- The matter part : the infinite number of poles of the SYK propagator indicates that there should be an infinite number of fields.
- However the residues at these poles are complicated functions : these fields do not have conventional kinetic terms.

CAN WE UNDERSTAND THE EMERGENCE OF THE HOLOGRAPHIC DIRECTION DIRECTLY FROM THE LARGE N DEGREES OF FREEDOM ?

The Bilocal Space

- In fact, as a special case of the suggestion of *S.R.D. and A. Jevicki (2003)* in the context of duality between Vasiliev theory and O(N) vector model – the center of mass and the relative coordinates $z = (t_1 - t_2)$ $t = (t_1 + t_2)$ can be thought of as Poincare coordinates in AdS_2
- The SL(2,R) transformations of the two points of the bilocal

$$\begin{aligned} \delta t_1 &= \epsilon_1, \qquad \delta t_1 = \epsilon_2 t_1 \qquad \delta t_1 = \epsilon_3 t_1^2 \\ \delta t_2 &= \epsilon_1, \qquad \delta t_2 = \epsilon_2 y \qquad \delta t_2 = \epsilon_3 t_2^2 \end{aligned}$$

- These become isometries of a Lorentzian AdS_2 space-time

$$\begin{aligned} \delta t &= \epsilon_1 \qquad \delta z = 0 \\ \delta t &= \epsilon_2 t \qquad \delta z = \epsilon_2 z \\ \delta t &= \frac{1}{2}\epsilon_3(t^2 + z^2) \qquad \delta z = \epsilon_3 t z \end{aligned}$$

$$ds^2 = \frac{1}{z^2}[-dt^2 + dz^2]$$

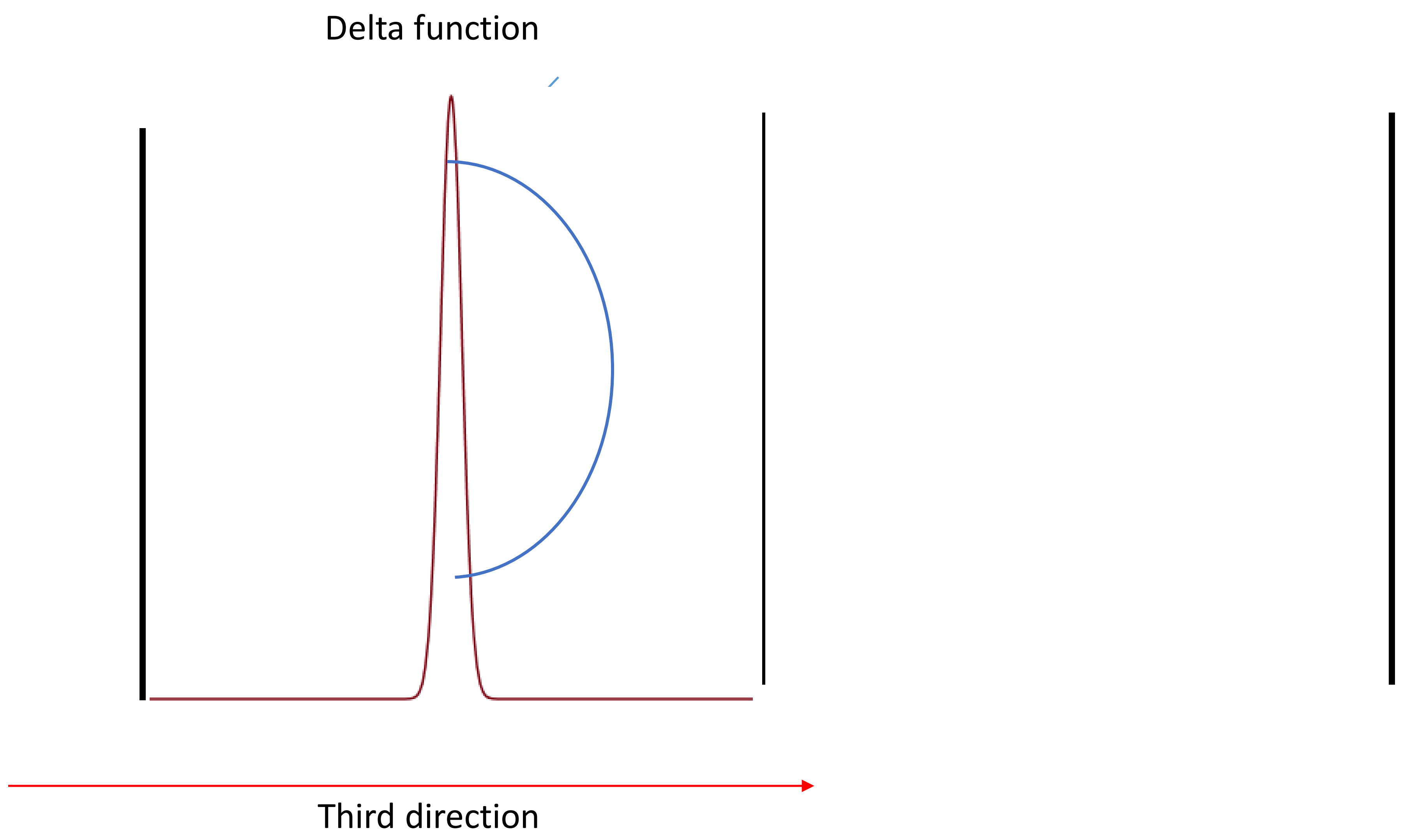
A Three dimensional view

(S.R. Das, A. Jevicki and K. Suzuki : JHEP 1709 (2017) 17
(S.R. Das, A. Ghosh, A. Jevicki and K. Suzuki : JHEP 1802 (2018) 162)

- It turns out that the infinite tower of fields can be interpreted as the KK tower of a Horava-Witten compactification of a 3 dimensional theory in a fixed background.
- For q=4 the background is $ds^2 = \frac{1}{z^2}[-dt^2 + dz^2] + (1 + \frac{a}{z})^2 dy^2 \qquad a \sim 1/J$
- The third direction is an interval S^1/Z_2 with Dirichlet boundary conditions
- There is a single scalar field which is subject to a delta function potential

$$S = \int dz dt \left[(\partial_t \phi)^2 - (\partial_z \phi)^2 + \frac{1}{4z^2} - \frac{1}{z^2}\{(\partial_y \phi)^2 + V(y)\phi^2\} \right] \qquad V(y) = V\delta(y)$$

Schrodinger problem in an infinite well with a delta function in the middle


Delta function
Third direction

- This reproduces the spectrum exactly.
- A rather non-standard three dimensional propagator between points which lie at y=0 has an exact agreement with the SYK bilocal propagator – including the enhanced contribution of the h=2 mode.
- Here the non-trivial residues which appear in the SYK answer come from non-trivial wavefunctions in the 3rd direction.
- This generalizes rather nontrivially to arbitrary q. The metric is now non-trivial

$$ds^2 = |x|^{\frac{4}{q}-1}\left[\frac{-dt^2+dz^2}{z^2}+\frac{dx^2}{4|x|(1-|x|)}\right]$$

- And there is a nontrivial potential

$$V(x) = \frac{1}{|x|^{\frac{4}{q}-1}}\left[4\left(\frac{1}{q}-\frac{1}{4}\right)^2 + m_0^2 + \frac{2V}{J(x)}\left(1-\frac{2}{q}\right)\delta(x)\right] \qquad J(x) = \frac{|x|^{\frac{2}{q}-1}}{2\sqrt{1-|x|}}$$

- It is significant that not just the spectrum, but the propagator is exactly reproduced by this three dimensional model.
- However there is no reason to expect that the theory has local interactions in these three dimensions – we should view this 3d picture as an unpacking of the infinite number of modes.

The Dual Space-time

- The bi-local space indeed provides a realization of AdS_2 - or $AdS_2 \times I$
- However

1. The wavefunctions which appear in the propagator are not the usual AdS_2 wavefunctions. The latter are Bessel functions with real positive order. May be possible to interpret in terms of dS_2
2. More significantly – we are actually working with Euclidean SYK model – in fact the bilocal propagator *does not have a factor of* i which should be there in Lorentzian signature.

   One would expect that the dual theory should have Euclidean signature as well.

- We will now argue that the bilocal fields are related to fields in $\mathrm{EAdS}_2 \times I$ by an integral transform. AdS_2
- The metric on the bilocal space can be written as

$$ds^2 = \frac{1}{\eta^2}[-dt^2 + d\eta^2] \qquad t \equiv \frac{t_1 + t_2}{2} \qquad \eta \equiv \frac{t_1 - t_2}{2}$$

EAdS_2

- The EAdS_2 metric will be taken as

$$ds^2 = \frac{d\tau^2 + dz^2}{z^2}$$

- The corresponding symmetry generators are

$$\hat{D}_{1+2} = t_1 p_1 + t_2 p_2\,, \qquad \hat{P}_{1+2} = -p_1 - p_2\,, \qquad \hat{K}_{1+2} = -t_1^2 p_1 - t_2^2 p_2$$

$$\hat{D}_{\mathrm{EAdS}} = \tau p_\tau + z p_z\,, \qquad \hat{P}_{\mathrm{EAdS}} = -p_\tau\,, \qquad \hat{K}_{\mathrm{EAdS}} = (z^2 - \tau^2)\, p_\tau - 2\tau z\, p_z$$

- Is there a canonical transformation which takes one to the other ?
- A priori this is an *overdetermined* problem. However the answer is YES.

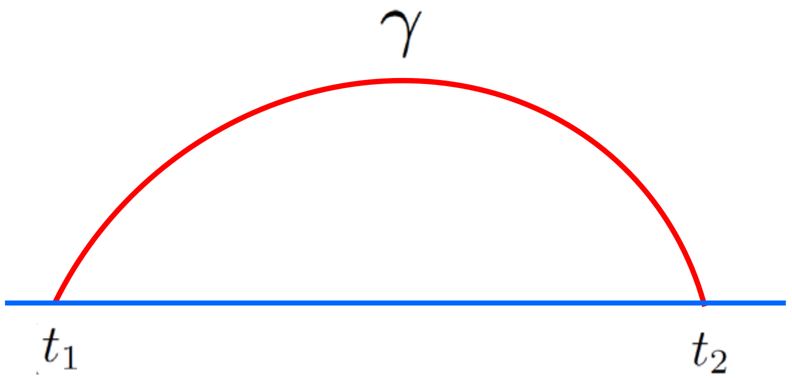
$$\tau = \frac{t_1 p_1 - t_2 p_2}{p_1 - p_2}, \quad p_\tau = p_1 + p_2, \quad z^2 = -\left(\frac{t_1 - t_2}{p_1 - p_2}\right)^2 p_1 p_2, \quad p_z^2 = -4 p_1 p_2$$

- This can be implemented as transformations on momentum space fields.
- It turns out that one of these – written in position space – is the X-ray (or radon) transformation acting on a function on EAdS_2

$$[\mathcal{R}f](\eta, t) = 2\eta \int_{t-\eta}^{t+\eta} d\tau \int_0^\infty \frac{dz}{z} \delta\left(\eta^2 - (\tau - t)^2 - z^2\right) f(\tau, z)$$

- In fact the bilocal space is somewhat similar to kinematic space defined by Czech et.al. for 1+1 dimensional CFT – however this is not on a single time slice.
- In the former context this has been proposed as a way to reconstruct the bulk (*Czech, Lamprou, McCandlish, Mosk and Sully*; *Bhowmick, Ray and Sen*). De Sitter makes a natural appearance there – as it does in related treatments *(de Boer, Haehl, Heller, Myers, Niemann)*

- The Radon transform of a function on is the integral of the function evaluated on a geodesic with end-points on the boundary.



$$\mathcal{R}f(\tau, z) = \int_\gamma ds f(\gamma)$$

$$t \equiv \frac{t_1 + t_2}{2} \qquad \eta . \equiv \frac{t_1 - t_2}{2}$$

- The radon transform intertwines between generators of $\mathrm{EAdS_2}$ isometries and those of $\mathrm{AdS_2}$

$$\nabla^2_{LAdS_2}\mathcal{R} = \mathcal{R}\nabla^2_{EAdS_2}$$

- A test of this proposal would be to check if the correct eigenfunctions of $\nabla^2_{EAdS_2}$ transform into the functions Z_ν

- Remarkably this is precisely what happens.
- The normalizable eigenfunctions of ∇^2_{EAdS}

$$\phi_{EAdS}(\tau, z) = z^{\frac{1}{2}} e^{-i\omega\tau} K_\nu(\omega z) \qquad \nu = ir$$

$$[\mathcal{R}\phi_{EAdS}](t,\eta) = -\frac{\pi^{3/2}}{\sin(\pi\nu)} \frac{\Gamma(\frac{1}{4}+\frac{\nu}{2})}{\Gamma(\frac{1}{4}+\frac{\nu}{2})} \eta^{\frac{1}{2}} e^{-i\omega t} \boxed{\left[J_\nu(\omega\eta) + \frac{\tan(\frac{\pi\nu}{2})+1}{\tan(\frac{\pi\nu}{2})-1} J_{-\nu}(\omega\eta) \right]} \qquad \nu = ir$$

$$Z_\nu(x) = J_\nu(x) + \xi_\nu J_{-\nu}(x)$$

- For the discrete series

$$\nu = \nu_n = 2n + 3/2$$

$$\mathcal{R}[\alpha'_{\nu_n} z^{1/2} e^{-ik\tau} I_{\nu_n}(|k|z)] = (2\nu_n\eta)^{1/2} e^{-ikx} J_{\nu_n}(|k|\eta)$$

$$\alpha'_{\nu_n} = \left(\frac{2\nu_n}{\pi}\right)^{\frac{1}{2}} \frac{\Gamma(\frac{3}{4}+\frac{\nu_n}{2})}{\Gamma(\frac{1}{4}+\frac{\nu_n}{2})}$$

- One might have thought that this would mean that the inverse Radon will transform the SYK propagator into the standard Euclidean propagator.
- This is *almost correct*, but not quite.

- Inverse radon transform on the SYK propagator

$$G(\tau, z; \tau', z')$$
$$= \frac{|zz'|^{\frac{1}{2}}}{2\pi J} \int_{-\infty}^{\infty} d\omega\, e^{-i\omega(\tau-\tau')} \left\{ \sum_{m=0}^{\infty} \frac{\Gamma(\frac{3}{4}+\frac{p_m}{2})\Gamma(\frac{3}{4}-\frac{p_m}{2})}{\Gamma(\frac{1}{4}+\frac{p_m}{2})\Gamma(\frac{1}{4}-\frac{p_m}{2})} \frac{p_m}{\widetilde{g}'(p_m)} K_{p_m}(|\omega|z^>) I_{p_m}(|\omega|z^<) \right.$$
$$\left. + \sum_{n=0}^{\infty} \frac{\Gamma^2(\frac{3}{4}+\frac{\nu_n}{2})}{\Gamma^2(\frac{1}{4}+\frac{\nu_n}{2})} \left(\frac{\nu_n}{\widetilde{g}(\nu_n)-1} \right) I_{\nu_n}(|\omega|z^<) \Big[2I_{\nu_n}(|\omega|z^>) - I_{-\nu_n}(|\omega|z^>) \Big] \right\}. \quad (4.13)$$

- Inverse radon transform on the SYK propagator

$$G(\tau, z; \tau', z')$$
$$= \frac{|zz'|^{\frac{1}{2}}}{2\pi J} \int_{-\infty}^{\infty} d\omega\, e^{-i\omega(\tau-\tau')} \left\{ \sum_{m=0}^{\infty} \frac{\Gamma(\frac{3}{4}+\frac{p_m}{2})\Gamma(\frac{3}{4}-\frac{p_m}{2})}{\Gamma(\frac{1}{4}+\frac{p_m}{2})\Gamma(\frac{1}{4}-\frac{p_m}{2})} \boxed{\frac{p_m}{\widetilde{g}'(p_m)}} K_{p_m}(|\omega|z^>) I_{p_m}(|\omega|z^<) \right.$$
$$\left. + \sum_{n=0}^{\infty} \frac{\Gamma^2(\frac{3}{4}+\frac{\nu_n}{2})}{\Gamma^2(\frac{1}{4}+\frac{\nu_n}{2})} \left(\frac{\nu_n}{\widetilde{g}(\nu_n)-1} \right) I_{\nu_n}(|\omega|z^<) \Big[2I_{\nu_n}(|\omega|z^>) - I_{-\nu_n}(|\omega|z^>) \Big] \right\}. \quad (4.13)$$

Residues of the poles – wavefunctions in the 3d picture

- Inverse radon transform on the SYK propagator

$$
\begin{aligned}
&G(\tau, z; \tau', z') \\
&= \frac{|zz'|^{\frac{1}{2}}}{2\pi J} \int_{-\infty}^{\infty} d\omega\, e^{-i\omega(\tau-\tau')} \left\{ \sum_{m=0}^{\infty} \frac{\Gamma(\frac{3}{4}+\frac{p_m}{2})\Gamma(\frac{3}{4}-\frac{p_m}{2})}{\Gamma(\frac{1}{4}+\frac{p_m}{2})\Gamma(\frac{1}{4}-\frac{p_m}{2})} \boxed{\frac{p_m}{\widetilde{g}'(p_m)}} \boxed{K_{p_m}(|\omega|z^>) I_{p_m}(|\omega|z^<)} \right. \\
&\quad \left. + \sum_{n=0}^{\infty} \frac{\Gamma^2(\frac{3}{4}+\frac{\nu_n}{2})}{\Gamma^2(\frac{1}{4}+\frac{\nu_n}{2})} \left(\frac{\nu_n}{\widetilde{g}(\nu_n)-1}\right) I_{\nu_n}(|\omega|z^<) \left[2I_{\nu_n}(|\omega|z^>) - I_{-\nu_n}(|\omega|z^>)\right] \right\}. \qquad (4.13)
\end{aligned}
$$

Residues of the poles – wavefunctions in the 3d picture

Usual Euclidean propagator for a field

- Inverse radon transform on the SYK propagator

$$G(\tau, z; \tau', z') = \frac{|zz'|^{\frac{1}{2}}}{2\pi J} \int_{-\infty}^{\infty} d\omega \, e^{-i\omega(\tau-\tau')} \left\{ \sum_{m=0}^{\infty} \frac{\Gamma(\frac{3}{4}+\frac{p_m}{2})\Gamma(\frac{3}{4}-\frac{p_m}{2})}{\Gamma(\frac{1}{4}+\frac{p_m}{2})\Gamma(\frac{1}{4}-\frac{p_m}{2})} \frac{p_m}{\widetilde{g}'(p_m)} K_{p_m}(|\omega|z^>) I_{p_m}(|\omega|z^<) + \sum_{n=0}^{\infty} \frac{\Gamma^2(\frac{3}{4}+\frac{\nu_n}{2})}{\Gamma^2(\frac{1}{4}+\frac{\nu_n}{2})} \left(\frac{\nu_n}{\widetilde{g}(\nu_n)-1}\right) I_{\nu_n}(|\omega|z^<) \left[2I_{\nu_n}(|\omega|z^>) - I_{-\nu_n}(|\omega|z^>)\right] \right\}. \quad (4.13)$$

Residues of the poles – wavefunctions in the 3d picture

Usual Euclidean propagator for a field

Additional “Leg Pole” factors. In 3d interpretation another transformation in the 3rd direction

- Similar factors appear in the c=1 matrix model

- Inverse radon transform on the SYK propagator

$$G(\tau, z; \tau', z') = \frac{|zz'|^{\frac{1}{2}}}{2\pi J} \int_{-\infty}^{\infty} d\omega\, e^{-i\omega(\tau-\tau')} \left\{ \sum_{m=0}^{\infty} \frac{\Gamma(\frac{3}{4}+\frac{p_m}{2})\Gamma(\frac{3}{4}-\frac{p_m}{2})}{\Gamma(\frac{1}{4}+\frac{p_m}{2})\Gamma(\frac{1}{4}-\frac{p_m}{2})} \frac{p_m}{\widetilde{g}'(p_m)} K_{p_m}(|\omega|z^>) I_{p_m}(|\omega|z^<) \right.$$

$$\left. + \sum_{n=0}^{\infty} \frac{\Gamma^2(\frac{3}{4}+\frac{\nu_n}{2})}{\Gamma^2(\frac{1}{4}+\frac{\nu_n}{2})} \left(\frac{\nu_n}{\widetilde{g}(\nu_n) - 1} \right) I_{\nu_n}(|\omega|z^<) \left[2I_{\nu_n}(|\omega|z^>) - I_{-\nu_n}(|\omega|z^>) \right] \right\}. \quad (4.13)$$

?? ????? Discrete states

Finite Temperature

S.R.D, A. Ghosh, A. Jevicki and K. Suzuki – in progress

- So far we have dealt with the zero temperature theory.
- The finite temperature theory is described as usual by compactifying the Euclidean time – this may be done by performing a reparamterization

$$t = \tan(\frac{\pi\theta}{\beta})$$

- One would have thought that this would transform the metric on the bilocal space to the metric in AdS_2 Rindler – or a AdS_2 black hole. This is NOT what happens. Rather

$$-\frac{4dt_1 dt_2}{|t_1 - t_2|^2} \to \boxed{\frac{-dt^2 + d\rho^2}{\sin^2\rho}} \qquad \mathrm{FTSYK} \qquad \rho = \frac{\pi}{\beta}(\theta_1 - \theta_2) \quad t = \frac{\pi}{\beta}(\theta_1 + \theta_2)$$

- As opposed to the AdS_2 black hole metric

$$\boxed{\frac{-dt^2 + d\rho^2}{\sinh^2\rho}}$$

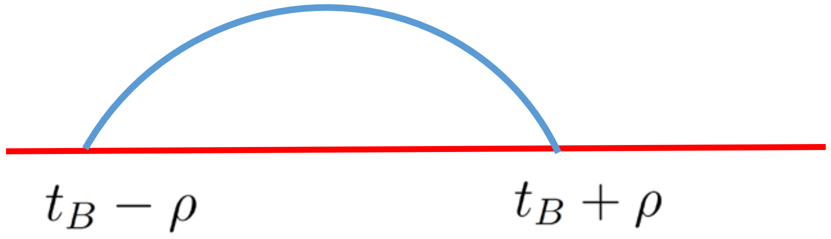
- The FTSYK IS global AdS_2 with identifications imposed by periodicities of the angles.

- The Euclidean continuation of this is

$$ds_B^2 = \frac{d\tau_B^2 + d\xi^2}{\sinh^2 \xi} \qquad\qquad -\pi \leq \tau_B \leq \pi$$

- This is actually diffeomorphic to the whole upper half plane (just as in flat space).
- In these coordinates a geodesic which joins two points on the boundary is

$$\cosh \xi = \sec \rho \ \cos(t_B - \tau_B)$$



- The radon transform of a function is

$$[\mathcal{R}f](t_B, \rho) = \int_{t_B-\rho}^{t_B+\rho} d\tau_B \int \frac{d\xi}{\sinh \xi} \delta(\cosh \xi - \sec \rho \ \cos(t_B - \tau_B)) f(\tau_B, \xi)$$

- This radon transform correctly intertwines the laplacians on Euclidean BH and on the lorentzian metric which appears in the space of bilocals at finite temperature

$$\nabla^2_{FTSYK}\mathcal{R} = \mathcal{R}\nabla^2_{EBH_2}$$

- The solution of the eigenvalue problem for the finite temperature kernel is not known for arbitrary finite q.
- However they are known for $q = \infty$ *(Maldacena and Stanford)*. The operator which needs to be diagonalized is precisely ∇^2_{FTSYK}

$$\left[\sin^2\rho(-\partial_t^2 + \partial_\rho^2) + \frac{1}{4}\right]\psi = \nu^2\psi$$

- The solutions which satisfy the periodicity conditions and regularity are (even n)

$$\psi_{n,\nu}(t,\rho) = d_{n,\nu}e^{-int}(\sin\rho)^{\nu+\frac{1}{2}}\,{}_2F_1[\frac{1}{2}(\frac{1}{2}+\nu-n), \frac{1}{2}(\frac{1}{2}+\nu+n); \frac{1}{2}; \cos^2\rho]$$

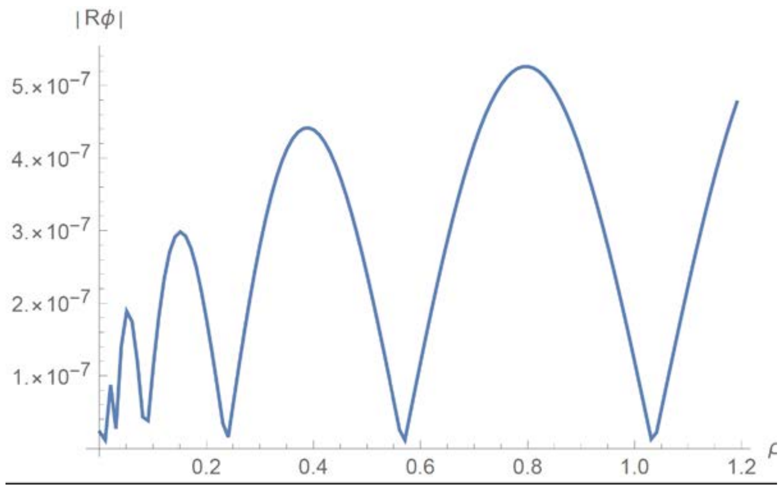
$$\nu = ir \qquad \nu = \frac{3}{2} + 2n$$

- In the zero temperature limit these reduce to the $Z_\nu(\omega z)$

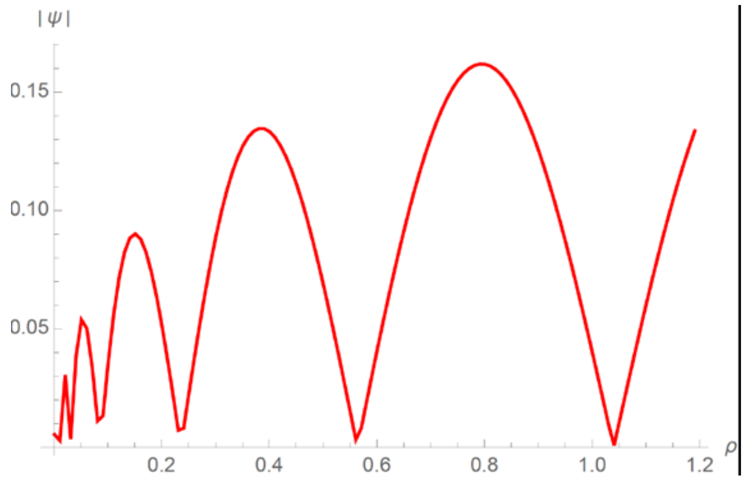
- On the other hand the eigenfunctions of $\nabla^2_{EBH_2}$ which are regular everywhere are, for $\nu = ir$ given by

$$\phi_{n,\nu} = c_{n,\nu} e^{-in\tau_B} (\sinh \xi)^{1/2} Q^{\nu}_{|n|-\frac{1}{2}}(\cosh \xi) \qquad \nu = ir$$

- We have calculated the radon transform of this numerically for several values of r and n and compared them with the eigenfunctions of ∇^2_{FTSYK} with the same values



Radon Transform of Euclidean BH eigenfunction for r=3 and n=5



eigenfunction of ∇^2_{FTSYK} for r=3 and n=5

- The necessity of an integral transform implies that to obtain quantities in a Lorentzian black hole background one needs to analytically continue the time τ_B - not the time t_B of the SYK model.
- The difference possibly goes away at long time scales – we are investigating this in detail at the moment.

THANK YOU

- We do not have analytic expressions for the normalization factors – so we compared the ratios of these two quantities for different values of ρ

$n = 2, r = 3 \; (\nu = ir, h = 1/2 + \nu)$:

ρ_1	ρ_2	$\frac{[\mathcal{R}\phi_{n,\nu}](\rho_1)}{[\mathcal{R}\phi_{n,\nu}](\rho_2)}$	$\frac{\psi_{n,h}(\rho_1)}{\psi_{n,h}(\rho_2)}$
0.3	0.7	-0.725792	-0.738056
0.2	0.5	-2.21603	-2.44764
0.4	0.1	-1.11162	-1.14059
0.7	0.2	-4.70768	-4.422

$n = 5, r = 2.5 \; (\nu = ir, h = 1/2 + \nu)$:

ρ_1	ρ_2	$\frac{[\mathcal{R}\phi_{n,\nu}](\rho_1)}{[\mathcal{R}\phi_{n,\nu}](\rho_2)}$	$\frac{\psi_{n,h}(\rho_1)}{\psi_{n,h}(\rho_2)}$
0.3	0.7	-0.833928	-0.842058
0.1	0.6	0.890446	0.919449
0.4	0.1	-1.32968	-1.34681
0.7	0.2	-4.90662	-5.07062

- The agreement gets substantially better with higher accuracy.
- We can say with great likelihood that the radon transform of the regular eigenfunctions of $\nabla^2_{EBH_2}$ are indeed the correct eigenfunctions of ∇^2_{FTSYK}