Intertwined order and the hair of the black hole.

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Nederlandse Organisatie voor Wetenschappelijk Onderzoek

Many body/bit Hilbert space.

Two qubits: Hilbert space dimension $2^2 = 4$

$$|00\rangle, |01\rangle, |10\rangle, |11\rangle$$

Three qubits: Hilbert space dimension $2^3 = 8$

$$|000\rangle, |001\rangle, |010\rangle, |011\rangle, |100\rangle, |101\rangle, |110\rangle, |111\rangle,$$

Physical world 10^{23} "qubits": Hilbert space dimension $2^{10^{23}}$

$$|\Psi_n\rangle = \sum_{config.i} C_i^n |config.i\rangle$$

Overwhelming amount of quantum information.

"The classical condensates: from crystals to Fermi-liquids."

States of matter that we understand are short ranged entangled product!

$$\left|\Psi_0\left\{\Omega_i\right\}\right\rangle = \Pi_i \hat{X}_i^+\left(\Omega_i\right)\left|vac\right\rangle$$

- Crystals: put atoms in real space wave packets $X_i^+\left(R_i^0\right) \propto e^{(R_i^0 - r)\wedge 2 / \sigma^2} \psi^+(r)$

- Magnets: put spins in generalized coherent state

$$X_i^+\left(\vec{n}_i\right) \propto e^{i\varphi_i/2}\cos\left(\theta_i/2\right)c_{i\uparrow}^+ + e^{-i\varphi_i/2}\sin\left(\theta_i/2\right)c_{i\downarrow}^+$$

- Superconductors/superfluids: put bosons/Cooper pairs in coherent superposition

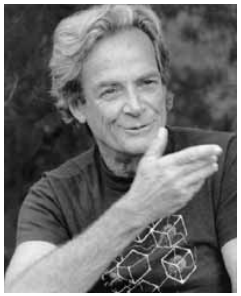
$$X_{k/i}{}^+ \propto u_k + v_k c_{k\uparrow}^+ c_{-k\downarrow}^+, \quad u_i + v_i e^{i\varphi_i} b_i^+$$

- Fermi gas/liquid: product state in momentum space (mod Pauli principle).

$$\left|\Psi_{FL}\right\rangle = \Pi_k^{k_F} c_k^+ \left|vac\right\rangle$$

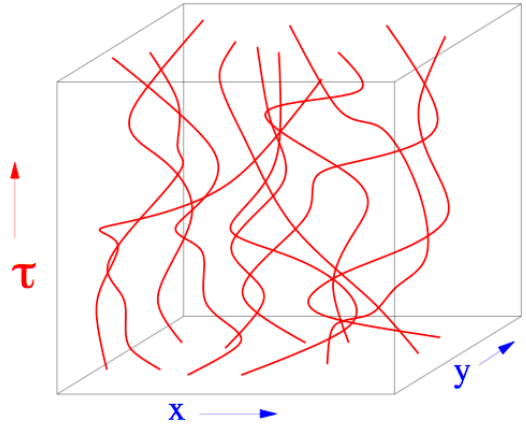
Fermions at a finite density: the sign problem.

Imaginary time first quantized path-integral formulation



$$\mathcal{Z} = \mathrm{Tr}\exp(-\beta\hat{\mathcal{H}}) = \int \mathrm{d}\mathbf{R}\rho(\mathbf{R},\mathbf{R};\beta)$$

$$\mathbf{R} = (\mathbf{r}_1,\ldots,\mathbf{r}_N) \in \mathbb{R}^{Nd}$$



$$\rho_{B/F}(\mathbf{R},\mathbf{R};\beta) = \frac{1}{N!}\sum_{\mathcal{P}}(\pm 1)^{\mathcal{P}}\rho_D(\mathbf{R},\mathcal{P}\mathbf{R};\beta)$$

$$= \frac{1}{N!}\sum_{\mathcal{P}}(\pm 1)^{\mathcal{P}}\int_{\mathbf{R}\to\mathcal{P}\mathbf{R}}\mathcal{D}\mathbf{R}(\tau)\exp\left\{-\frac{1}{\hbar}\int_0^{\hbar/T}\mathrm{d}\tau\left(\frac{m}{2}\dot{\mathbf{R}}^2(\tau)+V(\mathbf{R}(\tau))\right)\right\}$$

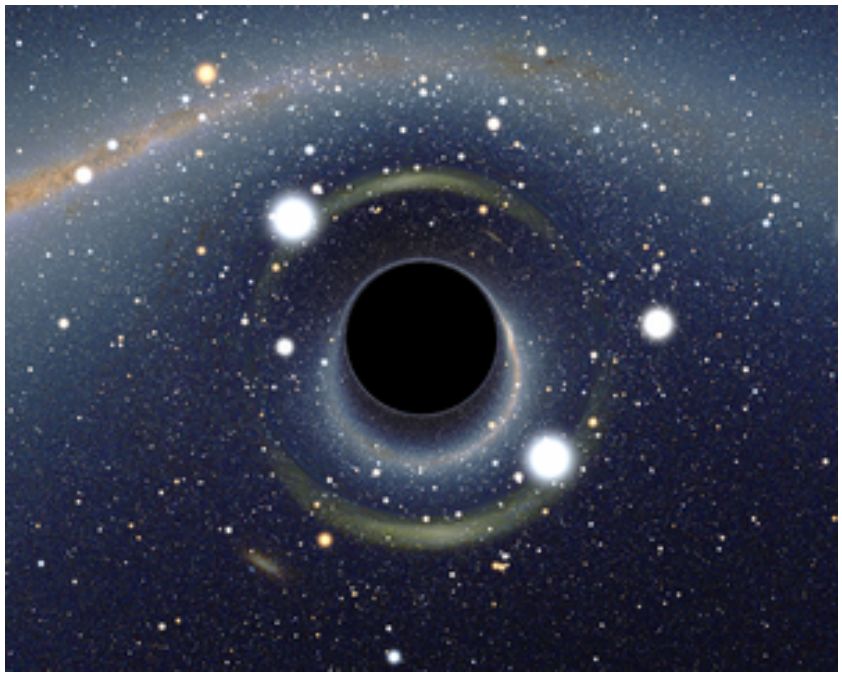
Boltzmannons or Bosons:

- integrand non-negative
- probability of equivalent classical system: (crosslinked) ringpolymers

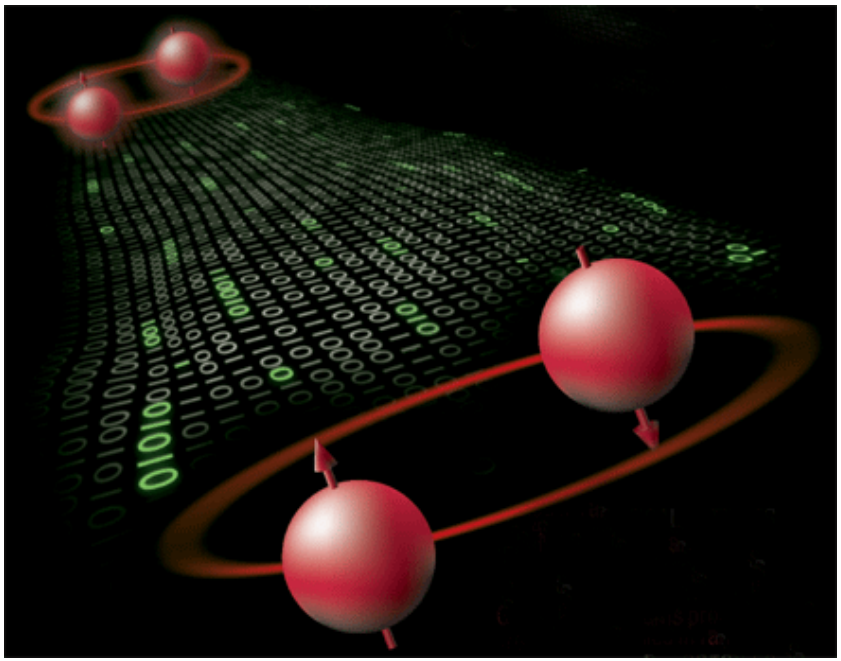
Fermions:

- negative Boltzmann weights
- non probablistic: NP-hard problem (Troyer, Wiese)!!!

Black holes as quantum computers !?

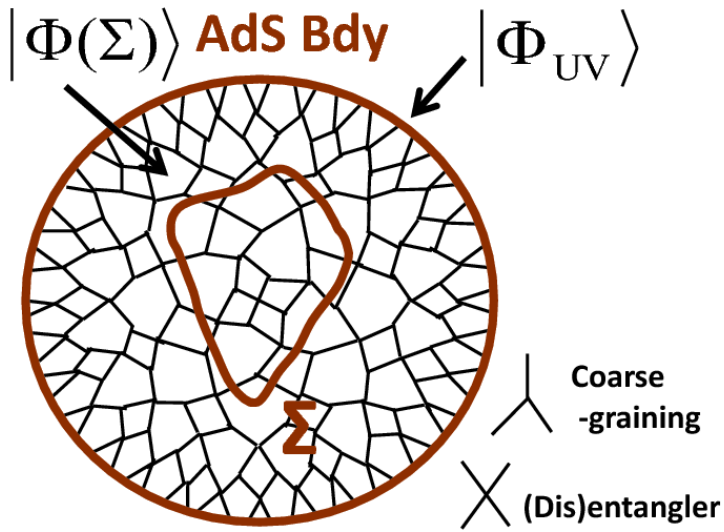


$\overset{?}{=}$



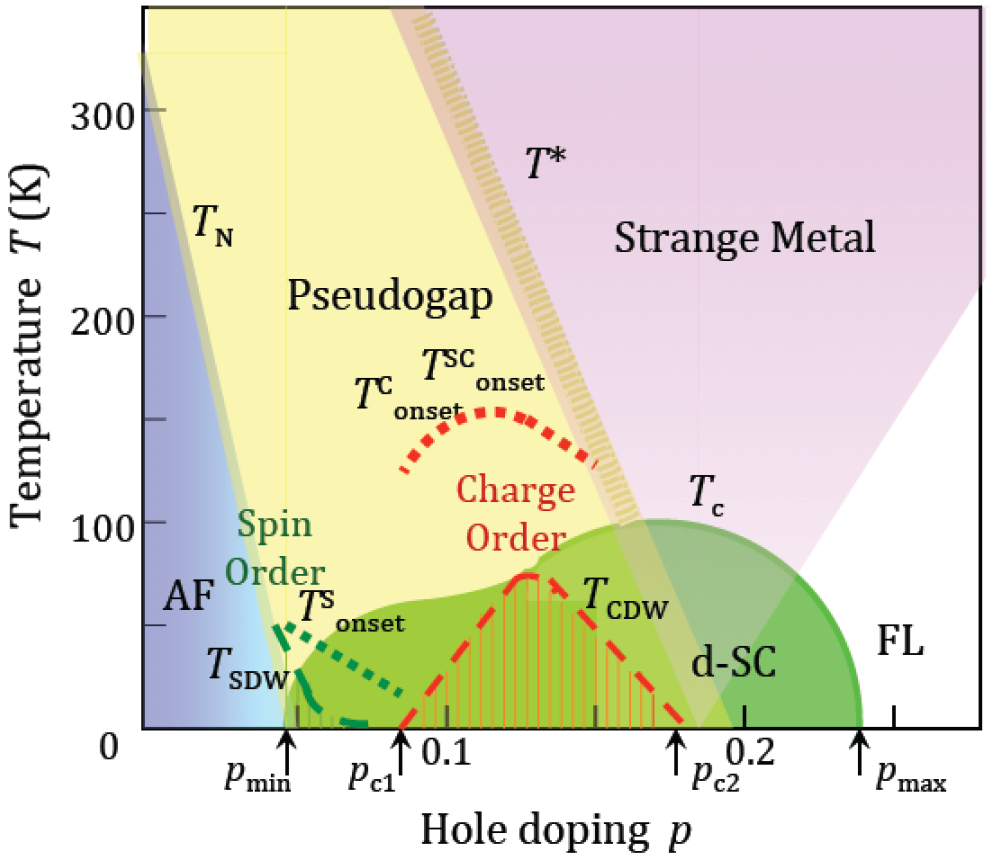
Holography: the mean field theory of "maximal entanglement"??.

Bulk geometry encodes for entanglement structure of the boundary state in terms of "quantum circuitry" (MERA: hierarchical tensor networks) !?



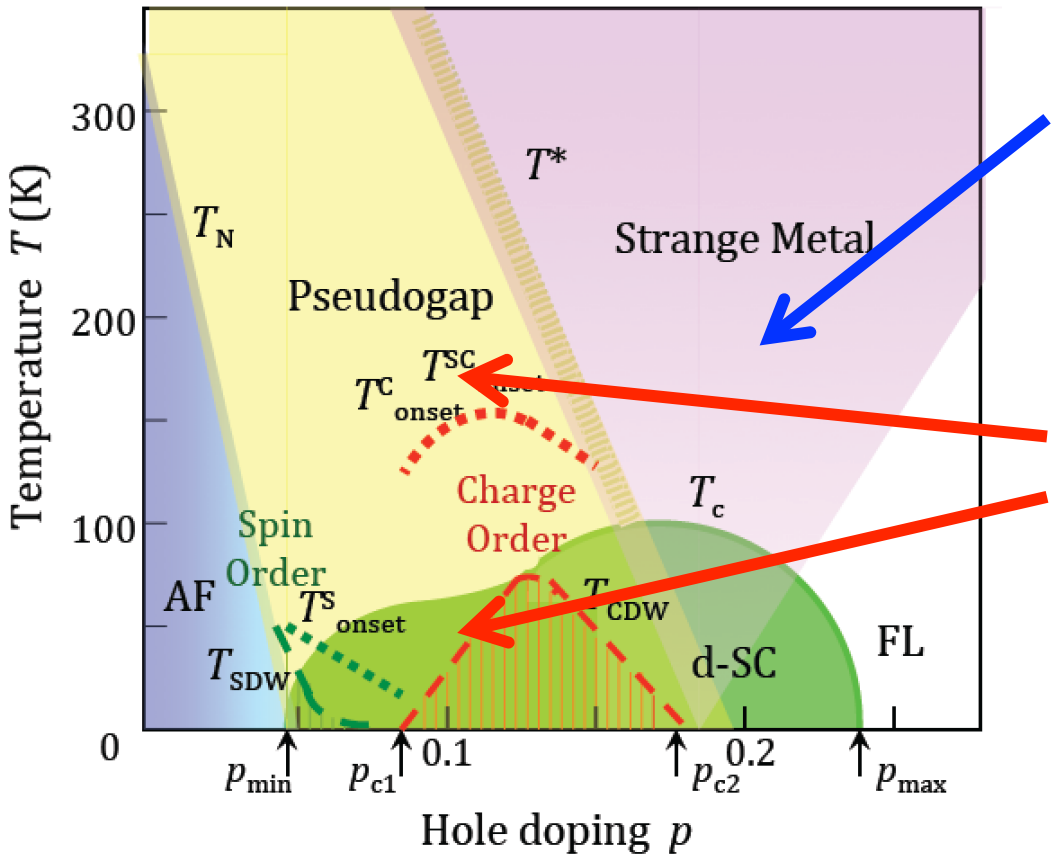
The "It from Qubit" program: Also "quantum error correction", "holeography", "kinematical space"…

A universe of mysteries ...



B. Keimer et al., Nature 518, 179 (2015)

A universe of mysteries ...



Holographic stronghold

But how about the "intertwined order"?

B. Keimer et al., Nature 518, 179 (2015)

Plan.

1. **Empiricisms: why are the condensed matter physicists obsessing about ordering phenomena in high Tc?**

2. **Holographic spontaneous symmetry breaking is remarkably "intertwined".**

3. **Doping the holographic Mott insulator.**

What is intertwined pseudogap order?

It is important: main focus point of the experimentalists during the last ten years.

It involves forms of symmetry breaking that are exotic, not seen elsewhere.

These compete but also work together (intertwined): like instruments in a symphony orchestra.

It is incomprehensible departing from conventional (=SRE product vacuum) mean field theory.

Next to high Tc superconductivity:

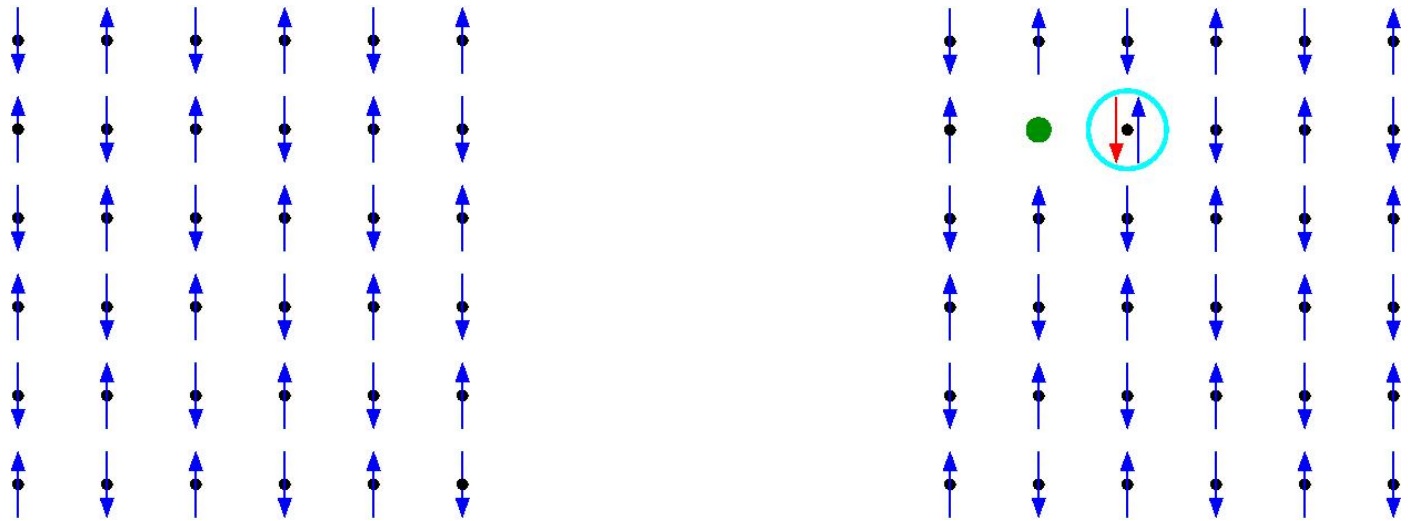
1. "Stripy" spin-charge order.

2. Pair density waves.

3. Loop currents.

4. Breaking of parity

The conventional Mott Insulator



Extreme Umklapp limit: tight binding = atomic orbitals.

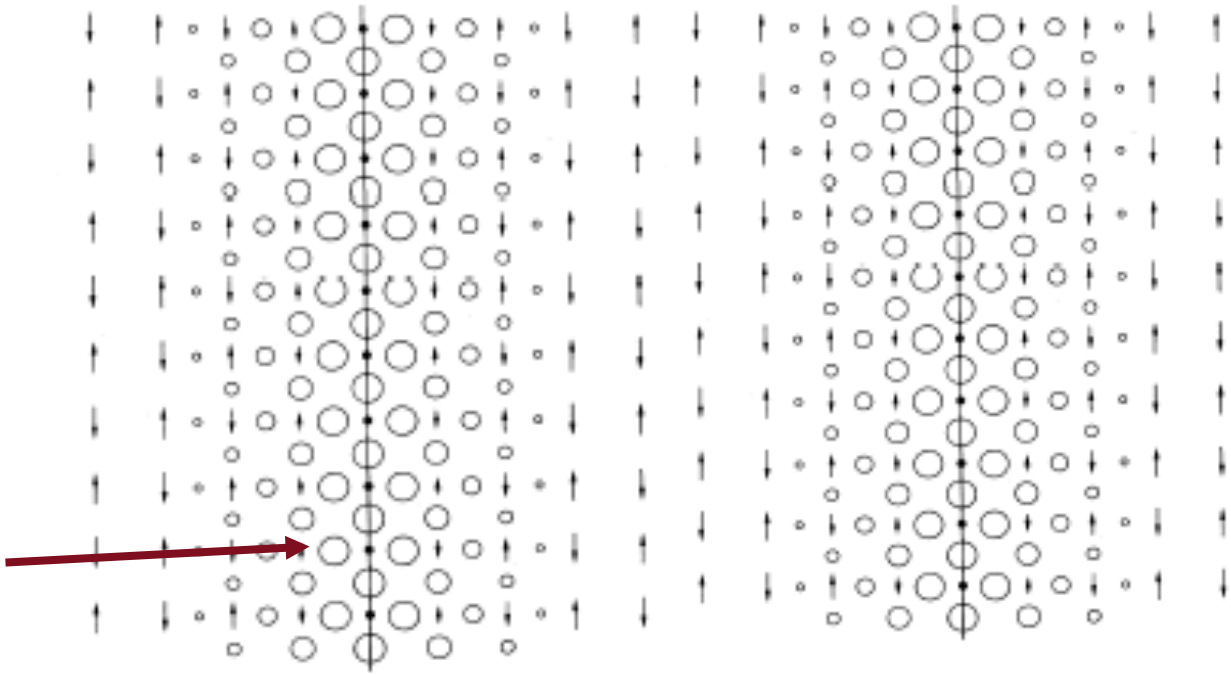
$$U = E_{ion}(n+1) + E_{ion}(n-1) - 2E_{ion}(n)$$

$$H_{Hubbard} = t\sum_{ij,\sigma} c^{\dagger}_{i\sigma} c_{j\sigma} + U\sum_{ij} n_{i\uparrow} n_{j\downarrow} \quad => \quad H_{eff} = \frac{t^2}{U}\sum_{ij} \vec{S}_i \cdot \vec{S}_j$$

Zaanen-Gunnarsson (1987)

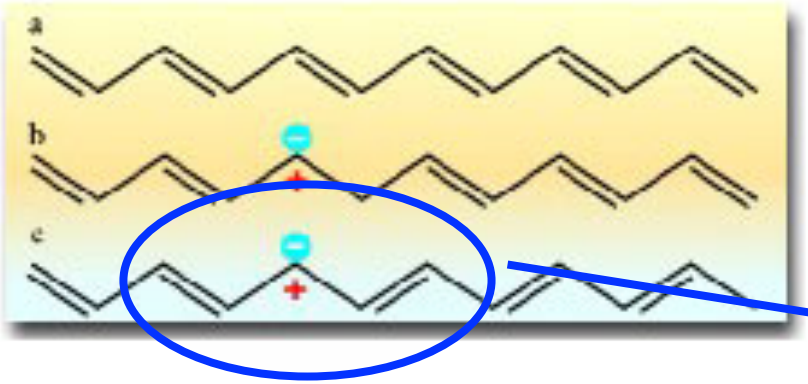
Unrestricted Hartree-Fock on 3-band model:

Hole density on oxygen

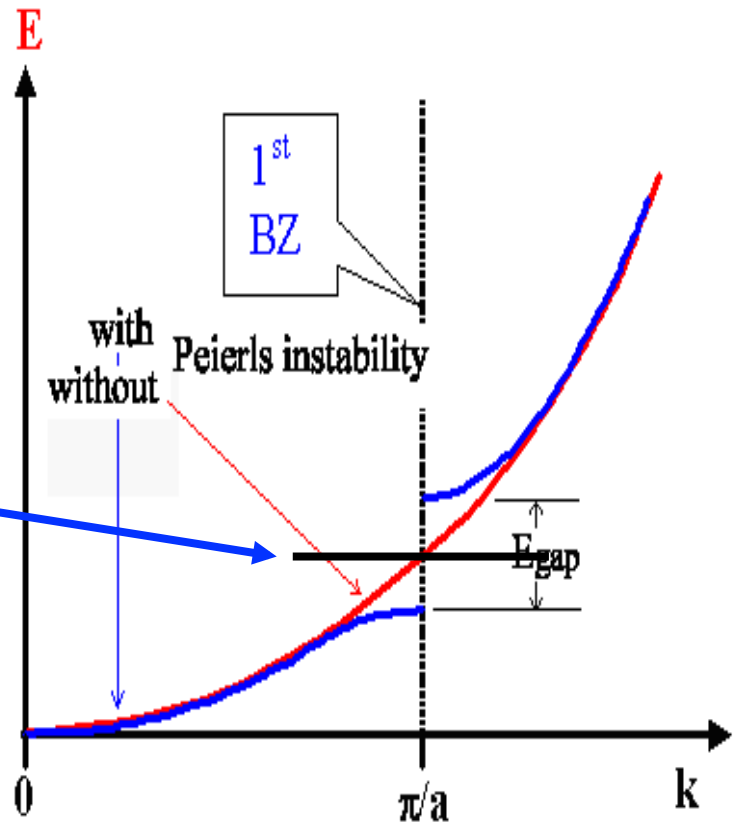


Su-Schrieffer-Heeger soliton = Jackiw-Rebbi zero mode.

polyacetelene



Domain 'wall'
('soliton')

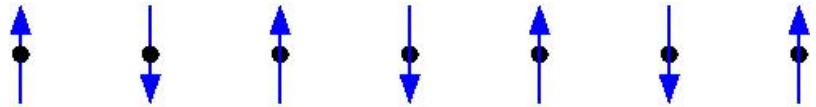


mid gap state!

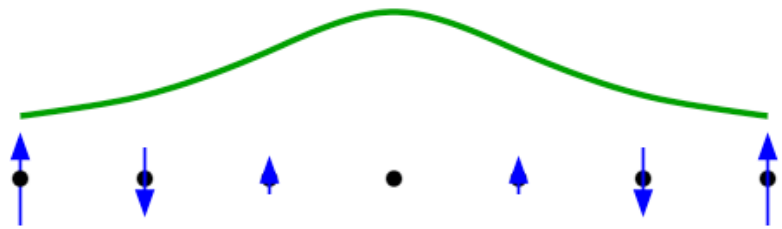
Large S holons

'Hartree-Fock' : $U\sum_i n_{i\uparrow}n_{i\downarrow} \approx U\sum_i \left[\langle n_{i\uparrow}\rangle n_{i\downarrow} + \langle n_{i\downarrow}\rangle n_{i\uparrow} - \langle n_{i\uparrow}\rangle\langle n_{i\downarrow}\rangle\right]$

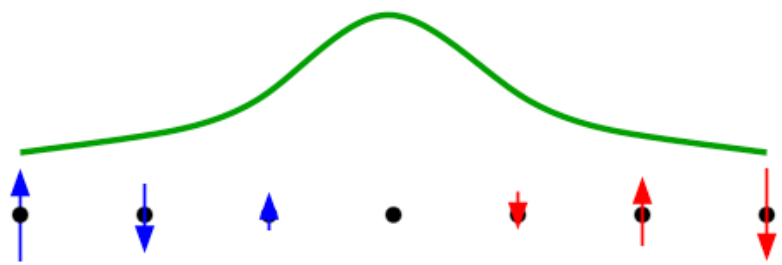
Uniform:

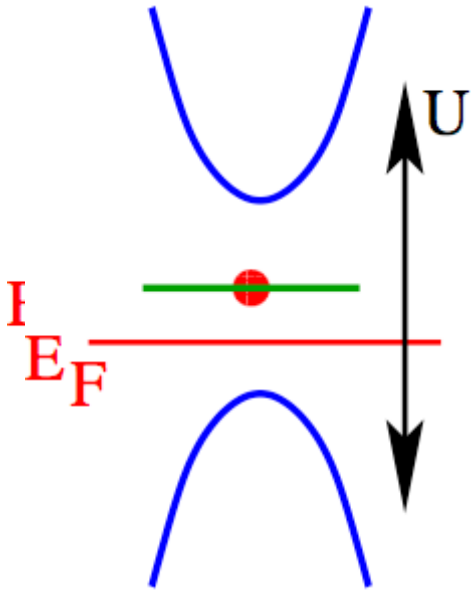


'Spin bag' :

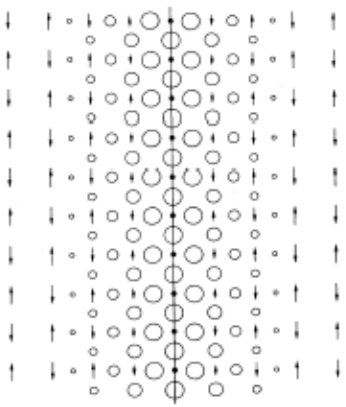


'Soliton' :

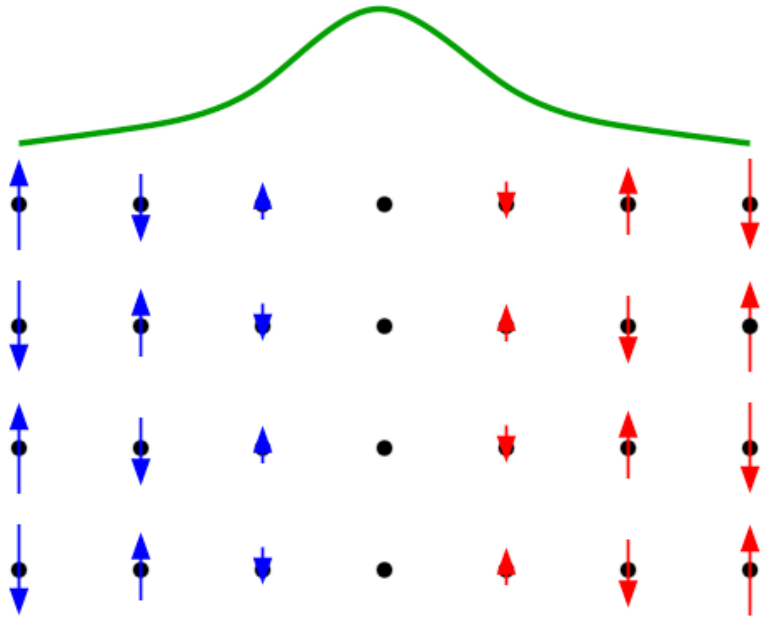




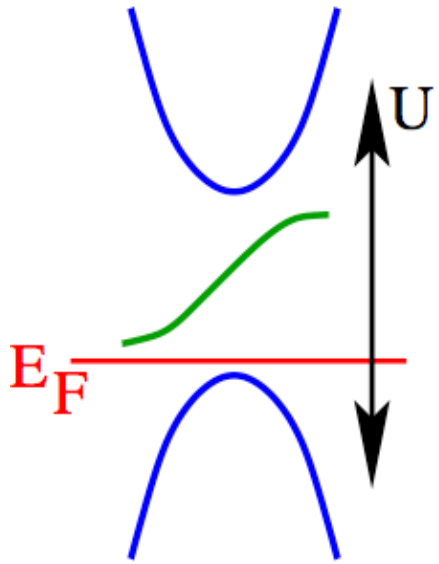
Zaanen-Gunnarsson stripes



'Holons on a row'

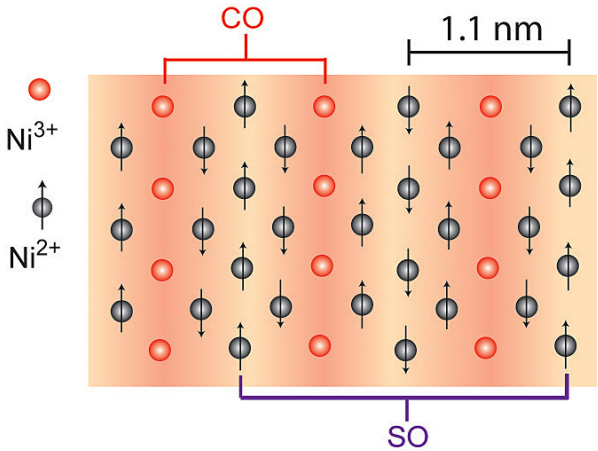


Mid-gap band

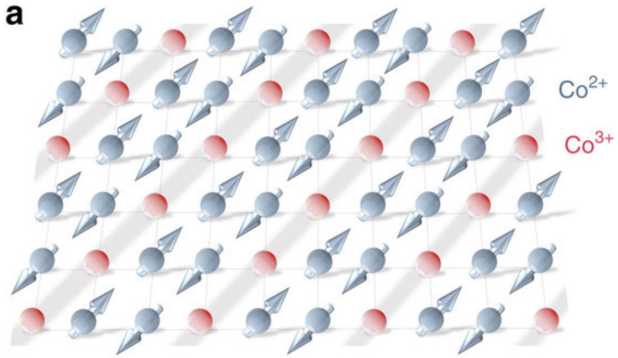


Hartree-Fock Stripes are ubiquitous in doped Mott Insulators ..

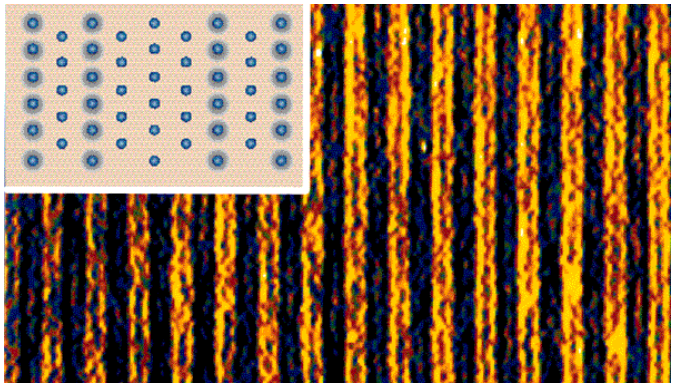
Nickelates



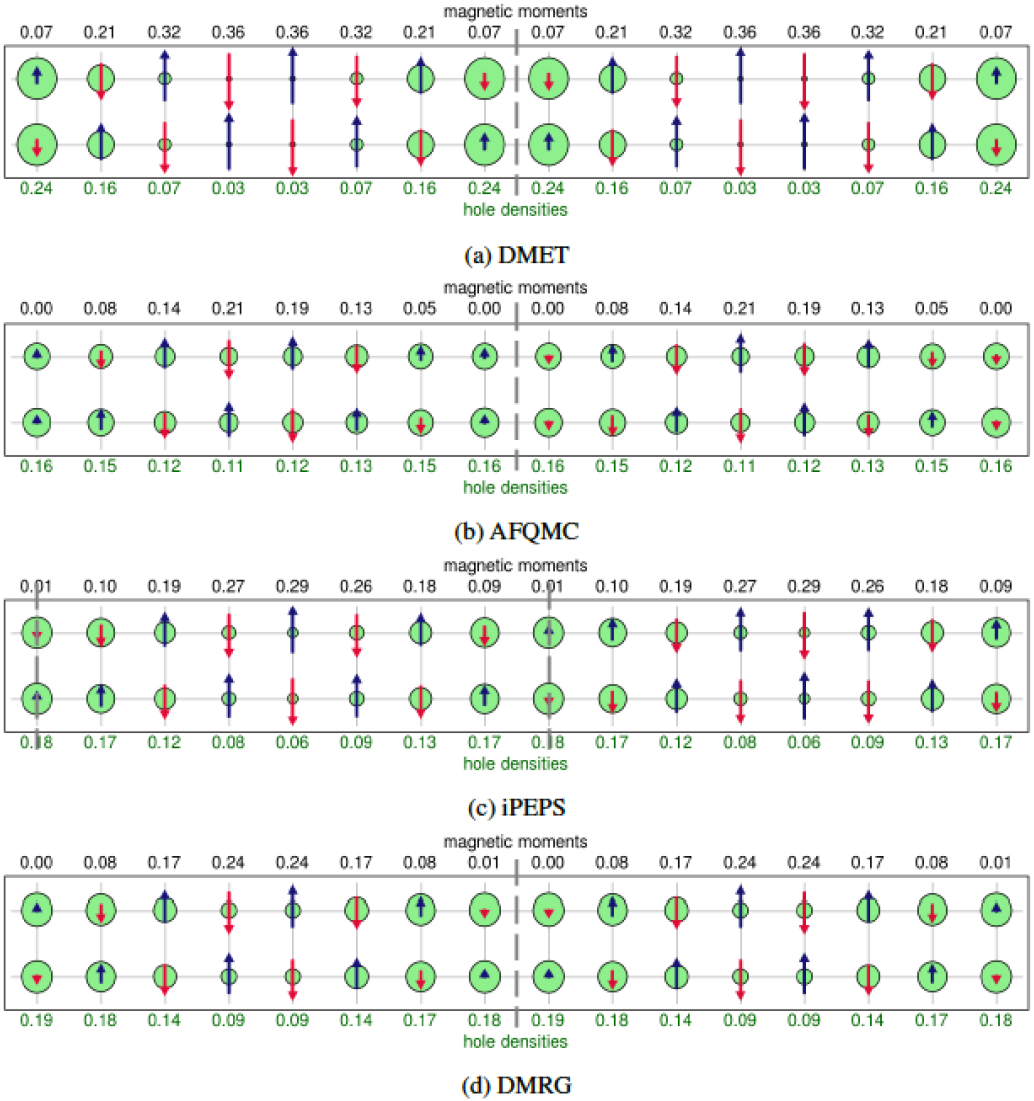
Cobaltates

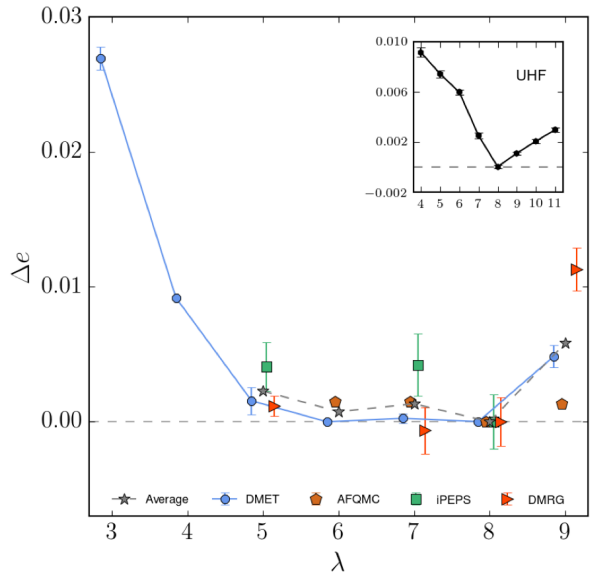


Manganites



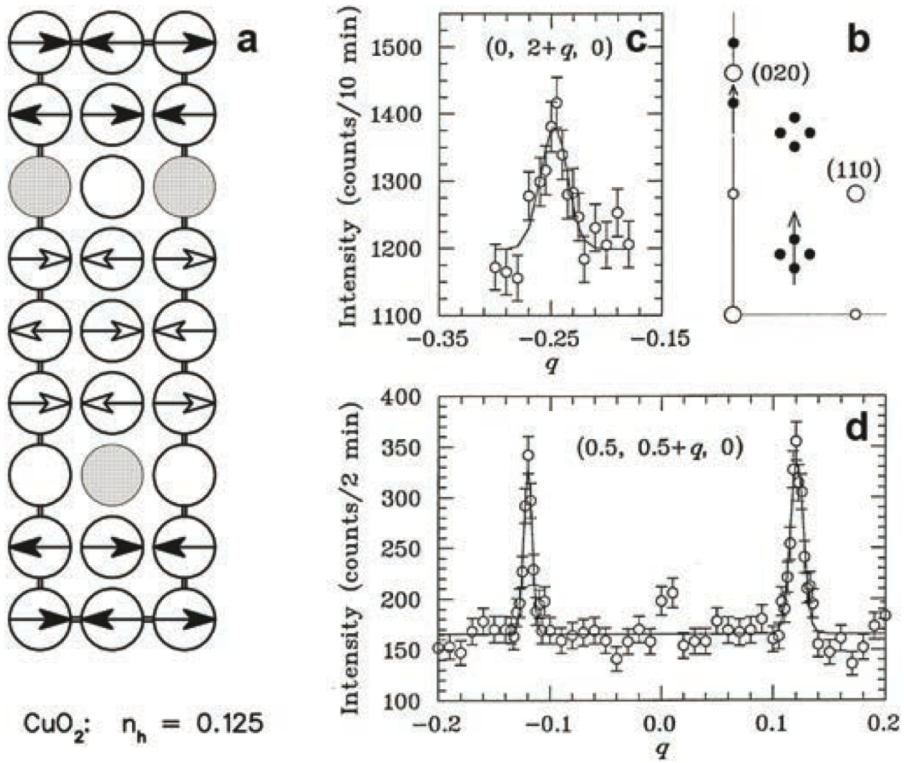
The BIG computational guns (2017).





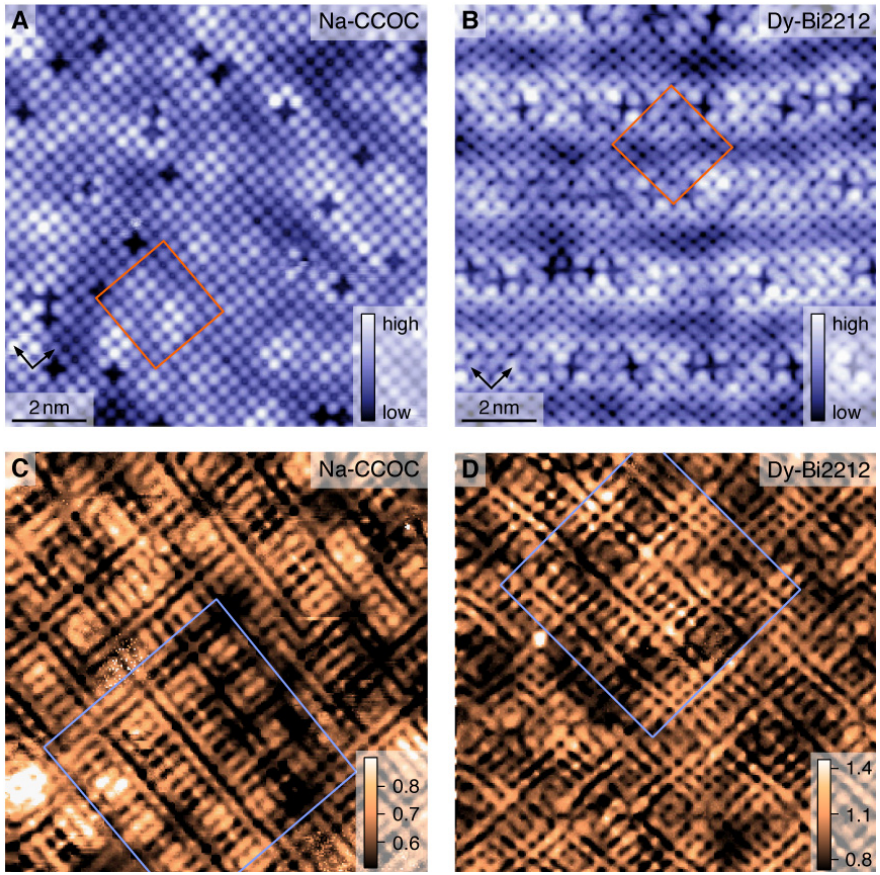
"Simons collaboration":
Hubbard model in 2D
x=1/8 U/t = 8
Science 358, 1155 (2017)

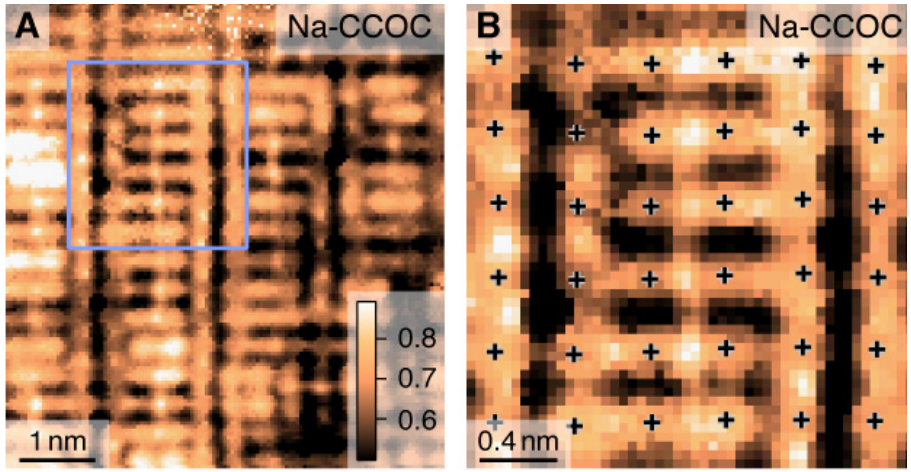
Static stripes in LTT 214.



Tranquada et al., 375, 561 (1995)

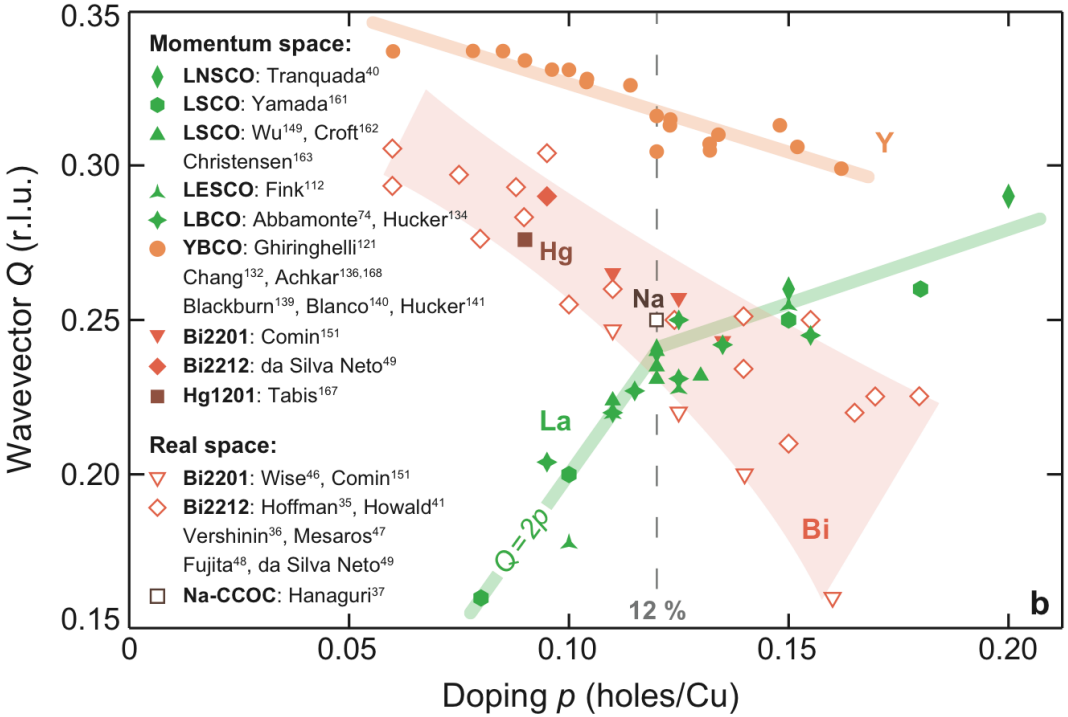
Probing rush hour in the electron world: scanning tunneling spectroscopy





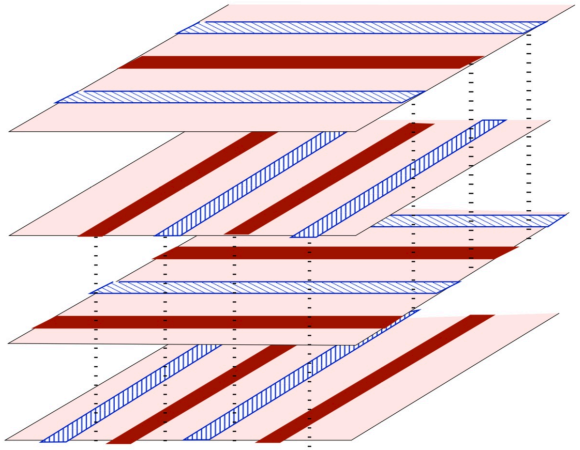
Seamus Davis et al.: Science, 2007

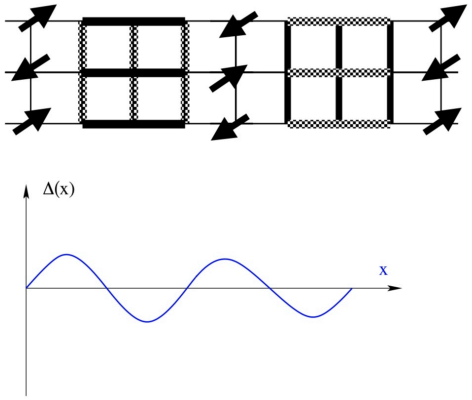
Doping dependence of charge order periodicity



Comin, Damascelli, An. Rev. Cond.Mat.Phys. 7, 369 (2016)

Pair density wave: evidences.





Tranquada et al. (2007): "spin stripes" in 214 are also finite momentum superconductors (complete decoupling along c-axis).

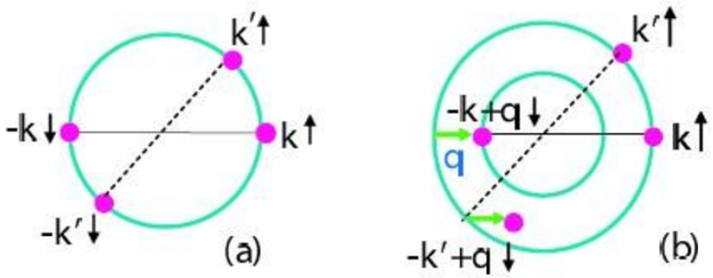
Seamus Davis et al. (2018): pair density wave in sync with charge order competes with uniform SC in 2212 (STS).

Pair density waves are troubling the SRE product state ...

Kivelson

- Never a ground state of bosons (local pairs): nodes cost energy!

- In fermionic mean field theory (BCS) it only has a chance to work in a magnetic field (FFLO).

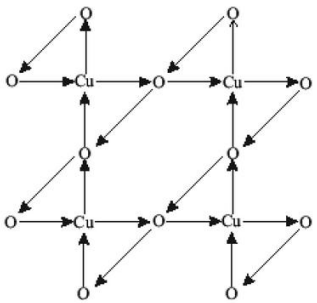


- No sign of it in "big machine" numerics.

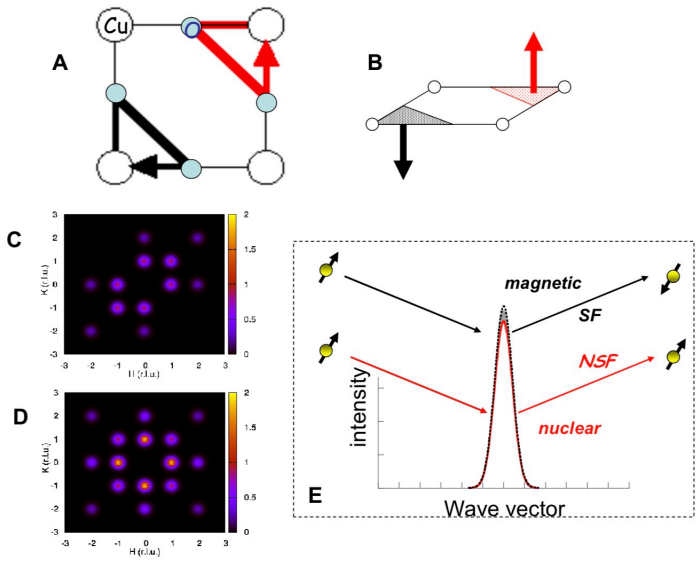
“Varma” loop currents.

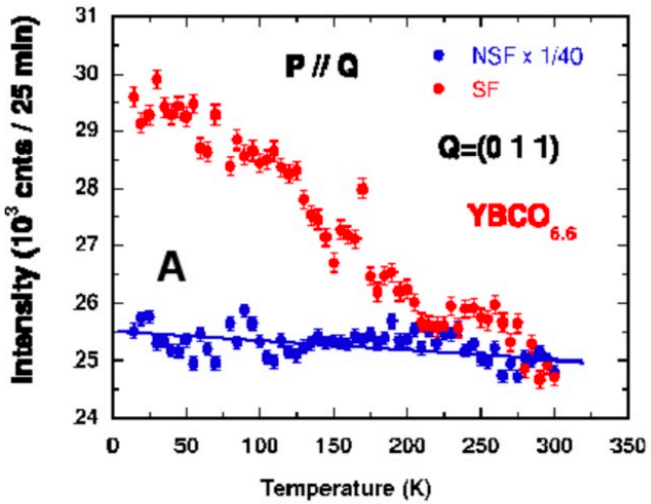
Bourges Varma

Spontaneous diamagnetic (p-h cannel) currents inside the unit cell (no precedent!):



Spin-flip neutron scattering evidence (Bourges et al.):





Loop currents are troubling the SRE product state ...

- Discovered in 1987 as mean-field solutions of weird slave theories (Affleck-Marston flux phase).

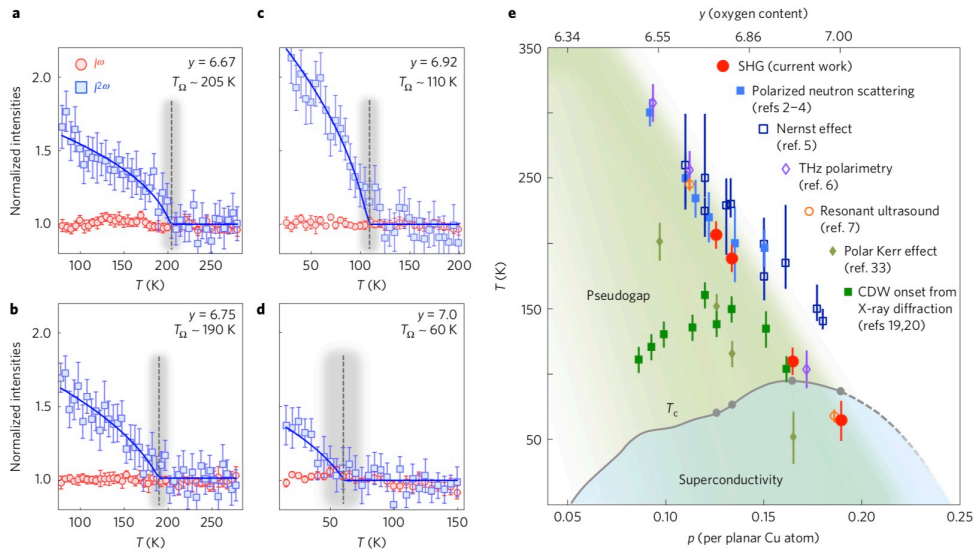
-"d-Density waves": natural particle-hole partners of particle-particle d-wave superconductors (Nayak et al.)

- Highly unnatural in fermionic mean field theory: never convincingly demonstrated for any model.

- No sign of it in "big machine" numerics.

Breaking parity at the pseudogap temperature .

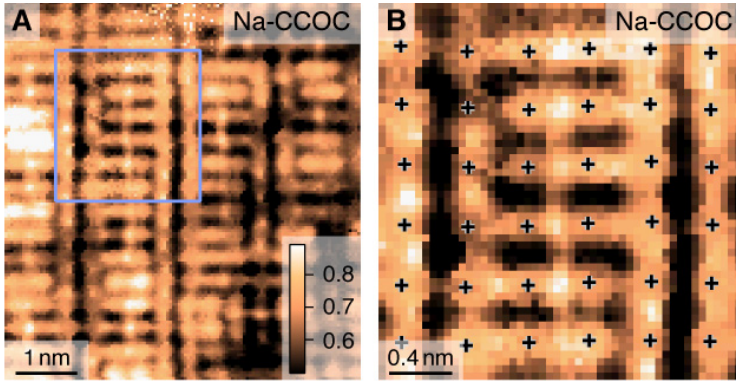
Second harmonic generation:



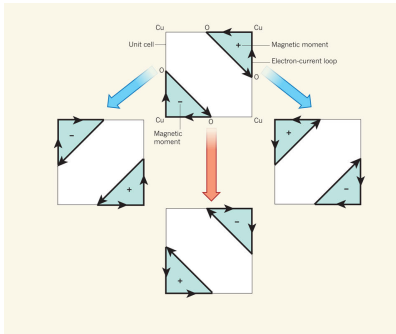
Breaking of parity as dominant order parameter: consistent with Varma currents.

L.Zhao et al., Nature Phys. 13, 250 (2016)

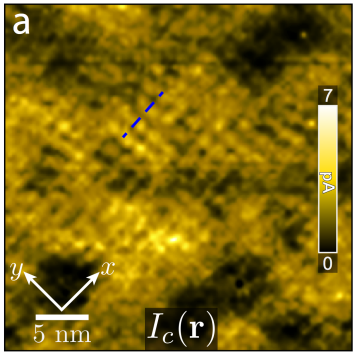
The intertwined Pseudogap order.



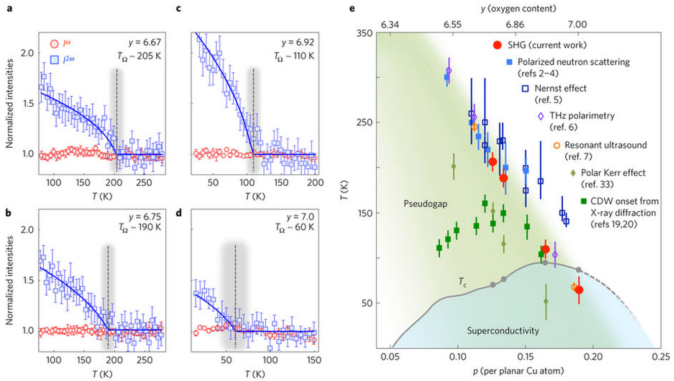
Stripy charge order



"Varma" loop currents



Pair density waves



Breaking of parity (second harm. generation)

Plan.

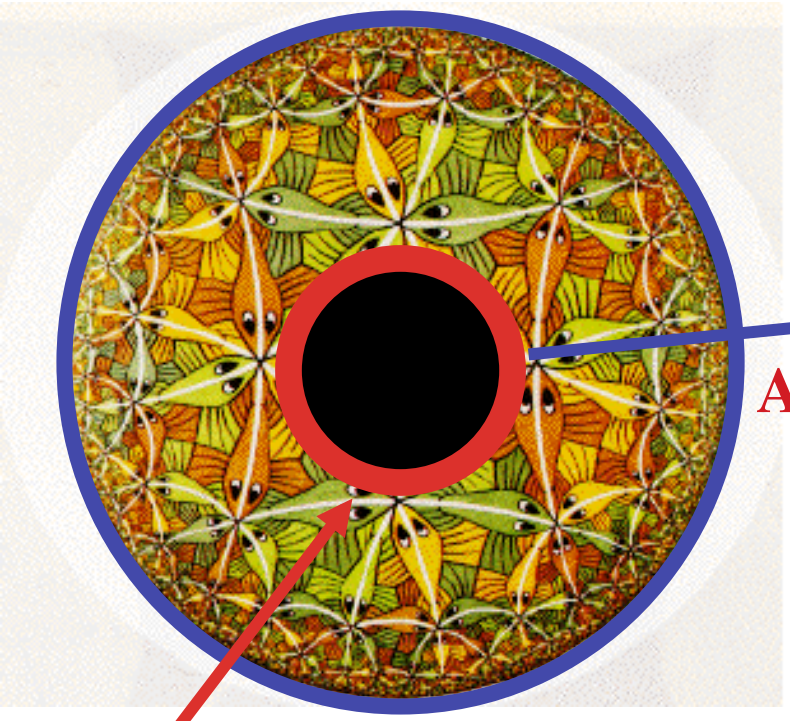
1. **Empiricisms: why are the condensed matter physicists obsessing about ordering phenomena in high Tc?**

2. **Holographic spontaneous symmetry breaking is remarkably "intertwined".**

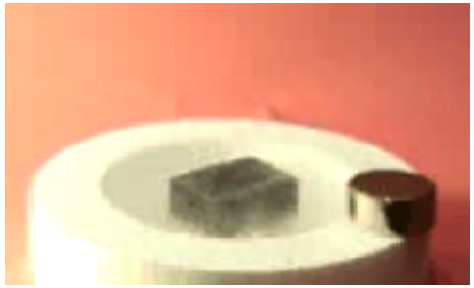
3. **Doping the holographic Mott insulator.**

The holographic superconductor

Hartnoll, Herzog, Horowitz, arXiv:0803.3295



Condensate (superconductor, …) on the boundary!



AdS-CFT

(Scalar) matter 'atmosphere'

"In AdS black holes can grow hair"

Holographic crystals (2D).

arXiv:1104.4478

Donos Gauntlett

Consistent truncation: $AdS_4 \times SE_7$ solutions of D=11 supergravity

$$S = \int d^4x \left[\mathcal{R} + 6 - \frac{1}{2}\partial_\mu \chi \partial^\mu \chi - \frac{Z(\chi)}{4} F_{\mu\nu} F^{\mu\nu} - V(\chi) - \boxed{\theta(\chi) \varepsilon_{\mu\nu\lambda\sigma} F^{\mu\nu} F^{\lambda\sigma}} \right]$$

Topological theta term is doing the work:

χ **condenses = breaking parity in the boundary.**

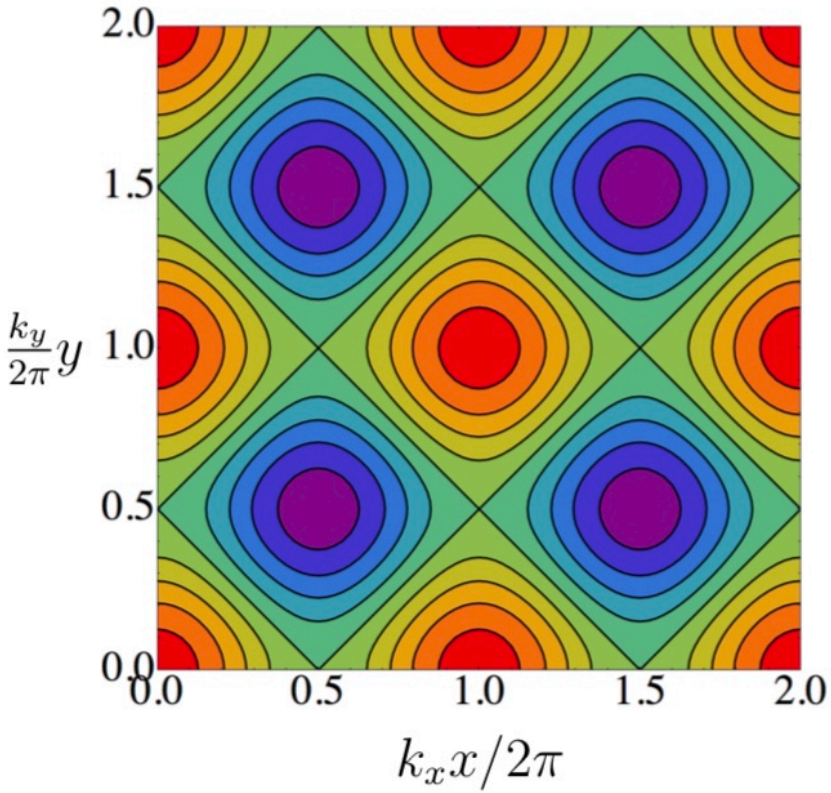
BF bound is violated first at finite momentum = spontaneous breaking of translations = crystallization in the boundary.

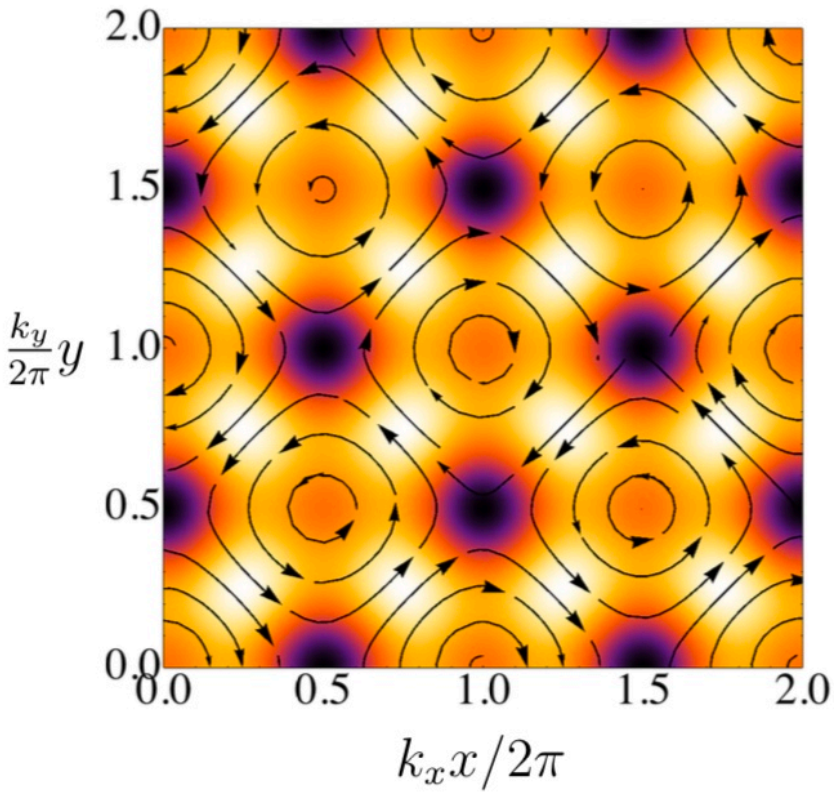
This automatically intertwines charge currents and density in the boundary!

Holographic d-density wave!

arXiv:1407.1085

Withers



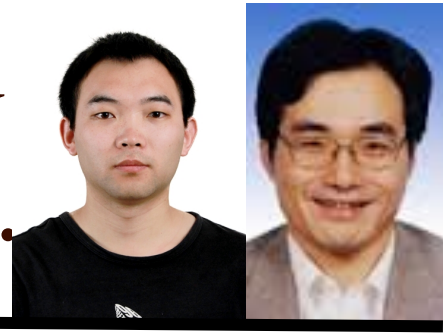


Effective field theory in the bulk.

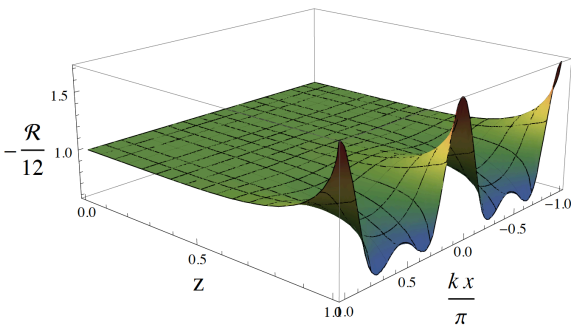
$$S = \int d^4x\sqrt{-g}\Big[\mathcal{R} + 6 - \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - \boxed{\mathcal{F}(\chi)(\partial_\mu\theta - A_\mu)^2} - \frac{Z(\chi)}{4}F^2 - V(\chi) - \vartheta(\chi)\epsilon_{\mu\nu\lambda\sigma}F^{\mu\nu}F^{\lambda\sigma}\Big].$$

Li Li: in principle we can add a Stueckelberg term to the bulk. This wires in holographic superconductivity in a minimal way ..

Complex horizons: intertwined charge-current-parity-SC order.

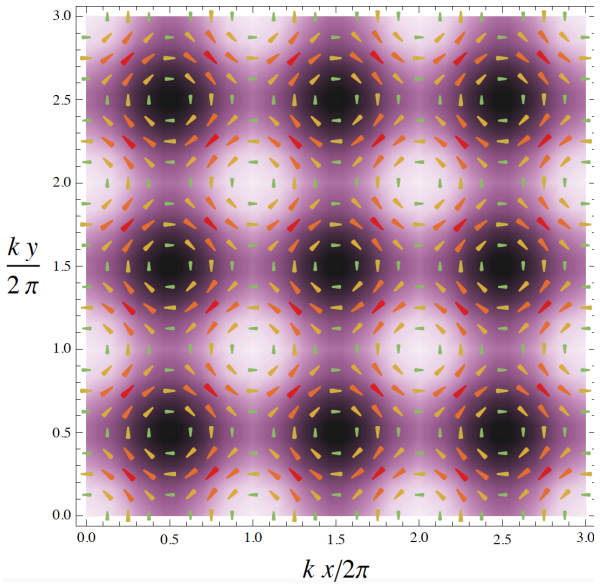


Rong-Gen Cai, Li Li, Yong-Qiang Wang, JZ, PRL 119, 181601 (2017), arXiv:1706.01470

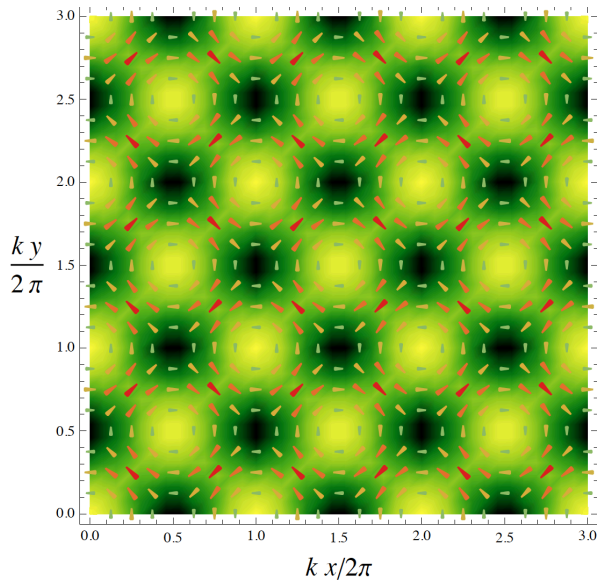


Black-hole hair, rasta style …

Charge and currents



Superconductivity



Charge order intertwined with parity breaking, spontaneous currents and pair density wave order.

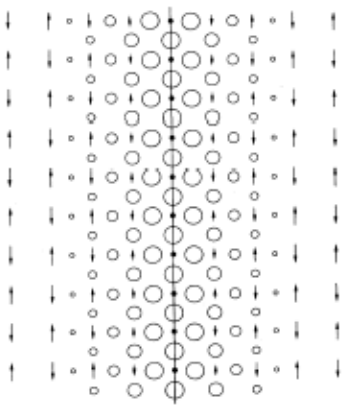
Plan.

1. **Empiricisms: why are the condensed matter physicists obsessing about ordering phenomena in high Tc?**

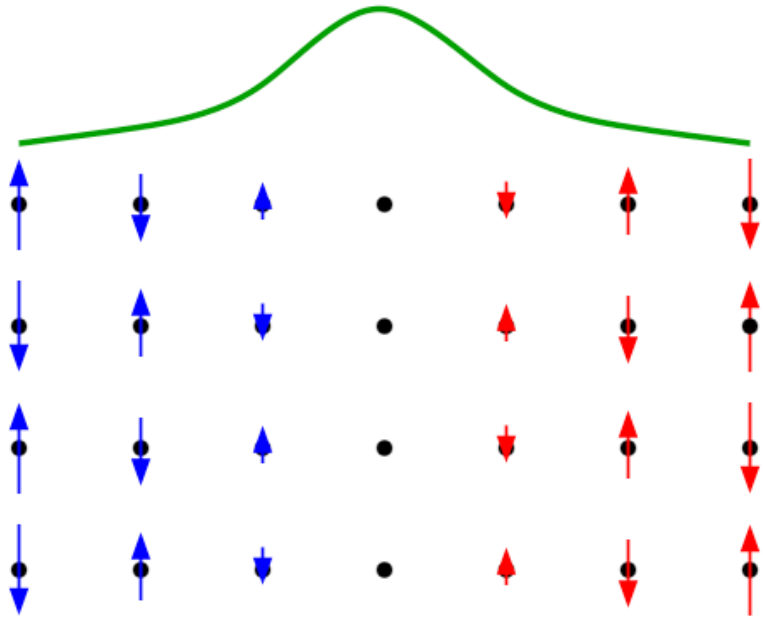
2. **Holographic spontaneous symmetry breaking is remarkably "intertwined".**

3. **Doping the holographic Mott insulator.**

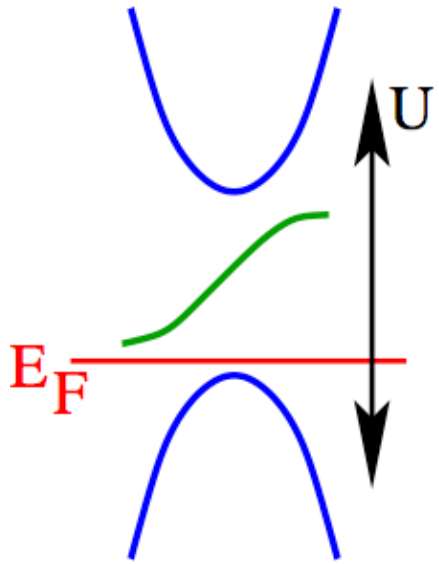
Zaanen-Gunnarsson stripes



'Holons on a row'



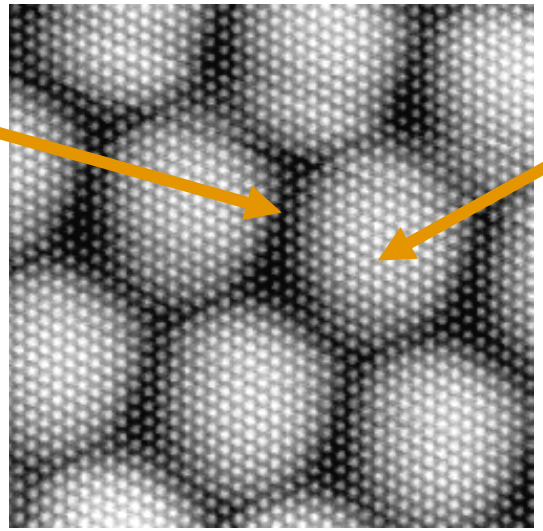
Mid-gap band



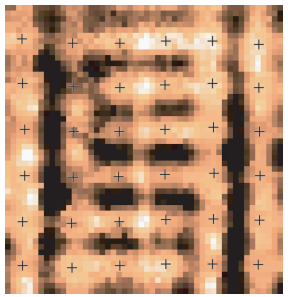
Why ‘Stripes’ ?

Xenon on graphite

Discommensurations or `stripes’

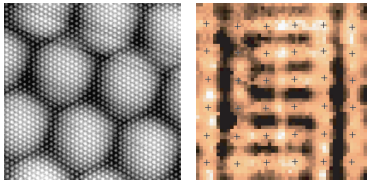


Commensurate domains

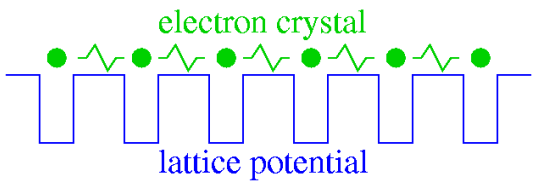


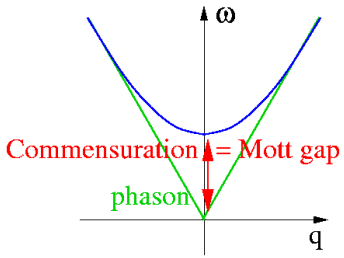
?

The Mott insulator as the charge commensurate charge density wave.

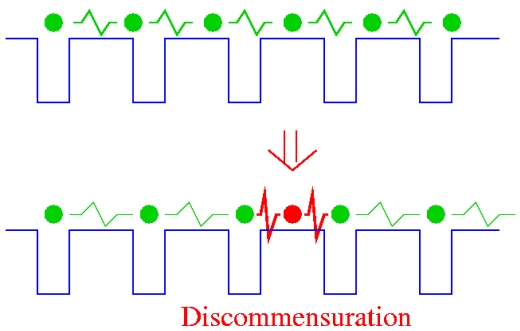


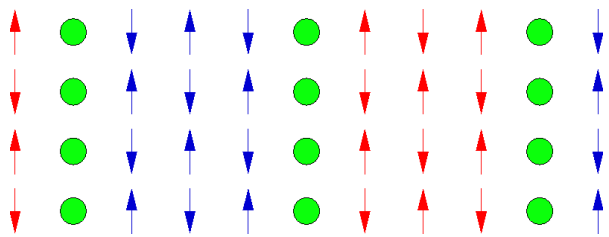
Mott insulator = commensurate charge density wave





Doped Mott-insulator: incommensurate charge density wave => discommensuration lattice = STRIPES





The holographic Mott insulator Nature Phys. (in press)

Krikun Andrade Schalm

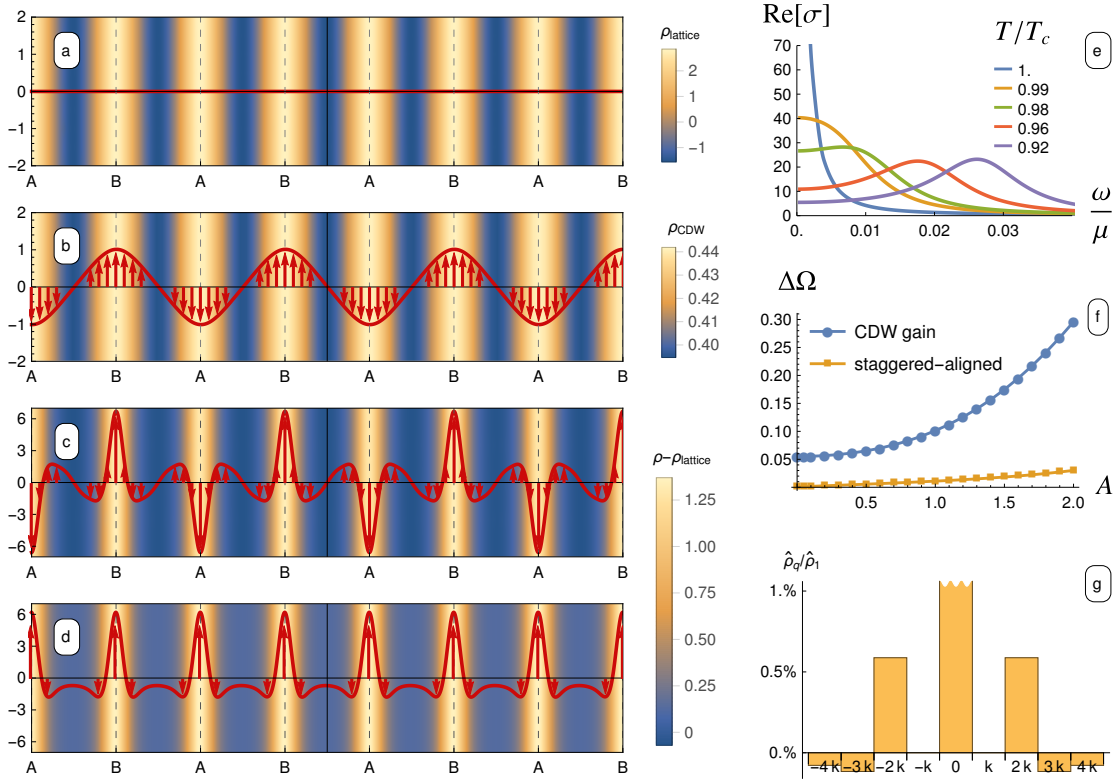
The commensurate case:

Ionic lattice

Spontaneous crystal

Pinned crystal staggered currents

Pinned crystal "uniform" currents



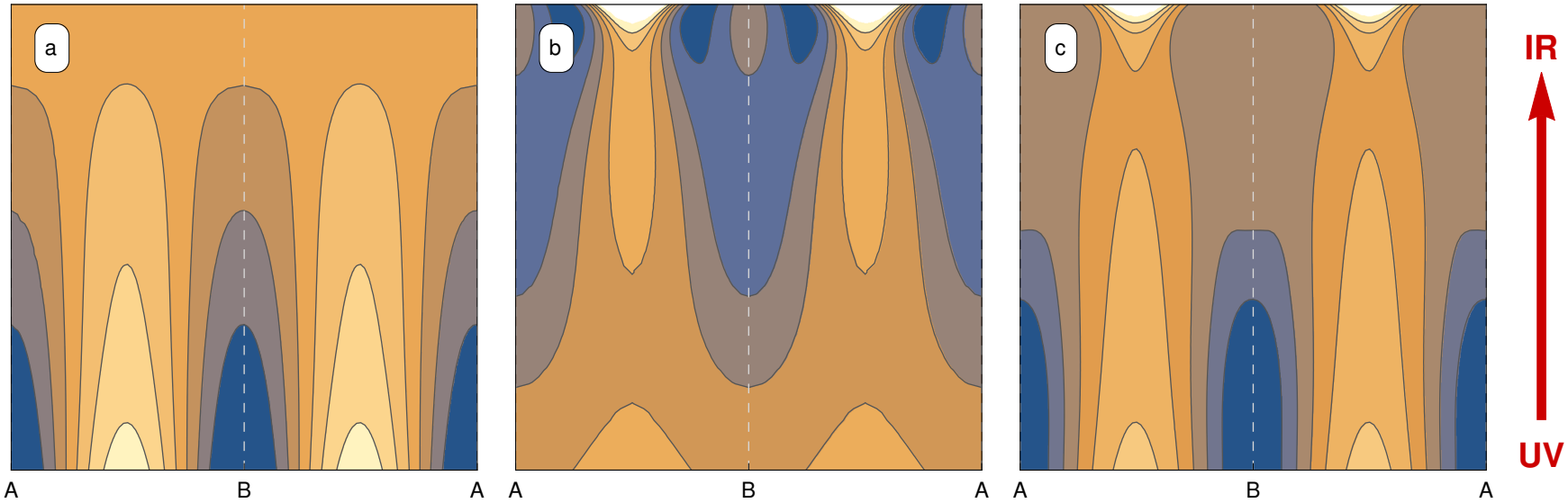
"Hubbard band"

"super-exchange"

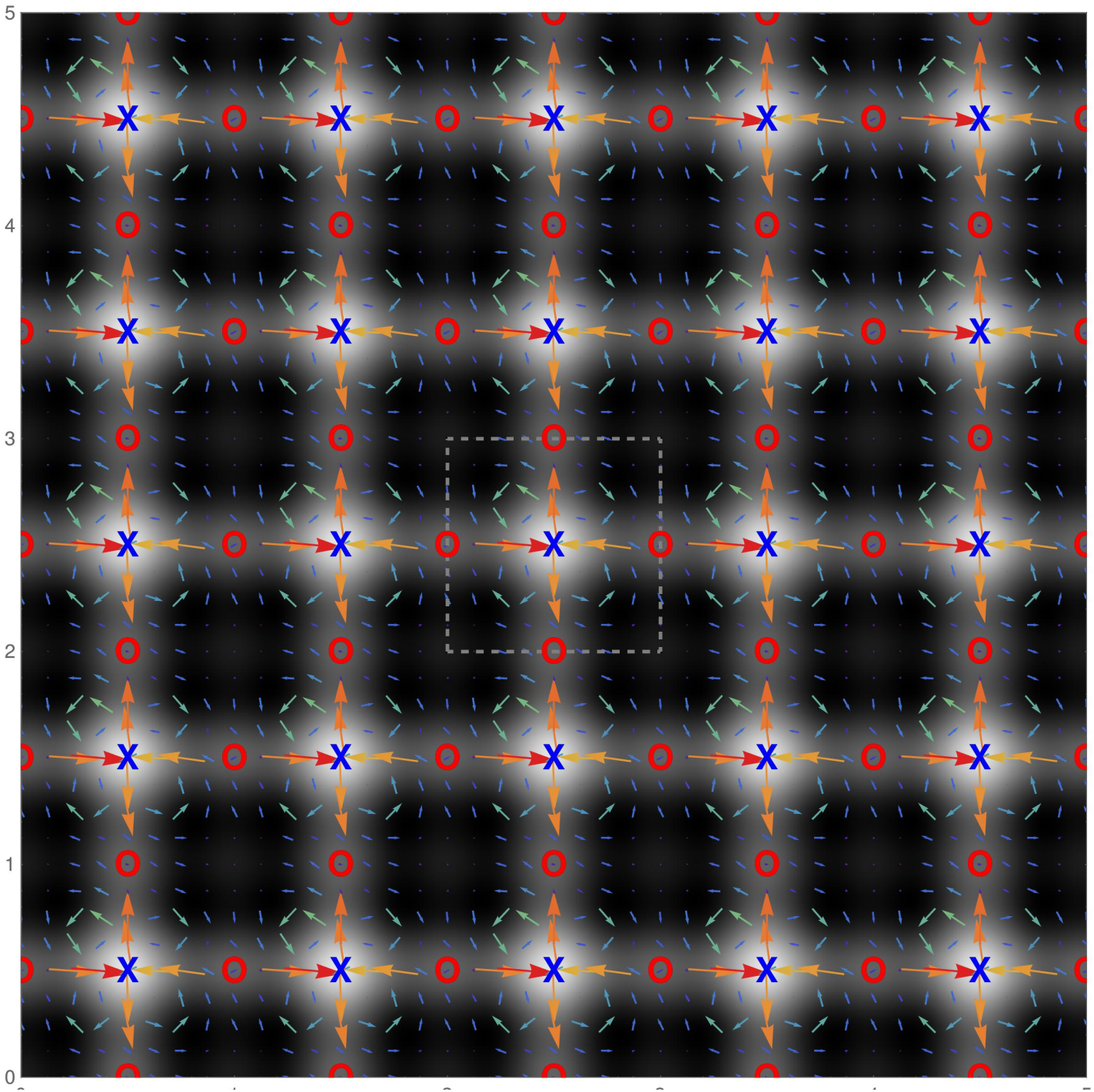
The holographic Mott insulator arXiv:1710.05791

Krikun Andrade Schalm

The RG flow of pinning:



External potential **Spontaneous crystal** **The Mott insulator**


5
4
3
2
1
0

The holographic stripes

arXiv:1710.05791

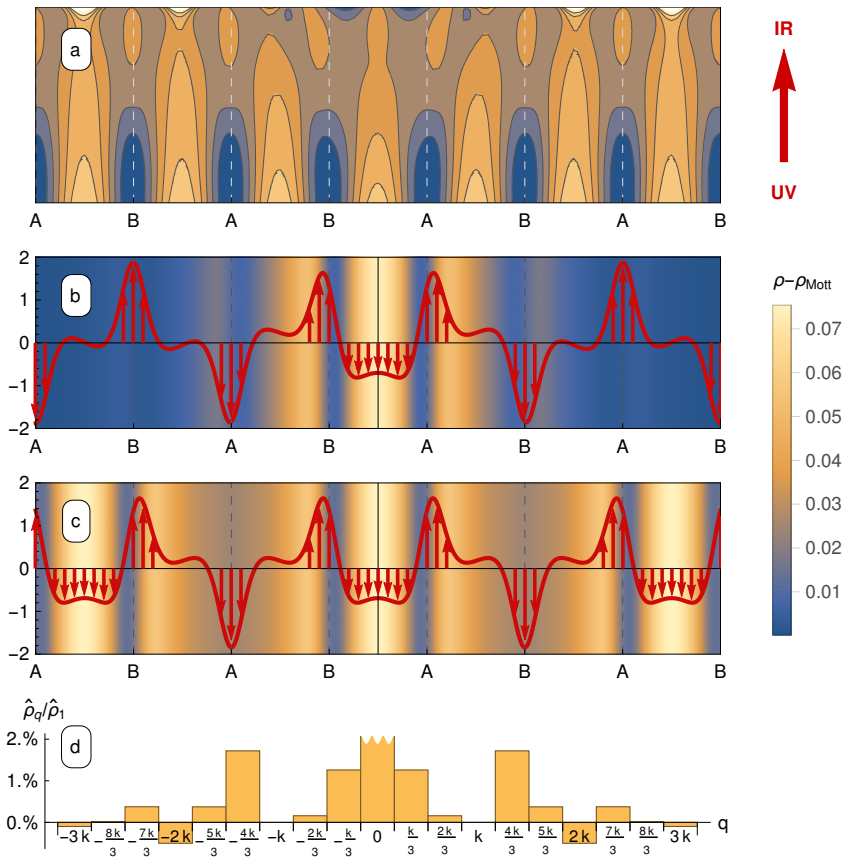
Krikun Andrade Schalm

Doping the holographic Mott insulator:

The bulk "RG flow":

Isolated discommensuration = domain wall in current order!

Discommensuration lattice = (current) stripe phase

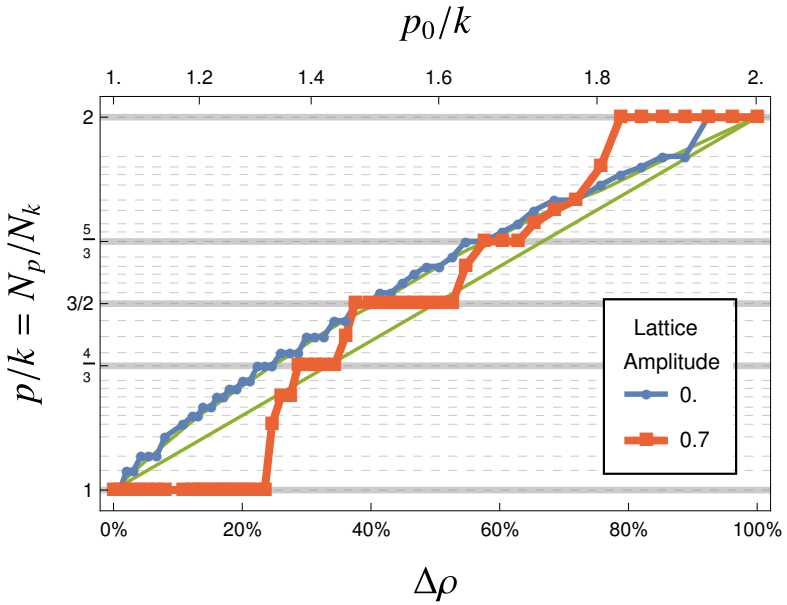


The ordering wave vector

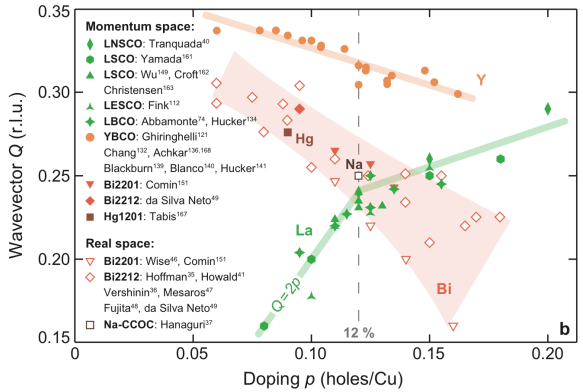
arXiv:1710.05791

Krikun Andrade Schalm

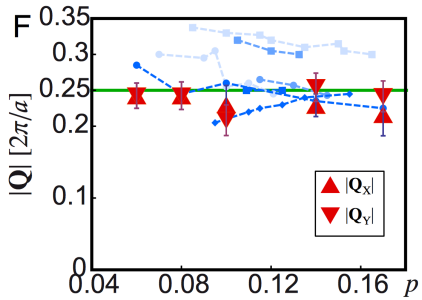
The doping dependence of the periodicity:



Lacking charge quantization no preferred density: "Devil's staircase" behavior with higher order commensurate plateaux.



Comin, Damascelli, An. Rev. Cond.Mat.Phys. 7, 369 (2016)

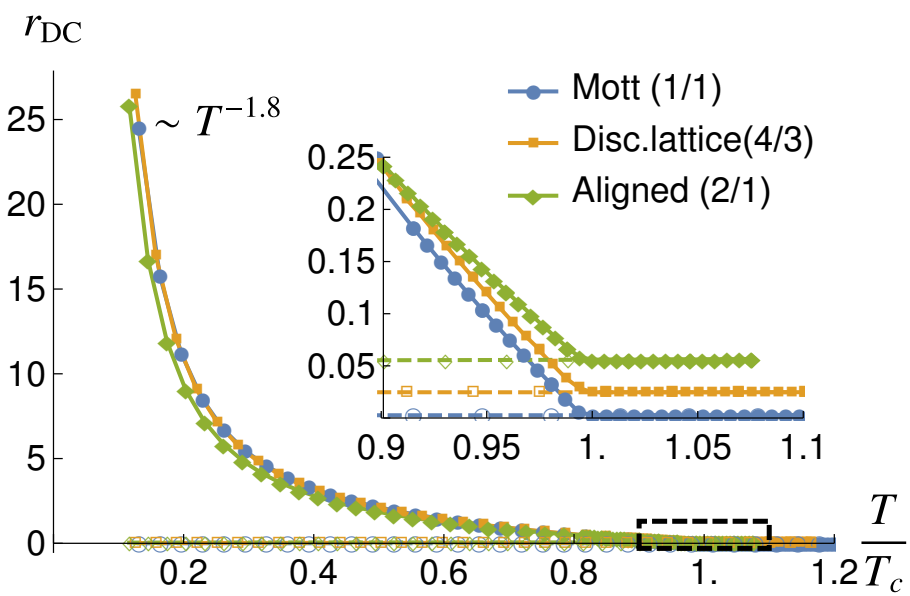


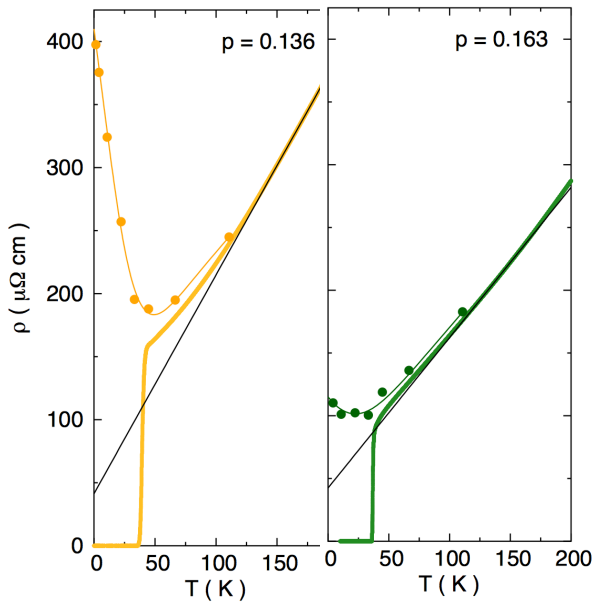
Mesaros et al, PNAS 113, 12661 (2016)

The insulator is algebraic! arXiv:1710.05791

Krikun Andrade Schalm

"Quantum critical infrared sector":





According to holography even deep in the MI presumably entangled stuff is left behind that is not localizable.

Challenge to experiment: origin of slow resistivity upturns in stripy 214?

Conclusion.

Holographic symmetry breaking works *way better* than conventional mean field theory in reproducing the salient features of the "intertwined orders" observed in Cuprates.

Parity + charge density + Spontaneous currents + pair density

Upon adding a background potential it reproduces literal "Mottness" physics.

Hubbard bands, superexchange, "current stripes", Devil staircases + explanation of transport of spin stripes.

Is this mere coincidence or does it reveal deep and general principle associated with the ordering quantum matter?