Gauge/Gravity Duality 2018

Holographic Striped Superconductors and Fermi Surfaces

Sera Cremonini
Lehigh University

Today:

- Holographic Realization of Intertwined Orders (Pair Density Wave)
  S.C., Li Li, Jie Ren arXiv:1612.04385 and arXiv:1705.05390

- Fermionic spectral functions in striped superconducting phases
  S.C., Li Li, Jie Ren 1808.xxxx

See also talks by Li Li and Jie Ren on Tuesday afternoon

Holography as a Theoretical Laboratory

Study solvable models that may be in the same universality class as strongly correlated QM phases

→ Can we understand **the basic mechanisms** underlying the dynamics and unconventional properties of these systems?

Draw qualitative and quantitative lessons → look for **universal features**

Solvable often implies working with overly simplified bottom-up **toy models**

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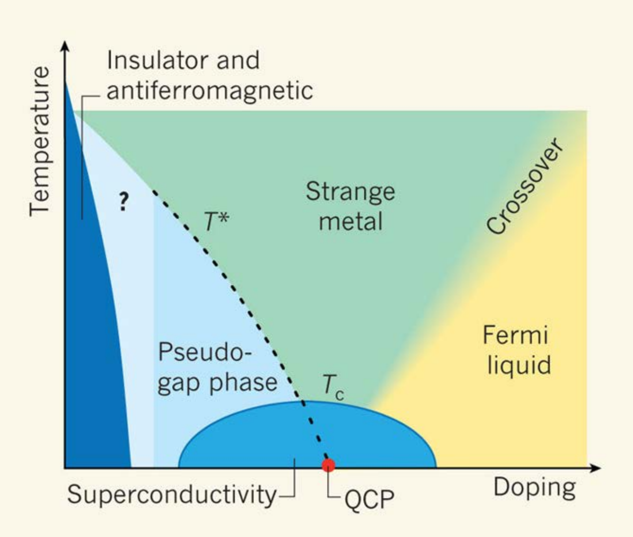
→ Can we understand **the basic mechanisms** underlying the dynamics and unconventional properties of these systems?

Challenges for understanding strongly coupled QM phases of matter

- Strong coupling, breakdown of Fermi-liquid theory, no quasiparticles
- An intrinsically complex phase diagram exhibiting a variety of orders
  - → Phases may compete but may also cooperate with each other
- Rich structure of emergent IR phases
- Different scales in the system, long-range entanglement, …



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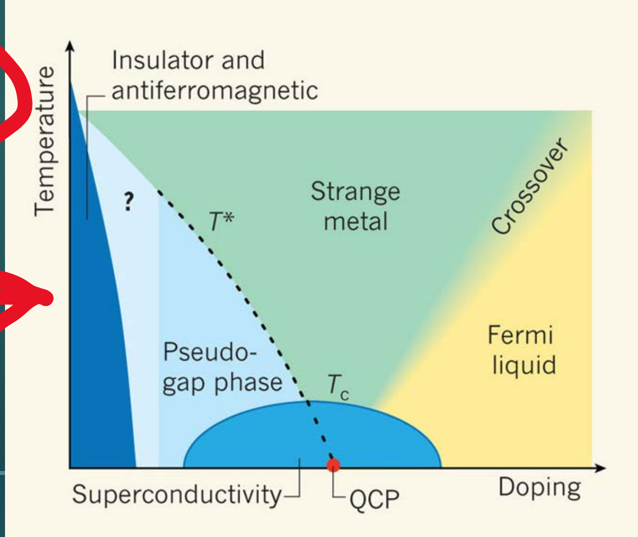

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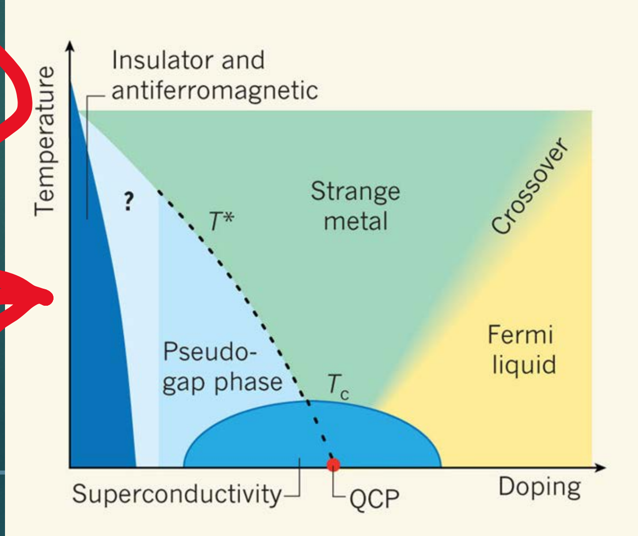
  → Phases may compete but may also cooperate with each other
- Rich structure
- Different scal

Attempt to identify generic imprints of these symmetry breaking mechanisms



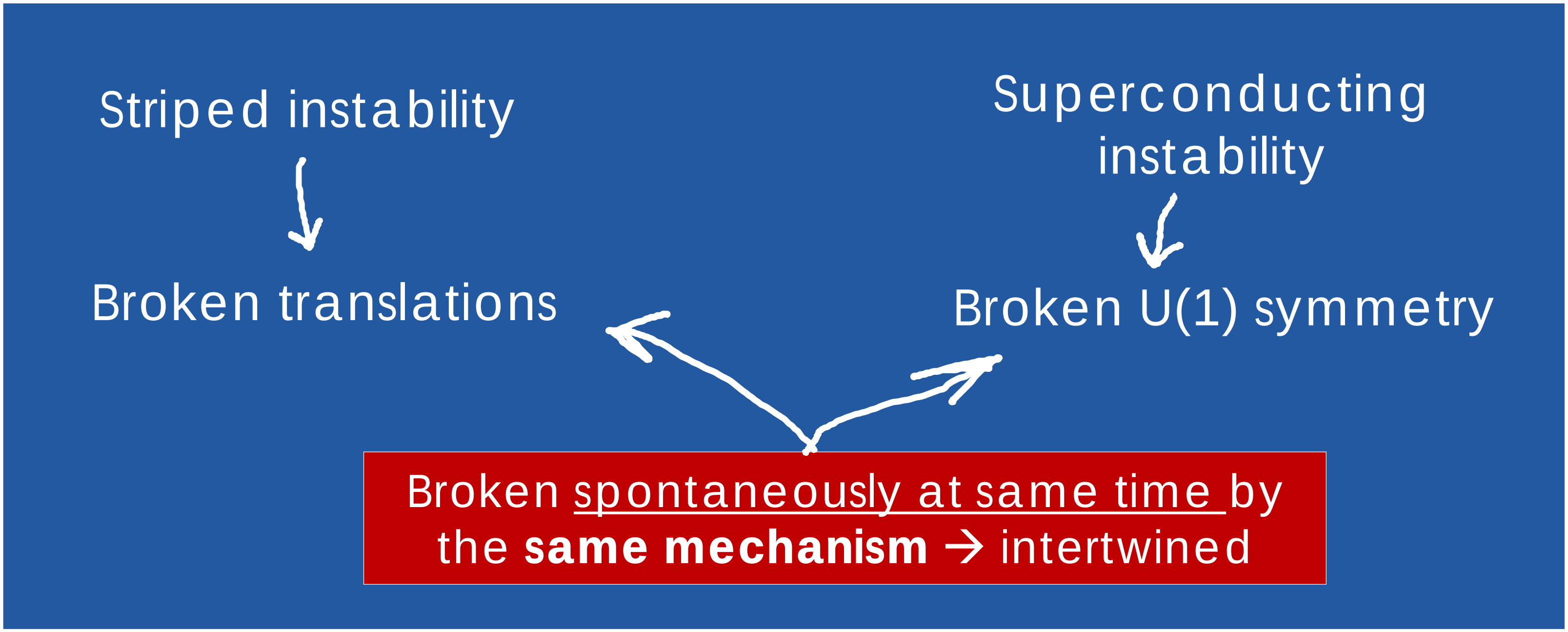
Holographic Realization of Intertwined Orders

Goal:

Break translational and U(1) symmetry spontaneously at same time (onset T)

- **parent phase gives rise to daughter phases**

In our model:



Striped instability

Broken translations

Superconducting instability

Broken U(1) symmetry

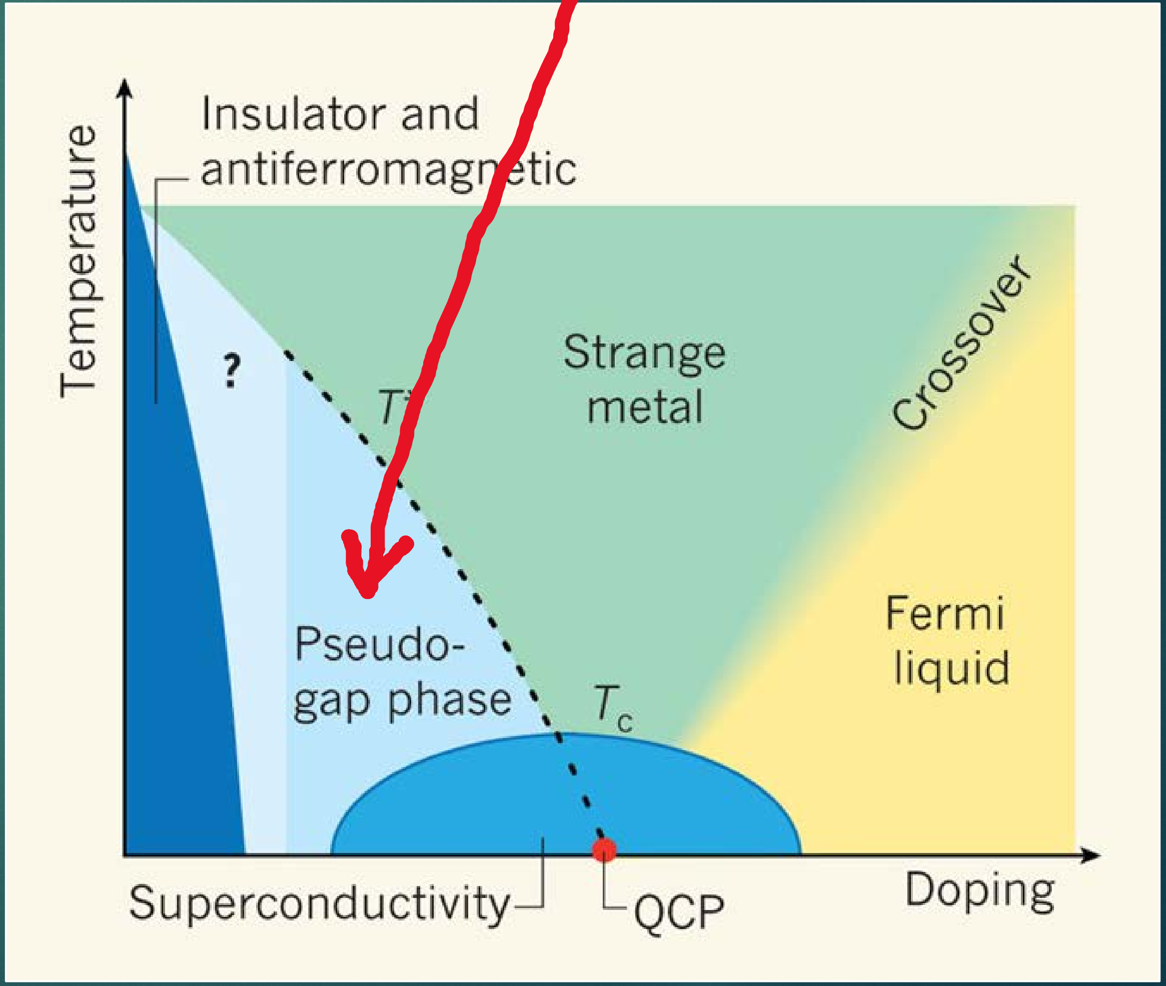
Broken <u>spontaneously at same time</u> by the **same mechanism** → intertwined

A particular type of striped superconductor

Motivation

Realize some of the features of **Pair Density Wave (PDW) order** of high temperature superconductors (cuprates)

Evidence for PDW:
in pseudo-gap of cuprate
high T_c superconductor
$La_{2-x}Ba_xCuO$ (LBCO)



Holographic Toy Model of PDW Order

Features of PDW order we focus on:

- Scalar condensate (superconducting order) is spatially modulated + its oscillations average out to zero (no homogeneous component)

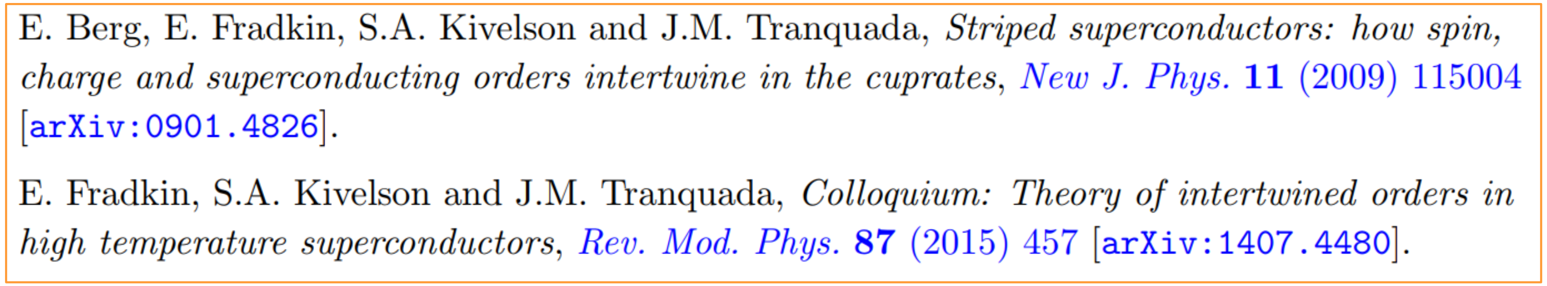
$$\langle O_\chi \rangle \propto \cos(k\,x)$$

- Charge density is modulated and oscillates at twice the frequency of the condensate

$$\rho(x) = \rho_0 + \rho_1 \cos(2k\,x)$$

Contrast to co-existing CDW + SC orders:

- Scalar condensate has a uniform component, and oscillates at the same frequency as the CDW



E. Berg, E. Fradkin, S.A. Kivelson and J.M. Tranquada, *Striped superconductors: how spin, charge and superconducting orders intertwine in the cuprates*, *New J. Phys.* **11** (2009) 115004 [`arXiv:0901.4826`].

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CAN REALIZE BOTH BUT FOCUS ON PDW TODAY for concreteness

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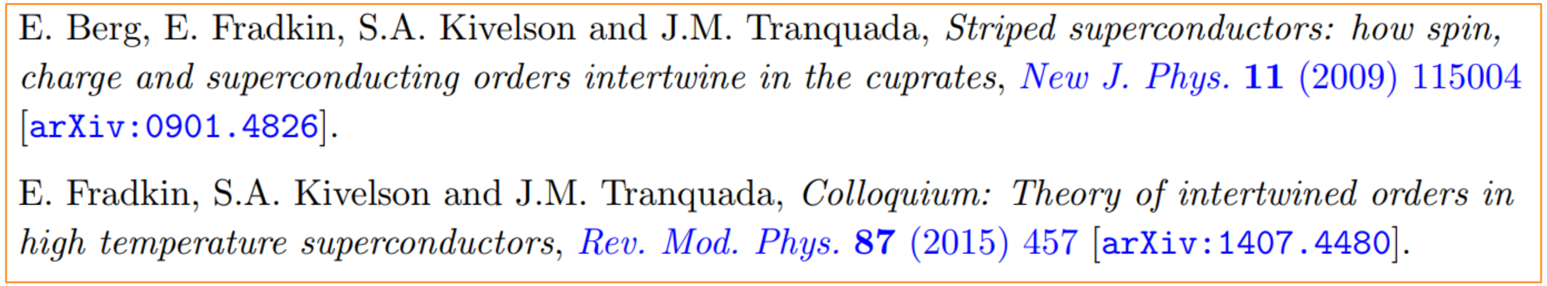

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The Holographic Model

S.C., L. Li, J. Ren (1612.04385, 1705.05390)

4D Bottom-up Model (2+1 dual QFT)

$$S = \frac{1}{2\kappa_N^2}\int d^4x\sqrt{-g}\,[\mathcal{R} - 2\Lambda + \mathcal{L}_m]$$

$$\mathcal{L}_m = -\frac{1}{2}\partial_\mu\chi\partial^\mu\chi - \frac{Z_A(\chi)}{4}F_{\mu\nu}F^{\mu\nu} - \frac{Z_B(\chi)}{4}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} - \frac{Z_{AB}(\chi)}{2}F_{\mu\nu}\tilde{F}^{\mu\nu}$$
$$-\mathcal{K}(\chi)(\partial_\mu\theta - q_A A_\mu - q_B B_\mu)^2 - V(\chi)\,,$$

Keep in mind:

Minimal model to get specific features we are after

Can be made more realistic (PDW order parameter) at the cost of adding a more complicated matter sector

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$$-\mathcal{K}(\chi)(\partial_\mu\theta - q_A A_\mu - q_B B_\mu)^2 - V(\chi)\,,$$

Two real scalars

Two U(1) vector fields

$F = dA \qquad \tilde{F} = dB$

Field content:

- Gravity
- Two real scalars χ and θ
- Two U(1) vector fields A_μ and B_μ with different physical interpretations:
  - A_μ → **charge density** of field theory
  - B_μ → **spectator field** or proxy for "spin" density or **second species of charge carriers**

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Stuckelberg Superconductor

Generalizes standard holographic SC
→ allows for more general couplings

Stuckelberg mechanism:
local gauge invariance encoded in

$$\theta \to \theta + \alpha(x^\mu)\,, \qquad A_\mu \to A_\mu + \frac{1}{q_A}\partial_\mu\alpha(x^\mu)$$

The Holographic Model

S.C., L. Li, J. Ren (1612.04385, 1705.05390)

Crucial coupling **for seeding spatially modulated instabilities**

c=0 → leading unstable mode is not striped

4D Bottom-up Model (2+1 dual QFT)

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Theory chosen so that symmetric phase $\chi = \theta = B_\mu = 0$ **is described by standard charged black hole in AdS**

$$Z_A(\chi) = 1 + \frac{a}{2}\chi^2 \qquad Z_B(\chi) = 1 + \frac{b}{2}\chi^2 \qquad \mathcal{K}(\chi) = \frac{\kappa}{2}\chi^2$$

$$Z_{AB}(\chi) = c\,\chi \qquad V(\chi) = \frac{1}{2}m^2\chi^2$$

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4D Bottom-up Model (2+1 dual QFT)

$$S = \frac{1}{2\kappa_N^2} \int d^4x \sqrt{-g} \left[\mathcal{R} - 2\Lambda + \mathcal{L}_m\right]$$

$$\begin{aligned}\mathcal{L}_m = & -\frac{1}{2}\partial_\mu \chi \partial^\mu \chi - \frac{Z_A(\chi)}{4} F_{\mu\nu}F^{\mu\nu} - \frac{Z_B(\chi)}{4} \tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} - \frac{Z_{AB}(\chi)}{2} F_{\mu\nu}\tilde{F}^{\mu\nu} \\ & - \mathcal{K}(\chi)(\partial_\mu \theta - q_A A_\mu - q_B B_\mu)^2 - V(\chi)\,,\end{aligned}$$

Note:

- At some critical temperature the system becomes unstable to the condensation of χ and B_μ
- Properties of condensate sensitive to whether q_A $q_B = 0$ or not (we will keep q_A non-zero since the charge density will be associated with A).
- Today **we focus on $q_B = 0$**.

Onset of Instabilities – Building Intuition

Analytical T=0 analysis gives some insight **(violation of BF bound)**

Working to leading order in perturbations:

$$\delta\chi = \varepsilon\, w(r)\cos(k\,x)\,, \quad \delta B_t = \varepsilon\, b_t(r)\cos(k\,x)$$

$$w'' + \# w' + c b_t' + m^2_{eff}(k)\, w = 0$$
$$b_t'' + \# b_t' \quad c w' + \tilde{m}^2_{eff}(k)\, b_t = 0$$

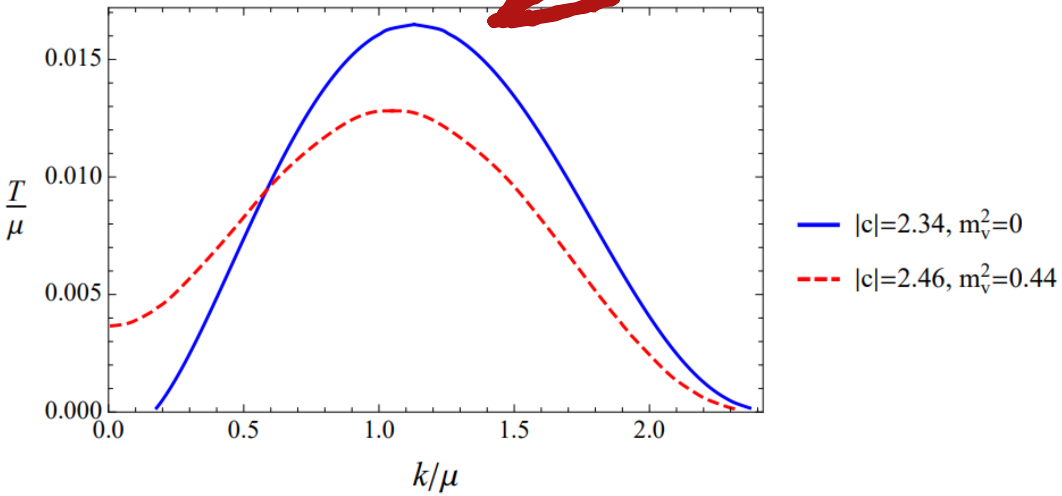
Coupled through
$Z_{AB} F \tilde{F}$

There are nonzero k values at which the **scaling dimension becomes imaginary** → instability

Critical Temperature for Instability

For a given k there will be a normalizable zero mode appearing at a particular T → T_c

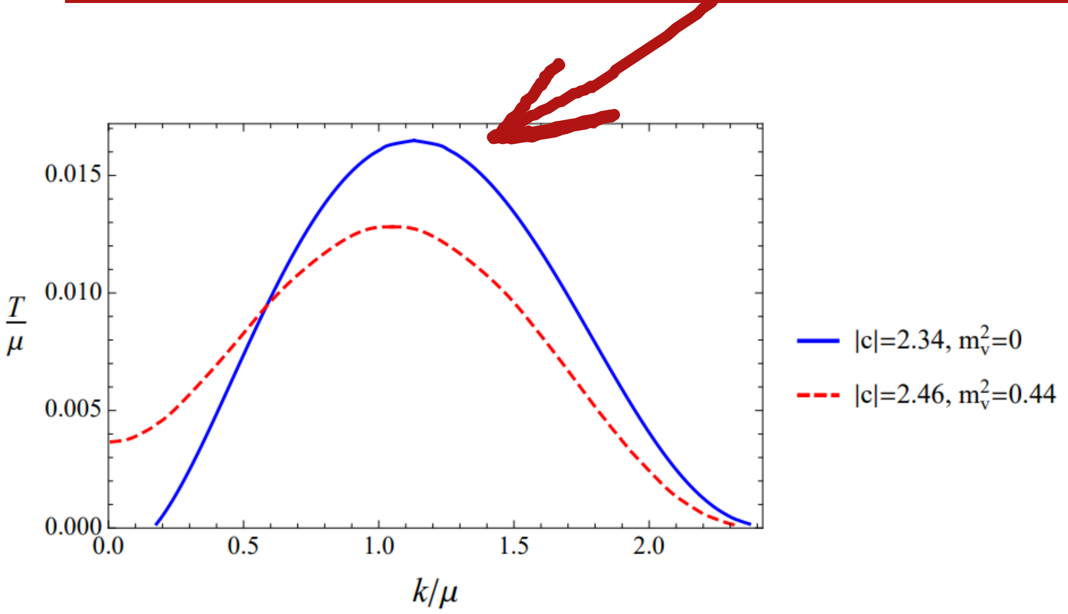
Condensate is driven by a spatially modulated mode



Minimize free energy →
thermodynamicall preferred solution

Critical Temperature for Instability

Simple observation: Condensation at nonzero k will always occur at higher T_c than in homogeneous phase (because of generic bell shape of instability curve)



spatial modulations “enhance” the superconducting critical temperature

→ Facilitate the transition

A Cartoon Picture

Gravity Side

Standard AdS-RN black hole supported by U(1) gauge field A_μ

$\chi = B_\mu = 0$

T

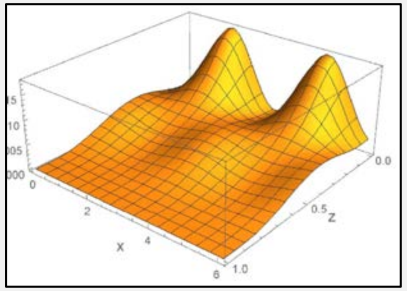
T_c

Dual Field Theory

Global U(1) symmetry

Spatially modulated instabilities
System unstable to condensation of χ and B_μ

New type of striped IR geometry



χ, B_μ nontrivial

U(1) and translational symmetry broken spontaneously at the same time

$$\langle O_\chi \rangle \sim \cos(k_c x)$$

$$\rho_B = \langle J_B^t \rangle \sim \cos(k_c x)$$

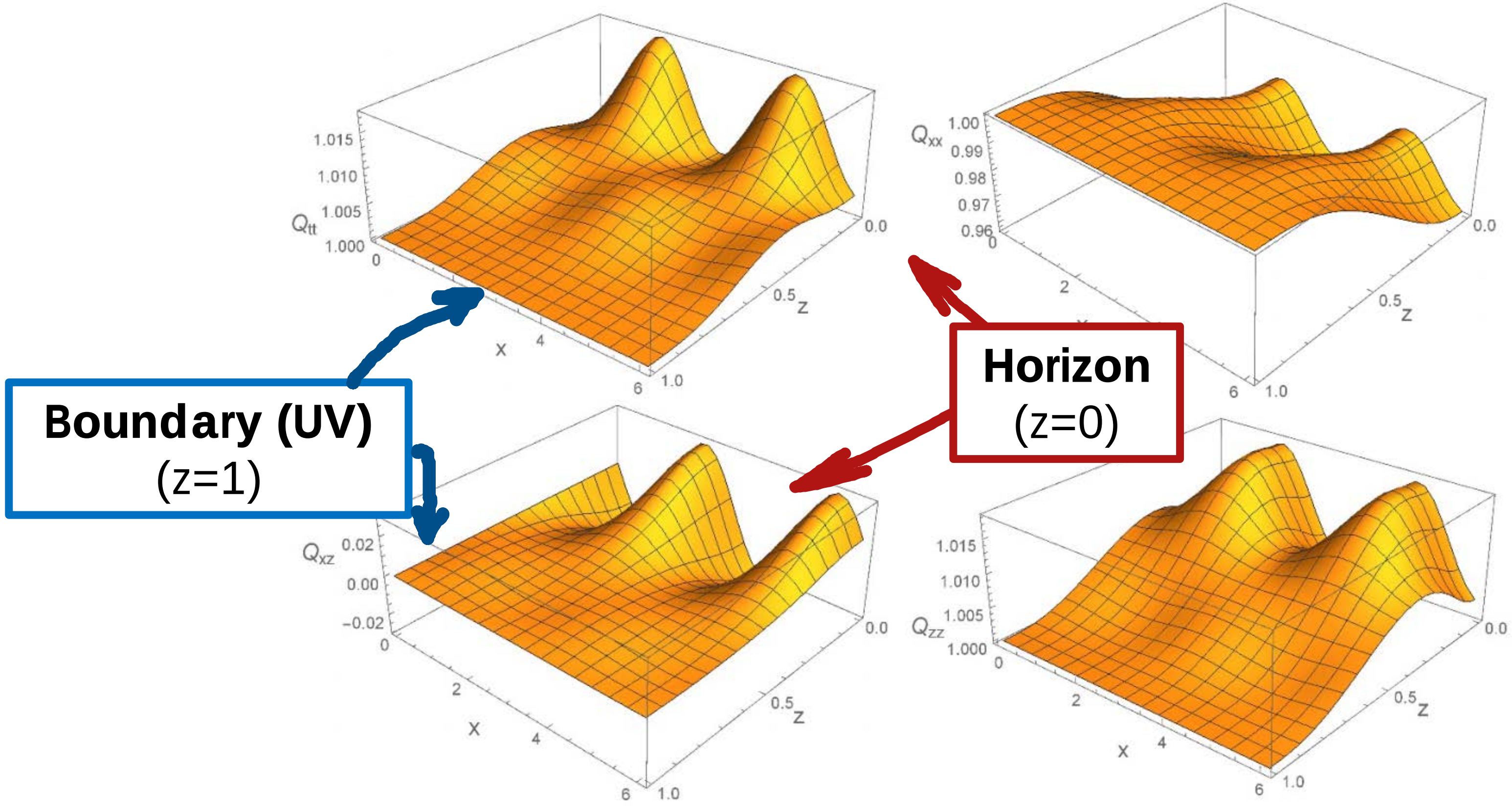
Modulated condensate and charge density

$$\rho_A = \langle J_A^t \rangle \sim \cos(2k_c x)$$

The Striped Geometry (PDW)

Black Hole Geometry

$$ds^2 = \frac{r_h^2}{L^2(1-z^2)^2}\left[-F(z)Q_{tt}\,dt^2 + \frac{4z^2L^4Q_{zz}}{r_h^2F(z)}\,dz^2 + Q_{xx}(dx - 2z(1-z^2)^2Q_{xz}dz)^2 + Q_{yy}\,dy^2\right]$$



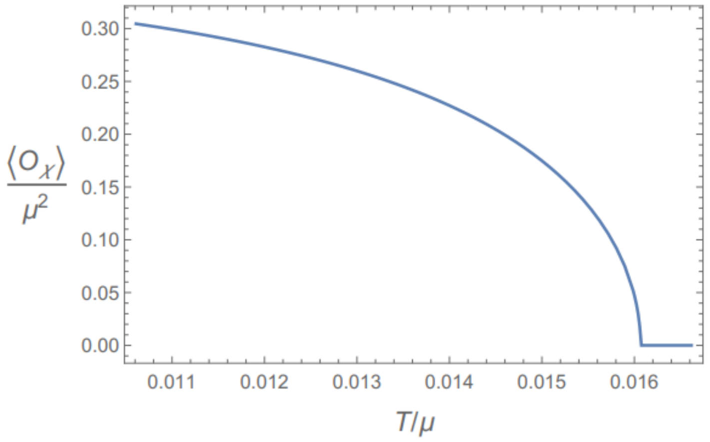
Boundary (UV)
(z=1)

Horizon
(z=0)

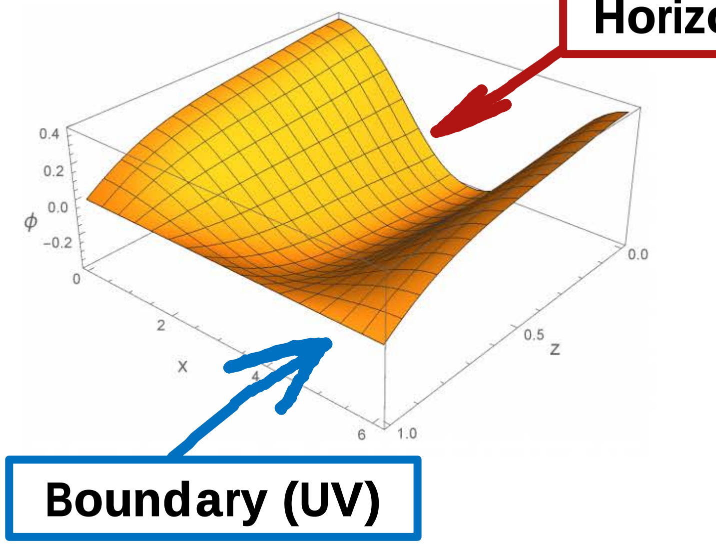
NOTE: spatial modulations are imprinted on the horizon (IR)

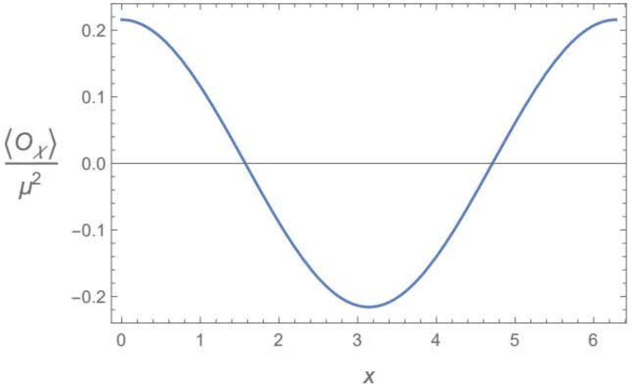
→ Stripes are relevant deformation of the UV CFT

The Scalar Field Condensate (PDW)



VEV of scalar condensate as a function of T at x=0



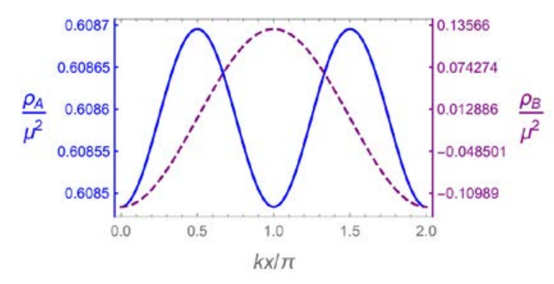


$$\langle O_\chi \rangle \sim \cos(k_c\, x)$$

Scalar condensate oscillations average out to zero

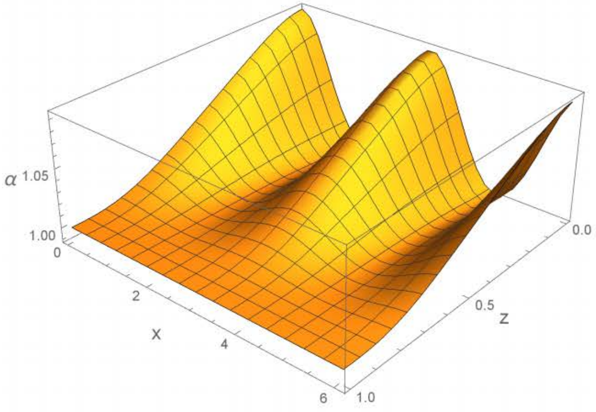
(PDW feature)

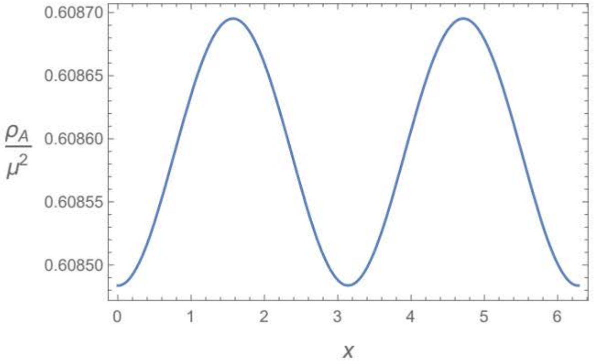
Vector Field Profiles (PDW)



Vector field A_t profile

$$\rho_A = \langle J^t_A \rangle \sim \cos(2k_c\, x)$$



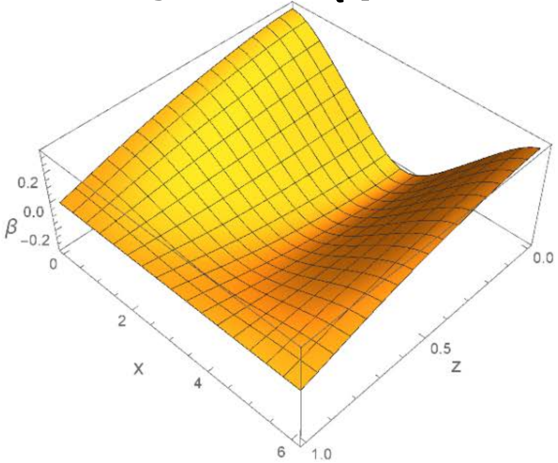


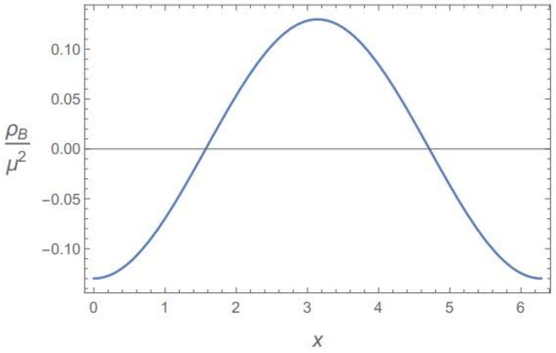
charge density oscillates twice as fast as scalar condensate

(PDW feature)

Auxiliary field B_t profile

$$\rho_B = \langle J^t_B \rangle \sim \cos(k_c\, x)$$





"charge" density (**proxy for SDW**?) oscillates at same frequency as scalar condensate

We have seen a concrete realization of spontaneously generated **intertwined** striped superconducting phases of various type (e.g. PDW but also **superconducting order co-existing with charge density wave order**)

Next:

Examine fermionic spectral functions in these phases (including effects of explicit breaking of translations)

Holographic Fermions in Striped Superconductors

SC, L. Li, J. Ren, arXiv:1808.xxxx

- A lot of work on fermionic response in holography (e.g. review by Iqbal, Liu and Mezei,1110.3814) but most studies focused on **cases with translational invariance or homogeneous lattices**

- To make contact with real materials important to include effects of **periodic lattices**
- Very few holographic studies on **fermions in inhomogeneous systems**

  Our work is motivated by and builds on:
  - **Y. Liu, K. Schalm, Y.W. Sun and J. Zaanen [1205.5227]**

    Perturbatively small periodic modulation of chemical potential, neglecting backreaction
  - **Y. Ling, C. Niu, J.P. Wu, Z.Y.Xian and H.B. Zhang [1304.2128]**

    Included backreaction
  - Among features identified: **anisotropic FS and appearance of a gap**

Holographic Fermions in Striped Superconductors

SC, L. Li, J. Ren, arXiv:1808.xxxx

- In the models of arXiv:1205.5227, arXiv:1304.2128 lattice is irrelevant in the IR

- Our main interest:

  Role of spontaneous vs. explicit translational symmetry breaking on fermionic spectral functions (in striped superconducting phases)

  Formation of a Fermi surface? Gap? Size of gap? "Destruction" of Fermi surface?

Setup:

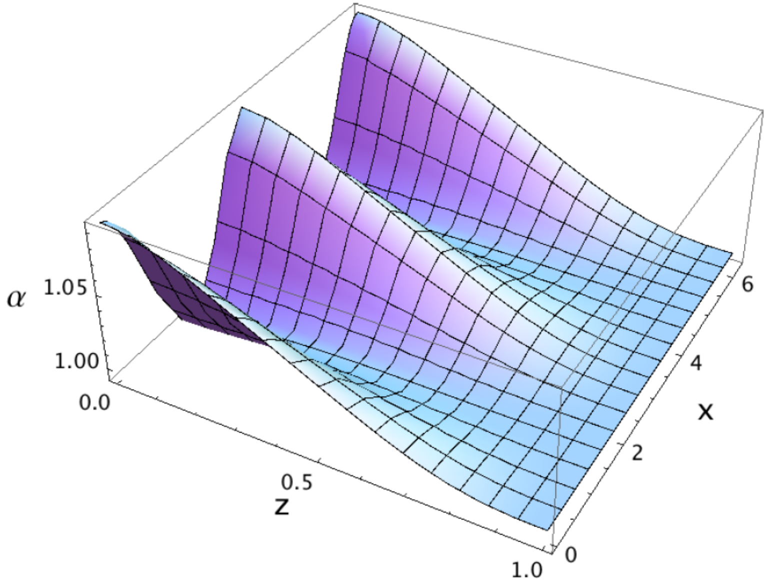
→ place a probe fermion in the spontaneously generated striped superconducting background, and include a source in the UV to break translations explicitly (ionic lattice)

→ Dirac equation solved numerically, recall geometry has periodic modulation so solutions will reflect this periodicity (**Bloch expansion**)

Breaking Translations – Explicit vs. Spontaneous

Gauge field profile

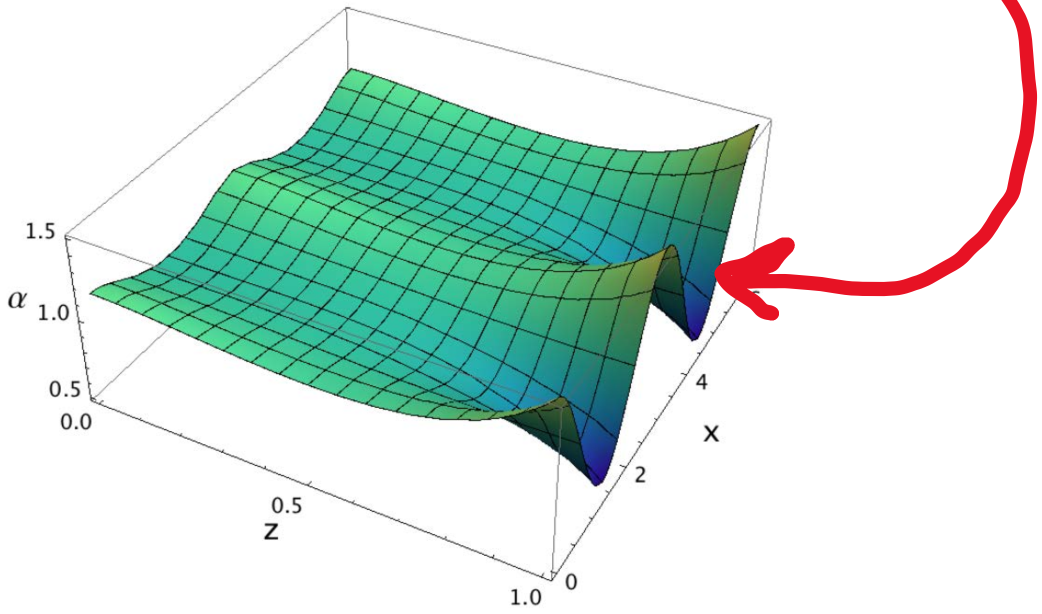
PDW order
(spontaneous breaking)



$$\rho_A = \langle J^t_A \rangle \sim \cos(2k_c\, x).$$

PDW + Ionic Lattice
(explicit breaking in UV, spontaneous in IR)

$$\mu(x) = A_t(1, x) = \mu[1 + a_0 \cos(p\, x)]$$

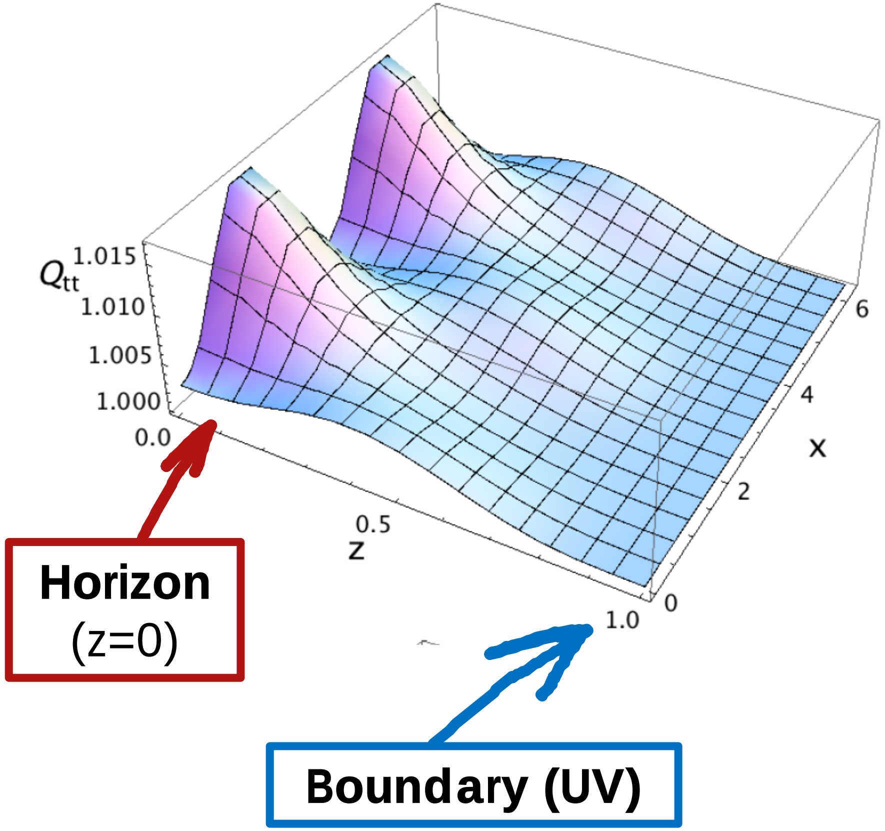


Note: periods chosen to be the same (p=2k)
→ lattice is commensurate with CDW

Breaking Translations – Spontaneous vs. Explicit

Geometry - typical metric profile

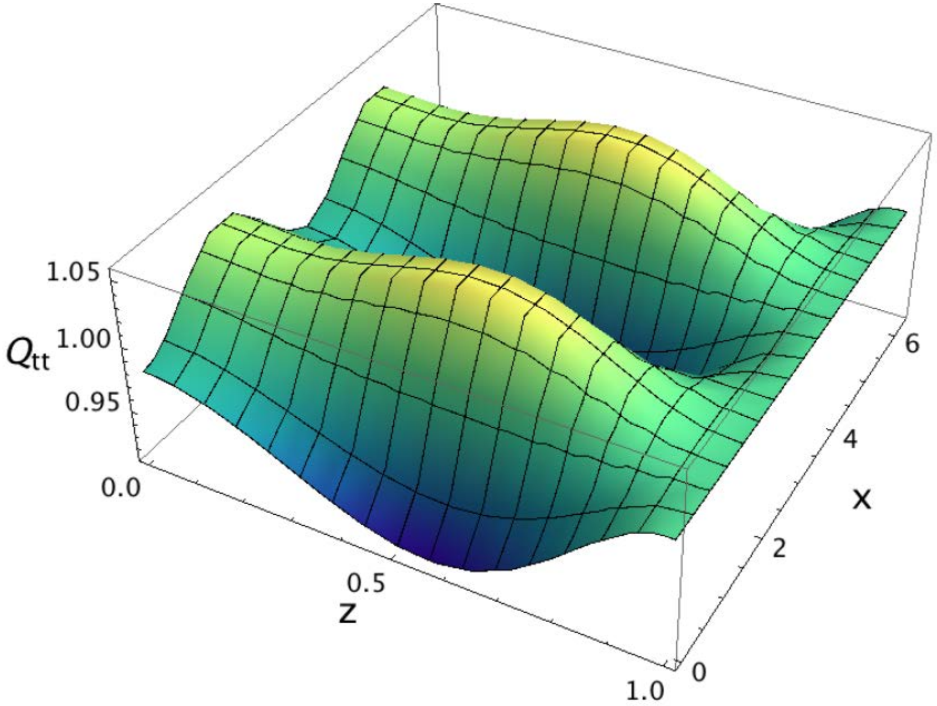
Pure PDW (spontaneous)



Horizon
(z=0)

Boundary (UV)

PDW + Ionic lattice



$$ds^2 = \frac{r_h^2}{L^2(1-z^2)^2}\left[-F(z)Q_{tt}\,dt^2 + \frac{4z^2L^4Q_{zz}}{r_h^2F(z)}\,dz^2 + Q_{xx}(dx - 2z(1-z^2)^2Q_{xz}dz)^2 + Q_{yy}\,dy^2\right]$$

$$\chi = (1-z^2)\phi\,, \qquad A_t = \mu\, z^2\alpha, \qquad B_t = z^2\beta\,, \qquad ($$

Probe fermion and criteria for Fermi surface

- Periodicity of spatially modulated background sets size of Umklapp vector K
- Solutions will reflect periodicity of background (**Bloch expansion**, periodic in x with period 2 π /K)

$$\Psi_\alpha = \int \frac{d\omega dk_x dk_y}{2\pi} \underbrace{\sum_{n=0,\pm1,\pm2,\cdots}}_{\text{momentum level}} \mathcal{F}_\alpha^{(n)}(z,\omega,k_x,k_y)e^{-i\omega t+i(k_x+\boxed{nK})x+ik_yy}$$

$$\boxed{k_x \in [-\frac{K}{2},\frac{K}{2}]}$$

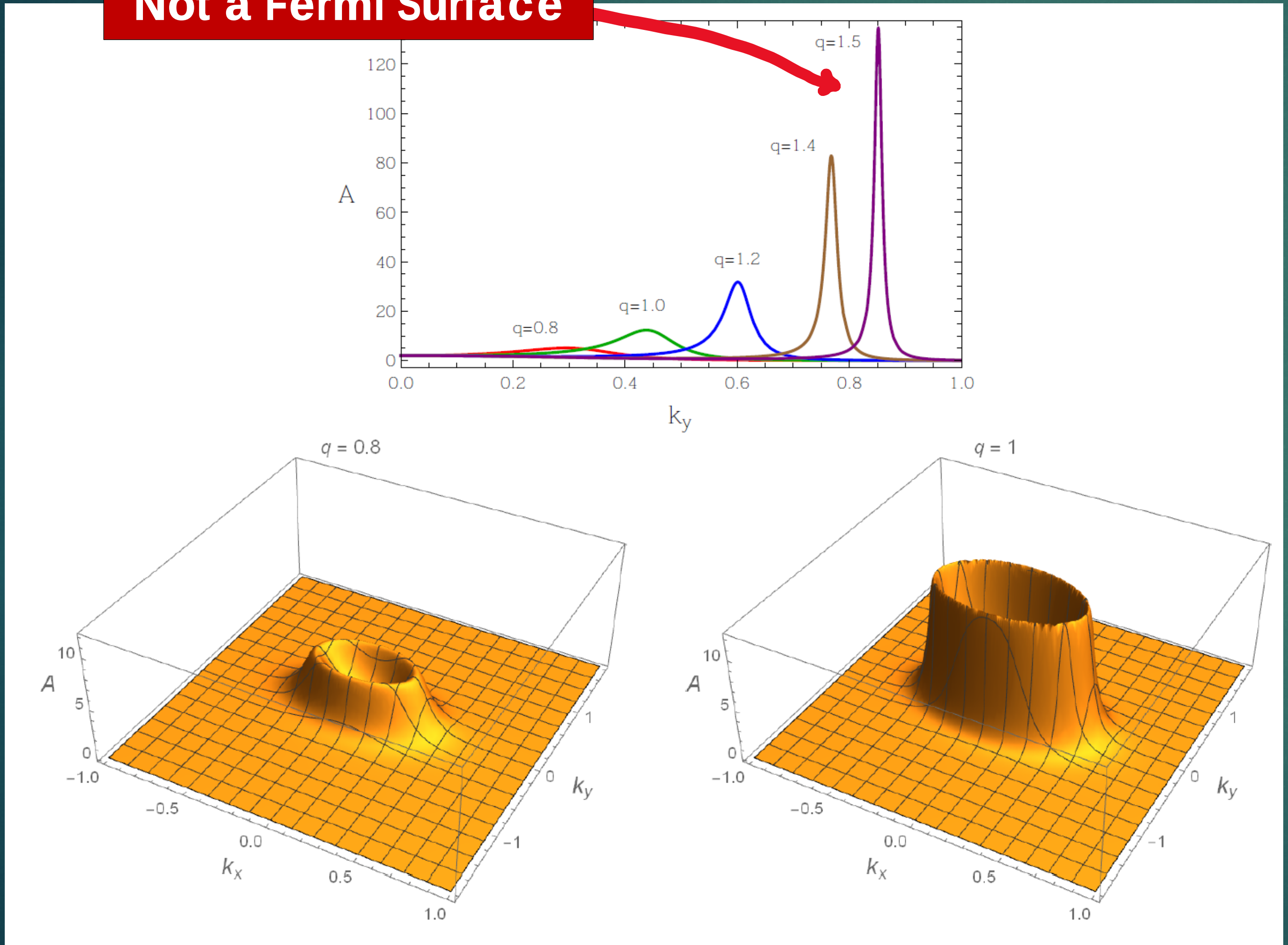
n: Brillouin zone
K: Umklapp vector

- Fermi surface: pole in spectral density at zero temperature as ω→ 0
- **Finite T criteria** to identify Fermi surface (<u>width, frequency and magnitude criteria</u>) introduced in Cosnier-Horeau & Gubser, arXiv:1411.5384
- Spectral function (diagonal momentum basis – **expect dominant response to be in <u>diagonal momentum channel</u>**)

$$A(\omega,k_x,k_y) = \sum_{n=0,\pm1,\pm2,\cdots} \mathrm{Tr}\ \mathrm{Im}[G^R_{\alpha,n;\alpha',n}(\omega,k_x,k_y)]$$

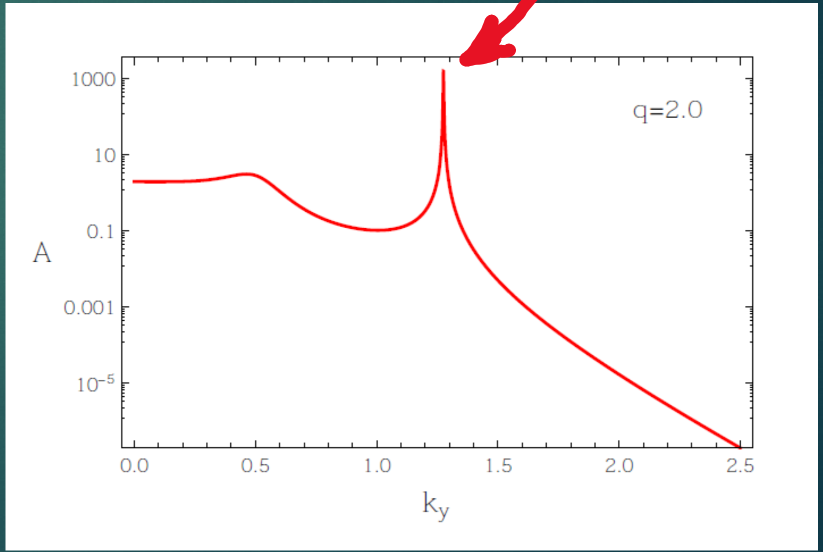
Fermi surface present when fermionic charge is large enough ✓

Not a Fermi Surface



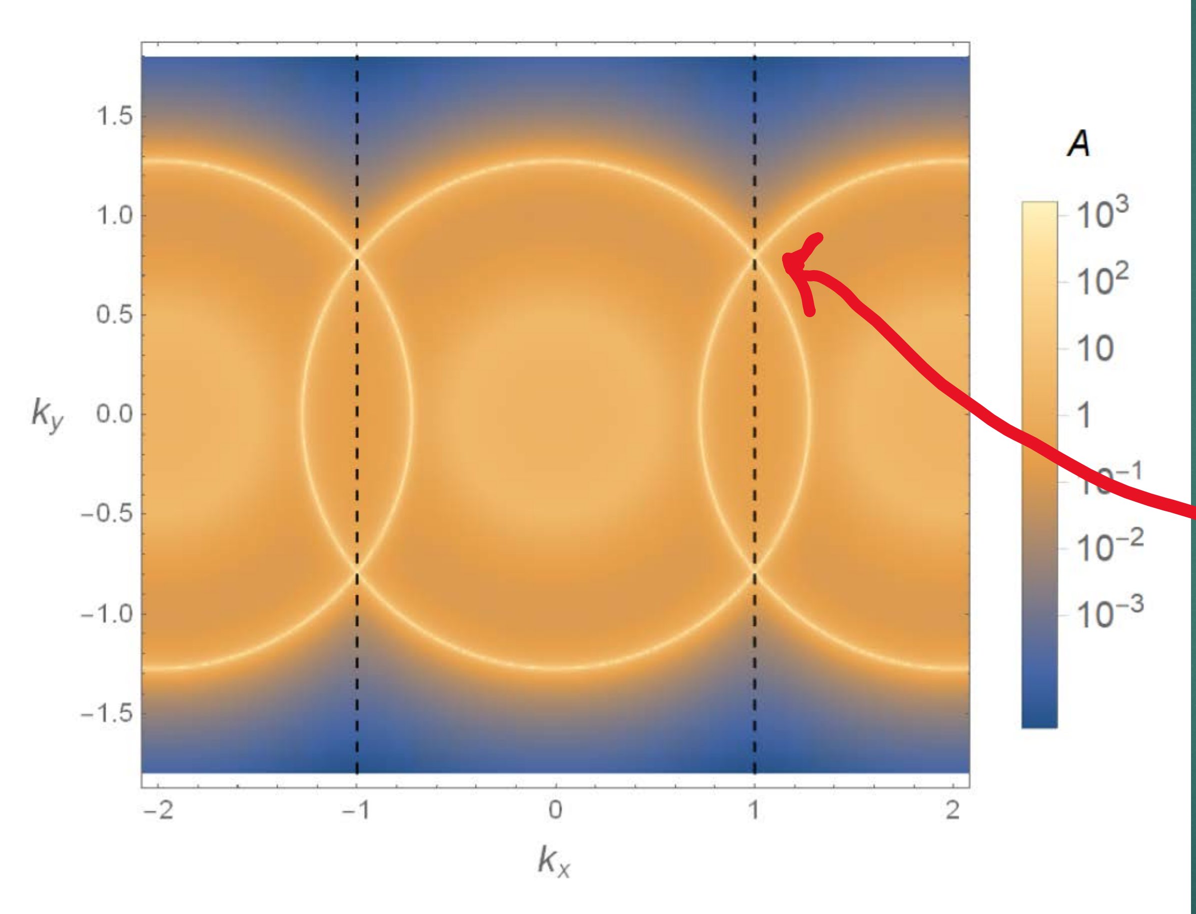
Peaks in spectral function become sharper and higher as q increases

Fermi Surface

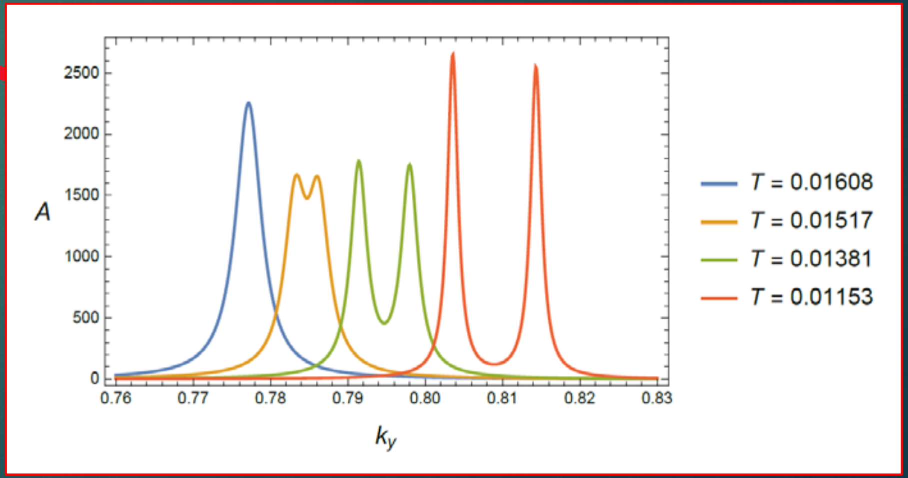


Spontaneous Case (pure PDW):

Gap opens up at T_c and increases as temperature is lowered

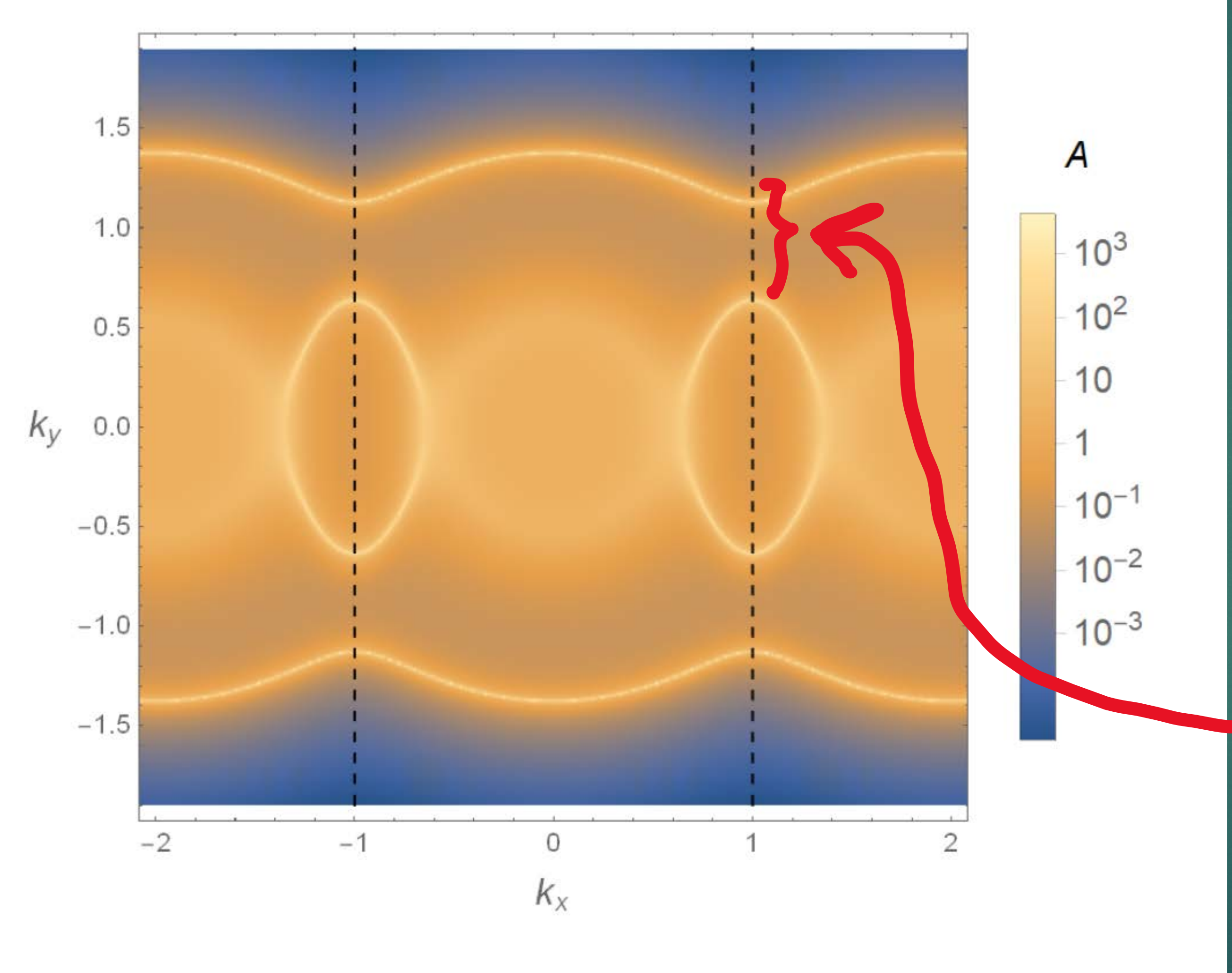


Shape becomes more anisotropic as strength of PDW modulation increases

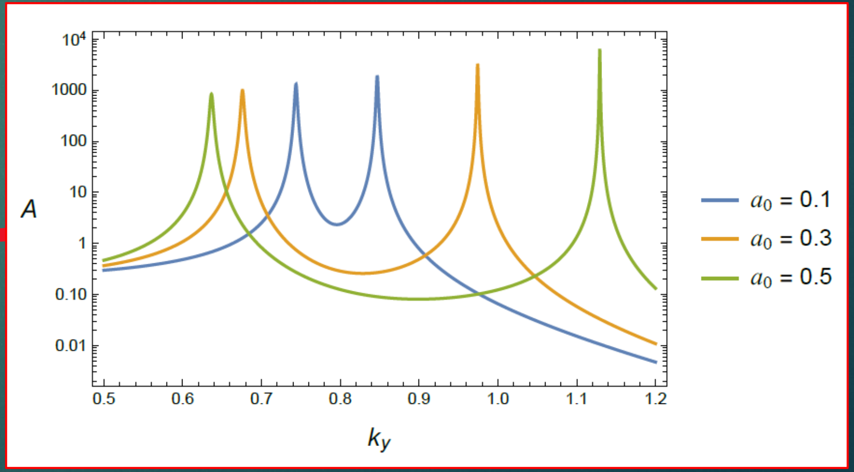


PDW + Ionic Lattice:

More pronounced anisotropy and larger gap



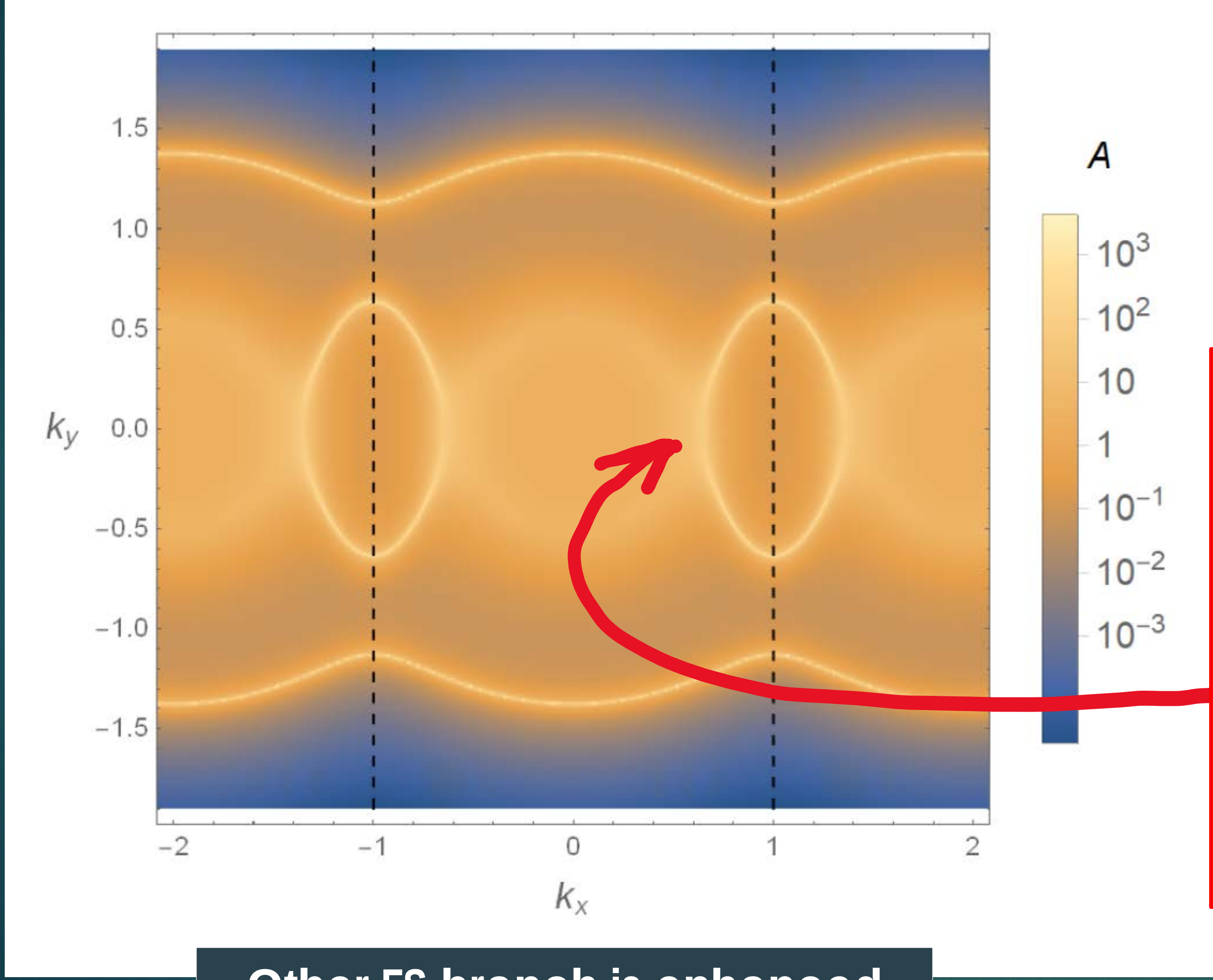
Larger gap at Brillouin zone boundary (increases with lattice amplitude)

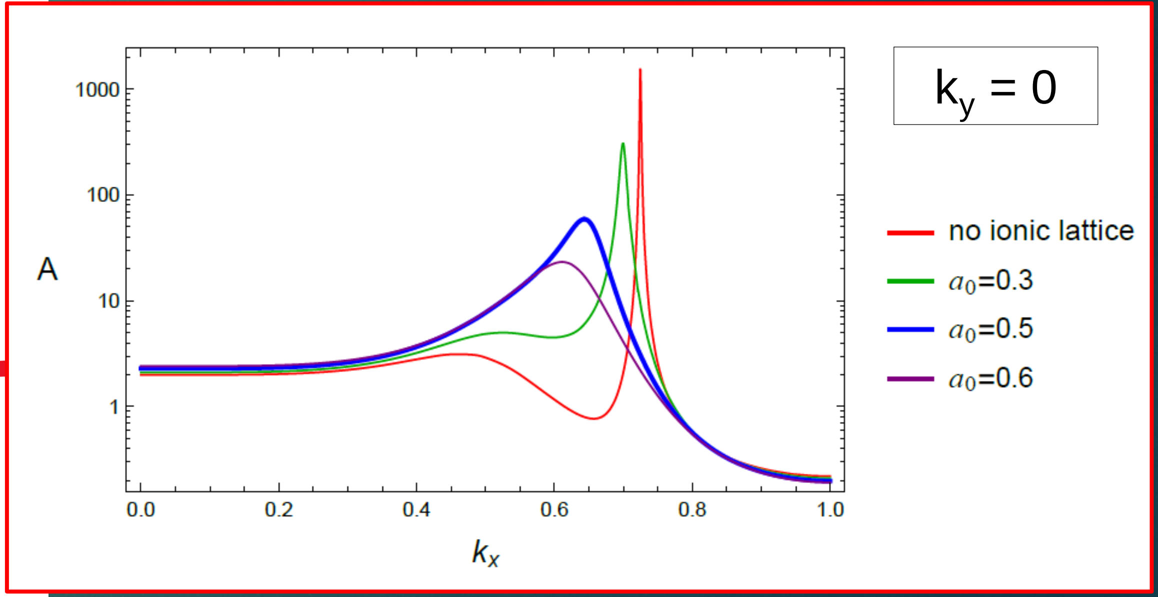


Interesting Feature:

Fermi surface gradually dissolves with strong lattice effects

spectral weight peaks suppressed with large lattice strength → Fermi surface gradually dissolves leaving behind detached segments





Other FS branch is enhanced

Laundry list of questions to address…

Our motivation today: **under which conditions does a Fermi surface form and dissolve in strongly correlated systems?**

- Suppression of spectral weight at strong ionic lattice in our analysis is a **UV effect**
  With **strong IR modulation** (spontaneous)? Generic result of strong inhomogeneity?
  To answer this question in our construction requires reaching **much lower temperatures**
- **Segmented pieces** are left over → related to Fermi arcs? Generic result of strong disorder?
- Incommensurate case? Role of two different scales of translational symmetry breaking?
- How does this compare to **expectations from PDW phases?**
- Fermionic response in **T=0 ground state**?
- Novel features from **coupling fermion to other orders**? (free fermion so far)

Thank you