Holographic Striped Superconductors and Fermi Surfaces

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Today:

- Holographic Realization of Intertwined Orders (Pair Density Wave)
  S.C., Li Li, Jie Ren  arXiv:1612.04385 and arXiv:1705.05390

- Fermionic spectral functions in striped superconducting phases
  S.C., Li Li, Jie Ren  1808.xxxx

See also talks by Li Li and Jie Ren on Tuesday afternoon
Holography as a Theoretical Laboratory

Study **solvable models** that may be in the same universality class as strongly correlated QM phases

→ Can we understand **the basic mechanisms** underlying the dynamics and unconventional properties of these systems?

Draw qualitative and quantitative lessons → look for **universal features**

**Solvable** often implies working with overly simplified bottom-up **toy models**
Holography as a Theoretical Laboratory

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**Challenges for understanding strongly coupled QM phases of matter**

- Strong coupling, breakdown of Fermi-liquid theory, no quasiparticles

- An intrinsically complex phase diagram exhibiting a variety of orders
  → Phases may compete but may also cooperate with each other

- Rich structure of emergent IR phases

- Different scales in the system, long-range entanglement, ...
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Attempt to identify generic imprints of these symmetry breaking mechanisms
Holographic Realization of Intertwined Orders

Goal:
Break **translational and U(1) symmetry spontaneously** at same time (onset T)
- parent phase gives rise to daughter phases

In our model:

- Striped instability
  - Broken translations

- Superconducting instability
  - Broken U(1) symmetry

**Broken spontaneously at same time by the same mechanism** \(\Rightarrow\) intertwined

A particular type of striped superconductor
Motivation

Realize some of the features of **Pair Density Wave (PDW) order** of high temperature superconductors (cuprates)

Evidence for PDW: in pseudo-gap of cuprate high $T_c$ superconductor $\text{La}_{2-x} \text{Ba}_x \text{CuO (LBCO)}$
Scalar condensate (superconducting order) is spatially modulated + its oscillations average out to zero (no homogeneous component)

$$\langle O(x) \rangle \propto \cos(kx)$$

Charge density is modulated and oscillates at twice the frequency of the condensate

$$\rho(x) = \rho_0 + \rho_1 \cos(2kx)$$

Contrast to co-existing CDW + SC orders:

Scalar condensate has a uniform component, and oscillates at the same frequency as the CDW


Features of PDW order we focus on:

- Scalar condensate (superconducting order) is spatially modulated + its oscillations average out to zero (no homogeneous component)

\[ \langle O(x) \rangle \propto \cos(kx) \]

- Charge density is modulated and oscillates at twice the frequency of the condensate

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The Holographic Model
S.C., L. Li, J. Ren (1612.04385, 1705.05390)

4D Bottom-up Model (2+1 dual QFT)

\[ S = \frac{1}{2\kappa_N^2} \int d^4 x \sqrt{-g} \left[ \mathcal{R} - 2\Lambda + \mathcal{L}_m \right] \]

\[ \mathcal{L}_m = -\frac{1}{2} \partial_\mu \chi \partial^\mu \chi - \frac{Z_A(\chi)}{4} F_{\mu\nu} F^{\mu\nu} - \frac{Z_B(\chi)}{4} \tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu} - \frac{Z_{AB}(\chi)}{2} F_{\mu\nu} \tilde{F}^{\mu\nu} \]

\[ -\mathcal{K}(\chi) (\partial_\mu \theta - q_A A_\mu - q_B B_\mu)^2 - V(\chi) , \]

Keep in mind:

Minimal model to get specific features we are after

Can be made more realistic (PDW order parameter) at the cost of adding a more complicated matter sector
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Field content:
- Gravity
- Two real scalars \( \chi \) and \( \theta \)
- Two U(1) vector fields \( A_\mu \) and \( B_\mu \) with different physical interpretations:
  - \( A_\mu \rightarrow \text{charge density} \) of field theory
  - \( B_\mu \rightarrow \text{spectator field} \) or proxy for “spin” density or \textit{second species of charge carriers}
Stuckelberg Superconductor

Generalizes standard holographic SC
→ allows for more general couplings

Stuckelberg mechanism:
local gauge invariance encoded in

\[ \theta \to \theta + \alpha(x^\mu), \quad A_\mu \to A_\mu + \frac{1}{q_A} \partial_\mu \alpha(x^\mu) \]
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- Eq. (1)

\[ \mathcal{L}_m = -\kappa(\chi)(\partial_\mu \theta - q_A A_\mu - q_B B_\mu)^2 - V(\chi), \]

- Eq. (2)

Theory chosen so that symmetric phase \( \chi = \theta = B_\mu = 0 \) is described by standard charged black hole in AdS

\[ Z_A(\chi) = 1 + \frac{a}{2} \chi^2 \quad Z_B(\chi) = 1 + \frac{b}{2} \chi^2 \quad \kappa(\chi) = \frac{\kappa}{2} \chi^2 \]

\[ Z_{AB}(\chi) = c \chi \quad V(\chi) = \frac{1}{2} m^2 \chi^2 \]

Crucial coupling for seeding spatially modulated instabilities

\( c = 0 \rightarrow \text{leading unstable mode is not striped} \)
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4D Bottom-up Model (2+1 dual QFT)

\[
S = \frac{1}{2\kappa_N^2} \int d^4 x \sqrt{-g} [\mathcal{R} - 2\Lambda + \mathcal{L}_m]
\]

\[
\mathcal{L}_m = -\frac{1}{2} \partial_{\mu} \chi \partial^{\mu} \chi - \frac{Z_A(\chi)}{4} F_{\mu \nu} F^{\mu \nu} - \frac{Z_B(\chi)}{4} \tilde{F}_{\mu \nu} \tilde{F}^{\mu \nu} - \frac{Z_{AB}(\chi)}{2} F_{\mu \nu} \tilde{F}^{\mu \nu}
- \mathcal{K}(\chi)(\partial_\mu \theta - q_A A_\mu - q_B B_\mu)^2 - V(\chi),
\]

Note:

- At some critical temperature the system becomes unstable to the condensation of $\chi$ and $B_\mu$.
- Properties of condensate sensitive to whether $q_A q_B = 0$ or not (we will keep $q_A$ non-zero since the charge density will be associated with $A$).
- Today we focus on $q_B = 0$. 
Analytical $T=0$ analysis gives some insight \textbf{(violation of BF bound)}

Working to leading order in perturbations:

$$\delta \chi = \varepsilon \, w(r) \cos(k \, x), \quad \delta B_t = \varepsilon \, b_t(r) \cos(k \, x)$$

There are nonzero $k$ values at which the \textbf{scaling dimension becomes imaginary} $\rightarrow$ instabilility
Critical Temperature for Instability

For a given $k$ there will be a normalizable zero mode appearing at a particular $T \rightarrow T_c$

Condensate is driven by a spatially modulated mode

Minimize free energy $\rightarrow$ thermodynamically preferred solution
Critical Temperature for Instability

Simple observation: Condensation at nonzero k will always occur at higher $T_c$ than in homogeneous phase (because of generic bell shape of instability curve)

Spatial modulations “enhance” the superconducting critical temperature → Facilitate the transition
A Cartoon Picture

Gravity Side

Standard AdS-RN black hole supported by U(1) gauge field $A_\mu$

$\chi = B_\mu = 0$

New type of striped IR geometry

$\chi, B_\mu$ nontrivial

Dual Field Theory

Global U(1) symmetry

Spatially modulated instabilities
System unstable to condensation of $\chi$ and $B_\mu$

U(1) and translational symmetry broken spontaneously at the same time

$\langle O_\chi \rangle \sim \cos(k_c x)$

$\rho_B = \langle J_B^t \rangle \sim \cos(k_c x)$

$\rho_A = \langle J_A^t \rangle \sim \cos(2k_c x)$

Modulated condensate and charge density
NOTE: spatial modulations are imprinted on the horizon (IR)

→ Stripes are relevant deformation of the UV CFT

The Striped Geometry (PDW)

\[
d s^2 = \frac{r_h^2}{L^2 (1 - z^2)^2} \left[ -F(z) Q_{tt} \, dt^2 + \frac{4 z^2 L^4 Q_{zz}}{r_h^2 F(z)} \, dz^2 + Q_{xx} (dx - 2 z (1 - z^2) Q_{xz} dz)^2 + Q_{yy} \, dy^2 \right]
\]
The Scalar Field Condensate (PDW)

Scalar condensate oscillations average out to zero (PDW feature)

\[ \langle O_X \rangle \sim \cos(k_c x) \]
Vector Field Profiles (PDW)

charge density oscillates twice as fast as scalar condensate (PDW feature)

"charge" density (proxy for SDW?) oscillates at same frequency as scalar condensate
We have seen a concrete realization of spontaneously generated intertwined striped superconducting phases of various type (e.g. PDW but also superconducting order co-existing with charge density wave order)

Next:

Examine fermionic spectral functions in these phases (including effects of explicit breaking of translations)
A lot of work on fermionic response in holography (e.g. review by Iqbal, Liu and Mezei, 1110.3814) but most studies focused on cases with translational invariance or homogeneous lattices.

To make contact with real materials important to include effects of periodic lattices.

Very few holographic studies on fermions in inhomogeneous systems.

Our work is motivated by and builds on:

- Among features identified: anisotropic FS and appearance of a gap
In the models of arXiv:1205.5227, arXiv:1304.2128 lattice is irrelevant in the IR

Our main interest:

Role of *spontaneous vs. explicit translational symmetry breaking* on fermionic spectral functions (in striped superconducting phases)


Setup:

- place a probe fermion in the *spontaneously generated* striped superconducting background, and include a source in the UV to break translations *explicitly* (ionic lattice)
- Dirac equation solved numerically, recall geometry has periodic modulation so solutions will reflect this periodicity (*Bloch expansion*)
Breaking Translations - Explicit vs. Spontaneous

Gauge field profile

**PDW order**
(s spontaneous breaking)

\[ \rho_A = \langle J_A^t \rangle \sim \cos(2k_c x) \]

**PDW + Ionic Lattice**
(explicit breaking in UV, spontaneous in IR)

\[ \mu(x) = A_t(1, x) = \mu[1 + a_0 \cos(px)] \]

Note: periods chosen to be the same (p=2k) → lattice is commensurate with CDW
Breaking Translations - Spontaneous vs. Explicit

Geometry - typical metric profile

Pure PDW (spontaneous)

PDW + Ionic lattice

Horizon (z=0)

Boundary (UV)

\[ ds^2 = \frac{r_h^2}{L^2(1-z^2)^2} \left[ -F(z)Q_{tt} \, dt^2 + \frac{4z^2L^4Q_{zz}}{r_h^2 F(z)} \, dz^2 + Q_{xx}(dx - 2z(1-z^2)Q_{zz}dz)^2 + Q_{yy} \, dy^2 \right] \]

\[ \chi = (1-z^2)\phi, \quad A_t = \mu z^2 \alpha, \quad B_t = z^2 \beta. \]
Periodicity of spatially modulated background sets size of Umklapp vector $K$

Solutions will reflect periodicity of background (Bloch expansion, periodic in $x$ with period $2\pi/K$)

Fermi surface: pole in spectral density at zero temperature as $\omega \to 0$

Finite $T$ criteria to identify Fermi surface (width, frequency and magnitude criteria) introduced in Cosnier-Horeau & Gubser, arXiv:1411.5384

Spectral function (diagonal momentum basis - expect dominant response to be in diagonal momentum channel)
Fermi surface present when fermionic charge is large enough.
Spontaneous Case (pure PDW):
Gap opens up at $T_c$ and increases as temperature is lowered.

Shape becomes more anisotropic as strength of PDW modulation increases.
PDW + Ionic Lattice:
More pronounced anisotropy and larger gap

Larger gap at Brillouin zone boundary (increases with lattice amplitude)
Interesting Feature:
Fermi surface gradually dissolves with strong lattice effects

spectral weight peaks suppressed with large lattice strength
→ Fermi surface gradually dissolves leaving behind detached segments

Other FS branch is enhanced
Our motivation today: **under which conditions does a Fermi surface form and dissolve in strongly correlated systems?**

- Suppression of spectral weight at strong ionic lattice in our analysis is a **UV effect**. With strong **IR modulation** (spontaneous)? Generic result of strong inhomogeneity? To answer this question in our construction requires reaching **much lower temperatures**.

- **Segmented pieces** are left over → related to **Fermi arcs**? Generic result of strong disorder?

- Incommensurate case? Role of two different scales of translational symmetry breaking?

- How does this compare to **expectations from PDW phases**?

- Fermionic response in **T=0 ground state**?

- Novel features from **coupling fermion to other orders**? (free fermion so far)
Thank you