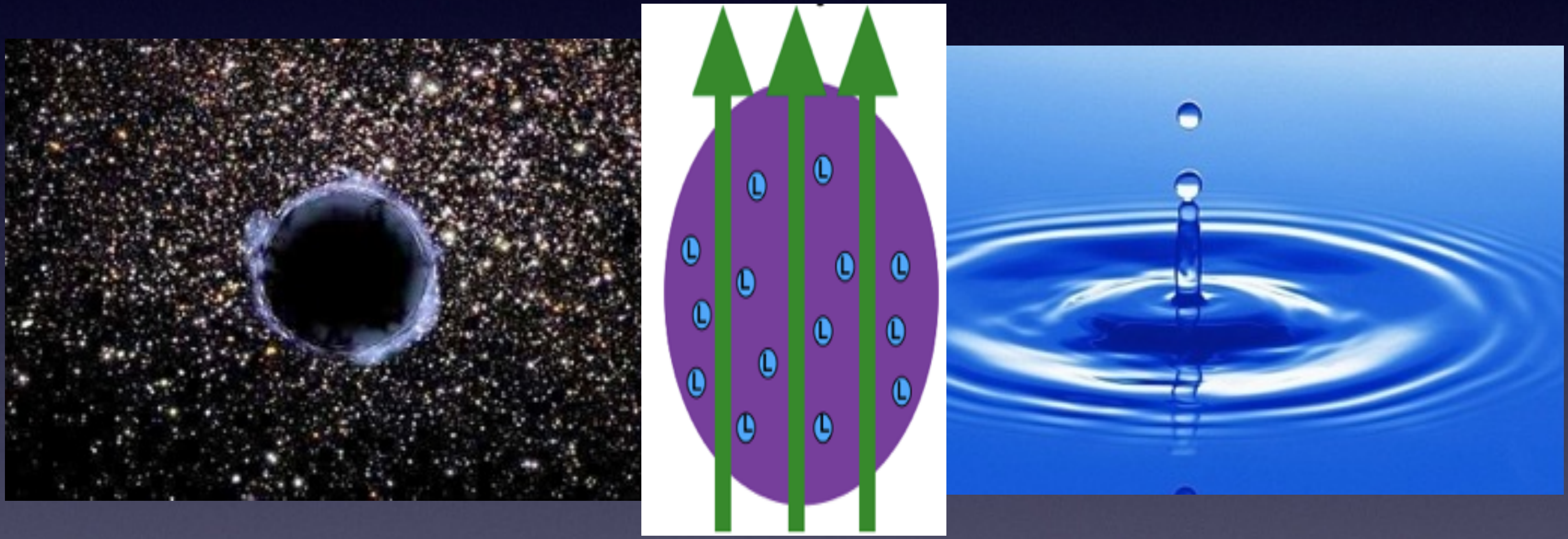
Chiral transport in strong magnetic fields from hydrodynamics & holography

Gauge/Gravity Duality 2018, Julius-Maximilians-Universität Würzburg

July 30th, 2018



Matthias Kaminski *(University of Alabama)*
in collaboration with
Juan Hernandez *(Perimeter Institute)*
Roshan Koirala, Jackson Wu *(University of Alabama)*
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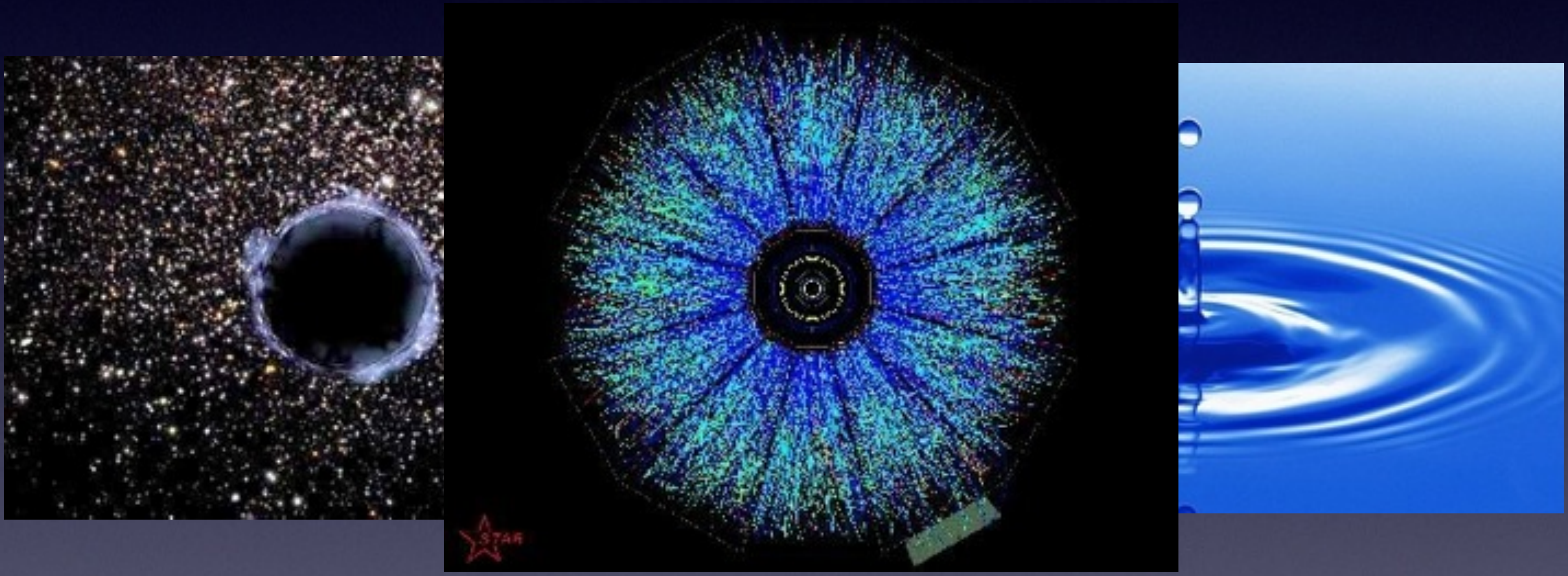

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Odd transport

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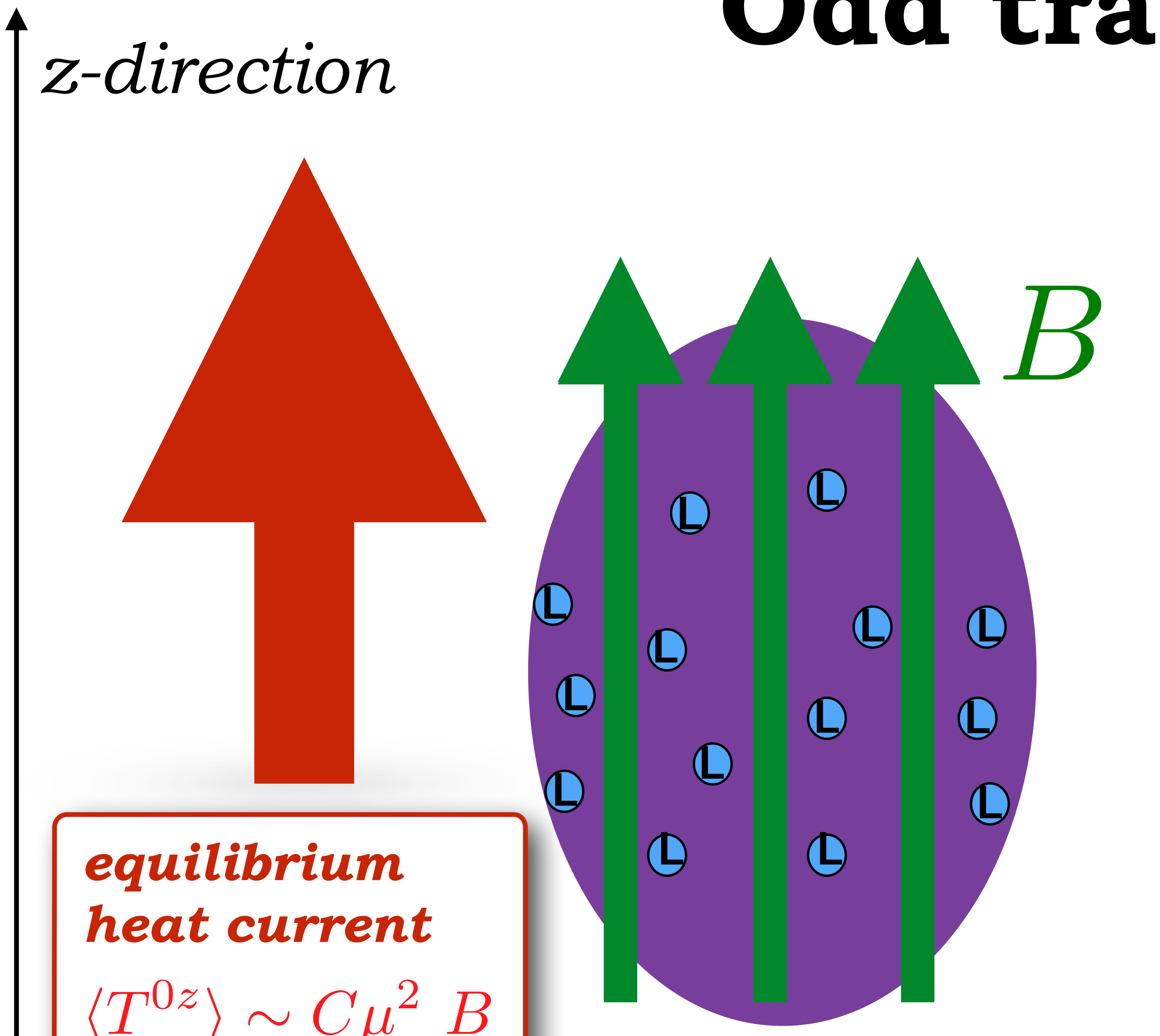
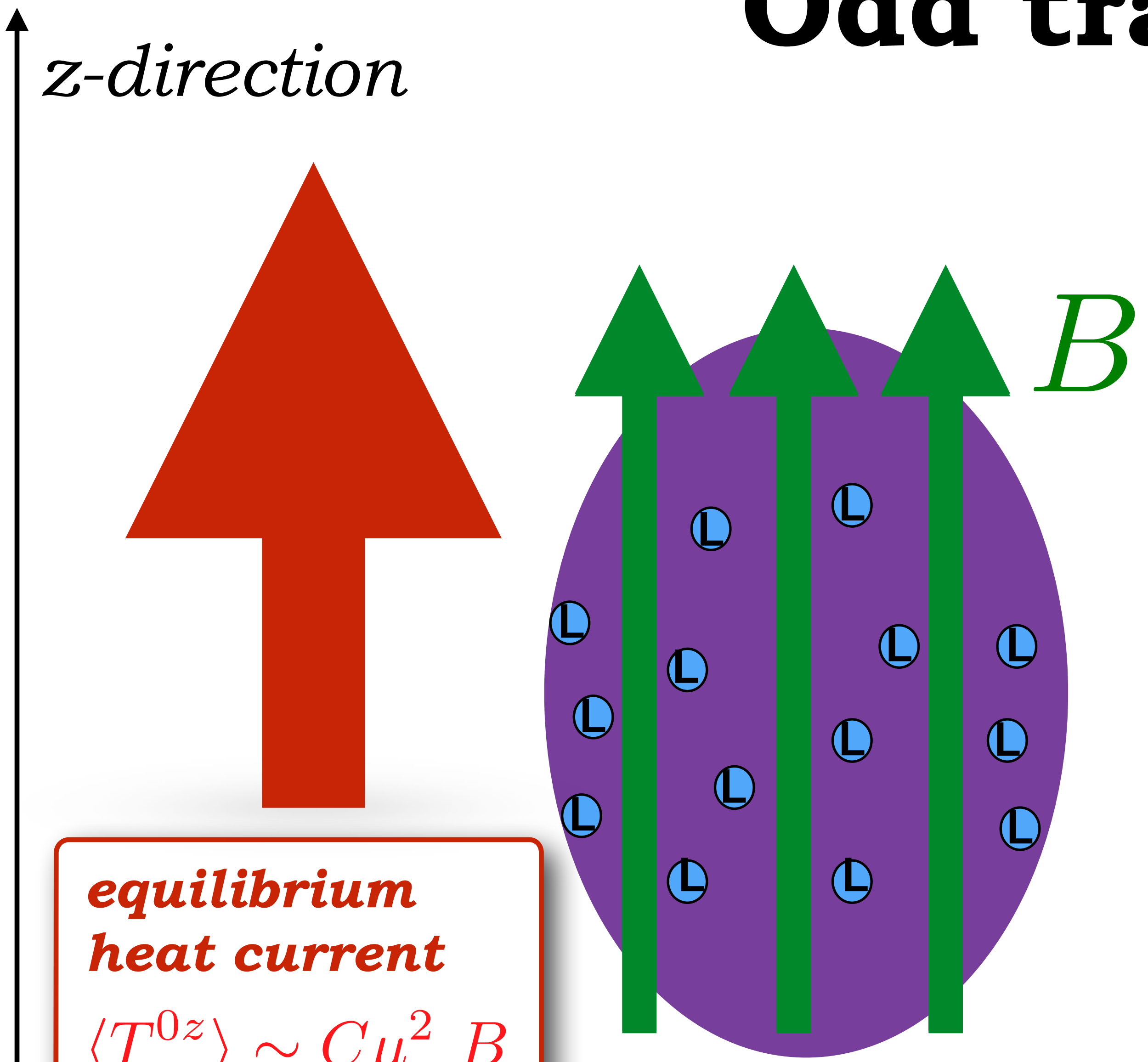


equilibrium heat current

$$\langle T^{0z}\rangle \sim \underbrace{C\mu^2}_{\sim\xi_V}\ B$$

[Ammon, Kaminski et al.; JHEP (2017)]
[Ammon, Leiber, Macedo; JHEP (2016)]

Odd transport



non-equilibrium parallel conductivity / perpendicular resistivity

$$\langle J^z J^z\rangle(\omega, \mathbf{k}=0) \sim \sigma_{||}$$

$$\langle J^x J^x\rangle(\omega, \mathbf{k}=0) \sim \rho_{\perp}$$

equilibrium heat current

$$\langle T^{0z}\rangle \sim \underbrace{C\mu^2}_{\sim\xi_V} B$$

[Ammon, Kaminski et al.; JHEP (2017)]
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non-equilibrium parity-odd transport

$$\langle J^x J^y\rangle(\omega, \mathbf{k}=0) \sim \frac{n}{B} - \omega^2 \frac{w^2}{B^4}\tilde{\rho}_{\perp} + \dots$$

Hall type

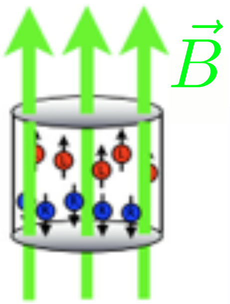
$$\langle J^x J^y\rangle(\omega=0, \mathbf{k}) \sim -ik \underbrace{\xi_B}_{C\mu}$$

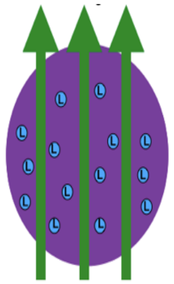
anomaly type

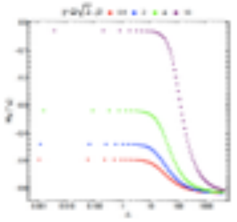
Outline

✓ Invitation: Odd transport

1. Review: hydrodynamics & holography
2. (Chiral magnetic) hydrodynamics
3. Holographic setup
4. Results
5. Discussion







1. Review: hydrodynamics & holography

Famous result: low shear viscosity over entropy density

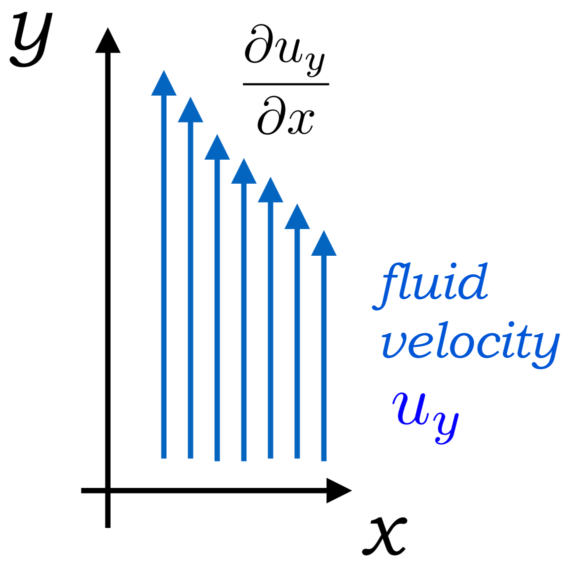
[Policastro, Son, Starinets; JHEP (2002)]

[RHIC measurement; (2004)]

KSS "bound": [Kovtun, Son, Starinets PRL (2005)]

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

Shear viscosity measures transverse momentum transport:



Kubo formula derived from hydrodynamics:

$$\eta = \lim_{\omega\to 0} \frac{1}{2\omega} \int dt\, d\boldsymbol{x}\, e^{i\omega t} \langle [T_{xy}(x),\, T_{xy}(0)] \rangle$$

from constitutive relation:

$$\langle T_{xy} \rangle \sim \eta\, \sigma_{xy}$$

$$\sim \eta(\nabla_x u_y + \nabla_y u_x)$$

1. Review: hydrodynamics & holography

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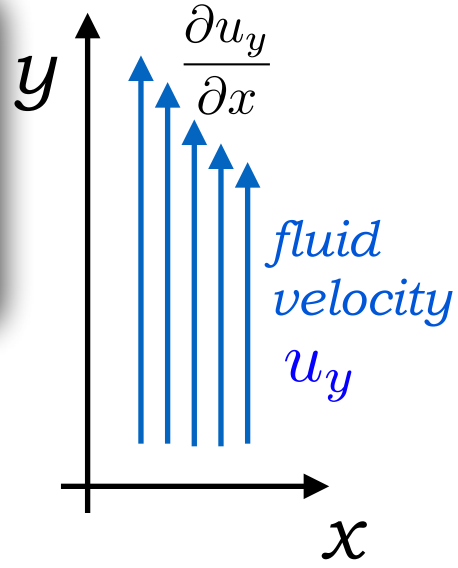

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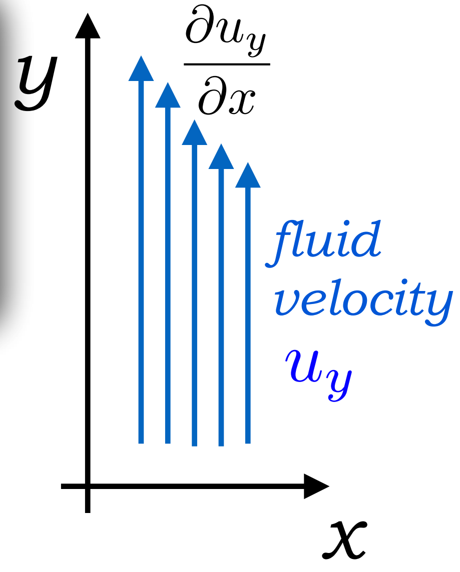

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Holographic calculation:

$$S = \frac{\pi^3 R^5}{2\kappa_{10}^2} \left[\int^1 du \int d^4x \sqrt{-g}\,(\mathcal{R} - 2\Lambda) + 2 \int d^4x \sqrt{-h}\, K \right]$$

$$ds_{10}^2 = \frac{(\pi T R)^2}{u}\left(-f(u)dt^2 + dx^2 + dy^2 + dz^2\right) + \frac{R^2}{4u^2 f(u)} du^2 + R^2 d\Omega_5^2 \quad \Rightarrow s = \frac{\pi^2}{2} N^2 T^3$$

$$f(u) = 1 - u^2$$

black brane metric

entropy density

Holographic correlation function: *[Son, Starinets; JHEP (2002)]*

$$G_{xy,xy}(\omega, \boldsymbol{q}) = -\frac{N^2 T^2}{16}\left(i\, 2\pi T\omega + q^2\right) \quad \Rightarrow \quad \eta = \frac{\pi}{8} N^2 T^3$$

shear viscosity

1. Review: hydrodynamics & holography

Famous result: low shear viscosity over entropy density

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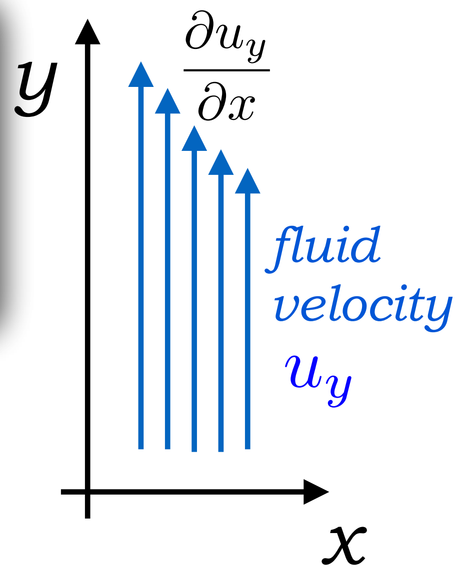

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$f(u) = 1 - u^2$ *black brane metric* *entropy density*

Holographic correlation function: *[Son, Starinets; JHEP (2002)]*

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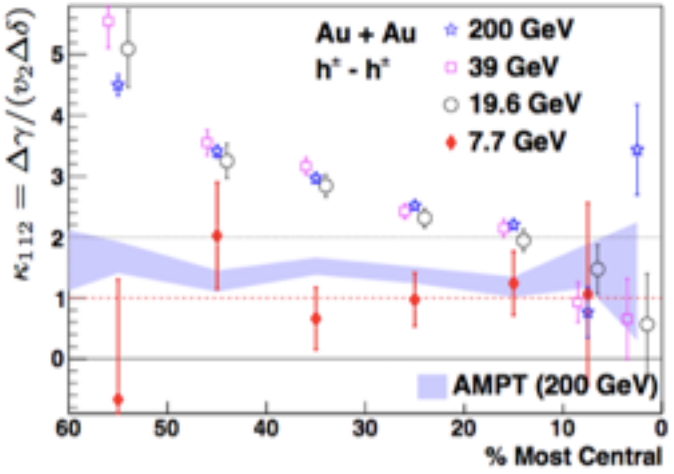
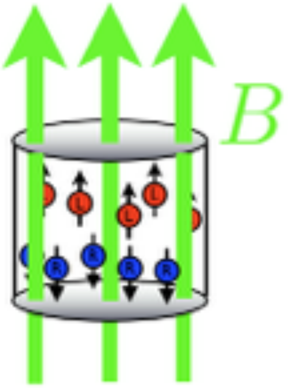
shear viscosity

2. Chiral magnetic hydrodynamics - Motivation

Chiral magnetic effect
- heavy ion collisions (HICs)

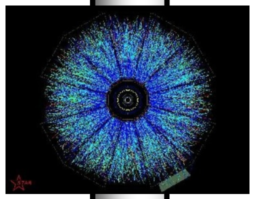


Beam Energy Scan;
Isobaric collisions: Zr / Ru
[RHIC STAR Collaboration; PoS (2018)]

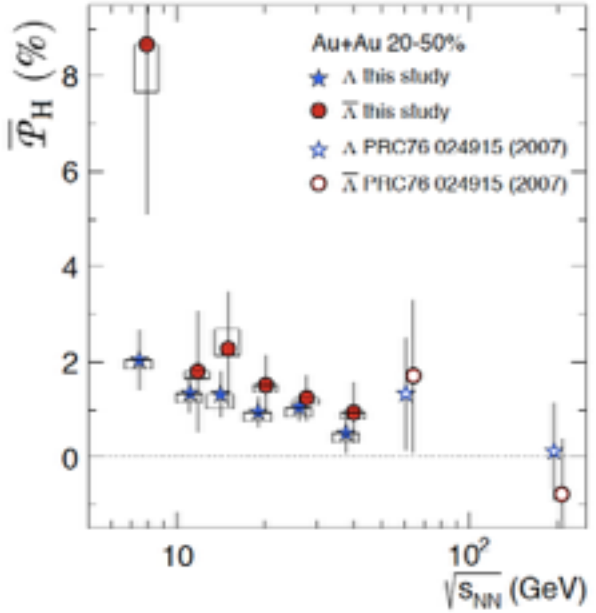


[Fukushima, Kharzeev, Warringa; PRD (2008)]
[Son,Surowka; PRL (2009)] **...**

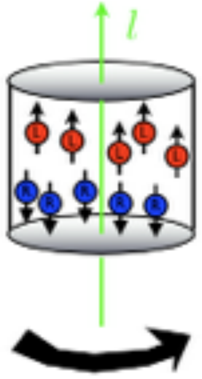
also ***cond-mat*** *and* ***plasma physics***



Most vortical fluid in HICs
- Lambda hyperon polarization



[RHIC STAR Collaboration; Nature (2017)]



vorticity

[Erdmenger, Haack, Kaminski, Yarom; JHEP (2008)]
[Banerjee et al.; JHEP (2011)]
[Landsteiner] [Son,Surowka; PRL (2009)] **...**

see Koenraad Schalm’s talk

Deriving chiral magnetic hydrodynamics

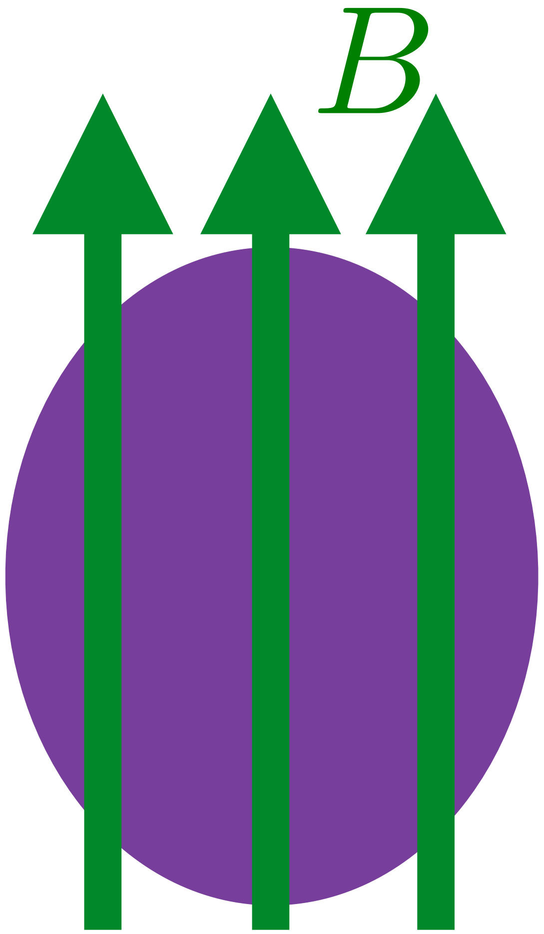
Consider a quantum field theory with a chiral anomaly, in a charged thermal plasma state, subjected to a strong external magnetic field

Hydro poles / eigenmodes, and QNMs: [Ammon, Kaminski et al.; JHEP (2017)]

Range of validity $B_0 \sim \mathcal{O}(1)$ $B_0 \ll T_0^2$

$\omega, k \ll T_0$

- equilibrium generating functional
  [Jensen, Kaminski, Kovtun, Meyer, Ritz, Yarom; PRL (2012)]
  [Kovtun; JHEP (2016)]
- equilibrium constitutive equations
  [Kovtun; JHEP (2016)]



$$W_s = \int d^4x\sqrt{-g}\left(p(T,\mu,B^2) + \sum_{n=1}^{5} M_n(T,\mu,B^2)s_n + O(\partial^2)\right)$$

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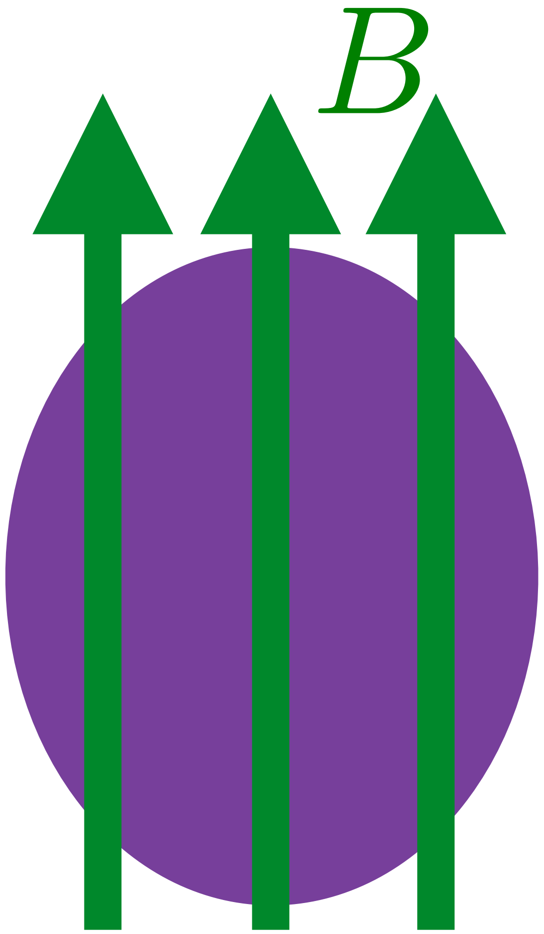
- equilibrium constitutive equations
  [Kovtun; JHEP (2016)]
- add time-dependent hydrodynamic terms
  [Kovtun, Hernandez; JHEP (2017)] $\Rightarrow$ **Kubo formulae**
- constrain through Onsager relations
  and
  entropy current

$$G^R_{\varphi_a\varphi_b}(\omega,\mathbf{k};\chi) = \eta_{\varphi_a}\eta_{\varphi_b}G^R_{\varphi_b^\dagger\varphi_a^\dagger}(\omega,-\mathbf{k};-\chi)$$

$$\nabla_\mu s^\mu \geq 0$$



Example relation for bulk viscosities:

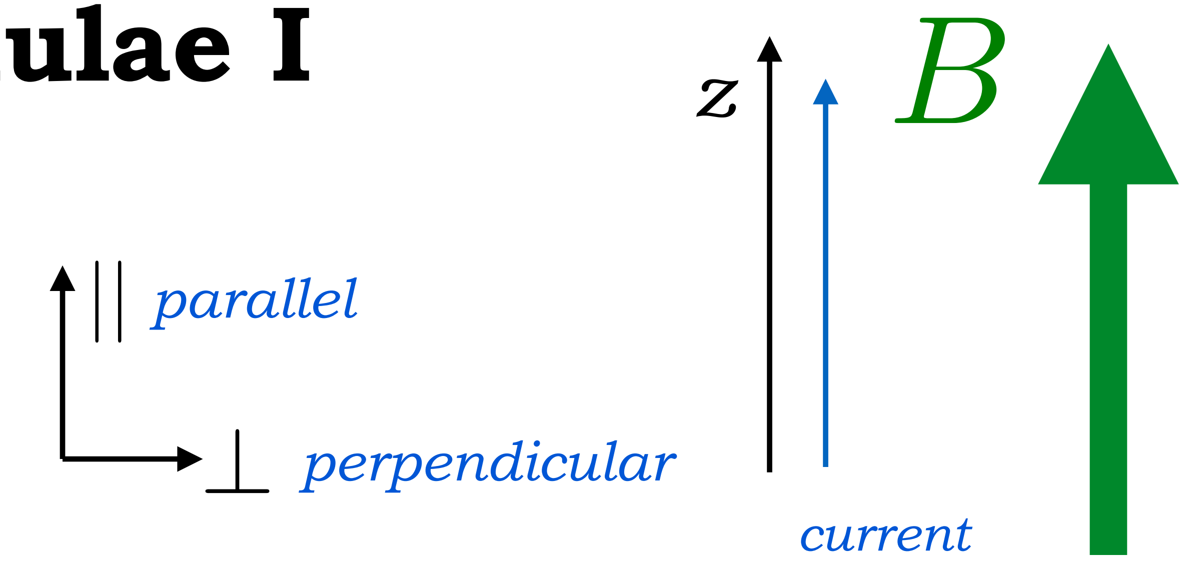
$$3\zeta_2 - 6\eta_1 - 2\eta_2 = 0$$

* thermodynamic frame
* consistent current

Kubo formulae I

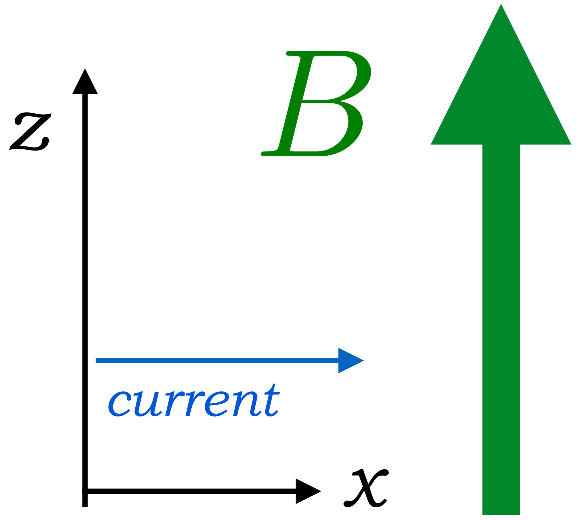
Parallel conductivity

$$\frac{1}{\omega}\mathrm{Im}\, G_{J^z J^z}(\omega, \mathbf{k}{=}0) = \sigma_{||} + \cdots$$



Perpendicular resistivity

$$\frac{1}{\omega}\mathrm{Im}\, G_{J^x J^x}(\omega, \mathbf{k}{=}0) = \omega^2 \rho_\perp \frac{w_0(w_0 - M_{5,\mu} B_0^2)}{B_0^4}$$



Magneto-vortical susceptibility

$$\frac{1}{k_z}\mathrm{Im}\, G_{T^{tx}T^{yz}}(\omega = 0, k_z \hat{k}) = -B_0\, M_5$$

$$W_S \sim M_5\, B \cdot \Omega$$

non-equilibrium parallel conductivity / perpendicular resistivity

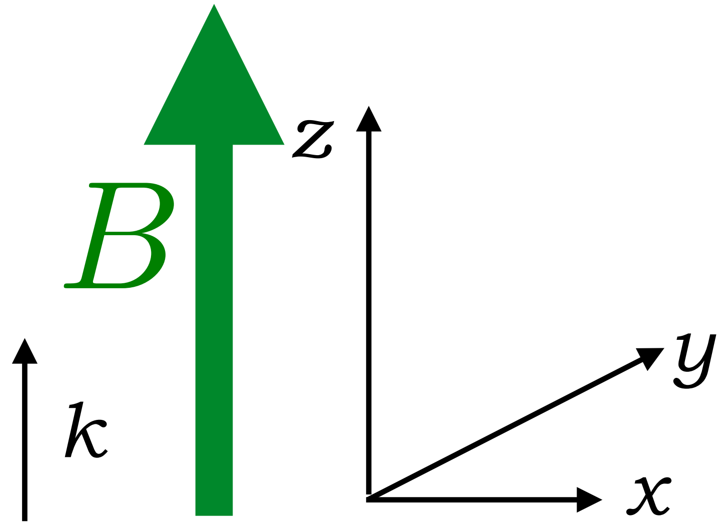
$$\langle J^z J^z \rangle(\omega, \mathbf{k} = 0) \sim \sigma_{||}$$

$$\langle J^x J^x \rangle(\omega, \mathbf{k} = 0) \sim \rho_\perp$$

Kubo formulae II

(Perpendicular) Hall resistivity

$$\frac{1}{\omega}\mathrm{Im}\, G_{J^x J^y}(\omega, \mathbf{k}{=}0) = \frac{n_0}{B_0} - \omega^2 \tilde{\rho}_\perp \frac{w_0(w_0 - M_{5,\mu} B_0^2)}{B_0^4}\,\mathrm{sign}(B_0)$$



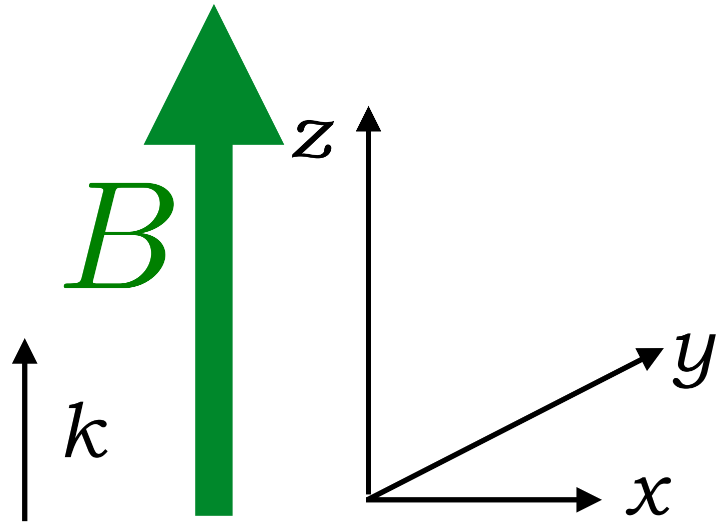
Chiral magnetic conductivity

$$\xi_B = \lim_{k\to 0} \frac{1}{-ik}\langle J^x J^y\rangle(\omega = 0, k) + \frac{1}{3} C\mu$$

Kubo formulae II

(Perpendicular) Hall resistivity

$$\frac{1}{\omega}\mathrm{Im}\,G_{J^x J^y}(\omega,\mathbf{k}{=}0) = \frac{n_0}{B_0} - \omega^2\tilde{\rho}_\perp \frac{w_0(w_0 - M_{5,\mu}B_0^2)}{B_0^4}\,\mathrm{sign}(B_0)$$



Chiral magnetic conductivity

$$\xi_B = \lim_{k\to 0}\frac{1}{-ik}\langle J^x J^y\rangle(\omega=0,k) + \frac{1}{3}C\mu$$

non-equilibrium
parity-odd transport

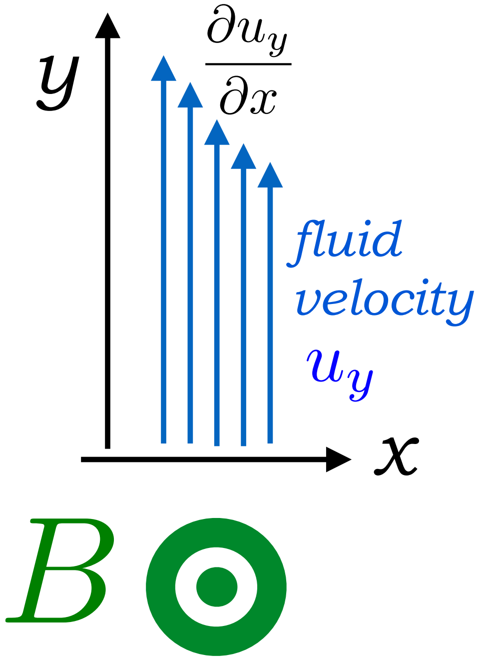
$$\langle J^x J^y\rangle(\omega,\mathbf{k}=0) \sim \frac{n}{B} - \omega^2\frac{w^2}{B^4}\tilde{\rho}_\perp + \dots$$

Hall type

$$\langle J^x J^y\rangle(\omega=0,\mathbf{k}) \sim -ik\,\underbrace{\xi_B}_{C\mu}$$

anomaly type

Kubo formulae III



Shear viscosity perpendicular

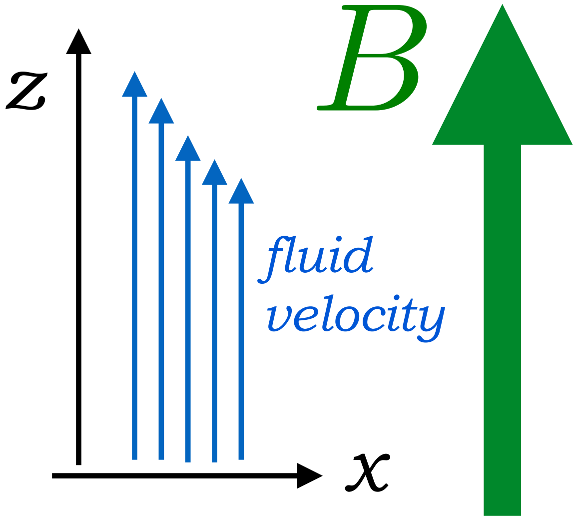
$$\frac{1}{\omega}\mathrm{Im}\, G_{T^{xy}T^{xy}}(\omega, \mathbf{k}{=}0) = \eta_\perp$$

Shear viscosity parallel

$$\frac{1}{\omega}\mathrm{Im}\, G_{T^{xz}T^{xz}}(\omega, \mathbf{k}{=}0) = \eta_{||} + (\bar{c}_8 c_{15} - c_{10}\bar{c}_{17})\rho_\perp - (\bar{c}_8 \bar{c}_{17} + c_{10} c_{15})\tilde{\rho}_\perp$$

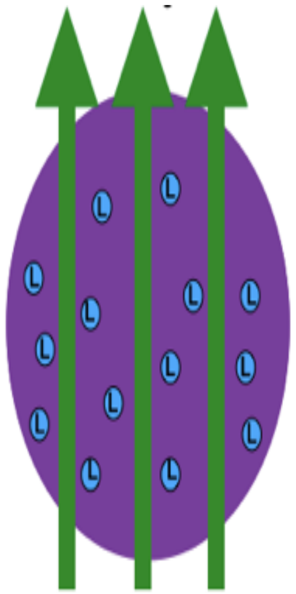
perpendicular resistivity

Hall resistivity



Holographic model values must satisfy:
➡ constraints
➡ consistency checks

3. Holographic setup



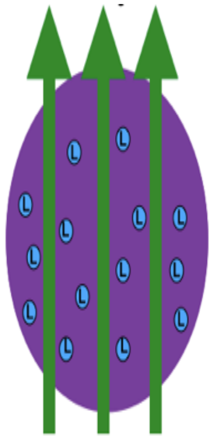
Action and background

Einstein-Maxwell-Chern-Simons action

chiral anomaly

$$S_{grav} = \frac{1}{2\kappa^2}\left[\int_{\mathcal{M}} d^5x\,\sqrt{-g}\left(R + \frac{12}{L^2} - \frac{1}{4}F_{mn}F^{mn}\right) - \frac{\gamma}{6}\int_{\mathcal{M}} A\wedge F\wedge F\right]$$

$$S_{bdy} = \frac{1}{\kappa^2}\int_{\partial\mathcal{M}} d^4x\,\sqrt{-\hat{g}}\left(K - \frac{3}{L} + \frac{L}{4}R(\hat{g}) + \frac{L}{8}\ln\left(\frac{\varrho}{L}\right)F_{\mu\nu}F^{\mu\nu}\right)$$



Magnetic black branes *[D'Hoker, Kraus; JHEP (2009)]*

- charged magnetic analog of RN black brane
- Asymptotically AdS5
- zero entropy density at vanishing temperature

$$ds^2 = \frac{1}{\varrho^2}\left[\left(-u(\varrho) + c(\varrho)^2\,w(\varrho)^2\right)\,dt^2 - 2\,dt\,d\varrho + 2\,c(\varrho)\,w(\varrho)^2\,dz\,dt\right.$$
$$\left.+v(\varrho)^2\,\left(dx^2 + dy^2\right) + w(\varrho)^2\,dz^2\right]\,,$$

$$F = \underbrace{A_t'(\varrho)\,d\varrho\wedge dt}_{\text{charge}} + \underbrace{B\,dx\wedge dy}_{\text{magnetic field}} + P'(\varrho)\,d\varrho\wedge dz\,,$$

Correlators from infalling fluctuations

see Richard Davison's talk

Problem: fluctuation equations are coupled (dual to operator mixing in QFT)

Numerical methods

- matrix method and shooting technique

[Kaminski, Landsteiner, Mas, Shock, Tarrio; JHEP (2010)]

$$G^{(ret)}(\mathbf{k}) = -2 \lim_{\epsilon \to 0} \mathcal{F}(\mathbf{k}, \epsilon)$$

$\Rightarrow$ frequency and momentum

find independent solutions to coupled systems (pure gauge solutions)

- one-point functions technique and spectral methods

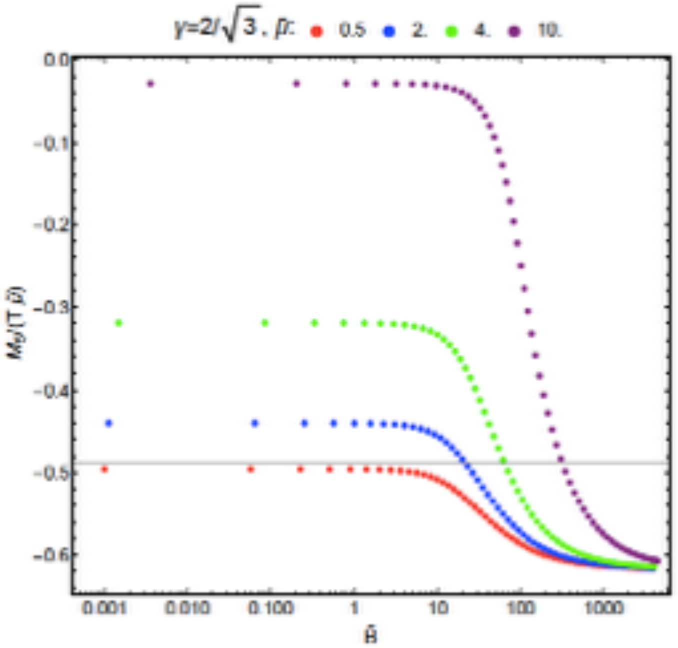
[Ammon, Grieninger, Hernandez, Kaminski, Koirala, Leiber, Wu; to appear]

$$\langle \mathcal{O}_A \, \mathcal{O}_B \rangle \sim \frac{\delta \langle \mathcal{O}_B \rangle}{\delta \phi_A}$$

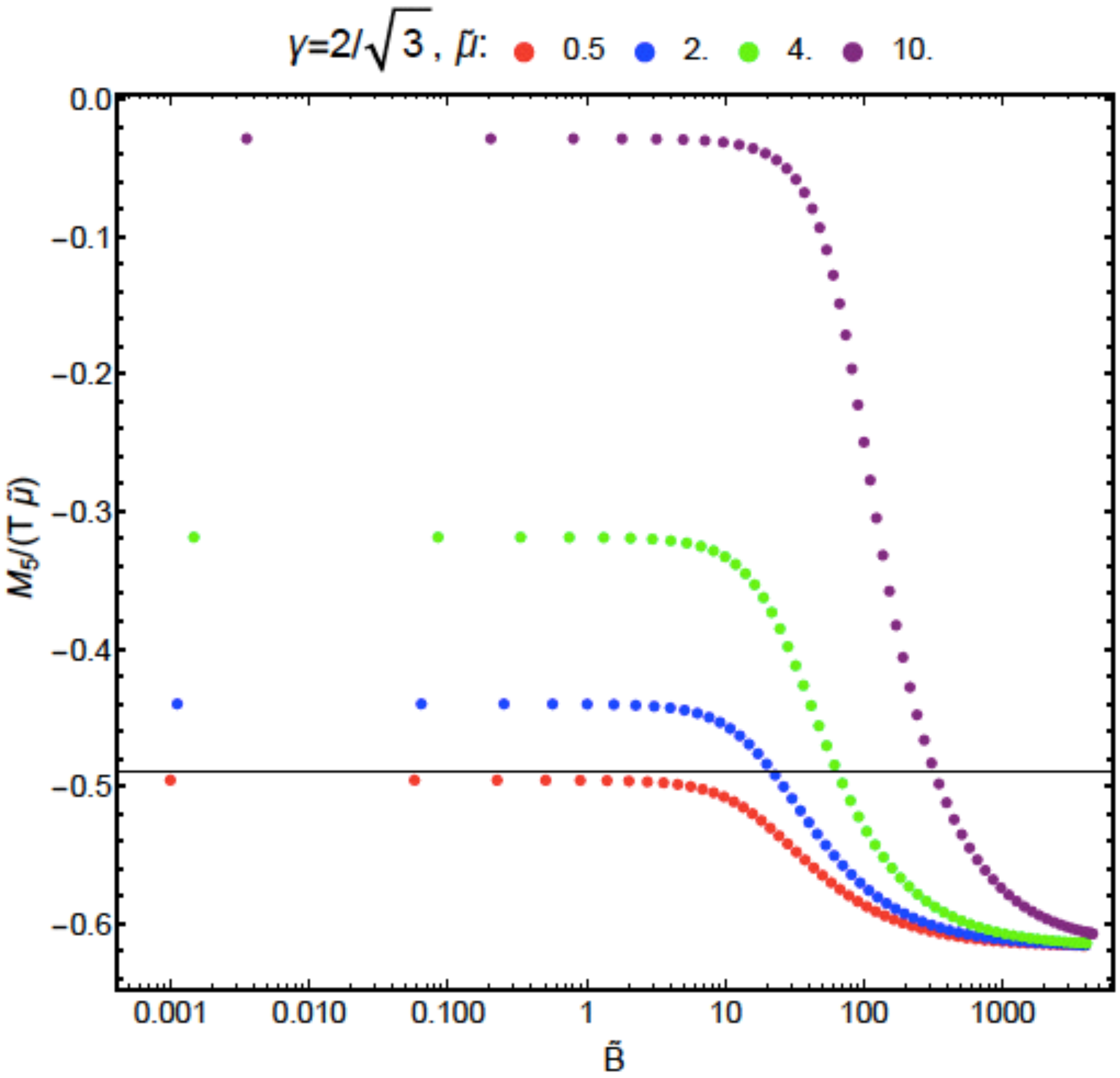
$\Rightarrow$ analytic relations

find independent solutions to coupled systems (no pure gauge solutions) preliminary

4. Results



Thermodynamic transport

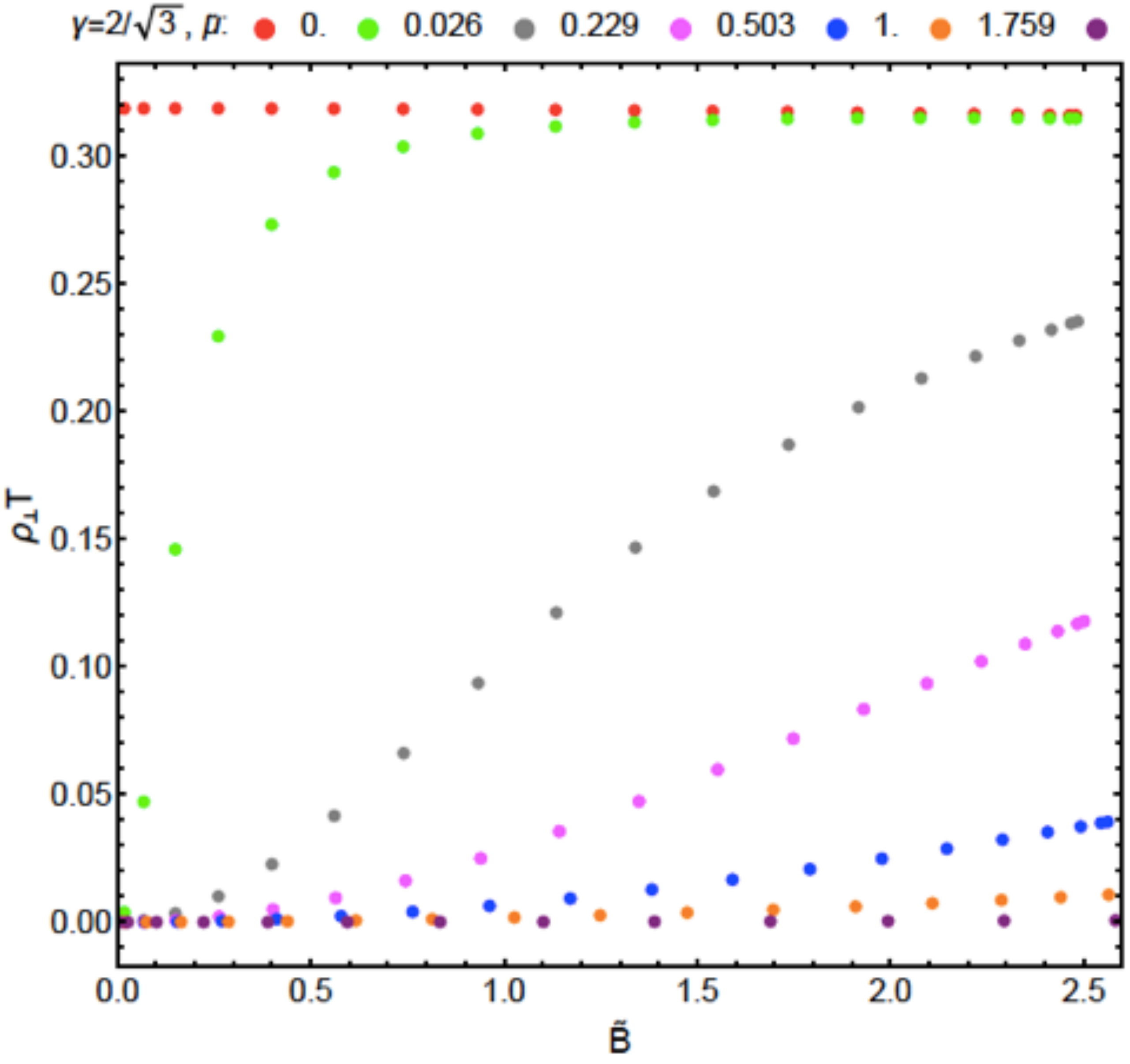


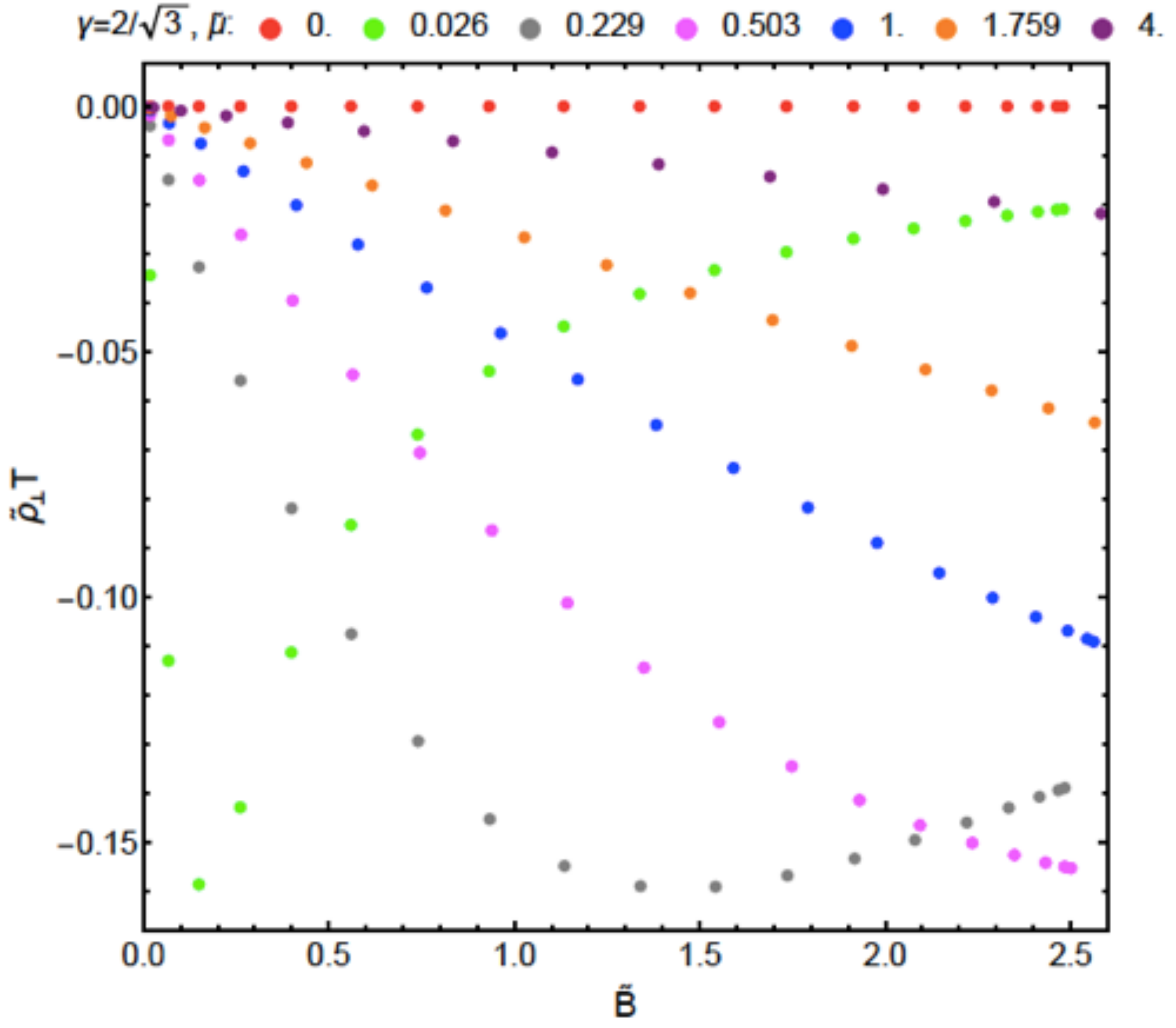
Magneto-vortical susceptibility

$$\frac{1}{k_z}\mathrm{Im}\, G_{T^{tx}T^{yz}}(\omega=0,k_z\hat{k}) = -B_0\, M_5$$

$$W_S \sim M_5\, B\cdot\Omega$$

Hydrodynamic transport

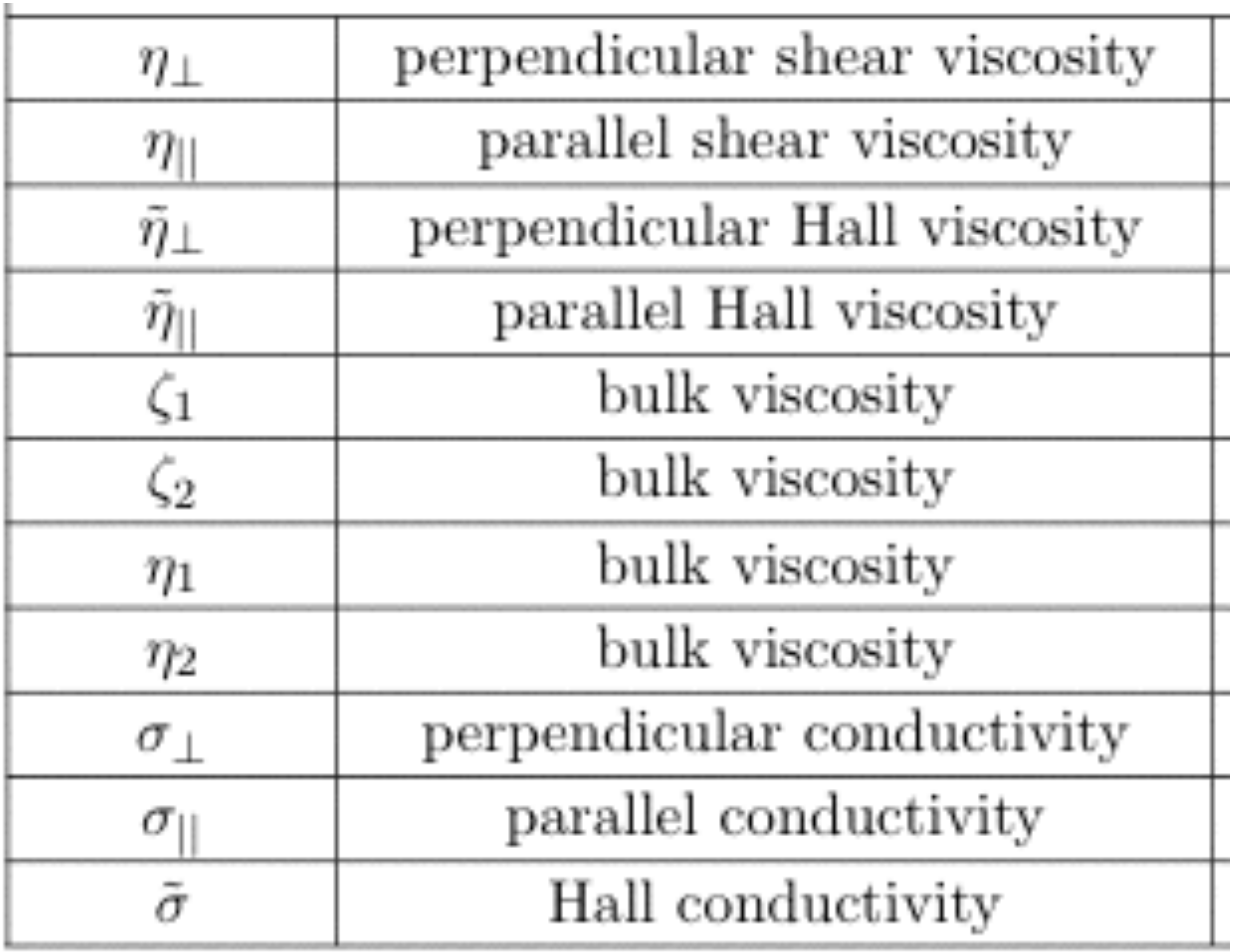




$$\frac{1}{\omega}\mathrm{Im}\,G_{J^xJ^x}(\omega,\mathbf{k}{=}0) = \omega^2\rho_\perp \frac{w_0(w_0 - M_{5,\mu}B_0^2)}{B_0^4}$$

$$\frac{1}{\omega}\mathrm{Im}\,G_{J^xJ^y}(\omega,\mathbf{k}{=}0) = \frac{n_0}{B_0} - \omega^2\tilde{\rho}_\perp \frac{w_0(w_0 - M_{5,\mu}B_0^2)}{B_0^4}\,\mathrm{sign}(B_0)$$

More transport coefficients



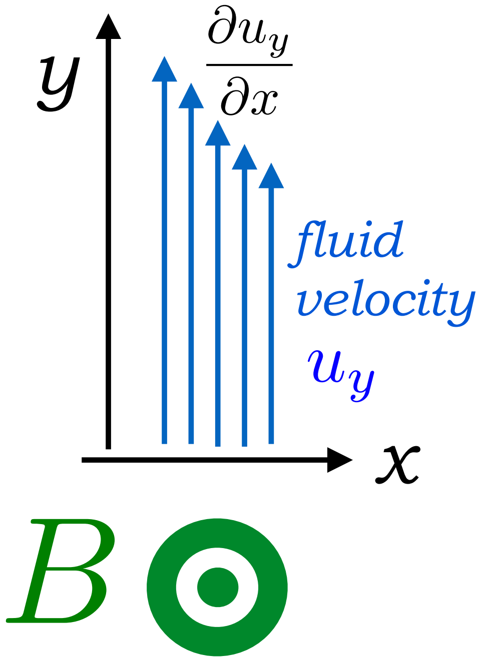
$\eta_\perp$	perpendicular shear viscosity
$\eta_{\|\|}$	parallel shear viscosity
$\tilde{\eta}_\perp$	perpendicular Hall viscosity
$\tilde{\eta}_{\|\|}$	parallel Hall viscosity
ζ_1	bulk viscosity
ζ_2	bulk viscosity
η_1	bulk viscosity
η_2	bulk viscosity
$\sigma_\perp$	perpendicular conductivity
$\sigma_{\|\|}$	parallel conductivity
$\tilde{\sigma}$	Hall conductivity

…

Analytic result from one-point function technique

Kubo formula: perpendicular shear viscosity

$$\frac{1}{\omega}\mathrm{Im}\, G_{T^{xy}T^{xy}}(\omega, \mathbf{k}{=}0) = \eta_\perp$$



Analytic result:

$$\eta_\perp = \lim_{\omega\to 0}\frac{1}{\omega}\mathrm{Im}\, G_{T^{xy}T^{xy}}(\omega, \mathbf{k}{=}0) = v(1)^2\, w(1)$$

$$s = 4\pi\, v(1)^2\, w(1)$$

$$\frac{\eta_\perp}{s} = \frac{1}{4\pi}$$

Discussion - Summary

- derived hydrodynamic transport coefficients & Kubo relations for QFT with chiral anomaly, in a charged thermal plasma state, within strong external B
- proof of existence within holographic model (EMCS)
- transport coefficients are nonzero and show non-trivial dependence on B, anomaly coefficient C, and chemical potential

  [Ammon, Grieninger, Hernandez, Kaminski, Koirala, Leiber, Wu; to appear]
- novel transport effects arise (e.g. perpendicular/parallel, unidentified)
- order zero CME (and CVE) *[Ammon, Kaminski et al.; JHEP (2017)]*

  [Ammon, Leiber, Macedo; JHEP (2016)]
- more motivation for strong B model: universal magneto response

  [Endrödi, Kaminski, Schäfer, Wu, Yaffe; arXiv:1806.09632]

Discussion - Outlook

- ➡ correlations far from equilibrium at high density and magnetic field with chiral anomaly
  *[Cartwright, Kaminski; to appear] see my **talk at HoloQuark2018***

- ➡ non-relativistic hydrodynamics & QNMs
  [Garbiso, Kaminski; to appear]
  [Davison, Grozdanov, Janiszewski, Kaminski; JHEP (2016)]
  [Janiszewski, Karch; PRL (2013)]
- ➡ dynamical electromagnetic fields - magnetohydrodynamics *[Kovtun, Hernandez; JHEP (2017)]*
- ➡ comparison to experimental data
  ([Endrödi, Kaminski, Schäfer, Wu, Yaffe; arXiv:1806.09632])

Collaborators

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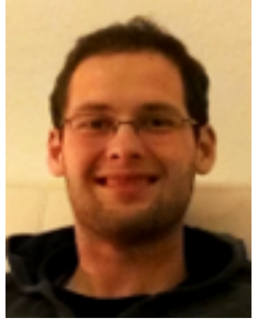

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APPENDIX

Charge, parity, time reversal

quantity	$\mathcal{C}$	$\mathcal{P}$	$\mathcal{T}$
t	+	+	-
x^i	+	-	+
r	+	+	+
T, h_{tt}, T^{tt}	+	+	+
μ_A, A_t, J^t	+	-	+
A_i, J^i	+	+	-
A_r	+	-	-
u^i, h_{ti}, T^{ti}	+	-	-
h_{ij}, T^{ij}	+	+	+
B^i	+	-	-
E^i	+	+	+
$dx^\mu \wedge dx^\nu \wedge dx^\rho \wedge dx^\sigma \wedge dx^\kappa$	+	-	-
$\int_i^f A \wedge F \wedge F$	+	+	+

Example: hydrodynamic correlators in 2+1

Simple (non-chiral) example in 2+1 dims:

$$j^\mu = n u^\mu + \sigma \left[E^\mu - T \Delta^{\mu\nu} \partial_\nu \left(\frac{\mu}{T} \right) \right]$$

$$\Delta^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$$

$$u^\mu = (1, 0, 0)$$

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sources $\quad A_t,\, A_x \propto e^{-i\omega t + ikx} \quad\quad u^\mu = (1, 0, 0)$

fluctuations $\quad n = n(t, x, y) \propto e^{-i\omega t + ikx}$ (fix T and u)

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fluctuations $\qquad n = n(t, x, y) \propto e^{-i\omega t + ikx} \qquad$ (fix T and u)

one point functions $\qquad \nabla_\mu j^\mu = 0$

$$\langle j^t \rangle = n(\omega, k) = \frac{ik\sigma}{\omega + ik^2 \frac{\sigma}{\chi}} (\omega A_x + k A_t)$$

$$\langle j^x \rangle = \frac{i\omega\sigma}{\omega + ik^2 \frac{\sigma}{\chi}} (\omega A_x + k A_t)$$

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$\Longrightarrow$ two point functions $\langle j^x j^x \rangle = \frac{\delta \langle j^x \rangle}{\delta A_x} = \frac{i\omega^2 \sigma}{\omega + iDk^2}$

$\Longrightarrow$ hydrodynamic poles in spectral function

Example: hydrodynamic correlators in 2+1

Simple (non-chiral) example in 2+1 dims:

$$j^\mu = nu^\mu + \sigma\left[E^\mu - T\Delta^{\mu\nu}\partial_\nu\left(\frac{\mu}{T}\right)\right]$$

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sources $A_t,\ A_x \propto e^{-i\omega t+ikx}$

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$\Longrightarrow$ hydrodynamic poles in spectral function

$\Longrightarrow$ Kubo formulae $\sigma = \lim_{\omega\to 0}\frac{1}{i\omega}\langle j^x j^x\rangle(\omega, k=0)$

Constitutive equations

Generic decomposition:

$$T^{\mu\nu} = \mathcal{E} u^\mu u^\nu + \mathcal{P}\Delta^{\mu\nu} + \mathcal{Q}^\mu u^\nu + \mathcal{Q}^\nu u^\mu + \mathcal{T}^{\mu\nu}$$

$$J^\mu = \mathcal{N} u^\mu + \mathcal{J}^\mu$$

Examples:

$$X = X_{\text{eq.}} + X_{\text{non-eq.}} + X_{\text{anomalous}}$$

$$\begin{aligned}
\mathcal{E}_{\text{eq.}} = & -p + T\, p_{,T} + \mu\, p_{,\mu} + \left(T M_{5,T} + \mu M_{5,\mu} - 2M_5\right) B{\cdot}\Omega \\
& + \left(T M_{1,T} + \mu M_{1,\mu} + 4B^2 M_{1,B^2} + T^4 M_{3,B^2} - M_1\right) s_1 \\
& + \left(T M_{2,T} + \mu M_{2,\mu} - M_2\right) s_2 \\
& + \frac{4B^2}{T^4}\left(M_1 - T M_{1,T} - \mu M_{1,\mu} - 4B^2 M_{1,B^2} - T^4 M_{3,B^2}\right) s_3 \\
& + \left(T M_{4,T} + \mu M_{4,\mu} + \frac{4B^2}{T^4} M_{1,\mu} + M_{3,\mu}\right) s_4 \,,
\end{aligned}$$

$$\begin{aligned}
\mathcal{N}_{\text{eq.}} = & \, p_{,\mu} + \nabla{\cdot}p - p{\cdot}a - m{\cdot}\Omega + \left(M_{1,\mu} - T^4 M_{4,B^2}\right) s_1 + M_{2,\mu} s_2 \\
& + \left(M_{3,\mu} + T M_{4,T} + \mu M_{4,\mu} + 4B^2 M_{4,B^2}\right) s_3 + M_{5,\mu} s_5 \,,
\end{aligned}$$

Anomalous parts:

$$\Delta T^{\mu\nu} = u^\mu(\xi_T\, \Omega^\nu + \xi_{TB}\, B^\nu) + u^\nu(\xi_T\, \Omega^\mu + \xi_{TB}\, B^\mu) \,,$$

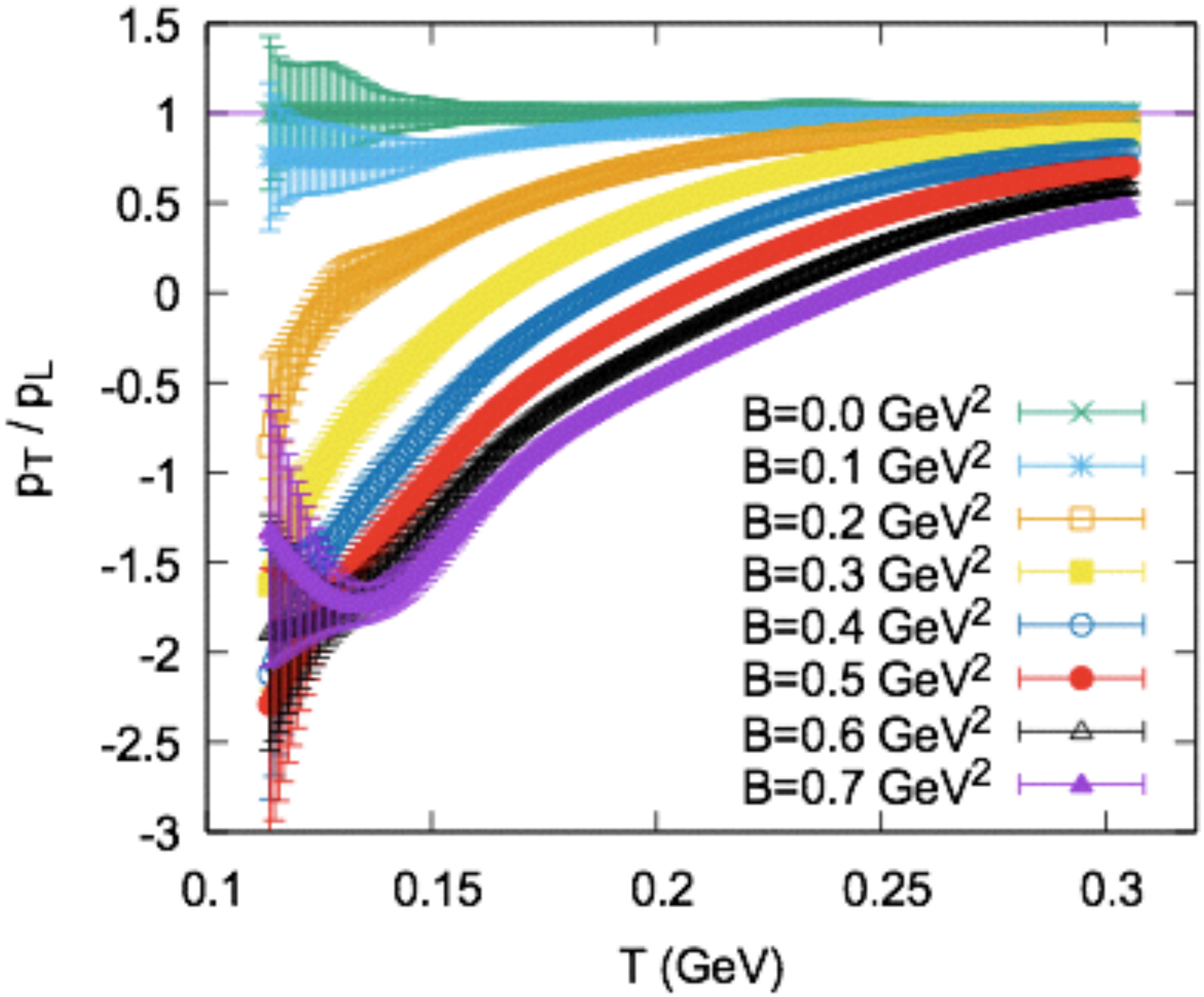
$$\Delta J^\mu_{cons} = \tfrac{1}{3} C B{\cdot}A u^\mu + \xi\, \Omega^\mu + \left(\xi_B - \tfrac{1}{3} C\mu\right) B^\mu + \tfrac{1}{3} C \epsilon^{\mu\nu\rho\sigma} A_\nu u_\rho E_\sigma \,,$$

$$\xi = \tfrac{1}{2} C\mu^2 + c_1 T^2 + 2c_2 T\mu \,, \quad \xi_B = C\mu + 2c_2 T \,,$$

$$\xi_T = \tfrac{1}{3} C\mu^3 + 2c_1 T^2 \mu + 2c_2 T\mu^2 \,, \quad \xi_{TB} = \tfrac{1}{2} C\mu^2 + c_1 T^2 + 2c_2 T\mu \,.$$

Universal magnetoresponse in QCD ...

[Endrödi, Kaminski, Schäfer, Wu, Yaffe; arXiv:1806.09632]



Lattice QCD with 2+1 flavors, dynamical quarks, physical masses

transverse pressure: $$p_{\mathrm{T}} = -\frac{L_{\mathrm{T}}}{V}\frac{\partial F_{\mathrm{QCD}}}{\partial L_{\mathrm{T}}}$$

longitudinal pressure: $$p_{\mathrm{L}} = -\frac{L_{\mathrm{L}}}{V}\frac{\partial F_{\mathrm{QCD}}}{\partial L_{\mathrm{L}}}$$

F_{QCD} ... *free energy*

L_{T} ... *transverse system size*

L_{L} ... *longitudinal system size*

V ... *system volume*

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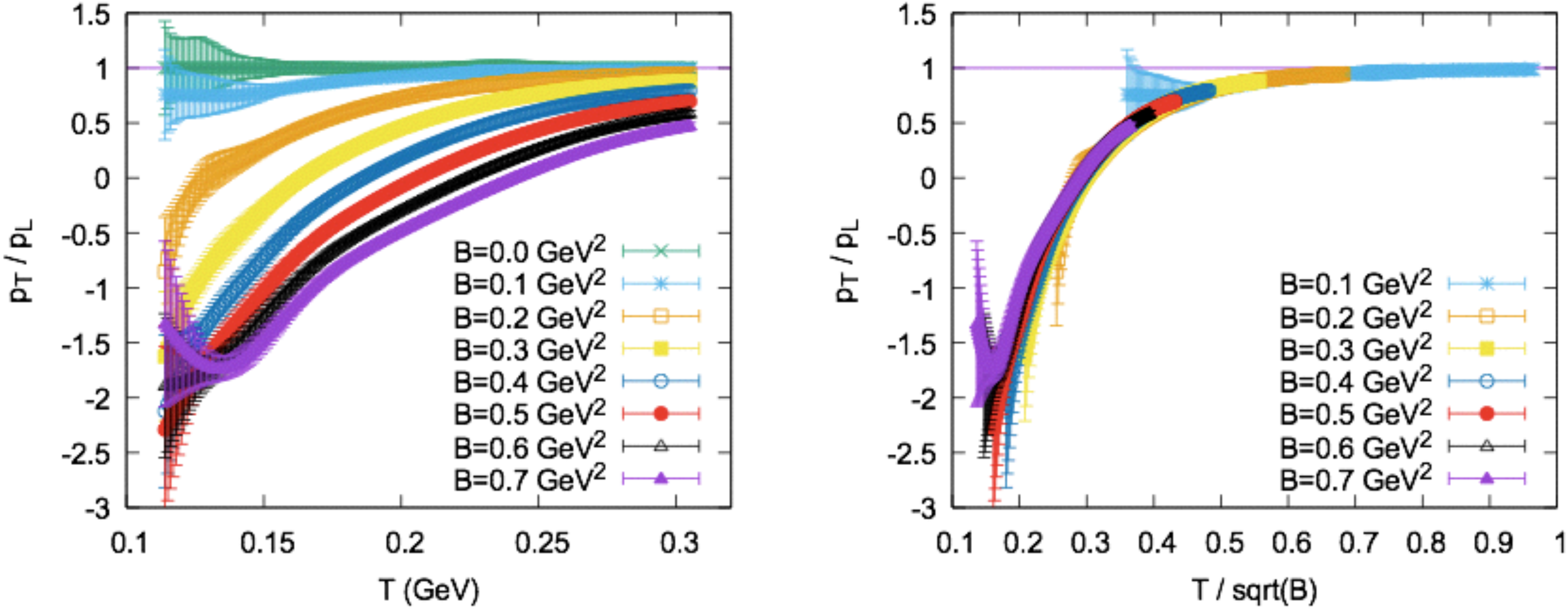

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… and $\mathcal{N}$=4 Super-Yang-Mills theory

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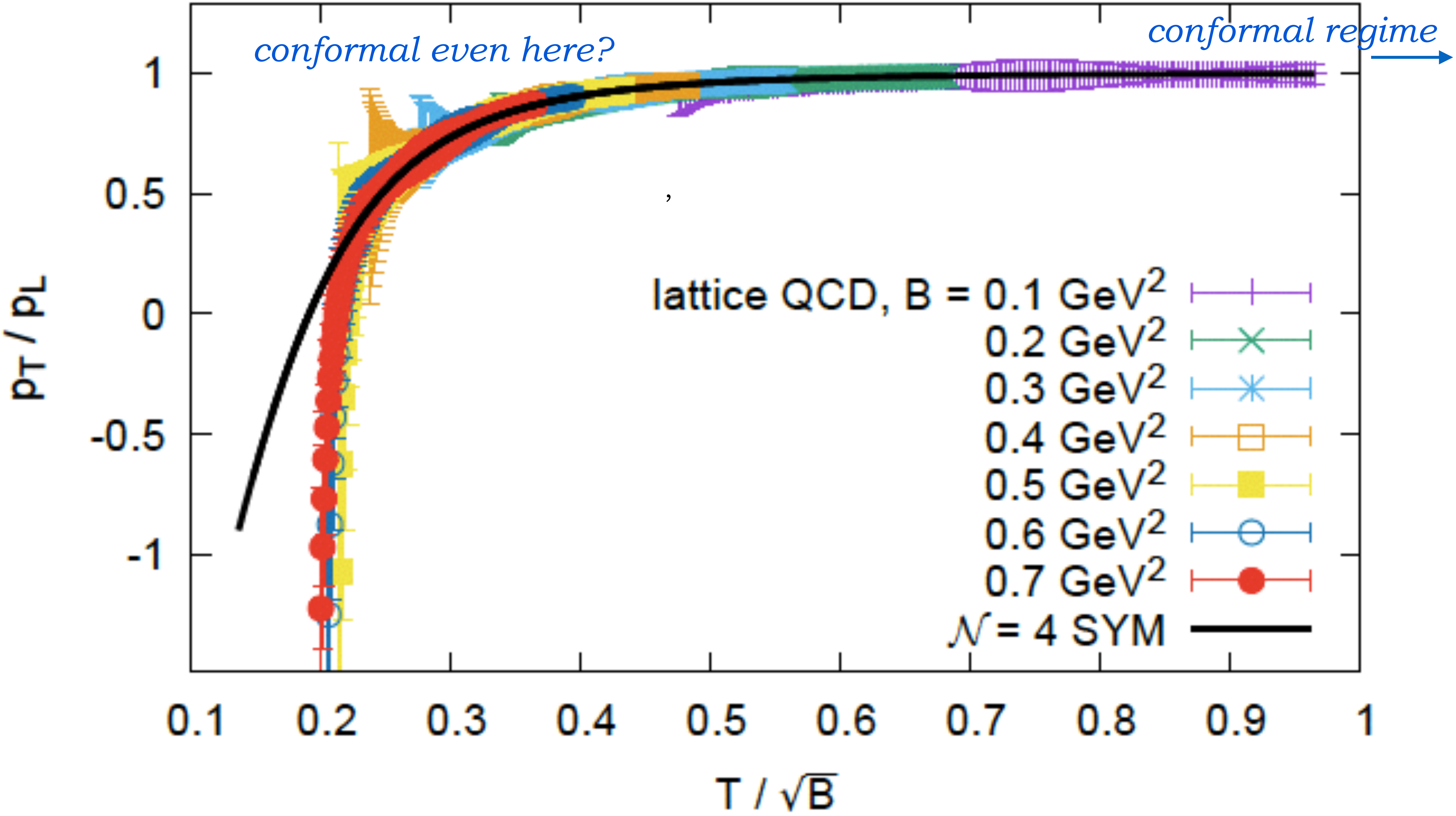
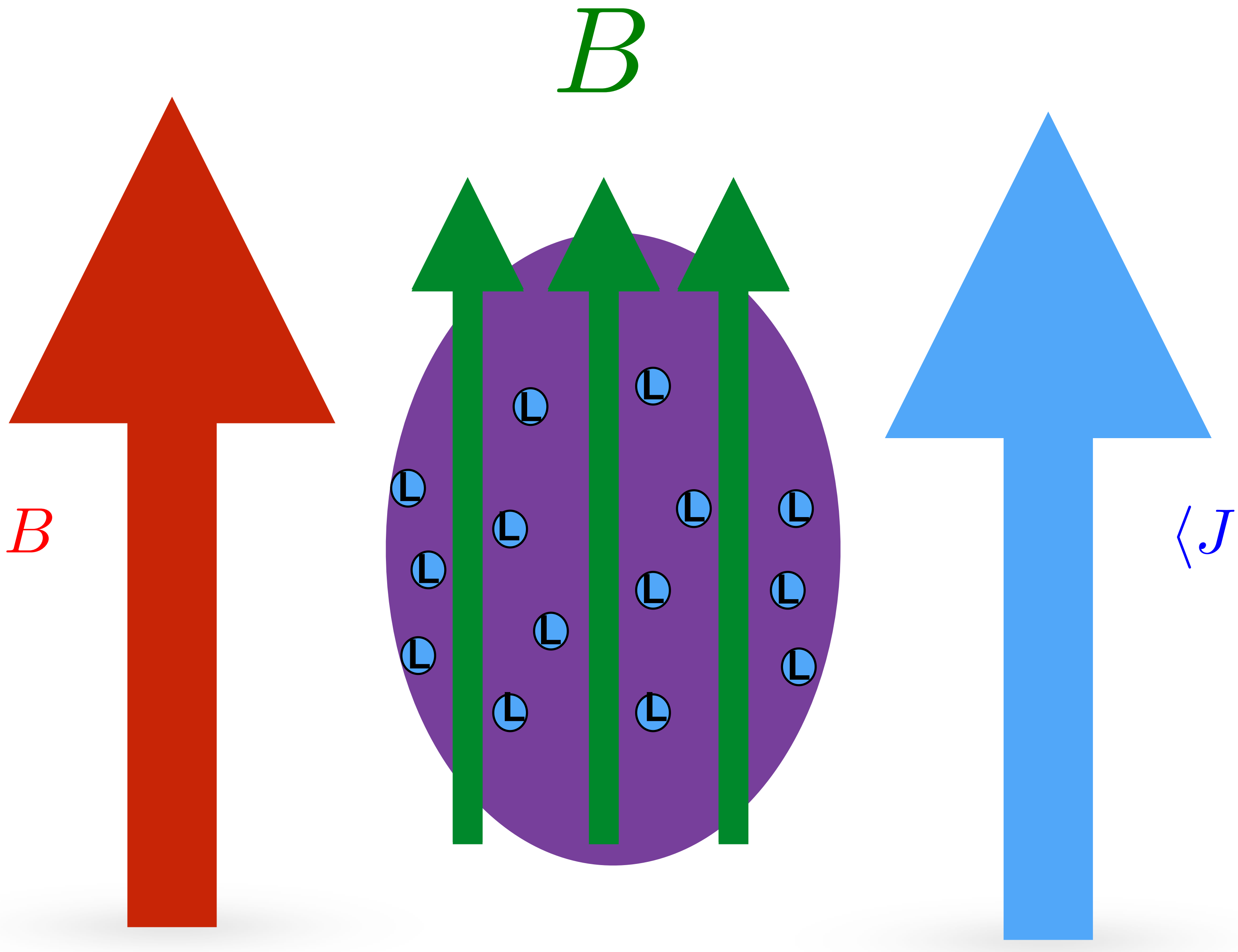


Zeroth order CME $\boxed{B \sim \mathcal{O}(1)}$ -thermodynamic chiral currents

[Ammon, Kaminski et al.; JHEP (2017)]
[Ammon, Leiber, Macedo; JHEP (2016)]



$\langle T^{0z} \rangle = \xi_V^{(0)} B$

heat current

$\langle J^z \rangle = \xi_B^{(0)} B$

axial current

Previous work: polarized matter at strong B

Generating functionals $W\sim P$ (pressure) for thermodynamics $\boxed{B \sim \mathcal{O}(1)}$

(i) No anomaly: *[Kovtun; JHEP (2016)]*

$$T^{\mu\nu} = Pg^{\mu\nu} + (Ts + \mu\rho)u^\mu u^\nu + T^{\mu\nu}_{\text{EM}}$$

$$J^\alpha = \rho u^\alpha - \underbrace{\nabla_\lambda M^{\lambda\alpha}}_{\textit{bound current}}$$

$$T^{\mu\nu}_{\text{EM}} = M^{\mu\alpha} g_{\alpha\beta} F^{\beta\nu} + u^\mu u^\alpha \left(M_{\alpha\beta} F^{\beta\nu} - F_{\alpha\beta} M^{\beta\nu} \right)$$

[Israel; Gen.Rel.Grav. (1978)]

Polarization tensor:

$$M_{\mu\nu} = p_\mu u_\nu - p_\nu u_\mu - \epsilon_{\mu\nu\rho\sigma} u^\rho m^\sigma \qquad M^{\mu\nu} = 2\frac{\partial P}{\partial F_{\mu\nu}}$$

Including vorticity:

$$W \sim M_\omega \, B \cdot \omega$$

[Kovtun, Hernandez; JHEP (2017)]

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Including vorticity:
$W \sim M_\omega \, B \cdot \omega$
[Kovtun, Hernandez; JHEP (2017)]

(ii) With anomaly: *[Jensen, Loganayagam, Yarom; JHEP (2014)]*

➡ opportunity: single framework allows for polarization, magnetization, external vorticity, *E*, *B*, *and* chiral anomaly

➡ opportunity: dynamical *E* and *B;* magnetohydrodynamics
[Kovtun, Hernandez; JHEP (2017)]

➡ opportunity: study equilibrium and near-equilibrium transport
[Ammon, Grieninger, Kaminski, Koirala, Leiber, Wu; to appear]

Holographic result: thermodynamics

[Ammon, Kaminski et al.; JHEP (2017)]

Background solution: charged magnetic black branes

[D'Hoker, Kraus; JHEP (2009)]

[Ammon, Leiber, Macedo; JHEP (2016)]

- **external magnetic field**
- **charged plasma**
- anisotropic plasma

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- **external magnetic field**
- **charged plasma**
- anisotropic plasma

Thermodynamics

$$\langle T^{\mu\nu}\rangle = \begin{pmatrix} -3\,u_4 & 0 & 0 & -4\,c_4 \\ 0 & -\frac{B^2}{4} - u_4 - 4\,w_4 & 0 & 0 \\ 0 & 0 & -\frac{B^2}{4} - u_4 - 4\,w_4 & 0 \\ -4\,c_4 & 0 & 0 & 8\,w_4 - u_4 \end{pmatrix}$$

$$\langle J^\mu\rangle = (\rho,\, 0,\, 0,\, p_1)\ .$$

$$\langle T^{\mu\nu}_{\text{EFT}}\rangle = \begin{pmatrix} \epsilon_0 & 0 & 0 & \xi_V^{(0)} B \\ 0 & P_0 - \chi_{BB}B^2 & 0 & 0 \\ 0 & 0 & P_0 - \chi_{BB}B^2 & 0 \\ \xi_V^{(0)} B & 0 & 0 & P_0 \end{pmatrix} + \mathcal{O}(\partial)$$

$$\langle J^\mu_{\text{EFT}}\rangle = \left(n_0,\, 0,\, 0,\, \xi_B^{(0)} B\right) + \mathcal{O}(\partial)$$

with near boundary expansion coefficients $u_4,\ w_4,\ c_4,\ p_1$

➡ **agrees in form with strong *B* thermodynamics from EFT**

EFT result II: weak B hydrodynamics

Weak B hydrodynamics - poles of 2-point functions $\langle T^{\mu\nu}\, T^{\alpha\beta}\rangle$, $\langle T^{\mu\nu}\, J^{\alpha}\rangle$, $\langle J^{\mu}\, T^{\alpha\beta}\rangle$, $\langle J^{\mu}\, J^{\alpha}\rangle$:

[Ammon, Kaminski et al.; JHEP (2017)]
[Abbasi et al.; PLB (2016)]
[Kalaydzhyan, Murchikova; NPB (2016)]

spin 1 modes under SO(2) rotations around *B*

$$\omega = \quad - ik^2 \frac{\eta}{\epsilon_0 + P_0} +$$

former momentum diffusion modes

$\mathfrak{s}_0 = s_0/n_0$
$\tilde{c}_P = T_0(\partial \mathfrak{s}/\partial T)_P$

EFT result II: weak B hydrodynamics

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former momentum diffusion modes

$$\mathfrak{s}_0 = s_0/n_0$$
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spin 0 modes under SO(2) rotations around *B*

$$\omega_0 = \underline{v_0\, k} - iD_0\, k^2 + \mathcal{O}(\partial^3)$$

former charge diffusion mode

$$\omega_+ = \underline{v_+\, k - i\Gamma_+\, k^2} + \mathcal{O}(\partial^3)$$

$$\omega_- = \underline{v_-\, k - i\Gamma_-\, k^2} + \mathcal{O}(\partial^3)$$

former sound modes

EFT result II: weak B hydrodynamics

Weak B hydrodynamics - poles of 2-point functions $\langle T^{\mu\nu}\, T^{\alpha\beta}\rangle$, $\langle T^{\mu\nu}\, J^{\alpha}\rangle$, $\langle J^{\mu}\, T^{\alpha\beta}\rangle$, $\langle J^{\mu}\, J^{\alpha}\rangle$:

[Ammon, Kaminski et al.; JHEP (2017)]
[Abbasi et al.; PLB (2016)]
[Kalaydzhyan, Murchikova; NPB (2016)]

spin 1 modes under SO(2) rotations around *B*

$$\omega = \mp\underline{\frac{Bn_0}{\epsilon_0 + P_0}} - ik^2\frac{\eta}{\epsilon_0 + P_0} + k\underline{\frac{Bn_0\xi_3}{(\epsilon_0 + P_0)^2} - \frac{iB^2\sigma}{\epsilon_0 + P_0}}$$

former momentum diffusion modes

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former charge diffusion mode

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$$\omega_- = \underline{v_-\, k - i\Gamma_-\, k^2} + \mathcal{O}(\partial^3)$$

former sound modes

➡ **a chiral magnetic wave**

[Kharzeev, Yee; PRD (2011)]

$$v_0 = \frac{2B\,T_0}{\tilde{c}_P n_0}\left(\tilde{C} - 3C\mathfrak{s}_0^2\right)$$

$$D_0 = \frac{w_0^2\,\sigma}{\tilde{c}_P n_0^3 T_0}$$

➡ **dispersion relations of hydrodynamic modes are heavily modified by anomaly and *B***