Susy Q and Boomerang RG Flows

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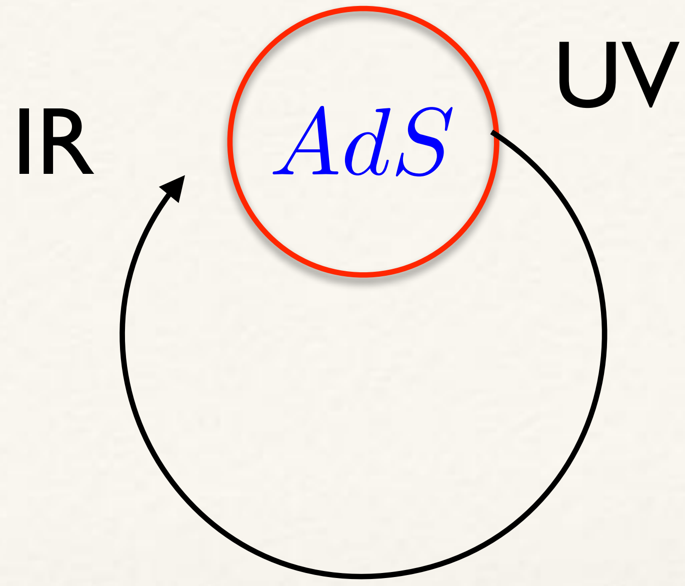
Holography provides powerful tools to study strongly coupled CFTs deformed by an operator that explicitly breaks translations

$$L_{CFT} \to L_{CFT} + \int dx \phi_s(x) \mathcal{O}(x)$$

Why study?

- Momentum can dissipate and hence finite DC conductivities
- RG flows lead to novel ground states in IR
  - Metals with Drude peaks [Hartnoll,Hoffman][Horowitz, Santos,Tong][…]
  - Novel metals without Drude peaks [Donos,JPG][Gouteraux][…..]
  - Insulators [Donos,Hartnoll][Donos,JPG][……]
- Defects/Interface/Janus [Bak,Gutperle,Hirano][……]

• “Boomerang” RG flows



with renormalisation of length scales from UV to IR

- Can have intermediate scaling
- Singularity resolving mechanism
- Can generate quantum phase transitions
- Can preserve supersymmetry

Q-Lattices [Donos,JPG]

- Exploit a bulk global symmetry to break translations leading to a system of ODEs rather than PDEs

- E.g. complex scalar field z in D=4

$$\mathcal{L} = R - \partial_\mu z \partial^\mu \bar{z} - \mathcal{V}(|z|) + \ldots$$

Use global symmetry $z \to e^{i\theta} z$ to construct ansatz

$$z = \rho e^{ikx}$$

$$ds^2 = e^{2A}(-dt^2 + dy^2) + e^{2V} dx^2 + N^2 dr^2$$

- Can generalise to break more translations and also to any D

Susy Q - a supersymmetric Q-lattice

- e.g. N=1 supergravity in D=4 with single chiral multiplet

$$\mathcal{L} = R - G\partial_\mu z \partial^\mu \bar{z} - \mathcal{V}$$

where

$$G = 2\partial\bar{\partial}\mathcal{K}(z,\bar{z})$$

$$\mathcal{V} = 4G^{-1}\partial\mathcal{W}\bar{\partial}\mathcal{W} - \frac{3}{2}\mathcal{W}^2 \quad \text{and} \quad \mathcal{W} = -2e^{\mathcal{K}/2}|W(z)|$$

- Q-lattice demands $\mathcal{K} = \mathcal{K}(|z|)$

- Susy Q demands $W = constant$

$$z = \rho e^{ikx}$$

$$ds^2 = e^{2A}(-dt^2 + dy^2) + e^{2V} dx^2 + N^2 dr^2$$

- BPS equations

$$N^{-1}\rho' + e^{\mathcal{K}/2} G^{-1}\mathcal{K}' W = k e^{-V}\rho$$
$$N^{-1}A' - e^{\mathcal{K}/2} W = 0$$
$$N^{-1}V' - e^{\mathcal{K}/2} W = -\frac{1}{2} e^{-V} k \rho \mathcal{K}'$$

- $k = 0$:1/2 BPS Poincare RG flow $\quad \Gamma^r \epsilon = -\epsilon$

- $k \neq 0$: 1/4 BPS Susy Q RG flow $\quad \Gamma^r \epsilon = -\epsilon$

$$\Gamma^{ty} \epsilon = -\epsilon\,,$$

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$$\Gamma^{ty} \epsilon = -\epsilon \,,$$

A Top-Down Example

- Consistent KK truncation

$$D = 11 \quad Sugra \quad \overset{S^7}{\longrightarrow} \quad N = 8 \quad SO(8) \quad Sugra$$

- Further truncate to $SO(4) \times SO(4) \subset SO(8)$ sector to get

$$N = 4 \quad SO(4) \quad Sugra$$

- Set $SO(4)$ gauge fields to zero to get N=1 Sugra in D=4

$$e^{-\mathcal{K}/2} = (1 - |z|^2)^{1/2} \qquad \text{and} \qquad W = constant$$

- Can explicitly uplift to D=11 solutions [Cvetic,Lu,Pope]

- Vacuum AdS_4 uplifts to $AdS_4 \times S^7$

- Can quotient by $\mathbb{Z}_q \subset U(1)_b$ with $U(1)_b \times SU(4) \subset SO(8)$

  and then dual to ABJM theory at level q

- Dual operators: $z = \mathcal{X} + i\mathcal{Y}$

  with

$$\Delta(\mathcal{O}_{\mathcal{X}}) = 1 \qquad \Delta(\mathcal{O}_{\mathcal{Y}}) = 2$$

achieved via holographic renormalisation:

[Cabo-Bizet,Kol,Pando Zayas,Papadimitriou,Rathee] [Freedman,Pufu]

$$S_{bdy} = \int d^3x \sqrt{-\gamma} \left(2K + 2\mathcal{W} + 4\left(r\mathcal{X}\partial_r\mathcal{X} + \mathcal{X}^2\right)\right)$$

$k = 0$ Susy Poincare invariant RG flows [Pope, Warner]

$$ds^2 = e^{2A}(-dt^2 + dx^2 + dy^2) + r^{-2}dr^2$$

- Analytic solution

$$e^{2A} = r^2 \left(1 - \frac{\mu^2}{r^2}\right)^2 \qquad z = \frac{2\mu r}{r^2 + \mu^2} e^{i\theta}$$

singular at $r = \mu$

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- $\theta = 0$ $\langle \mathcal{O}^{\Delta=1} \rangle \neq 0$ and no sources

Coulomb branch RG flow - distribution of membranes

- $\theta \neq 0$ Switches on source for $\mathcal{O}^{\Delta=2}$

Dielectric RG flow - membranes puff up into fivebranes

- Various operators gapped for these $k = 0$ flows

e.g. massless scalar field - dual to marginal operator

$$\delta h(t, r) = h(r) e^{-i\omega t}$$

Retarded Greens function

$$G^R(\omega) \propto \omega^2 \mu \sqrt{1 - \frac{\omega^2}{4\mu^2}}$$

is real for $\omega \leq 2\mu$

$k \neq 0$ Susy Q Boomerang RG flows

$$ds^2 = e^{2A}(-dt^2 + dy^2) + e^{2V} dx^2 + r^{-2} dr^2$$

- UV boundary conditions to BPS equations

$$e^{2A} = r^2 + \dots \qquad e^{2V} = r^2 + \dots \qquad \rho = \Lambda \frac{1}{r} - k\Lambda \frac{1}{r^2} + \dots$$

Specified by dimensionless parameter $\frac{\Lambda}{k}$

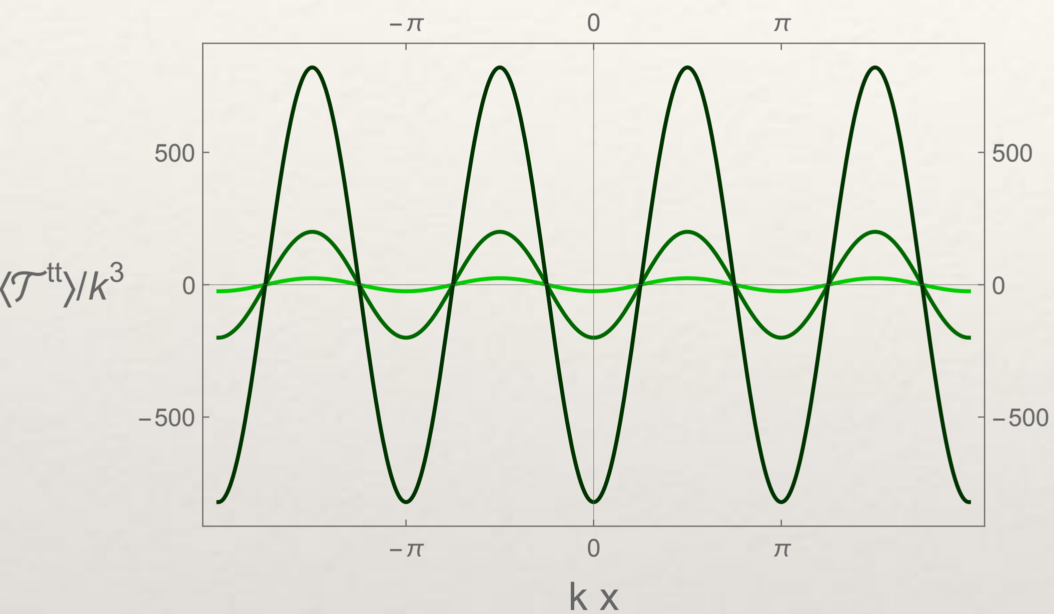
- UV sources and vevs

$$\mathcal{X}_s = 4k\Lambda \cos kx \qquad \mathcal{Y}_s = \Lambda \sin kx$$

$$\langle \mathcal{O}_{\mathcal{X}} \rangle = \Lambda \cos kx \qquad \langle \mathcal{O}_{\mathcal{Y}} \rangle = -4k\,\Lambda \sin kx$$

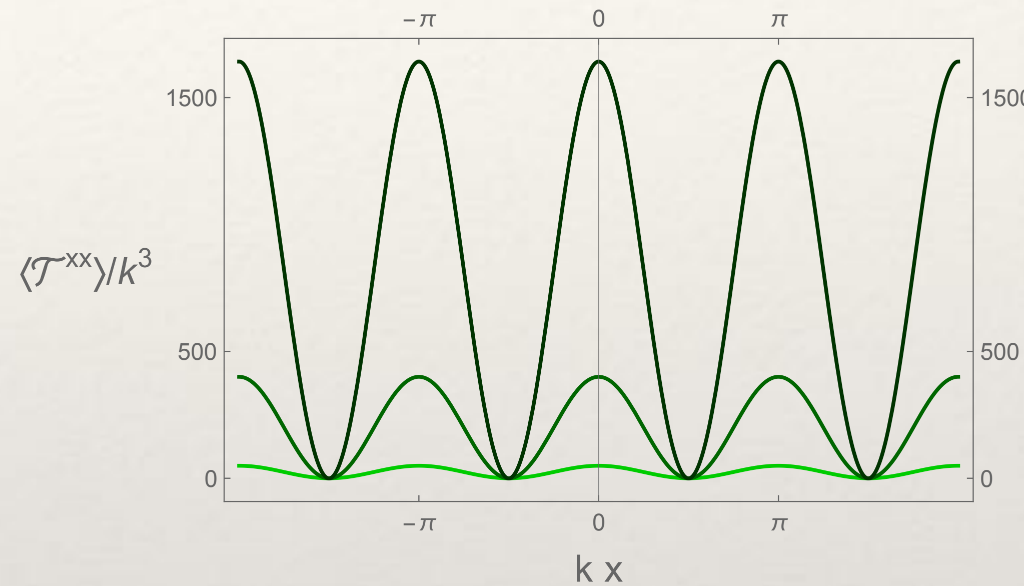
- Stress tensor - spatially modulated

$$\langle \mathcal{T}^{tt} \rangle / k^3 = -2 \left(\frac{\Lambda}{k} \right)^2 \cos 2kx$$



kx

$$\langle \mathcal{T}^{xx} \rangle / k^3 = 4 \left(\frac{\Lambda}{k} \right)^2 \cos^2 kx$$



kx

- Zero average energy density $\langle \overline{\mathcal{T}^{tt}} \rangle = 0$

- RG flows perturbative in $\frac{\Lambda}{k}$

$$ds^2 = e^{2A}(-dt^2 + dy^2) + e^{2V}dx^2 + r^{-2}dr^2$$

Solve scalar in AdS_4 background: $\rho = \frac{\Lambda}{r}e^{-k/r} + \ldots$

Back reacts on metric

$$e^{2A} = r^2\left[1 + \frac{\Lambda^2}{4k^2}\left(-1 + \frac{(2k+r)}{r}e^{-2k/r}\right) + \ldots\right]$$

$$e^{2V} = r^2\left[1 + \frac{\Lambda^2}{4k^2}\left(1 - \frac{(4k^2 + 2kr + r^2)}{r^2}e^{-2k/r}\right) + \ldots\right]$$

We have a boomerang flow

Generic for Q-lattices with relevant operators

- Refractive index

$$ds^2 = e^{2A}(-dt^2 + dy^2) + e^{2V}dx^2 + r^{-2}dr^2$$

For AdS_4 speed of light e^{A-V}

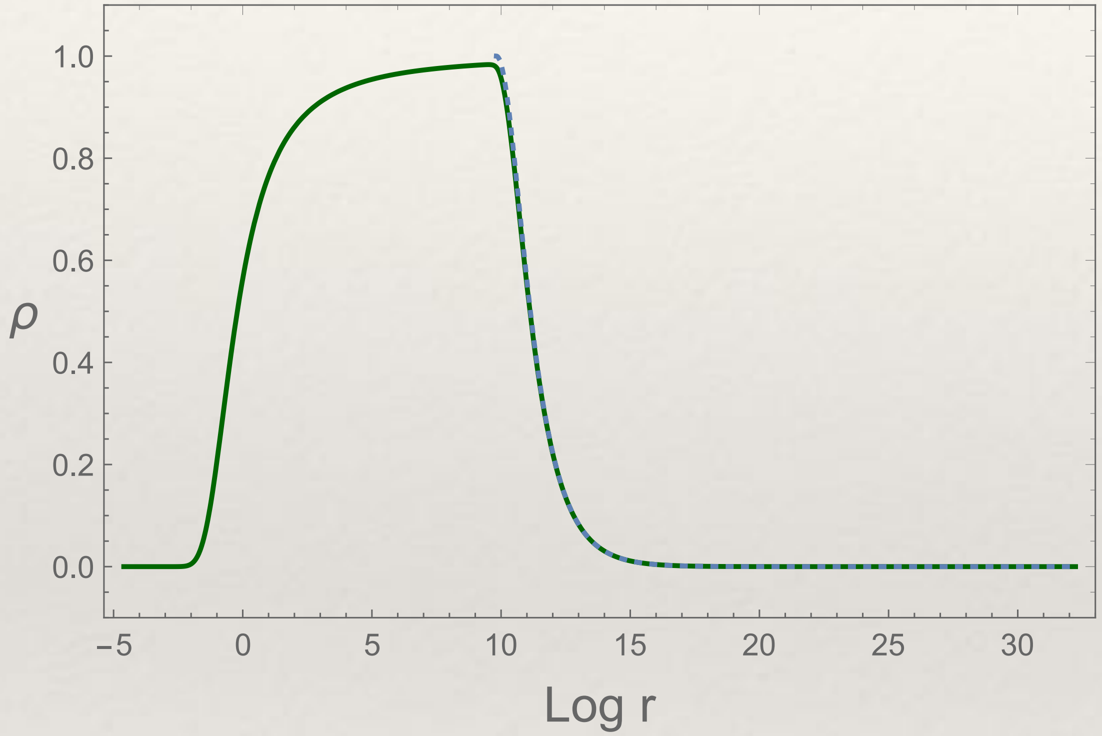
Refractive index $n_x = \frac{e^{A-V}|_{UV}}{e^{A-V}|_{IR}}$

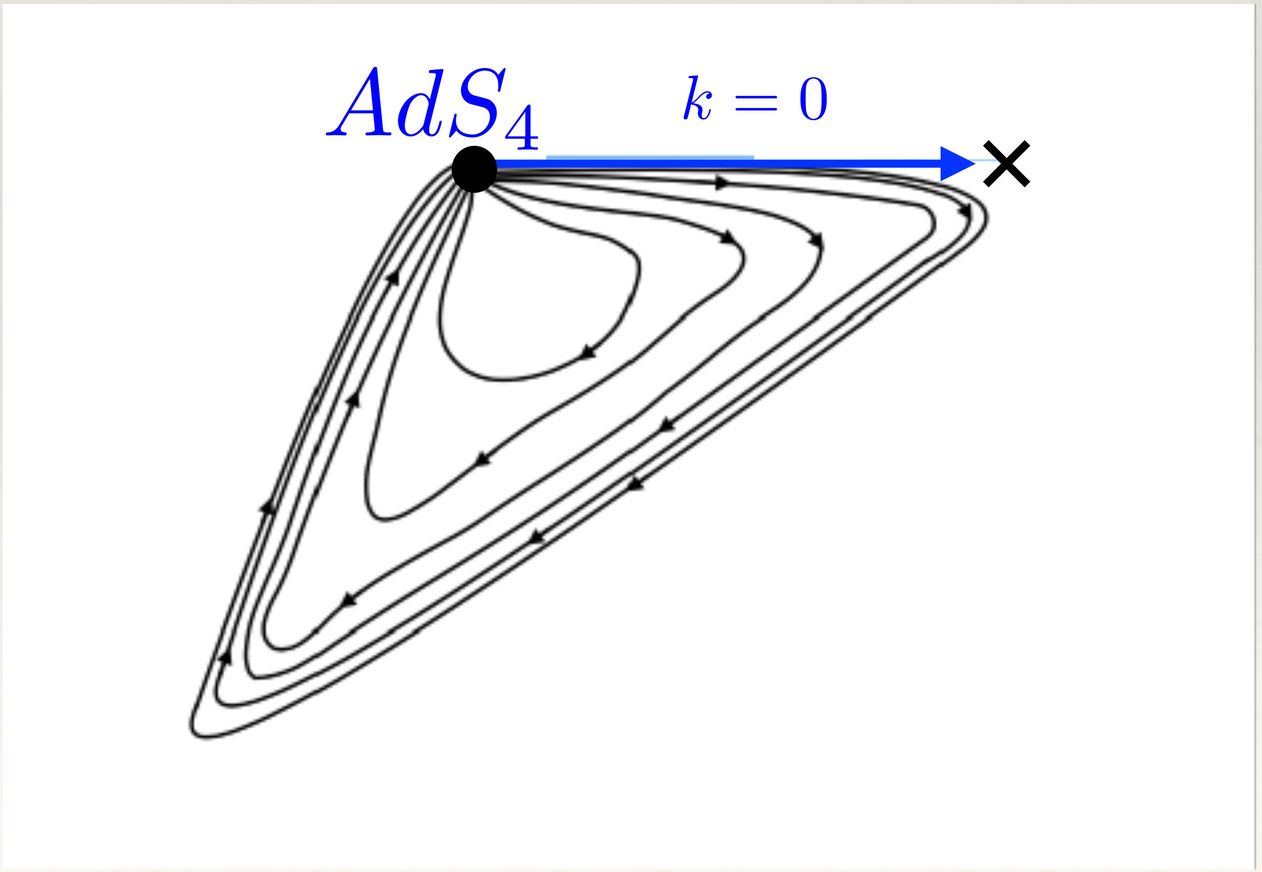
BPS equations $\Rightarrow$ $n_x \geq 1$

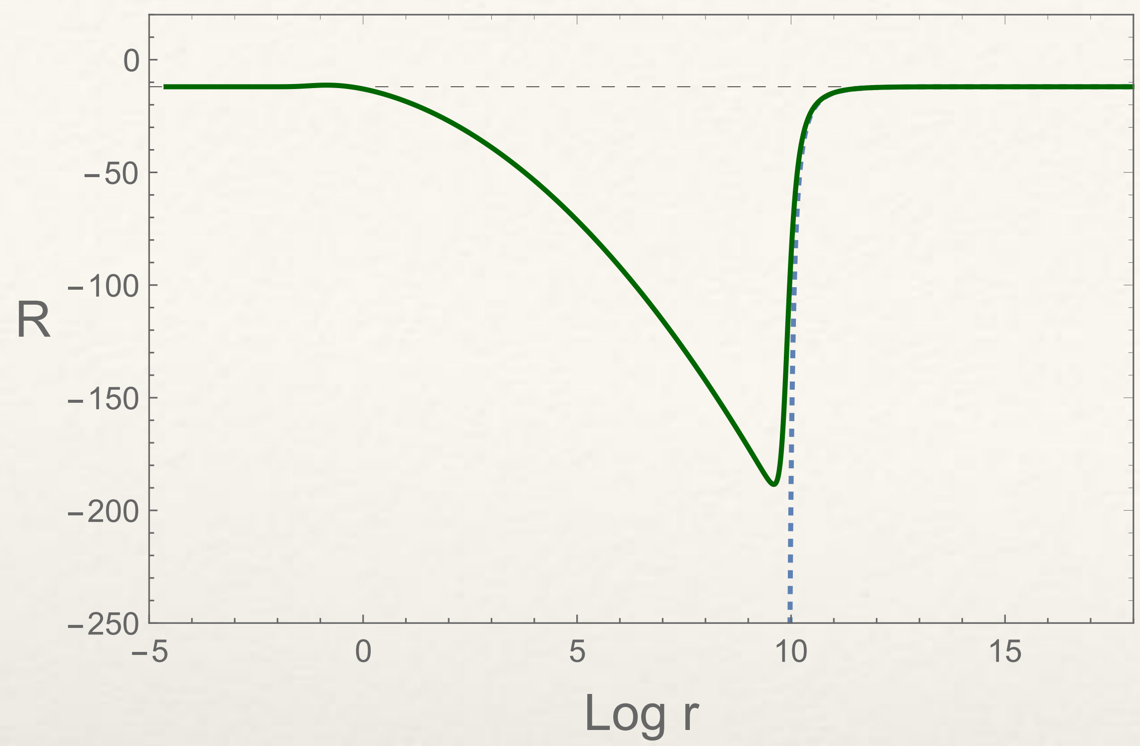
Perturbative boomerang flows $n^x = 1 + \frac{1}{4}\left(\frac{\Lambda}{k}\right)^2$

- RG flows non-perturbative in $\frac{\Lambda}{k}$

Exist for arbitrary large $\frac{\Lambda}{k}$ (not guaranteed!)



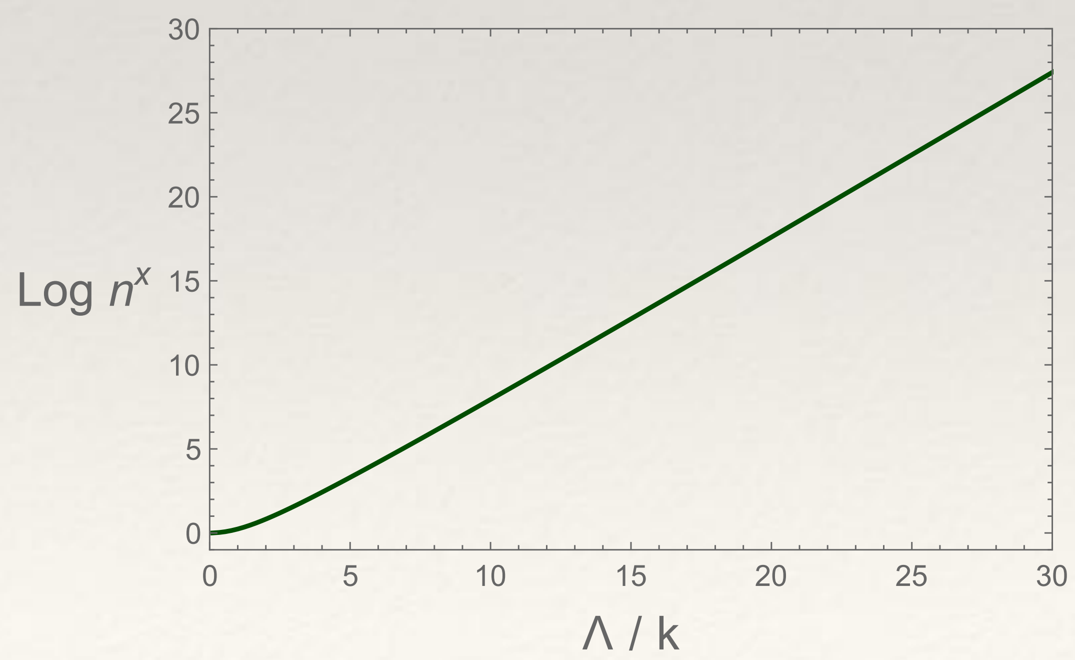




- These boomerang flows resolve singularity

  Resolves hard gap in spectral functions

- Index of refraction



ABJM theory

$$AdS_4 \times S^7/\mathbb{Z}_q$$

$$U(N)_q \times U(N)_{-q}$$

bosons Y^A $\mathbf{4_0}$

fermions ψ_A $\bar{\mathbf{4}}_0$

$$SU(4) \times U(1)_b$$

- $q > 2$ $\mathcal{N} = 6$ supersymmetry
- $q = 1, 2$ $\mathcal{N} = 8$ supersymmetry
- Deformation in ABJM theory $M_A{}^B = \mathrm{diag}(1, 1, -1, -1)$

$$\rho \cos kx \quad \longleftrightarrow \quad \mathcal{O}^{\Delta=1} \sim M_A{}^B \mathrm{Tr}\left(Y^A Y_B^\dagger\right)$$

$$\rho \sin kx \quad \longleftrightarrow \quad \mathcal{O}^{\Delta=2} \sim M_A{}^B \mathrm{Tr}\left(\psi^{\dagger\, A}\psi_B + \frac{8\pi}{q} Y^C Y^\dagger_{[C} Y^A Y^\dagger_{B]}\right)$$

[Kim,Kwon]

Deformations of ABJM with $m = m(x)$

$$\Delta\mathcal{L} = m' M_A{}^B \mathrm{Tr}\left(Y^A Y_B^\dagger\right) + m M_A{}^B \mathrm{Tr}\left(i\psi^{\dagger\, A}\psi_B + \frac{8\pi}{q} Y^C Y^\dagger_{[C} Y^A Y^\dagger_{B]}\right) + m^2 \mathrm{Tr}\left(Y^A Y_A^\dagger\right)$$

preserve $\mathcal{N} = 3$ susy

- We have found the gravity dual for $m(x) = \sin kx$

  Corollary: for $q = 1, 2$ should preserve $\mathcal{N} = 4$ susy

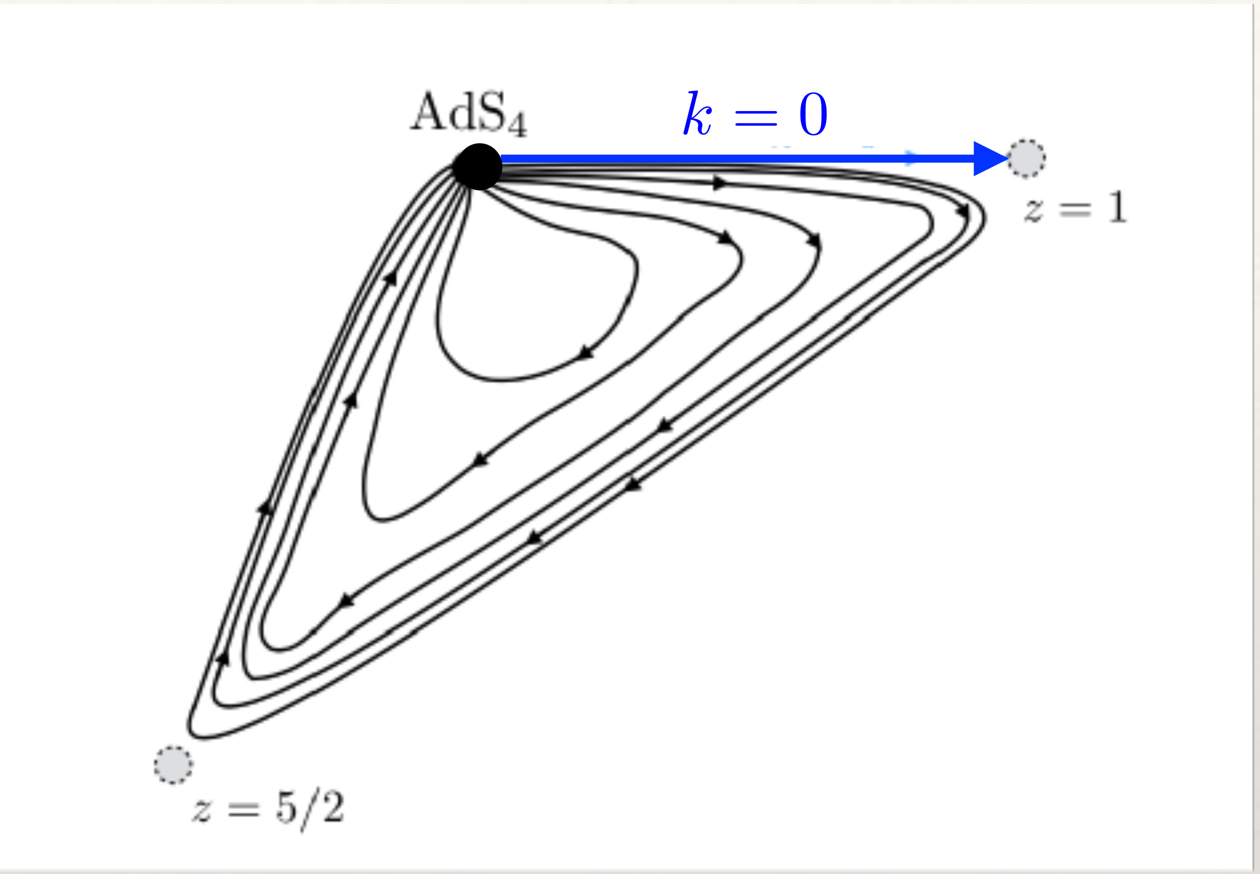
- Gravity duals for other $m(x)$?

  In general PDEs but a Janus solution is known

[D'Hoker,Estes,Gutperle,Krym] [Bobev,Pilch,Warner]

- Secondary intermediate scaling?

Isotropic flow in M-Theory [Donos,JPG,Rosen,Sosa-Rodriguez



For Susy Q? $\mathcal{L} = R - \frac{1}{2}(\partial\lambda)^2 - \frac{1}{2}\sinh^2\lambda(\partial\sigma)^2 + 2(2+\cosh\lambda)$

$$\mathcal{L} \approx R - \frac{1}{2}(\partial\lambda)^2 - \frac{1}{8}e^{2\lambda}(\partial\sigma)^2 + e^{\lambda}$$

Has novel $AdS_3 \times \mathbb{R}$ solution for arbitrary λ_0

Final Comments

- Susy Q is possible
- Susy boomerang RG flows for ABJM theory

- Some questions

Other top down Susy Q from D=4?

Not for truncations of $\mathcal{N} = 8$ that have scalars in $SL(2)/SO(2)$ classified by [Bobev,Pilch,Warner]

Isotropic Susy Q?

D=5 Susy Q?

What other susy $m(x)$ are possible for SCFTs?