Susy Q and Boomerang RG Flows

Jerome Gauntlett

C. Rosen
A. Donos, C. Rosen, O. Sosa-Rodriguez
Suzie Q and Boomerang RG Flows
Holography provides powerful tools to study strongly coupled CFTs deformed by an operator that explicitly breaks translations

\[ L_{CFT} \rightarrow L_{CFT} + \int dx \phi_s(x) \mathcal{O}(x) \]

**Why study?**

- Momentum can dissipate and hence finite DC conductivities
  - RG flows lead to novel ground states in IR
    - Metals with Drude peaks [Hartnoll, Hoffman][Horowitz, Santos, Tong][…]
    - Novel metals without Drude peaks [Donos, JPG][Gouteraux][…]
    - Insulators [Donos, Hartnoll][Donos, JPG][…]
    - Defects/Interface/Janus [Bak, Gutperle, Hirano][…]

- Why study?
• “Boomerang” RG flows

with renormalisation of length scales from UV to IR

• Can have intermediate scaling
• Singularity resolving mechanism
• Can generate quantum phase transitions
• Can preserve supersymmetry
Q-Lattices

- Exploit a bulk global symmetry to break translations leading to a system of ODEs rather than PDEs

- E.g. complex scalar field $z$ in $D=4$

$$\mathcal{L} = R - \partial_\mu z \partial^\mu \bar{z} - V(|z|) + \ldots$$

Use global symmetry $z \rightarrow e^{i\theta} z$ to construct ansatz

$z = \rho e^{ikx}$

$$ds^2 = e^{2A}(-dt^2 + dy^2) + e^{2V} dx^2 + N^2 dr^2$$

- Can generalise to break more translations and also to any D
Susy $Q$ - a supersymmetric Q-lattice

- e.g. $\text{N}=1$ supergravity in $\text{D}=4$ with single chiral multiplet

$$\mathcal{L} = R - G\partial_{\mu}z\partial^{\mu}\bar{z} - \mathcal{V}$$

where

$$G = 2\partial\bar{\partial}\mathcal{K}(z, \bar{z})$$

$$\mathcal{V} = 4G^{-1}\partial W\bar{\partial}W - \frac{3}{2}W^2$$

and

$$W = -2e^{\mathcal{K}/2}|W(z)|$$

- Q-lattice demands

$$\mathcal{K} = \mathcal{K}(|z|)$$

- Susy $Q$ demands

$$W = \text{constant}$$
\[ z = \rho e^{ikx} \]
\[ ds^2 = e^{2A}(-dt^2 + dy^2) + e^{2V} dx^2 + N^2 dr^2 \]

- **BPS equations**

\[ N^{-1} \rho' + e^{\mathcal{K}/2}G^{-1}\mathcal{K}'W = ke^{-V}\rho \]
\[ N^{-1} A' - e^{\mathcal{K}/2}W = 0 \]
\[ N^{-1} V' - e^{\mathcal{K}/2}W = -\frac{1}{2}e^{-V}k\rho\mathcal{K}' \]

- \( k = 0 \): \(1/2\) BPS Poincare RG flow
  \( \Gamma^r\epsilon = -\epsilon \)

- \( k \neq 0 \): \(1/4\) BPS Susy Q RG flow
  \( \Gamma^r\epsilon = -\epsilon \)
  \( \Gamma^{ty}\epsilon = -\epsilon \),
\[ z = \rho e^{ikx} \]
\[ ds^2 = e^{2A}(-dt^2 + dy^2) + e^{2V} dx^2 + N^2 dr^2 \]

- **BPS equations**

\[ N^{-1} \rho' + e^{K/2} G^{-1} K' W = ke^{-V} \rho \]
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  \[ \Gamma^r \epsilon = -\epsilon \]
  \[ \Gamma^{ty} \epsilon = -\epsilon \]
A Top-Down Example

• Consistent KK truncation

\[ D = 11 \quad \text{Sugra} \quad \rightarrow \quad N = 8 \quad SO(8) \quad \text{Sugra} \]

• Further truncate to \( SO(4) \times SO(4) \subset SO(8) \) sector to get

\[ N = 4 \quad SO(4) \quad \text{Sugra} \]

• Set \( SO(4) \) gauge fields to zero to get \( N=1 \) Sugra in \( D=4 \)

\[ e^{-\kappa/2} = (1 - |z|^2)^{1/2} \quad \text{and} \quad W = \text{constant} \]

• Can explicitly uplift to \( D=11 \) solutions \[\text{[Cvetic,Lu,Pope]}\]
• Vacuum $AdS_4$ uplifts to $AdS_4 \times S^7$

• Can quotient by $\mathbb{Z}_q \subset U(1)_b$ with $U(1)_b \times SU(4) \subset SO(8)$

and then dual to ABJM theory at level $q$

• Dual operators: $z = \mathcal{X} + i\mathcal{Y}$

with

$$\Delta(\mathcal{O}_\mathcal{X}) = 1 \quad \Delta(\mathcal{O}_\mathcal{Y}) = 2$$

achieved via holographic renormalisation:

[Cabo-Bizet, Kol, Pando Zayas, Papadimitriou, Rathee] [Freedman, Pufu]

$$S_{\text{bdy}} = \int d^3 x \sqrt{-\gamma} \left( 2K + 2\mathcal{W} + 4 \left( r \mathcal{X} \partial_r \mathcal{X} + \mathcal{X}^2 \right) \right)$$
$k = 0 \quad \text{Susy Poincare invariant RG flows}$

\[
ds^2 = e^{2A}(-dt^2 + dx^2 + dy^2) + r^{-2}dr^2
\]

- Analytic solution

\[
e^{2A} = r^2 \left(1 - \frac{\mu^2}{r^2}\right)^2 \quad z = \frac{2\mu r}{r^2 + \mu^2} e^{i\theta}
\]

singular at $r = \mu$
\[ k = 0 \quad \text{Susy Poincare invariant RG flows} \quad \text{[Pope, Warner]} \]

\[ ds^2 = e^{2A} (-dt^2 + dx^2 + dy^2) + r^{-2} dr^2 \]

- **Analytic solution**

\[ e^{2A} = r^2 \left( 1 - \frac{\mu^2}{r^2} \right)^2 \]

\[ z = \frac{2\mu r}{r^2 + \mu^2} e^{i\theta} \]

singular at \( r = \mu \)

- \( \theta = 0 \quad \langle \mathcal{O}^{\Delta=1} \rangle \neq 0 \quad \text{and no sources} \)

  Coulomb branch RG flow - distribution of membranes

- \( \theta \neq 0 \quad \text{Switches on source for} \quad \mathcal{O}^{\Delta=2} \)

  Dielectric RG flow - membranes puff up into fivebranes
Various operators gapped for these $k = 0$ flows

e.g. massless scalar field - dual to marginal operator

$$\delta h(t, r) = h(r)e^{-i\omega t}$$

Retarded Greens function

$$G^R(\omega) \propto \omega^2 \mu \sqrt{1 - \frac{\omega^2}{4\mu^2}}$$

is real for $\omega \leq 2\mu$
\[ k \neq 0 \quad \text{Susy Q Boomerang RG flows} \]

\[ ds^2 = e^{2A}(-dt^2 + dy^2) + e^{2V} dx^2 + r^{-2} dr^2 \]

- UV boundary conditions to BPS equations

\[ e^{2A} = r^2 + \ldots \quad e^{2V} = r^2 + \ldots \quad \rho = \frac{1}{r} - k\Lambda \frac{1}{r^2} + \ldots \]

Specified by dimensionless parameter \( \frac{\Lambda}{k} \)

- UV sources and vevs

\[ \mathcal{X}_s = 4k\Lambda \cos kx \]
\[ \mathcal{Y}_s = \Lambda \sin kx \]
\[ \langle \mathcal{O}_X \rangle = \Lambda \cos kx \]
\[ \langle \mathcal{O}_Y \rangle = -4k \Lambda \sin kx \]
• **Stress tensor** - spatially modulated

\[
\langle \mathcal{T}^{tt} \rangle / k^3 = -2 \left( \frac{\Lambda}{k} \right)^2 \cos 2kx
\]

\[
\langle \mathcal{T}^{xx} \rangle / k^3 = 4 \left( \frac{\Lambda}{k} \right)^2 \cos^2 kx
\]

• **Zero average energy density**

\[
\langle \mathcal{T}^{tt} \rangle = 0
\]
• RG flows perturbative in \( \frac{\Lambda}{k} \)

\[
ds^2 = e^{2A}(-dt^2 + dy^2) + e^{2V} dx^2 + r^{-2} dr^2
\]

Solve scalar in \( AdS_4 \) background:

\[
\rho = \frac{\Lambda}{r} e^{-k/r} + \ldots
\]

Back reacts on metric

\[
e^{2A} = r^2 \left[ 1 + \frac{\Lambda^2}{4k^2} \left( -1 + \frac{(2k + r)}{r} e^{-2k/r} \right) + \ldots \right]
\]

\[
e^{2V} = r^2 \left[ 1 + \frac{\Lambda^2}{4k^2} \left( 1 - \frac{(4k^2 + 2kr + r^2)}{r^2} e^{-2k/r} \right) + \ldots \right]
\]

We have a boomerang flow

Generic for Q-lattices with relevant operators
Refractive index

\[ ds^2 = e^{2A}(-dt^2 + dy^2) + e^{2V} dx^2 + r^{-2} dr^2 \]

For \( AdS_4 \) speed of light \( e^{A-V} \)

Refractive index \( n_x = \frac{e^{A-V}|_{UV}}{e^{A-V}|_{IR}} \)

BPS equations \( \Rightarrow \) \( n_x \geq 1 \)

Perturbative boomerang flows \( n^x = 1 + \frac{1}{4} \left( \frac{\Lambda}{k} \right)^2 \)
• RG flows non-perturbative in $\frac{\Lambda}{k}$

Exist for arbitrary large $\frac{\Lambda}{k}$ (not guaranteed!)

Graph showing the relationship between $\rho$ and $\text{Log } r$.

Diagram illustrating $\text{AdS}_4$ for $k = 0$. 
- These boomerang flows resolve singularity
- Resolves hard gap in spectral functions

- Index of refraction
ABJM theory

\[ U(N)_q \times U(N)_{-q} \quad \text{bosons} \quad Y^A_{4_0} \]

\[ SU(4) \times U(1)_b \quad \text{fermions} \quad \psi_A_{\bar{4}_0} \]

- \( q > 2 \) \quad \mathcal{N} = 6 \quad \text{supersymmetry}

- \( q = 1, 2 \) \quad \mathcal{N} = 8 \quad \text{supersymmetry}

• Deformation in ABJM theory \[ M_{A}^{\ B} = \text{diag}(1, 1, -1, -1) \]

\[ \rho \cos kx \quad \longleftrightarrow \quad \mathcal{O}^{\Delta=1} \sim M_{A}^{\ B} \text{Tr} \left( Y^A Y^\dagger_B \right) \]

\[ \rho \sin kx \quad \longleftrightarrow \quad \mathcal{O}^{\Delta=2} \sim M_{A}^{\ B} \text{Tr} \left( \psi^\dagger_A \psi_B + \frac{8\pi}{q} Y^C Y^\dagger_{[C} Y^A Y^\dagger_{B]} \right) \]
Deformations of ABJM with \( m = m(x) \)

\[
\Delta \mathcal{L} = m'M_A^B \Tr (Y^A Y_B^\dagger) + mM_A^B \Tr \left( i\psi^A \psi_B + \frac{8\pi}{q} Y_C^B Y_C^B Y_D^A Y_D^A \right) + m^2 \Tr (Y^A Y_A^\dagger)
\]

preserve \( \mathcal{N} = 3 \) susy

- We have found the gravity dual for \( m(x) = \sin kx \)

Corollary: for \( q = 1, 2 \) should preserve \( \mathcal{N} = 4 \) susy

- Gravity duals for other \( m(x) \)?

In general PDEs but a Janus solution is known

[Kim, Kwon]

[D’Hoker, Estes, Gutperle, Krym] [Bobev, Pilch, Warner]
• Secondary intermediate scaling?

Isotropic flow in M-Theory [Donos, JPG, Rosen, Sosa-Rodriguez]

For Susy Q:

\[ \mathcal{L} = R - \frac{1}{2} (\partial \lambda)^2 - \frac{1}{2} \sinh^2 \lambda (\partial \sigma)^2 + 2(2 + \cosh \lambda) \]

\[ \mathcal{L} \approx R - \frac{1}{2} (\partial \lambda)^2 - \frac{1}{8} e^{2\lambda} (\partial \sigma)^2 + e^\lambda \]

Has novel \( AdS_3 \times \mathbb{R} \) solution for arbitrary \( \lambda_0 \)
Final Comments

• Susy Q is possible

• Susy boomerang RG flows for ABJM theory

• Some questions

  Other top down Susy Q from D=4?

  Not for truncations of $\mathcal{N} = 8$ that have scalars in $SL(2)/SO(2)$ classified by [Bobev,Pilch,Warner]

  Isotropic Susy Q?

D=5 Susy Q?

What other susy $m(x)$ are possible for SCFTs?