A nAttractor Mechanism for $nAdS_2/nCFT_1$ Holography

Finn Larsen

University of Michigan and Leinweber Center for Theoretical Physics

Gauge/Gravity Duality 2018

Würzburg, August 3, 2018

Ubiquity of AdS$_2$

- ***Extremal*** black holes are important: they have the smallest possible mass for given charges so they are ***ground states***.

- Spherically symmetric asymptotically flat extremal black holes in $D = 4$ have ***horizon geometry AdS$_2 \times S^2$***.

- In fact ***all*** extremal black holes include an AdS$_2$ factor (a theorem).

- This motivates interest in ***AdS$_2$ quantum gravity***.

AdS_2/CFT_1 Holography?

- AdS_{d+1}/CFT_d correspondence is confusing for $d = 1$.

- No finite energy excitations possible in AdS_2:

  Their ***backreaction spoil asymptotic*** AdS_2 boundary conditions.

- Also many other (related) unpleasantries.

- So AdS_2/CFT_1 holography is not yet well understood.

nAdS$_2$/nCFT$_1$ Holography.

- Recently developed version AdS$_2$/CFT$_1$ holography: duality between ***nearly*** AdS$_2$ geometry and ***nearly*** CFT$_1$.

- Conformal symmetry is ***broken spontaneously*** (by boundary conditions) and ***broken dynamically*** (by an anomaly).

- Interesting nCFT$_1$'s realize the symmetry breaking pattern: SYK,....

- This talk: ***gravitational aspects***, especially (nearly) extreme 4D black holes with spherically symmetry.

A Canonical Setting

- 4D $\mathcal{N} = 2$ ungauged SUGRA with n_V vector multiplets.

- The black hole parameters are:

  Mass M not an independent variable (extremality!)

  Charges (p^I, q_I), $I = 0, ...n_V$

  Asymptotic value of complex scalars z^i_∞, $i = 1, ...n_V$.

- ***Extreme*** black holes in this setting have been studied extensively.

The Extremal Attractor

- A radial ***flow***: the scalars z^i ***evolve*** from infinity to the horizon.
- The ***attractor mechanism***: scalar fields ***at the horizon*** are independent of the asymptotic value of the scalar fields.
- So the horizon theory is ***universal***: independent of moduli, including the coupling constants,....
- Many other versions of the extremal attractor mechanism:

  rotation, asymptotically AdS, 5D, multicenter solutions, nonBPS branch, higher derivative corrections, extended black objects, more SUSY, ...
- This talk: an attractor mechanism for ***nearly*** extreme black holes. ***A nAttractor***.

Near Extreme Black Holes

- The “near” of ***n***$\text{AdS}_2/nCFT_1$ appears in ***two ways***.

- Black holes only ***nearly*** extremal so scalars at the horizon depart from their extremal attractor value.

- Also: $\text{nAdS}_2/\text{nCFT}_1$ considers the entire ***near*** horizon region so scalars are ***not constant***.

- The corresponding symmetry breakings introduce ***new scale***(s).

- These ***scales are fundamental*** for $\text{nAdS}_2/nCFT_1$ holography.

A Scale: the Specific Heat

- The ***extremal*** black hole entropy is a ground state entropy

$$S_0 = \frac{4\pi \ell_2^2}{4G_4} = \frac{2\pi}{\kappa_2^2}$$

  There is ***no scale, just a large dimensionless number***.

- The ***nearly*** extreme black hole entropy:

$$S = S_0 + CT$$

- The ***specific heat*** $C = 2L$ is the ***symmetry breaking scale***.

- Literature: is the symmetry breaking scale universal, essentially the AdS_2 scale ℓ_2?

$$C \sim \frac{\ell_2}{\kappa_2^2}$$

  This talk: ***"no"***.

Preview: nAttractor Mechanism

- The nAdS$_2$ region introduces ***many scales*** that are related to the black hole parameters (p^I, q_I), z^i_∞.

- A ***nAttractor mechanism***: these scales are computed by a generalization of the ***extremal*** attractor mechanism

- There is no need to construct and analyze non extremal black hole solutions.

Non-Extreme Black Holes

- General ***non-extreme*** black hole depends on a single ***radial function*** $R(r)$:

$$ds_4^2 = -\frac{r(r-2m)}{R^2(r)}dt^2 + \frac{R^2(r)}{r(r-2m)}dr^2 + R^2(r)d\Omega_2^2$$

- There is a horizon at $r = 2m$.

- Entropy and temperature are encoded in the radial function:

$$S = \frac{\pi R^2(2m)}{G_4} \ .$$

$$T = \frac{m}{2\pi R^2(2m)} \ .$$

- The extremal limit is $m \to 0$ ***with charges and moduli fixed***.

Near-Extreme Black Holes

- The radial function $R(r)$ depends r and ***also on*** M.
- The entropy depends ***only on*** M.
- Near extremality (change mass M ***and*** position r):

$$\Delta S = \frac{\partial S}{\partial M}\Delta M = \frac{\pi}{G_4}\left(\frac{\partial R^2}{\partial M}\Delta M + \frac{\partial R^2}{\partial r}\Delta r\right)$$

- Estimate 1: $\partial_M S \sim T^{-1}$ so $\Delta M \sim T^2$ and $\Delta S \sim T$.
  Estimate 2: $\Delta r \sim m \sim T$.
- So $\partial_M R^2$ is ***subleading***: at linear order ΔS follows from R^2 ***at extremality*** but at a new position $r = 2m$.
- This is a ***major simplification*** (addressing one aspect of “near”).

The Symmetry Breaking Scale

- Therefore, the ***heat capacity*** follows from the radial function ***of the extremal black hole***:

$$L = \frac{1}{2} C = \frac{2\pi^2}{G_4} R^2 \left. \frac{\partial R^2}{\partial r} \right|_{\rm hor} \ .$$

- The symmetry breaking scale only depends on ***moving away from the horizon***.

- Moreover, the dependence is extremely simple: just a radial derivative.

The Extremal Attractor

- Consider the ***horizon*** attractor: the $\mathrm{AdS}_2 \times S^2$ solution.

- For fixed charges, the $F_{\mu\nu}F^{\mu\nu}$-type terms in the Lagrangian subject the scalars z^i to the ***effective potential***

$$V = 2G_4^2 \left(p^I \ q_I\right) \begin{pmatrix} \nu_{IJ} & \mu_{IK}(\nu^{-1})^{KJ} \\ (\nu^{-1})^{IK}\mu_{KJ} & (\nu^{-1})^{IJ} + \mu_{IK}(\nu^{-1})^{KL}\mu_{LJ} \end{pmatrix} \begin{pmatrix} p^J \\ q_J \end{pmatrix}$$

  μ_{IJ}, ν_{IJ} are functions of z^i determined by special geometry.

- The scalars z^i are ***constant*** on the $\mathrm{AdS}_2 \times S^2$ attractor geometry.

- So the ***effective potential*** V ***is extremized***: $\partial_i V = 0$

- The extremum value of the potential gives: $R^4(0) = G_4^2 V_{\mathrm{ext}}^2$

Results of Extremization

- Notation for the resulting radial function on $\mathrm{AdS}_2 \times S^2$:

$$R^4(0) = I_4(P^I, Q_I)$$

- The ***generating*** function I_4 is ***quartic*** in the charges.

- Example ($\mathcal{N} = 4$ SUGRA): $I_4(p^I, q_I) = \vec{p}^2\vec{q}^2 - (\vec{p}\vec{q})^2$.

- The ***scalar*** values at the horizon are ***also encoded in I_4***:

$$\begin{pmatrix} X^I_{\text{hor}} \\ F_I^{\text{hor}} \end{pmatrix} = \begin{pmatrix} p^I \\ q_I \end{pmatrix} - i \begin{pmatrix} -\partial_{q_I} \\ \partial_{p^I} \end{pmatrix} I_4^{1/2}(p^I, q_I)$$

  Symplectic section (X^I, F_I) represents scalars projectively: $z^i = X^i/X^0$.

Moving Away from the Horizon

- The radial function ***at the horizon*** depends only on charges.

- It depends on ***scalars at infinity*** away from the horizon.

- Parametrize scalars at infinity through "charges" p^I_∞, q^∞_I:

$$\begin{pmatrix} X^I_\infty \\ F^\infty_I \end{pmatrix} = \begin{pmatrix} p^I_\infty \\ q^\infty_I \end{pmatrix} - i \begin{pmatrix} -\partial_{q^\infty_I} \\ \partial_{p^I_\infty} \end{pmatrix} I_4^{1/2}(p^I_\infty, q^\infty_I)$$

- So: parametrize scalars ***at infinity*** using the charge/scalar relation determined ***at the horizon***.

- The ***full attractor flow*** has the radial function

$$R^4(r) = I_4(P^I + r p^I_\infty, Q_I + r q^\infty_I)$$

The Symmetry Breaking Scale

- The radial derivative of R^2 gives the symmetry breaking scale:

$$L = \frac{\pi}{G_4}\left(p^I_\infty \frac{\partial}{\partial P^I} + q_I^\infty \frac{\partial}{\partial Q_I}\right) I_4(P^I, Q_I)\ .$$

- The radial derivative is equivalent to ***a derivative in charge space***.

- So the ***nAttractor behavior*** follows from ***attractor geometry***.

- The derivative replaces a charge by its corresponding modulus.

- I_4 is quartic in the charges; L is ***cubic in charges*** and linear in moduli.

A Flow of Many Fields

- "The" breaking scale is (essentially) the radial derivative of R^2.

- Other scalar fields ***approach*** their fixed value $z^i_{\rm hor}$ at the horizon.

- Their radial derivatives from differentiation in charge space:

$$\frac{dz^i}{dr} = \left(p^I_\infty \frac{\partial}{\partial P^I} + q_I^\infty \frac{\partial}{\partial Q_I} \right) z^i_{\rm hor}$$

- In general ***each scalar field introduces a scale***.

Explicit Example: The STU Model

- Eg.: $F = \frac{X^1 X^2 X^3}{X^0}$, simplify charges so $p^0 = 0, q_1 = q_2 = q_3 = 0$.
- The effective potential

$$V = \frac{1}{8y^1 y^2 y^3} \left(q_0^2 + (p^1 y^2 y^3)^2 + (p^2 y^3 y^1)^2 + (p^3 y^1 y^2)^2 \right) .$$

  p^i are M5-brane numbers, q_0 is momentum quantum number
  $y^i = -\mathrm{Im} z^i$ are volumes of 4-cycles (in string units).

- The ***extremal*** attractor gives scalar fields y^i ***at the horizon*** as

$$y^i_{\mathrm{hor}} = \sqrt{\frac{q_0}{p^1 p^2 p^3}}\, p^i$$

  independently of their asymptotic values.

- The extremal entropy

$$S = 4\pi V_{\mathrm{hor}} = 2\pi \sqrt{q_0 p^1 p^2 p^3}$$

The nAttractor Mechanism

- Present moduli ***at infinity*** as “charges” by inverting

$$y^i_\infty = \sqrt{\frac{q_0^\infty}{p^1_\infty p^2_\infty p^3_\infty}}\, p^i_\infty$$

- The ***symmetry breaking scale***/specific heat:

$$L = \frac{\pi^2}{G_4}\left(p^i_\infty \frac{\partial}{\partial P^i} + q_0^\infty \frac{\partial}{\partial Q_0}\right) I_4$$

$$= \pi^2 q_0 p^1 p^2 p^3 R_{11}\left(\frac{1}{q_0} + \frac{1}{p^1 y^2_\infty y^3_\infty} + \frac{1}{p^2 y^3_\infty y^1_\infty} + \frac{1}{p^3 y^1_\infty y^2_\infty}\right)$$

- It ***depends on moduli at infinity***.

- It depends on ***non-trivial combinations of charges***.

The Long String Scale

- In the ***dilute gas regime*** the momentum charge is ***small compared to background*** charges (M5-branes).

- Then the symmetry breaking scale is

$$L_{\text{long}} = 2\pi p^1 p^2 p^3 R_{11}$$

- This is the ***long string scale*** known from microscopic black hole models.

- Physics: low energy excitations "live" on a circle of length L_{long} rather than on a circle of radius R_{11}.

- The scales for spatial dependence of the y^i is similar, but with charges permuted.

Summary

- $\mathrm{nAdS}_2/\mathrm{nCFT}_1$ holography describes the ***near*** horizon region of ***nearly*** extreme black holes.

- The ***nAttractor*** mechanism computes ***near*** horizon scalars and ***near*** extreme heat capacity in terms of the ***extreme*** attractor.

- The nAttractor introduces new parameters: the moduli, i.e. the asymptotic values of the scalars.

- It computes the ***intrinsic scales*** of $\mathrm{nAdS}_2/\mathrm{nCFT}_1$.

This research was supported by the US DoE.