# Probing holographic dualities through Penrose limits and dual spin chains

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and
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## Summary:

- 0. **Introduction**
- 1. **GJV duality and 2+1d CS-SYM in IR**
- 2. **Penrose limits of GJV and string quantization**
- 3.**Spin chains in CS-SYM field theory IR limit**
- 4. **Penrose limits of Abelian and Nonabelian T-duals of** $AdS_5 \times S^5$
- 5. **Operators in field theory, RG flow and deconstruction**
- 6. **Conclusions.**

# 0. Introduction

•Penrose limit of gravity dual background: pp wave $\leftrightarrow$ a large $J$ charge limit.

•$AdS_5 \times S^5 \rightarrow$ max. susy pp wave $\leftrightarrow$ BMN operators

•Pp wave calculable for strings $\leftrightarrow$ lightcone Hamiltonian $H$ $\leftrightarrow$ $\Delta - J$ calculable for BMN operators

•Want to use the same logic for less symmetric, and more complicated holographic dual pairs.

•We use it for GJV duality, and for T-duals of $AdS_5 \times S^5$.

•GJV duality: warped, squashed $AdS_4 \times S^6$ vs. 2+1d $\mathcal{N} = 2$ SYM-CS theory *in the IR*.

•T-duals of $AdS_5 \times S^5 \rightarrow$ less susy *for the background*.

•Abelian: known, but dual field theory only conjectured: quiver field theory.

•Nonabelian: recent, less known about it $\rightarrow$ also a quiver.

•Find spin chains in the Penrose limit, but mismatch in conformal dimensions. Fixable for GJV, not so much for T-duals: field theory issue here?

# 1. GJV duality and 2+1d CS-SYM in the IR

• GJV find geometry (in string frame)

$$\begin{aligned}
\mathrm{d}s^2 &= e^{\phi/2+2A}\left(\mathrm{d}s^2_{AdS_4}+\frac{3}{2}\mathrm{d}\alpha^2+\frac{6\sin^2\alpha}{(3+\cos 2\alpha)}\mathrm{d}s^2_{\mathbb{CP}^2}+\frac{9\sin^2\alpha}{(5+\cos 2\alpha)}\eta^2\right),\\
&\equiv L^2_{\text{string}}\left(\mathrm{d}s^2_{AdS_4}+\frac{3}{2}\mathrm{d}\alpha^2+\Xi\mathrm{d}s^2_{\mathbb{CP}^2}+\Omega\eta^2\right)\\
e^{\phi} &= e^{\phi_0}\frac{(5+\cos 2\alpha)^{3/4}}{(3+\cos 2\alpha)},\quad B=-\frac{6L^2e^{\phi_0/2}\sqrt{2}\sin^2\alpha\cos\alpha}{(3+\cos 2\alpha)}\mathcal{J}-\frac{3L^2e^{\phi_0/2}}{\sqrt{2}}\sin\alpha\mathrm{d}\alpha\wedge\eta,\\
\widetilde{F}_0 &= \frac{1}{\sqrt{3}L\,e^{5\phi_0/4}},\\
\widetilde{F}_2 &= -\frac{\sqrt{6}L}{e^{3\phi_0/4}}\left(\frac{4\sin^2\alpha\cos\alpha}{(3+\cos 2\alpha)(5+\cos 2\alpha)}\mathcal{J}+\frac{3(3-\cos 2\alpha)}{(5+\cos 2\alpha)^2}\sin\alpha\,\mathrm{d}\alpha\wedge\eta\right),\\
\widetilde{F}_4 &= \frac{L^3}{e^{\phi_0/4}}\Bigg(6\mathrm{vol}(AdS_4)-12\sqrt{3}\frac{(7+3\cos 2\alpha)}{(3+\cos 2\alpha)^2}\sin^4\alpha\,\mathrm{vol}(\mathbb{CP}^2)\\
&\qquad +18\sqrt{3}\frac{(9+\cos 2\alpha)\sin^3\alpha\cos\alpha}{(3+\cos 2\alpha)(5+\cos 2\alpha)}\mathcal{J}\wedge\mathrm{d}\alpha\wedge\eta\Bigg)
\end{aligned}$$

where $\eta=d\psi+\omega, d\omega=2\mathcal{J}$, $\omega=\frac{1}{2}\sin^2\lambda(\mathrm{d}\sigma+\cos\theta\mathrm{d}\phi)$, and

$$\mathrm{d}s^2_{\mathbb{CP}^2}=\mathrm{d}\lambda^2+\frac{1}{4}\sin^2\lambda\left\{\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\phi^2+\cos^2\lambda(\mathrm{d}\sigma+\cos\theta\mathrm{d}\phi)^2\right\}$$

•Quantizing changes, $n_p = \int \hat{F}_p$, we find that the parameters of the solutions are $n_6 = N$ and $n_0 = k = 2\pi l_s m$, and that

$$L = \frac{\pi^{3/8}\ell_s}{2^{7/48}3^{7/24}}(kN^5)^{1/24}; \qquad e^{\phi_0} = \frac{2^{11/12}\pi^{1/2}}{3^{1/6}}\frac{1}{(k^5N)^{1/6}} \Rightarrow$$

$$L^2_{\text{string}} = \frac{2^{1/6}\pi}{3^{2/3}}\left(\frac{N}{k}\right)^{1/3}\ell_s^2\sqrt{5+\cos 2\alpha}$$

•Field theory: $\mathcal{N}=2$ SYM-CS in the IR:

-chiral superfield $\Phi = \phi + \sqrt{2}\theta\psi + \theta\theta F$

-vector superfield (in WZ gauge)

$\mathcal{V}(x) = 2i\theta\bar{\theta}\sigma + 2\theta\gamma^\mu\bar{\theta}A_\mu + i\sqrt{2}\theta^2\bar{\theta}\bar{\chi} - i\sqrt{2}\bar{\theta}^2\theta\chi + \theta^2\bar{\theta}^2 D$

•Is dimensional reduction of $\mathcal{N}=4$ SYM in 3+1d. Then, $\exists$ superpotential

$$\mathcal{W} = g\text{Tr}\,(\Phi_1[\Phi_2,\Phi_3]) = \frac{g}{12}\epsilon_{ijk}f^{abc}\Phi_i^a\Phi_j^b\Phi_k^c$$

•Action (in the IR), CS + matter + superpotential,

$$
\begin{aligned}
S_{CS} &= \frac{k}{4\pi}\int \mathrm{d}^3x \mathrm{Tr}\left[\epsilon^{\mu\nu\rho}\left(A_\mu\partial_\nu A_\rho + \frac{2i}{3}A_\mu A_\nu A_\rho\right) + i\bar{\chi}\chi - 2D\sigma\right] \\
\mathcal{L}_m &= -\mathrm{Tr}\left[(D_\mu\phi)^{i\dagger}D^\mu\phi^i + i\bar{\psi}^i\gamma^\mu D_\mu\psi^i - \bar{F}^i F^i + \phi^{i\dagger}D\phi^i + \phi^{i\dagger}\sigma^2\phi^i - i\bar{\psi}^i\sigma\psi^i \right. \\
&\quad \left. + i\phi^{i\dagger}\chi\psi^i + i\bar{\psi}^i\bar{\chi}\phi^i\right] \\
\mathcal{L}_{sp} &= -\int \mathrm{d}^2\theta\mathcal{W}(\Phi) - \int \mathrm{d}^2\bar{\theta}\overline{\mathcal{W}(\Phi)} \\
&= -\mathrm{Tr}\left(\frac{\partial\mathcal{W}(\phi)}{\partial\phi^i}F^i + \frac{\partial\overline{\mathcal{W}(\phi)}}{\partial\bar{\phi}^i}\bar{F}^i - \frac{1}{2}\frac{\partial^2\mathcal{W}(\phi)}{\partial\phi^i\partial\phi^j}\psi^i\psi^j - \frac{1}{2}\frac{\partial^2\overline{\mathcal{W}(\phi)}}{\partial\bar{\phi}^i\partial\bar{\phi}^j}\bar{\psi}^i\bar{\psi}^j\right)
\end{aligned}
$$

•Solve for $\sigma^a$, $D$ and find interaction potential terms

$$
\begin{aligned}
V_{\mathrm{conf}} &= \frac{4\pi^2}{k^2}\mathrm{Tr}\left([[\phi^{i\dagger},\phi^i],\phi^{k\dagger}][[\phi^{j\dagger},\phi^j],\phi^k]\right) \\
V_{\mathrm{sp}} &= \frac{g^2}{2}\mathrm{Tr}\left([\phi_i,\phi_j][\phi^{i\dagger},\phi^{j\dagger}]\right)
\end{aligned}
$$

•Obs.: in the IR, SYM subleading to CS $\rightarrow$ drop it.

•Moreover, in the IR, conformal term in the potential remains, while nonconformal one is in the vacuum.

•Indeed, classical conformal dimensions: $[g] = 1/2, [\phi] = 1/2 = [\Phi]$, $([V] = 3, [W] = 2)$, $[\psi] = 1, [\theta] = -1/2$.

•Then, $V_{\text{sp}}$ must be put to 0 (vacuum) in the IR $\Rightarrow [\phi_i, \phi_j] = 0$, $\forall i \neq j$.

•Obs: one still has $[\phi_i, \bar{\phi}_j] \neq 0$.

•Alternative explanation: Gaiotto+Yin: D2/D6 in massive IIA. D2:$\Phi_1 \perp$D6, $\Phi_2, \Phi_3 ||$D6, $Q, \tilde{Q}$ : D2-D6 coords. Then *in the IR*,

$$\mathcal{W} = \mathrm{Tr}\,[\Phi_1[\Phi_2, \Phi_3]] + \tilde{Q}\Phi_1 Q$$

•$[\Phi_1] = 1$, $[Q] = [\tilde{Q}] = [\Phi_{2,3}] = 1/2$. Why? $\Phi_1$ is auxiliary (no $\int d^4\theta\bar{\Phi}_1\Phi_1$). Can add $\epsilon\mathrm{Tr}\,\Phi_1^2/2 \Rightarrow$

$$\mathcal{W} = \frac{1}{2\epsilon}\mathrm{Tr}\,[([\Phi_3, \Phi_3] + \tilde{Q}Q)^2]$$

•In our symmetric case (GJV): $[\Phi_1] = [\Phi_2] = [\Phi_3] = 2/3$; then only $\partial\mathcal{W}/\partial\phi^i F^i$ in action ($[F] = 5/3$), so $\delta S/\delta F^i \Rightarrow$ again $\partial\mathcal{W}/\partial\phi_i = 0 \Rightarrow [\phi_i, \phi_j] = 0$.

## 2. Penrose limits of GJV and their quantization

•Penrose limit: close to null geodesic $\Rightarrow$ for geodesic parametrized by $\lambda$, in direction $x^\lambda$, need no $\perp$ acceleration, ${\Gamma^i}_{\lambda\lambda} = 0$. For isometry direction,

$$g^{ij}\partial_j g_{\lambda\lambda} = 0$$

•Also $ds^2 = 0$ (null).

•GJV: $\sigma, \psi, \phi$ isometries: $\partial_\sigma g_{\mu\nu} = \partial_\phi g_{\mu\nu} = \partial_\psi g_{\mu\nu} = 0$.

•Motion in $\psi$: $\alpha = \pi/2, \rho = 0, \lambda = 0$.
•Motion in $\sigma$: $\lambda = \pi/4, \alpha = \pi/2, \theta = \pi/2, \rho = 0$.
•Motion in $\phi$: $\lambda = \pi/2,\ \alpha = \pi/2,\ \theta = \pi/2,\ \rho = 0$.
•Motion in $\phi + \sigma$: $\theta = 0, \lambda = \pi/4, \alpha = \pi/2, \rho = 0, \psi = 0$.

•**Motion in** $\psi$: pp wave:

$$
\begin{aligned}
\mathrm{d}s^2_{pp} &= -4\mathrm{d}x^+\mathrm{d}x^- + \mathrm{d}u^2 + \sum_{i=1}^{3}(\mathrm{d}y_i)^2 + \sum_{j=1}^{2}\mathrm{d}w_j\mathrm{d}\bar{w}_j - \left(u^2 + \sum_{i=1}^{3} y_i^2 + \frac{1}{4}\sum_{j=1}^{2}|w_j|^2\right)(\mathrm{d}x^+)^2 \\
e^{\phi} &= \sqrt{2}e^{\phi_0}, \quad H_3 = 0, \\
\widetilde{F}_0 &= 0, \quad \widetilde{F}_2 = -\frac{e^{-\phi_0}}{\sqrt{2}}\mathrm{d}u\wedge\mathrm{d}x^+, \quad \widetilde{F}_4 = \frac{3e^{-\phi_0}}{\sqrt{2}}\mathrm{d}x^+\wedge\mathrm{d}y_1\wedge\mathrm{d}y_2\wedge\mathrm{d}y_3
\end{aligned}
$$

•Symmetries

$$U(1)_{\pm}\times SO(3)_r\times U(1)_u\times SO(3) \to U(1)_R\times SU(2)_r\times U(1)_u\times SU(2)_L$$

•Closed string in background find Hamiltonian

$$H = \sum_{n=-\infty}^{\infty}\left\{\sum_{A=1}^{4} N_n^{(A)}\sqrt{1+\frac{n^2}{(\alpha' p^+)^2}} + \sum_{B=5}^{8} N_n^{(B)}\sqrt{\frac{1}{4}+\frac{n^2}{(\alpha' p^+)^2}}\right\}$$

and eigenenergies at $n/(\alpha' p^+) \ll 1$

$$E^A \simeq 1 + \frac{1}{2}\frac{n^2}{(\alpha' p^+)^2}, \quad E^B \simeq \frac{1}{2} + \frac{n^2}{(\alpha' p^+)^2}$$

•**Motion in** $\sigma$: pp wave

$$
\begin{aligned}
\mathrm{d}s^2_{pp} &= -4\mathrm{d}x^+\mathrm{d}x^- + \sum_{i=1}^{3}\mathrm{d}x_i^2 + \sum_{k=4}^{8}\mathrm{d}y_k^2 \\
&\quad - \left(\frac{1}{2}\sum_{i=1}^{3}x_i^2 + \frac{2y_4^2}{3} + \frac{y_5^2+y_6^2}{8} + \frac{y_7^2+y_8^2}{6}\right)(\mathrm{d}x^+)^2 \\
&\quad -\frac{1}{6}\left[y_5\cos\left(\frac{\sqrt{2}x^+}{4}\right) + y_6\sin\left(\frac{\sqrt{2}x^+}{4}\right)\right]^2(\mathrm{d}x^+)^2 \\
B &= -\sqrt{\frac{2}{3}}\left[y_5\cos\left(\frac{\sqrt{2}x^+}{4}\right) + y_6\sin\left(\frac{\sqrt{2}x^+}{4}\right)\right]\mathrm{d}y_4\wedge\mathrm{d}x^+\ , \\
\widetilde{F}_4 &= \frac{3e^{-\phi_0}}{2}\mathrm{d}x^+\wedge\mathrm{d}x_1\wedge\mathrm{d}x_2\wedge\mathrm{d}x_3 \\
&\quad + \frac{e^{-\phi_0}}{\sqrt{3}}\mathrm{d}x^+\wedge\left[\cos\left(\frac{\sqrt{2}x^+}{4}\right)\mathrm{d}y_5 + \sin\left(\frac{\sqrt{2}x^+}{4}\right)\mathrm{d}y_6\right]\wedge\mathrm{d}y_7\wedge\mathrm{d}y_8
\end{aligned}
$$

•**Motion in** $\phi$: pp wave

$$
\begin{aligned}
\mathrm{d}s^2 &= -2\mathrm{d}x^+\mathrm{d}x^- + \mathrm{d}\rho^2 + \rho^2\mathrm{d}s^2(S^2) + \mathrm{d}v^2 + \mathrm{d}x^2 + x^2\mathrm{d}\sigma^2 + \mathrm{d}w_1^2 + \mathrm{d}w_2^2 \\
&- \left[\frac{1}{3}v^2 + \frac{1}{4}\rho^2 + \frac{1}{12}x^2 + \frac{1}{12}\left(\cos\frac{x^+}{4}w_1 - \sin\frac{x^+}{4}w_2\right)^2 + \frac{1}{16}(w_1^2 + w_2^2)\right](\mathrm{d}x^+)^2, \\
e^\phi &= \sqrt{2}e^{\phi_0}, \\
\widetilde{F}_4 &= \frac{3}{2\sqrt{2}e^{\phi_0}}\mathrm{d}x^+ \wedge \rho^2\mathrm{d}\rho \wedge \mathrm{vol}(S^2) + \frac{1}{\sqrt{6}e^{\phi_0}}\mathrm{d}x^+ \wedge x\mathrm{d}x \wedge \mathrm{d}z \wedge \mathrm{d}\sigma, \\
H_3 &= \frac{1}{\sqrt{3}}\mathrm{d}x^+ \wedge \left(\cos\frac{x^+}{4}\mathrm{d}w_1 - \sin\frac{x^+}{4}\mathrm{d}w_2\right) \wedge \mathrm{d}v
\end{aligned}
$$

•**Motion in** $\phi+\sigma$: pp wave is (modulo rescalings and signs) the same as for motion in $\sigma$.

## Open string quantization

.

•Open strings attached to D4-brane wrapping $\mathbb{R}_t \times \mathbb{CP}^2$: $(t, \lambda, \theta, \phi, \sigma)$ $\rightarrow (x^{\pm}, x, y, z) \rightarrow x^{\pm}, Y_7, Y_8, \left[Y_5 \frac{\sqrt{2}\sigma^0}{4} + Y_6 \sin \frac{\sqrt{2}\sigma^0}{4}\right]$. String action is

$$\begin{aligned} S_{pp} &= -\frac{1}{4\pi\alpha'}\int d\sigma^0 \int_0^{\pi\alpha' p^+} d\sigma^1 \left[\eta^{ab}(\partial_a X_i \partial_b X_i + \partial_a Y_k \partial_b Y_k) + \mu^2 \left(\frac{X_i^2}{2} + \frac{2Y_4^2}{3}\right.\right. \\ &\quad \left. + \frac{Y_5^2 + Y_6^2}{8} + \frac{Y_7^2 + Y_8^2}{6} + \frac{1}{6}\left[Y_5 \frac{\sqrt{2}\sigma^0}{4} + Y_6 \sin \frac{\sqrt{2}\sigma^0}{4}\right]^2\right) \\ &\quad \left. + 2\sqrt{\frac{2}{3}}\mu \left[Y_5 \frac{\sqrt{2}\sigma^0}{4} + Y_6 \sin \frac{\sqrt{2}\sigma^0}{4}\right] \partial_1 Y_4 \right] \end{aligned}$$

•$X_4, Y_5, Y_6 \rightarrow$ hard. $X_i, Y_7, Y_8$ simple. Eigenmodes

$$\omega_n^{(i)} = \sqrt{\frac{\mu^2}{2} + \frac{n^2}{(\alpha' p^+)^2}}\,, \quad \omega_n^{(I')} = \sqrt{\frac{\mu^2}{6} + \frac{n^2}{(\alpha' p^+)^2}}, \quad I' = 7, 8.$$

and same for $\phi, \sigma$, or $\phi + \sigma$ pp waves.

# 3. Spin chains in CS-SYM field theory IR limits

•Interactions in IR involving scalars $\phi_i$: $[\phi_i,\phi_j]=0$, and

$$H_{\text{int},1}=\frac{4\pi^2}{k^2}\text{Tr}\left([[\phi^{i\dagger},\phi^i],\phi^{k\dagger}][[\phi^{j\dagger},\phi^j],\phi^k]\right)$$

•Closed string spin chain: pick out $Z$ among $\phi_i$ $\rightarrow$ rest $\phi^m$, $m=1,2$. $Z\rightarrow e^{i\alpha}Z$ corresponds to $U(1)$ of pp wave in $\psi$. Define $\Delta - J = 0$ for $Z \Rightarrow J_Z = 1/2$. Then,

| | $Z$ | $\bar{Z}$ | $\phi^m$ | $\bar{\phi}^m$ | $A_\mu$ | $D_\mu$ |
|---|---|---|---|---|---|---|
| $\Delta$ | 1/2 | 1/2 | 1/2 | 1/2 | 1 | 1 |
| $J$ | 1/2 | −1/2 | 0 | 0 | 0 | 0 |
| $\Delta - J$ | 0 | 1 | 1/2 | 1/2 | 1 | 1 |

•Unique object with $\Delta - J = 0$, $Z$, defines the vacuum

$$|0,p^+\rangle \leftrightarrow \frac{1}{\sqrt{J}N^{J/2}}\text{Tr}\,[Z^J]$$

•Oscillators: trickier. Since $[\phi_i, \phi_j] = 0$, one possibility is $[\phi^m, \bar{\phi}^m]$. Also $(\phi_i \phi_j)$ (like for ABJM) doesn't work.

•Only possibility for oscillators

$$\Phi_M = \{\phi^m, \bar{\phi}^m, \bar{Z}, D_a\}$$

such that

$$a_n^{\dagger M}|0, p^+\rangle \sim \sum_{l=0}^{J-1} e^{\frac{2\pi i n l}{J}} \mathrm{Tr}\,[Z^l \Phi^M Z^{J-l}]$$

•Then classical dimensions match $n = 0$ pp wave results: 4 with $\Delta - J = 1$ and 4 with $\Delta - J = 1/2$.

•Insertion of $\Phi_M = \bar{Z} \Rightarrow$ Feynman diagrams give factor $f_{\bar{Z}}(p) = 8\sin^2\frac{p}{2}\left(1 - 2\sin^2\frac{p}{2}\right)$

$\rightarrow$ string Hamiltonian $2(\phi')^2 - (\phi'')^2$.

•Insertion of $\Phi_M = \bar{\phi}^m \Rightarrow f_{\bar{\phi}}(p) = 8\sin^2\frac{p}{2}\left(5 - 6\sin^2\frac{p}{2}\right)$

$\rightarrow$ string Hamiltonian $10(\phi')^2 - 3(\phi'')^2$.

•Insertion of $\Phi_M = \phi^m \Rightarrow f_{\phi}(p) = \left[16\sin^2\frac{p}{2}\left(1 - \frac{2}{3}\sin^2\frac{p}{2}\right)\right]$

$\rightarrow$ string Hamiltonian $4(\phi')^2 - \frac{2}{3}(\phi'')^2$.

•Feynman diagram factor is

$$\frac{\mathcal{F}(x)}{\mathcal{F}^{\text{tree}}(x)} = 1 + f_i(p)\frac{N^2}{2k^2}\ln|x|\Lambda + \text{finite} = 1 + f_i(p)\frac{\lambda^2}{2}\ln|x|\Lambda + \text{finite}$$

to be compared with

$$\frac{\mathcal{F}(x)}{\mathcal{F}^{\text{tree}}(x)} = (1 + \text{finite})\frac{|x|^{\Delta_{\text{tree}}}}{|x|^{\Delta(\lambda)}} \simeq 1 - \delta\Delta(\lambda)\ln|x|\Lambda + \text{finite}$$

•Thus

$$\Delta - J \simeq (\Delta - J)(\text{tree}) + \delta\Delta(\lambda) = (\Delta - J)(\text{tree}) - f_i(p)\frac{\lambda^2}{2}$$

•In $\mathcal{N} = 4$ SYM, we had no $f(\lambda) \to$ max. susy. In $\mathcal{N} = 6$ ABJM (3/4 maximal susy) $\exists$ one $f(\lambda)$.

•Now, $f_i(\lambda)$ for each insertion, and at small coupling, $f_i(\lambda)g(p) = f_i(p)\lambda^2$.

•Moreover, valid only at small $p$.

•At strong coupling,

$$\frac{n^2}{(\alpha' p^+)^2} = \left(\frac{L^2_{\text{string}}}{\alpha'}\right)^2 \frac{n^2}{J^2} \propto \lambda^{2/3}\frac{n^2}{J^2}$$

•$\bar{Z}$ insertion:

$$\Delta - J \simeq 1 - 4\lambda^2 \sin^2\frac{p}{2}\left(1 - 2\sin^2\frac{p}{2}\right) \quad \text{vs.} \quad \sqrt{1 - f_{\bar{Z}}(\lambda)\sin^2 p/2} \simeq 1 - \frac{1}{2} f_{\bar{Z}}(\lambda)\sin^2\frac{p}{2}$$

gives $f\bar{Z}(\lambda) \simeq 8\lambda^2$ to $f_{\bar{Z}}(\lambda) \propto \lambda^{2/3}$.

•$\bar{\phi}^m$ insertion:

$$\Delta - J \simeq 1 - 4\lambda^2 \sin^2\frac{p}{2}\left(1 - 2\sin^2\frac{p}{2}\right) \quad \text{vs.} \quad \sqrt{1 - f_{\bar{Z}}(\lambda)\sin^2 p/2} \simeq 1 - \frac{1}{2} f_{\bar{Z}}(\lambda)\sin^2\frac{p}{2}$$

gives $f_{\bar{\phi}}(\lambda) \simeq 40\lambda^2$ vs. $f_{\bar{\phi}}(\lambda) \propto 2\lambda^{2/3}$.

•$\phi^m$ insertion:

$$\Delta - J \simeq \frac{1}{2} - 8\lambda^2 \sin^2\frac{p}{2}\left(1 - \frac{2}{3}\sin^2\frac{p}{2}\right), \quad \text{vs.} \quad \sqrt{\frac{1}{4} - \frac{1}{2} f_{\phi}(\lambda)\sin^2\frac{p}{2}} \simeq \frac{1}{2} - \frac{1}{2} f_{\phi}(\lambda)\sin^2\frac{p}{2}$$

gives $f_{\phi}(\lambda) \simeq 16\lambda^2$ vs. $f_{\phi}(\lambda) \propto 2\lambda^{2/3}$.

•**Open strings on D-branes**: not even $\Delta - J$ at $n = 0$ works out.

# 4. Penrose limits of Abelian and Nonabelian T-duals of $AdS_5 \times S^5$

- Apply the same technique to understand field theories AdS/CFT dual to T-duals of $AdS_5 \times S^5$.
- Abelian T-dual on $\tilde{\psi}$ of $AdS_5 \times S^5/\mathbb{Z}_k$: coordinate acted on by $\mathbb{Z}_k : \tilde{\psi}$.

$$
\begin{aligned}
ds^2 &= 4\,L^2\,ds^2(\mathrm{AdS}_5) + 4\,L^2\,d\Omega_2^2(\alpha,\beta) + \frac{L^2 d\psi^2}{\cos^2\alpha} + L^2\cos^2\alpha\,d\Omega_2^2(\chi,\xi)\ ,\\
B_2 &= L^2\,\psi\,\sin\chi\,d\chi\wedge d\xi\ , \qquad F_4 = \frac{8\,L^4}{g_s\sqrt{\alpha'}}\cos^3\alpha\,\sin\alpha\,\sin\chi\,d\alpha\wedge d\beta\wedge d\chi\wedge d\xi\\
e^{-2\Phi} &= \frac{L^2\cos^2\alpha}{g_s^2\alpha'}
\end{aligned}
$$

Here we rescaled $\tilde{\psi} = \frac{L^2}{\alpha'}\psi$, meaning $\psi = \frac{\alpha'}{L^2}\tilde{\psi} \in \left[0, 2\pi k\alpha'/L^2\right]$.
Also define $g_s = \frac{L}{\sqrt{\alpha'}}\tilde{g}_s$.

- For Penrose limits, $\exists$ 3 isometries: $\xi, \beta, \psi$.
- For geodesic in $\xi$: at $\chi = \pi/2$, $\alpha = 0, r = 0, \psi = 0$.

$$
\begin{aligned}
ds^2 &= 4\,dx^+dx^- + d\bar{r}^2 + \bar{r}^2 d\Omega_3^2 + dx^2 + x^2 d\beta^2 + dz^2 + dy^2 - \left(\bar{r}^2 + x^2 + 4\,z^2\right)(dx^+)^2\\
B_2 &= 2\,y\,dz\wedge dx^+\ , \qquad e^{-2\Phi} = \frac{1}{\tilde{g}_s^2} \qquad F_4 = \frac{4\,x}{\tilde{g}_s}dx\wedge d\beta\wedge dz\wedge dx^+.
\end{aligned}
$$

•Geodesic in $\beta$: at $\alpha = \pi/2, r = 0, \psi = \psi_0, \chi = \chi_0, \xi = \xi_0$,

$$
\begin{aligned}
ds^2 &= 4\,dx^+dx^- + d\bar{r}^2 + \bar{r}^2 d\Omega_3^2 + 4\,dy^2 + y^2 d\Omega_2^2(\chi,\xi) + \frac{\alpha'^2 d\tilde{\psi}^2}{y^2} - (\bar{r}^2 + 4\,y^2)(dx^+)^2 \\
B_2 &= \alpha' \tilde{\psi} \sin\chi \, d\chi \wedge d\xi \ , \qquad e^{-2\phi} = \frac{y^2}{\alpha' g_s^2} \\
F_4 &= \frac{8\,y^3}{g_s\sqrt{\alpha'}} \sin\chi \, dx^+ \wedge dy \wedge d\chi \wedge d\xi
\end{aligned}
$$

doesn't test T-duality.

•Is not in Brinkmann form $\to$ unclear.

•In $\psi$: at $\alpha = 0, \chi = \pi/2$. But motion just in $\psi$ is pathologic.

•Geodesic must move in $\psi$ AND $\xi$, with $\dot{\xi} = \frac{d\xi}{du} = -J$ (at $dt/du = 1/4$).

•From null condition, find

$$
\dot{\psi}^2 = \frac{1}{4}\left(1 - 4\,J^2\right) \quad \Longrightarrow \quad \psi = \frac{\sqrt{1 - 4\,J^2}}{2}\,u
$$

which means $J \leq 1/2$.

•Pp wave is

$$ds^2 = 2\,du\,dv + d\bar{r}^2 + \bar{r}^2\,d\Omega_3^2 + dz^2 + dx^2 + x^2\,d\beta^2 + dw^2 - \left[\frac{\bar{r}^2}{16} + \frac{8J^2-1}{16}\,x^2 + J^2\,z^2\right] du^2$$

$$e^{2\,\Phi.} = g_s^2\frac{\alpha'}{L^2} \equiv \tilde{g}_s^2 \;, \qquad B_2 = \frac{u}{2}\,dz\wedge dw \;, \qquad F_4 = \frac{2\,J\,x}{\tilde{g}_s}\,du\wedge dz\wedge dx\wedge d\beta$$

•Non-tachyonic mode $\Rightarrow 8J^2 - 1 \geq 0 \Rightarrow \frac{1}{2\sqrt{2}} \leqslant J \leqslant \frac{1}{2}$.

•Closed string in pp wave gives $(\tilde{\kappa}_1 + \tilde{\kappa}_2 = 1)$

$$S = -\frac{1}{4\,\pi\,\alpha'}\int d\tau\,d\sigma\Bigg[\partial X^i\cdot\partial X^i + \frac{\left(X^1\right)^2 + \left(X^2\right)^2 + \left(X^3\right)^2 + \left(X^4\right)^2}{16}$$
$$+\frac{\left(X^5\right)^2 + \left(X^6\right)^2}{16}\,(8J^2-1) + J^2\left(X^7\right)^2 - \left(\tilde{\kappa}_1\,X^7\,\partial_\sigma X^8 - \tilde{\kappa}_2\,X^8\,\partial_\sigma X^7\right)\Bigg]$$

and frequencies

$$\omega_{n,i}^2 = n^2 + \frac{1}{16}\;, \qquad i = 1,\ldots,4$$
$$\omega_{n,i}^2 = n^2 + \frac{8\,J^2-1}{16}\;, \qquad i = 5,6$$
$$\omega_{n,\pm}^2 = n^2 + \frac{J^2}{2} \pm \frac{1}{2}\sqrt{n^2 + J^4}$$

•Nonabelian T-dual of $AdS_5 \times S^5/\mathbb{Z}_k$ on direction becoming $\tilde{\rho} \in [0, 2\pi k]$ is

$$
\begin{aligned}
ds^2 &= 4\,L^2\,ds^2(\mathrm{AdS}_5) + 4\,L^2\,d\Omega_2^2(\alpha,\beta) + \frac{\alpha'^2 d\tilde{\rho}^2}{L^2\cos^2\alpha} + \frac{\alpha'^2 L^2 \tilde{\rho}^2 \cos^2\alpha}{\alpha'^2\tilde{\rho}^2 + L^4\cos^4\alpha} d\Omega_2^2(\chi,\xi) \\
B_2 &= \frac{\alpha'^3\tilde{\rho}^3}{\alpha'^2\tilde{\rho}^2 + L^4\cos^4\alpha} \sin\chi\, d\chi \wedge d\xi\ , \quad e^{-2\Phi} = \frac{L^2\cos^2\alpha}{g_s^2\,\alpha'^3}\left(\alpha'^2\tilde{\rho}^2 + L^4\cos^4\alpha\right) \\
F_2 &= \frac{8\,L^4}{g_s\,\alpha'^{3/2}} \sin\alpha\,\cos^3\alpha\, d\alpha \wedge d\beta\ , \\
F_4 &= \frac{8\,\alpha'^{3/2}\,L^4}{g_s} \frac{\tilde{\rho}^3\cos^3\alpha}{\alpha'^2\tilde{\rho}^2 + L^4\cos^4\alpha} \sin\alpha\,\sin\chi\, d\alpha\wedge d\beta \wedge d\chi \wedge d\xi
\end{aligned}
$$

•For Penrose limits, $\exists$ 2 isometries: on $\beta$ and $\xi$.

•For geodesic on $\beta$, at $\alpha = \pi/2, \tilde{\rho} = \tilde{\rho}_0, \chi = \chi_0, \xi = \xi_0$, is

$$
\begin{aligned}
ds^2 &= 4\,dx^+\,dx^- + d\vec{r}^2 + \vec{r}^2\,d\Omega_3^2 + dy^2 - (\vec{x}_4^2 + y^2)\,dx^{+2} \\
&\quad + 4\frac{\alpha'^2}{y^2}\,d\tilde{\rho}^2 + \frac{4\,\alpha'^2\tilde{\rho}^2 y^2}{16\,\alpha'^2\tilde{\rho}^2 + y^4} d\Omega_2^2(\chi,\xi) \\
B_2 &= \frac{16\,\alpha'^3\tilde{\rho}^3}{16\,\alpha'^2\tilde{\rho}^2 + y^4} \sin\chi\,d\chi\wedge d\xi\ , \qquad e^{-2\Phi} = g_s^{-2}\,\frac{y^2\left(16\,\alpha'^2\tilde{\rho}^2 + y^4\right)}{64\alpha'^3} \\
g_s\,F_2 &= \frac{y^3}{2\,\alpha'^{3/2}} dx^+ \wedge dy\ , \qquad g_s\,F_4 = \frac{8\,\alpha'^{3/2} y^3 \tilde{\rho}^3}{16\,\alpha'^2\tilde{\rho}^2 + y^4} \sin\chi\,dx^+ \wedge dy \wedge d\chi \wedge d\xi
\end{aligned}
$$

•For geodesic in $\xi$, find no solution $\rightarrow$ ned also motion in $\tilde{\rho}$.

•Define $\tilde{\rho} = \frac{L^2}{\alpha'}\rho$ and redefine also coupling as $\tilde{\tilde{g}}_s = g_s \frac{\alpha'^{3/2}}{L^3}$. Then pp wave near geodesic at $\chi = \pi/2, \alpha = 0, r = 0$ is

$$\begin{aligned} L^{-2}ds^2 &= 4\,ds^2(\mathrm{AdS}_5) + 4\,d\Omega_2^2(\alpha,\beta) + \frac{d\rho^2}{\cos^2\alpha} + \frac{\rho^2\cos^2\alpha}{\rho^2+\cos^4\alpha}d\Omega_2^2(\chi,\xi) \\ B_2 &= \frac{L^2\rho^3}{\rho^2+\cos^4\alpha}\sin\chi\,d\chi\wedge d\xi\ , \qquad e^{-2\Phi} = \frac{\cos^2\alpha}{\tilde{\tilde{g}}_s^2}(\rho^2+\cos^4\alpha) \\ F_2 &= \frac{8\,L}{\tilde{\tilde{g}}_s}\sin\alpha\,\cos^3\alpha\,d\alpha\wedge d\beta\ , \qquad F_4 = \frac{8\,L^3}{\tilde{\tilde{g}}_s}\frac{\rho^3\cos^3\alpha}{\rho^2+\cos^4\alpha}\sin\alpha\,\sin\chi\,d\alpha\wedge d\beta\wedge d\chi\wedge d\xi \end{aligned}$$

•Particle on the null geodesic $\Rightarrow \dot{t} = 1/4$ and $\frac{p_\xi}{L^2} = \frac{\rho^2}{\rho^2+1}\dot{\xi} = -J$. Then, find

$$\dot{\rho}^2 = \frac{1}{4} - \frac{\rho^2+1}{\rho^2}J^2$$

which means again $J \leq 1/2$.

•Moreover, $\tilde{\rho} \geqslant \frac{L^2}{\alpha'}\frac{2J}{\sqrt{1-4J^2}}$ needs to fit in $(0, 2\pi k]$, meaning $k \sim L^2/\alpha'$.

.

•Pp wave becomes

$$
\begin{aligned}
ds^2 &= 2\,du\,dv + d\bar{r}^2 + \bar{r}^2\,d\Omega_3^2 + dx^2 + x^2\,d\beta^2 + dz^2 + dw^2 \\
&\quad - \left[\frac{\bar{r}^2}{16} + \frac{x^2}{16}(8J^2-1) + \frac{(\rho^2+1)^2}{\rho^4}J^2 z^2 - F_z\,z^2 - F_w\,w^2\right] du^2 \\
H &= dB_2 = \frac{1}{2}\frac{\rho^2+3}{\rho^2+1}\,du\wedge dz\wedge dw \\
e^{-2\Phi} &= \frac{\rho^2+1}{\tilde{\tilde{g}}_s^2} \\
F_4 &= \frac{2\,J\,x\,\sqrt{\rho^2+1}}{\tilde{\tilde{g}}_s}\,du\wedge dx\wedge dz\wedge d\beta\ , \quad F_2 = 0
\end{aligned}
$$

which means that non-tachyonic $\Rightarrow 8J^2 - 1 \geq 0 \Rightarrow \frac{1}{2\sqrt{2}} \leqslant J \leqslant \frac{1}{2}$.

•Closed string on pp wave

$$S = -\frac{1}{4\,\pi\,\alpha'}\int d\tau\, d\sigma\Bigg[\partial X^i\cdot\partial X^i + \Bigg(\frac{\left(X^1\right)^2+\left(X^2\right)^2+\left(X^3\right)^2+\left(X^4\right)^2}{16}$$

$$+\frac{\left(X^5\right)^2+\left(X^6\right)^2}{16}\left(8J^2-1\right)+\frac{(\rho^2+1)^2}{\rho^4}J^2\left(X^7\right)^2 - F_z\left(X^7\right)^2 - F_w\left(X^8\right)^2\Bigg]$$

$$-\frac{\rho^2+3}{\rho^2+1}\left(\kappa_1\,X^7\,\partial_\sigma X^8 - \kappa_2\,X^8\,\partial_\sigma X^7\right)\Bigg]$$

•Frequencies are

$$\omega^2_{n,i} = n^2+\frac{1}{16}\,, \qquad i=1,\ldots,4\,, \quad \omega^2_{n,i} = n^2+\frac{8\,J^2-1}{16}\,, \qquad i=5,6$$

•$n \gg 1$: Abelian T-dual

•$n \ll 1$ gives

$$\omega^2_{n,i} = n^2+\frac{1}{16}\,, \qquad i=1,\ldots,4$$
$$\omega^2_{n,i} = n^2+\frac{1-2\,a}{16}\,, \qquad i=5,6$$
$$\omega^2_{n,\pm} = n^2+\frac{1}{16}\left(a^2+2\right)\pm\frac{1}{8}\left(a+1\right)\sqrt{16\,n^2+\left(a-1\right)^2}$$

•Abelian and Nonabelian pp waves both preserve just the minimum (for pp waves) 1/2 susy.

# 5. Operators in field theory, RG flow and deconstruction

## AdS/CFT map

•Conjecture for field theories: quivers: Abelian: $SU(N)^k \to \mathcal{N} = 2$ susy. Each node: $\mathcal{N} = 2$ vector multiplet of $SU(N)$, each 2 adjoining nodes: $\mathcal{N} = 2$ bifundamental hypers.

•Nonabelian: infinitely long quiver, $SU(N) \times SU(2N) \times ... \times SU(kN) \times$ .... Terminates only for a completion of the background.

•Quiver dual $AdS_5 \times S^5/\mathbb{Z}_k$ and its Penrose limit was considered before [Alishahiha+Sheikh-Jabbari 2002 & Mukhi+Rangamani+Verlinde 2002].

## Field theory limit

• Limit is different than theirs.

• $\psi$ or $\rho \in [0, 2\pi k]$ gives $k \sim L^2/\alpha' = \sqrt{4\pi g_s^B N}$, meaning

$$\frac{g_{YM}^2 N}{k} = \frac{4\pi g_s^B N}{k} \sim k \to \infty$$

unlike the case in previous papers. T-duality acts as $g_s^A = g_s^B \frac{\sqrt{\alpha'}}{L}$, and since $g_s^A = \frac{L}{\sqrt{\alpha'}} \tilde{g}_s$, we find $\tilde{g}_s \sim k/N \ll 1$.

• Nonabelian case: $g_s = L^3/\alpha'^{3/2} \tilde{\tilde{g}}_s$, so we find $\tilde{\tilde{g}}_s \sim 1/N \ll 1$.

• Then strings on pp waves are classical $\to$ need only compute eigenvalues.

•Field theory has superpotential

$$W = \sum_{i=1}^{k} \int d^2\theta \, \mathrm{Tr}_{\,i+1}[V_i X_i W_i]$$

and kinetic terms

$$L_{kin} = \sum_{i=1}^{k} \int d^2\theta \, d^2\bar{\theta} \, \mathrm{Tr}_{\,i}[\bar{V}_i e^{-2V} V_i + W_i e^{+2V} \bar{W}_i + X_i^\dagger e^V X_i]$$

with symmetries:

-$SU(2)_R$ acting on $V_i$ and $\overline{W}_i$

-$U(1)_R$ acting on $X^i$ and $d^2\theta$ as $X_i \to e^{i\alpha} X_i \, , \quad d^2\theta \to e^{-i\alpha} d^2\theta$.

-non-R $U(1)$ acting on $V_i, W_i$ as $V_i \to e^{i\alpha} V_i \, , \quad W_i \to e^{-i\alpha} W_i$.

•Same in dual: $SU(2)_{\chi,\xi} \times U(1)_\beta \times U(1)_\psi$.

•Charge assignment

| | $X$ | $V$ | $W$ | $\overline{X}$ | $\overline{V}$ | $\overline{W}$ |
|---|---|---|---|---|---|---|
| $\Delta$ | 1 | 1 | 1 | 1 | 1 | 1 |
| $J_1$ | 0 | $\frac{1}{2}$ | $\frac{1}{2}$ | 0 | $-\frac{1}{2}$ | $-\frac{1}{2}$ |
| $kJ_2$ | 0 | $\frac{1}{2}$ | $-\frac{1}{2}$ | 0 | $-\frac{1}{2}$ | $\frac{1}{2}$ |
| $H$ | 1 | 0 | 1 | 1 | 2 | 1 |

•Large charge (BMN) operators for $U(1)_\psi = U(1)_{\text{extra}}$, but also $U(1)_\xi \subset SU(2)_R$, with

$$\left(\frac{J'}{J}\right)_{\text{them}} = \left(\frac{J_1}{J_2}\right)_{\text{us}} = \frac{\dot{\xi}}{\dot{\psi}} = \frac{2J}{\sqrt{1-4J^2}}\bigg|_{\text{us}}$$

•We can't vary $J_1/J_2$, and $J_1/J_2 = 1$, for $J = 1/2\sqrt{2}$, is only possibility.

•Vacuum, T-dual to one of Mukhi et al. $\to$ winds around the quiver

$$|p=1, m=0\rangle_{\text{them}} = |m=0, p=1\rangle_{\text{us}} = \mathcal{O}_k = \frac{1}{\sqrt{\mathcal{N}}}\text{Tr}\,[V_1 V_2 \ldots V_k]$$

•Oscillators of energy $H = 1$: $D_a$, $a = 0, 1, 2, 3$, $W_i, \bar{W}_i, X_i, \bar{X}_i$,

$$\mathcal{O}_{D_p} = a^\dagger_{D,0}|m = 0, p = 1\rangle_{\text{us}} = \frac{1}{\sqrt{Nk}}\frac{1}{\sqrt{N}}\sum_{i=1}^{k} \text{Tr}\left[V_1 ... V_{i-1}(D_a V_i) ... V_k\right]$$

$$\mathcal{O}_{X_p} = a^\dagger_{X,0}|m = 0, p = 1\rangle_{\text{us}} = \frac{1}{\sqrt{Nk}}\frac{1}{\sqrt{N}}\sum_{i=1}^{k} \text{Tr}\left[V_1 ... V_{i-1} X_i V_i ... V_k\right]$$

$$\mathcal{O}_{\bar{X}_p} = a^\dagger_{\bar{X},0}|m = 0, p = 1\rangle_{\text{us}} = \frac{1}{\sqrt{Nk}}\frac{1}{\sqrt{N}}\sum_{i=1}^{k} \text{Tr}\left[V_1 ... V_{i-1} \bar{X}_i V_i ... V_k\right]$$

$$\mathcal{O}_{W,0} = a^\dagger_{W,0}|m = 0, p = 1\rangle_{\text{us}} = \frac{1}{\sqrt{N^2 k}}\frac{1}{\sqrt{N}}\sum_{i=1}^{k} \text{Tr}\left[V_1 ... V_{i-1} V_i W_i V_i ... V_k\right]$$

$$\mathcal{O}_{\bar{W},0} = a^\dagger_{\bar{W},0}|m = 0, p = 1\rangle_{\text{us}} = \frac{1}{\sqrt{k}}\frac{1}{\sqrt{N}}\sum_{i=1}^{k} \text{Tr}\left[V_1 ... V_{i-1} \bar{W}_i V_{i+1} ... V_k\right]$$

$$a^\dagger_{X,n}|m, p = 1\rangle_{\text{us}} = \frac{1}{\sqrt{Nk}}\frac{1}{\sqrt{N}}\sum_{l=1}^{k} \text{Tr}\left[V_1 ... V_{l-1} X_l V_l ... V_k\right] e^{\frac{2\pi i l n}{k}}$$

•Note that now, momentum $m = \sum_i n_i$.

•Eigenenergies: for pp waves, should be

$$\omega_{0,a} = \frac{1}{4}\ , \ \ a = 1,2,3,4\ , \ \ \omega_{0,i} = \frac{\sqrt{8J^2-1}}{4}\ , \ \ i = 5,6\ , \ \ \omega_{0,+} = J, \ \ \omega_{0,-} = 0$$

•$D_a$ matches: $H = 1 \to \omega_{0,a} = 1/4$ (rescale). But $X, \bar{X}, W, \bar{W}$ give $H = 1$ also.

•Origin: extra interactions in $\frac{g^2_{\rm YM} N}{k} \sim k \to \infty$ limit.

•From interaction $V \sim g^2_{YM} \mathrm{Tr}\,_i |W_i V_i|^2 = \mathrm{Tr}\,_i(\bar{W}_i W_i V_i \bar{V}_i)$ mixing $\mathcal{O}_{W,0}$ and $\mathcal{O}_{\bar{W},0}$ and $V \sim g^2_{YM} \mathrm{Tr}\,_k [\bar{X}_i \bar{V}_i V_i X_i]$ mixing $\mathcal{O}_{X,0}$ and $\mathcal{O}_{\bar{X},0}$.

•Nonabelian case: RG flow in $\rho$. Match it to Abelian case if $\frac{\rho_0}{\tilde{\tilde{g}}_s} = \frac{1}{\tilde{g}_s}$.

•Eigenenergies $\omega = \omega_k(u)$ "flow" in lightcone time $u$, between $u = 0$ and $u = \infty$. $\rightarrow$ flow generic in Gaiotto-Maldacena backgrounds.

•Conformal symmetry broken in dual by winding modes of strings?

•Or dual to conconformal theory in higher dimensions via *deconstruction*:

-Wilson loops show deviation from conformal behaviour.

-pp wave limit of Janus solution (dual to defect CFT) has similarity with Nonabelian case.

# Conclusions

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- We can use Penrose limits to probe difficult holographic dualities

- For GJV, Penrose limit with closed strings matches BMN sectors in field theory, but only to leading order. Open strings don't match.

- We find several functions $f_i(\lambda)$ between weak and strong coupling.

- For abelian and nonabelian T-duals of $AdS_5 \times S^5/\mathbb{Z}_k$, BMN-type operators are clear, but naive calculations don't match $\rightarrow \exists$ large contributions from Feynman diagrams that can't be neglected, unlike $\mathcal{N} = 4$ SYM or ABJM cases.

- Take pp wave calculations as predictions that probe unknown field theories.