Type IIB holographic duals of 5d SCFTs

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In this talk I review progress in the last two years in constructing and investigating new IIB supergravity solutions which are dual to 5dim SCFTs

- Local solutions D'Hoker, Uhlemann, MG: arXiv:1606.01254
- Global regular solutions D'Hoker, Uhlemann, MG: arXiv:1703.08186
- Inclusion of 7 branes D'Hoker, Uhlemann, MG: arXiv:arXiv:1706.00433
- Entanglement entropy and on shell action Marasinou, Trivella, Uhlemann, MG: arXiv:1705.01561, arXiv:1802.07274
- Spin 2 excitations Varela, Uhlemann,MG: arXiv:1805.11914
- Precision tests via localization Fluder, Uhlemann: arXiv:1806.08374
- Stringy operators Bergman, Rodriguez-Gomez, Uhlemann: arXiv:1806.07898

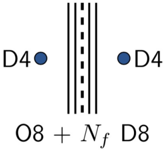
Five dimensional SCFT

Five dimensional SCFT superconformal theories are different than their cousins in $d = 2, 3, 4, 6$.

- The maximal super-conformal theories have 16 not 32 supersymmetries
- Unique superconformal algebra $F(4)$
- Bosonic sub-algebra: $SO(2,5) \times SU(2)_R$
- 5dim theories non renormalizable, weakly coupled gauge theory in IR.
- UV conformal fixed points are strongly coupled, non Lagrangian.
- No simple holographic near horizon limit of D-branes or M2/5 branes.

Previosuly known AdS_6 supergravity solutions

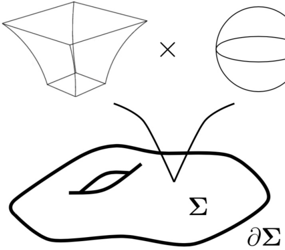
There are several other supergravity solutions with AdS_6 factors:



- Massive type IIA theory: D8/D4 near O_8^+: Realizes $USp(N)$ theories with hypermultiplets A. Brandhuber and Y. Oz, hep-th/9905148

- Generalization of these solutions to quiver gauge theories using D4 branes near orbifold singularities. Oren Bergman, Diego Rodriguez-Gomez, arXiv:1206.3503
- Type IIB supergravity solutions can be obtained from the type IIA ones using (non-belian) T-duality and have been investigated by several groups: Lozano et al, arXiv:1212.1043 ; F. Apruzzi et al,arXiv:1406.0852; Kim et al., arXiv:1406.0852, but involve T-duality of circle which shrinks to zero size.

Supergravity Ansatz and BPS equations



Ansatz for type IIB supergravity solution realizes $SO(2,5) \times SU(2)_R$ as isometries of a warped product of $AdS_6 \times S^2$ over two dimensional Riemann surface Σ, with boundary $\partial\Sigma$.

$$ds^2 = f_2^2(z,\bar{z})ds^2_{AdS_6} + f_6^2(z,\bar{z})ds^2_{S^2} + \rho^2(z,\bar{z})dz \otimes d\bar{z}$$

The complex three form field strength (NS-NS and RR 2 form potential) takes the form

$$G = g_z e^z \wedge \omega_{S^2} + g_{\bar{z}} e^{\bar{z}} \wedge \omega_{S^2}$$

and the dilaton ϕ and axion χ only depend on coordinates $z, \bar{z}$ of Σ.

Constructing BPS solutions

- Solve BPS conditions

$$\begin{aligned}\delta\lambda &= i(G\cdot P)B^{-1}\epsilon^* - \frac{i}{24}(G\cdot G)\epsilon \\ \delta\psi_M &= D_M\epsilon - \frac{1}{96}\left(\Gamma_M(G\cdot G) + 2(G\cdot G)G_M\right)B^{-1}\epsilon^*\end{aligned}$$

- Work with a basis of Killing spinors on $AdS_6\times S^2$ and express 10d spinor ϵ in this basis $\zeta_{\eta_1\eta_2}$ 2d spinors on Σ

$$\epsilon = \sum_{\eta_1,\eta_2=\pm1}\chi^{\eta_1,\eta_2}\otimes\zeta_{\eta_1,\eta_2}$$

- Discrete symmetries restrict the BPS equation to a single 2d spinor (α,β), instead of $\zeta_{\eta_1\eta_2}$
- BPS equations can be solved in terms of two holomorphic functions $A_\pm$ on Σ

Local solutions

The local solution is completely determined (up to one additional constant of integration) by two holomorphic functions $A_\pm(w)$.

- From $A_\pm(w)$. we can form two functions κ^2 and G

$$\kappa^2 = -|\partial_w A_+|^2 + |\partial_w A_-|^2, \quad G = |A_+|^2 - |A_-|^2 + \mathcal{B} + \bar{\mathcal{B}}$$

  where

$$\partial_w \mathcal{B} = A_+ \partial_w A_- - A_- \partial_w A_+$$

- The metric factors is given by

$$f_6^2 = \frac{c_6^2\, \kappa^2 (1+R)}{\rho^2\, (1-R)}, \quad f_2^2 = \frac{c_6^2\, \kappa^2 (1-R)}{9\, \rho^2 (1+R)}, \quad \rho^2 = \frac{(R+R^2)^{\frac{1}{2}}\, (\kappa^2)^{\frac{3}{2}}}{|\partial_w G|\, (1-R)^{\frac{3}{2}}}$$

- R is determined by

$$R + \frac{1}{R} = 2 + 6\, \frac{\kappa^2\, G}{|\partial_w G|^2}$$

Local solutions

- Self-dual five form F_5 has to vanish due to symmetry of ansatz
- axion/dilaton $\tau = \chi + ie^{-\phi}$ is given by

$$B = \frac{1 + i\tau}{1 - i\tau} = \frac{\kappa_+ \partial_{\bar w} G - \kappa_- R\, \partial_w G}{\bar\kappa_+ R\, \partial_w G - \bar\kappa_- \partial_{\bar w} G}$$

- Complex three form field strength G is related to closed three form F with $dF = 0$ by

$$G = \frac{1}{\sqrt{1 - |B|^2}} \left(F_{(3)} - B \bar F_{(3)} \right)$$

- field strength $F_{(3)} = dC_{(2)}$ and the potential $C_{(2)}$ is given by

$$C_{(2)} = \frac{4ic_6^2}{9} \left(\frac{\partial_{\bar w} \bar A_- \, \partial_w G}{\kappa^2} - 2R\, \frac{\partial_w G\, \partial_{\bar w} \bar A_- + \partial_{\bar w} G\, \partial_w A_+}{(R+1)^2\, \kappa^2} - \bar A_- - 2A_+ \right)$$

Global solutions

Local solutions in general are geodesically incomplete, complex or have unphysical singularities. The conditions for the existence of regular solutions are

- Inside Σ there are 2 positivity conditions

$$\kappa^2 > 0, \quad G > 0$$

  guarantees regularity inside Σ as $|R| < 1$.

- On the boundary of Σ we have vanishing conditions (except for isolated poles of κ^2)

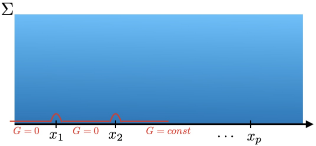
$$\kappa^2|_{\partial\Sigma} = 0, \quad G|_{\partial\Sigma} = 0$$

  It follows that $R = 1$ on Σ this guarantees that $Vol(S^2) \to 0$ on the boundary of Σ closing space off.

Global solutions

- A large class of regular solutions can be constructed from the following ansatz: Σ is the upper half plane, $\partial_w A_\pm$ have L simple poles on the real line with complex residues.

$$A_\pm(w) = A^0_\pm + \sum_{\ell=1}^{L} Z^\ell_\pm \ln(w - p_\ell), \quad \overline{Z^\ell_\pm} = -Z^\ell_\mp, \quad \sum_{\ell=1}^{L} Z^\ell_\pm = 0$$



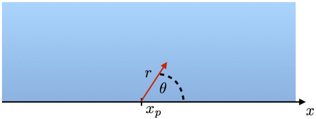
Vanishing of G on $\partial\Sigma$ implies L jump conditions (one for each pole) with $Z^{[\ell,\ell']} = Z^\ell_+ Z^{\ell'}_- - Z^{\ell'}_+ Z^\ell_-$

$$A^0 Z^k_- + \bar{A}^0 Z^k_+ + \sum_{\ell \neq k} Z^{[\ell,k]} \ln|p_\ell - p_k| = 0, \quad k = 1, 2, \cdots L$$

- Number of moduli of our solutions: 2L-2 free real parameters, they can be chosen to be $L-1$ complex residues Z^+_p correspond to (p,q) 5-brane charges.

Behavior of solution near poles

- Solution is singular near poles at $x = p_m$. Expanding near the m-th pole $w = p_m + r\, e^{i\theta}$



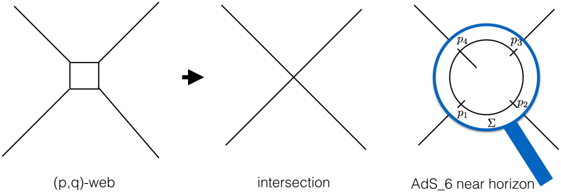
- Near pole solution matches exactly the near brane solution for a (p, q) 5-brane solution with the following identification Roy and Lu, hep-th/9802080

$$p = \frac{8}{3} Re(Z_+^m), \qquad q = -\frac{8}{3} Im(Z_+^m)$$

- $2(L-1)$ moduli: (p, q) five brane charges.

5-brane intersection

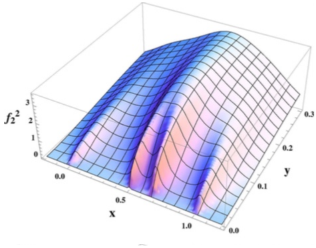
- The poles are remnants of semi-infinite fivebranes of a 5-brane intersection

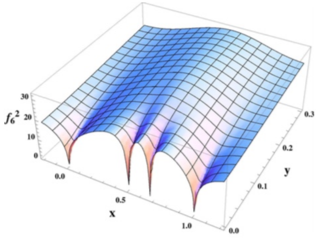


- $L \geq 3$ - Minimum 3 (p,q) 5-brane intersection.
- $L - 1$ (p,q) 5-brane charges completely specify intersection and sugra solution.
- Z_+^l cannot all be real or imaginary: Charges cannot all be parallel (no brane web with only D5 or NS5 branes)

4 pole example

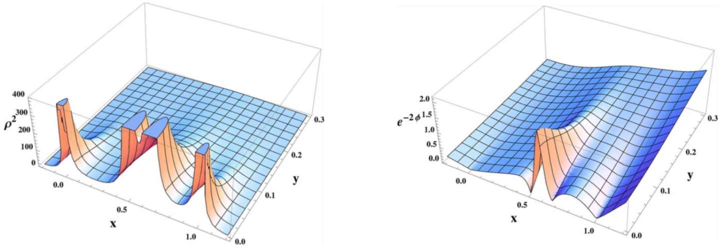
- Solution with four poles. One can choose $Z_+^1 = -Z_+^3$ real, $Z_+^2 = -Z_+^4$ imaginary.
- metric functions for S^2 and AdS_6



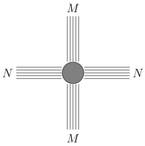


4 pole example

- metric factor for Σ and dilaton



- Solution corresponds to an junction of N D5 and M NS 5-branes



4 pole example

- Brane web corresponding to this solutions studied before Aharony et al hep-th/9710116.
- When external five branes are terminated on seven branes the corresponding quiver theory is

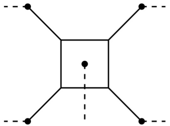
$$N - \underbrace{SU(N) \times \cdots \times SU(N)}_{SU(N)^{M-1}} - N$$

- Holographic calculations should provide evidence for this conjectured relation

Inclusion of 7 branes

Solution has single valued fields, can introduce $SL(2, Z)$ monodromy. Corresponds to inclusion of 7 branes.

	0	1	2	3	4	5	6	7	8	9
D5	×	×	×	×	×	×				
NS5	×	×	×	×	×		×			
D7	×	×	×	×	×			×	×	×



(1) terminate (p, q) 5branes on $[p, q]$ 7-branes

(2) add D7 branes into faces of 5-brane web

- In near horizon limit, we can include (2) by modifying the supergravity solution

$$\partial_w A_\pm \rightarrow \partial_w A_\pm + f \times (\partial_w A_+ - \partial_w A_-)$$

- the function f is given by

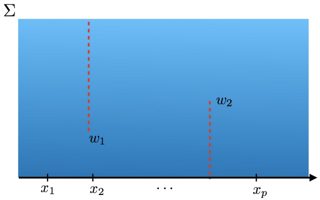
$$f = \sum_i \frac{n_i^2}{4\pi} \ln \left(\gamma_i \frac{w - w_i}{w - \bar{w}_i} \right)$$

Inclusion of 7 branes

- w_i location of D7-branes with monodromy

$$f(w_i + e^{2\pi i}(w - w_i)) = f(w) + \frac{i}{2} n_i^2$$

- Corresponds to $SL(2,R)$ axion monodromy $\tau \to \tau + n_i^2$, i.e. n_i^2 D7 branes (at present we don't know how to include noncommuting monodromies).
- Regularity conditions can be solved for the new solution with monodromy, constraining some of the new parameters.



On shell action and entanglement entropy 1705.01561

- Holographic entanglement entropy of a spherical region in the 5d SCFT: 8 dimensional RT surface wrapping S^2, Σ and a 4dim minimal surface γ_4 in AdS_6

$$S_{EE} = \frac{1}{4G_N}\int_{\gamma_8} vol(\gamma_8) = \frac{1}{4G_N} vol(S^2)\,\mathcal{I}\,\mathrm{Area}(\gamma_4)$$

- I is integral over Σ

$$I = 4\int_\Sigma d^2w\ f_6^2 f_2^2 \rho^2 = \frac{8}{3}\int_\Sigma d^2w\ \kappa^2 G$$

- Area of γ_4 divergent

$$Area(\gamma_4) = Vol(S_3)\left(\frac{r_0^3}{3\epsilon^3} - \frac{r_0}{\epsilon} + \frac{2}{3} + o(\epsilon)\right)$$

but finite piece is universal and corresponds to finite part of EE.

On shell action and entanglement entropy

- type IIB action with $C_4 = 0$ can be written as a total derivative Okuda and Trancanelli arxiv:0806.4191

$$S_{IIB} = \frac{1}{64\pi G_N} \int d\Big(\frac{1}{2} f^2 (1+|B|^2) \bar{C}_2 \wedge * dC_2 - f^2 \bar{B} C_2 \wedge * dC_2 + c.c.\Big)$$

where

$$B = \frac{1+i\tau}{1-i\tau}, \quad f = 1/\sqrt{1-|B|^2}, \quad F = dC^1 + i dC^2 \tag{1}$$

- In 1705.01561 we evaluated both S_{EE} and S_{IIB} and showed that the finite regularized pieces agree Casini, Huerta,Myers arXiv:1102.0440
- No contribution to S_{EE} from region close to poles.
- For the four pole example of above we find that $S_{IIB} \sim \zeta(3) N^2 M^2$

Massive spin two fluctuations with C. Uhlemann and O. Varela, arXiv:1805.11914

- KK reduction of solution gives operator spectrum of 5d SCFT
- Very hard for warped solution, complicated dependence on Σ, and mutual coupling of supergravity fields.
- Bachas and Estes arXiv:1103.2800 showed that transverse traceless spin 2 excitations on AdS_6 decouple. For type IIA AdS_6 solution Passias and Richmond, arxiv:1804.09728
- perturbation of metric

$$ds^2 = f_6^2\left(ds^2_{AdS_6} + h_{\mu\nu}dx^\mu dx^\nu\right) + \ f_2^2 ds^2_{S^2} + 4\rho^2|dw|^2 \ ,$$

with

$$h_{\mu\nu}(x,y) = h^{[tt]}_{\mu\nu}(x)\psi(y), \quad \Box_{AdS_6} h^{[tt]}_{\mu\nu} = m^2 h^{[tt]}_{\mu\nu} \ .$$

Massive spin two fluctuations

- Equation for ψ on $S^2 \times \Sigma$ becomes

$$\frac{1}{f_6^4 f_2^2 \rho^2}\partial_a\left(f_6^6 f_2^2 \eta^{ab}\partial_b\psi\right) + \frac{f_6^2}{f_2^2}\nabla^2_{S^2}\psi + m^2\psi = 0 \ ,$$

- expand ψ in spherical harmonics on S^2 $\psi(y) = \phi_\ell(w,\bar{w})Y_{\ell m}(S^2)$

$$6\partial_a\left(G^2\eta^{ab}\partial_b\phi_\ell\right) - \ell(\ell+1)\left(9\kappa^2 G + 6|\partial G|^2\right)\phi_\ell + m^2\kappa^2 G\phi_\ell = 0 \ .$$

- looks horrible but there are two simple solutions simple solution

$$\phi_\ell = G^\ell, \qquad m^2 = 3\ell(3\ell+5)$$

$$\phi_\ell = G^\ell(A_+ - \bar{A}_-), \qquad m^2 = 3\ell(3\ell+6)$$

- This works because κ and G satisfy $\partial_w\partial_{\bar{w}}G = -\kappa^2$ and $A_\pm$ are holomorphic

Massive spin two fluctuations

- Using $m^2 = \Delta(\Delta - 5)$ we see that this solution is dual to a spin two operators of dimension

$$\Delta_{B_2} = 5 + 3\ell, \qquad \Delta_{A_4} = 6 + 3\ell$$

- The spin 2 operators are Q^4 descendants in a short multiplets denoted B_2 and A_4 in the notation of Cordova, Dumitrescu and Intriligator arXiv:1612.00809
- The dimension of the scalar primaries in the two multiplets are

$$\Delta_{B_2} = 3 + 3\ell \ , \qquad \Delta_{A_4} = 4 + 3\ell$$

Universally present for all IIB solutions (no matter how complicated)

Other checks of duality

- Dual field theory for for IIB supergravity solutions with N $(1,0),(0,1)$ and $(1,1)$ branes: T_N quiver theory.
- Five sphere partition function can be calculated using localization and extrapolated to large N
- Results match ! Similar results for system involving N D5 and M NS5 branes. Fluder, Uhlemann: arXiv:1806.08374 .
- Operators in field theory with $o(N)$ scaling dimensions can be matched to string (junctions) in the IIB background Bergman, Rodriguez-Gomez, Uhlemann: arXiv:1806.07898 .

Conclusions

- Constructed new type IIB supergravity solutions which are dual to 5d SCFTs
- warped product of $AdS_6 \times S^2$ over Riemann surface with boundary
- Preserve 16 susy $F(4)$ superalgebra.
- Relation to (p,q) 5-brane webs (including 7 branes).
- Some holographic observables have been calculated: entanglement entropy, free energy, spectrum of spin 2 excitations.
- Matches give confidence in duality and identification of field theory.
- Many things still need to be done: operator spectrum, other holographic observables, correlators, probe branes, any role for singularities.