Type IIB holographic duals of 5d SCFTs

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In this talk I review progress in the last two years in constructing and investigating new IIB supergravity solutions which are dual to 5dim SCFTs

- **Local solutions** D’Hoker, Uhlemann, MG: arXiv:1606.01254
- **Global regular solutions** D’Hoker, Uhlemann, MG: arXiv:1703.08186
- **Spin 2 excitations** Varela, Uhlemann, MG: arXiv:1805.11914
- **Precision tests via localization** Fluder, Uhlemann: arXiv:1806.08374
- **Stringy operators** Bergman, Rodriguez-Gomez, Uhlemann: arXiv:1806.07898
1. Five dimensional SCFT
2. Constructing BPS solutions
3. Local solutions
4. Global Solutions
5. Applications and checks of duality
6. Conclusions
Five dimensional SCFT superconformal theories are different than their cousins in $d = 2, 3, 4, 6$.

- The maximal super-conformal theories have 16 not 32 supersymmetries
- Unique superconformal algebra $F(4)$
- Bosonic sub-algebra: $SO(2, 5) \times SU(2)_R$
- 5dim theories non renormalizable, weakly coupled gauge theory in IR.
- UV conformal fixed points are strongly coupled, non Lagrangian.
- No simple holographic near horizon limit of D-branes or M2/5 branes.
Previosuly known $AdS_6$ supergravity solutions

There are several other supergravity solutions with $AdS_6$ factors:

- Massive type IIA theory: D8/D4 near $O_8^+$: Realizes $USp(N)$ theories with hypermultiplets  
  A. Brandhuber and Y. Oz, hep-th/9905148

- Generalization of these solutions to quiver gauge theories using D4 branes near orbifold singularities.  

- Type IIB supergravity solutions can be obtained from the type IIA ones using (non-belian) T-duality and have been investigated by several groups:  
Constructing BPS solutions

Supergravity Ansatz and BPS equations

Ansatz for type IIB supergravity solution realizes $SO(2,5) \times SU(2)_R$ as isometries of a warped product of $AdS_6 \times S^2$ over two dimensional Riemann surface $\Sigma$, with boundary $\partial \Sigma$.

\[
\text{ds}^2 = f_2^2(z, \bar{z}) \text{ds}^2_{AdS_6} + f_6^2(z, \bar{z}) \text{ds}^2_{S^2} + \rho^2(z, \bar{z}) \text{dz} \otimes \text{d\bar{z}}
\]

The complex three form field strength (NS-NS and RR 2 form potential) takes the form

\[
G = g_z e^z \wedge \omega_{S^2} + g_{\bar{z}} e^{\bar{z}} \wedge \omega_{S^2}
\]

and the dilaton $\phi$ and axion $\chi$ only depend on coordinates $z, \bar{z}$ of $\Sigma$. 

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Constructing BPS solutions

- Solve BPS conditions

\[ \delta \lambda = i (G \cdot P) B^{-1} \epsilon^* - \frac{i}{24} (G \cdot G) \epsilon \]

\[ \delta \psi_M = D_M \epsilon - \frac{1}{96} (\Gamma_M (G \cdot G) + 2(G \cdot G) G_M) B^{-1} \epsilon^* \]

- Work with a basis of Killing spinors on \( AdS_6 \times S^2 \) and express 10d spinor \( \epsilon \) in this basis \( \zeta_{\eta_1 \eta_2} \) 2d spinors on \( \Sigma \)

\[ \epsilon = \sum_{\eta_1, \eta_2 = \pm 1} \chi^{\eta_1, \eta_2} \otimes \zeta_{\eta_1, \eta_2} \]

- Discrete symmetries restrict the BPS equation to a single 2d spinor \( (\alpha, \beta) \), instead of \( \zeta_{\eta_1 \eta_2} \)

- BPS equations can be solved in terms of two holomorphic functions \( A_{\pm} \) on \( \Sigma \)
Local solutions

The local solution is completely determined (up to one additional constant of integration) by two holomorphic functions $A_{\pm}(w)$.

- From $A_{\pm}(w)$, we can form two functions $\kappa^2$ and $G$

$$
\kappa^2 = -|\partial_w A_+|^2 + |\partial_w A_-|^2, \quad G = |A_+|^2 - |A_-|^2 + B + \bar{B}
$$

where

$$
\partial_w B = A_+ \partial_w A_- - A_- \partial_w A_+
$$

- The metric factors is given by

$$
 f_6^2 = \frac{c_6^2 \kappa^2 (1 + R)}{\rho^2 (1 - R)}, \quad f_2^2 = \frac{c_6^2 \kappa^2 (1 - R)}{9 \rho^2 (1 + R)}, \quad \rho^2 = \frac{(R + R^2)^{\frac{1}{2}} (\kappa^2)^{\frac{3}{2}}}{|\partial_w G| (1 - R)^{\frac{3}{2}}}
$$

- $R$ is determined by

$$
R + \frac{1}{R} = 2 + 6 \frac{\kappa^2 G}{|\partial_w G|^2}
$$
Local solutions

- Self-dual five form $F_5$ has to vanish due to symmetry of ansatz
- Axion/dilaton $\tau = \chi + i e^{-\phi}$ is given by
  \[ B = \frac{1 + i \tau}{1 - i \tau} = \frac{\kappa_+ \partial \bar{w} G - \kappa_- R \partial w G}{\kappa_+ R \partial w G - \kappa_- \partial \bar{w} G} \]
- Complex three form field strength $G$ is related to closed three form $F$ with $dF = 0$ by
  \[ G = \frac{1}{\sqrt{1 - |B|^2}} \left( F^{(3)} - B \bar{F}^{(3)} \right) \]
- Field strength $F^{(3)} = dC^{(2)}$ and the potential $C^{(2)}$ is given by
  \[ C^{(2)} = \frac{4i c_6^2}{9} \left( \frac{\partial \bar{w} \bar{A}_- \partial w G}{\kappa^2} - 2R \frac{\partial w G \partial \bar{w} \bar{A}_- + \partial \bar{w} G \partial w A_+}{(R + 1)^2 \kappa^2} - \bar{A}_- - 2A_+ \right) \]
Global solutions

Local solutions in general are geodesically incomplete, complex or have unphysical singularities. The conditions for the existence of regular solutions are

- Inside $\Sigma$ there are 2 positivity conditions

\[
\kappa^2 > 0, \quad G > 0
\]

 guarantees regularity inside $\Sigma$ as $|R| < 1$.

- On the boundary of $\Sigma$ we have vanishing conditions (except for isolated poles of $\kappa^2$)

\[
\kappa^2 |_{\partial \Sigma} = 0, \quad G |_{\partial \Sigma} = 0
\]

It follows that $R = 1$ on $\Sigma$ this guarantees that $Vol(S^2) \to 0$ on the boundary of $\Sigma$ closing space off.
Global solutions

A large class of regular solutions can be constructed from the following ansatz: $\Sigma$ is the upper half plane, $\partial_w A_\pm$ have $L$ simple poles on the real line with complex residues.

$$A_\pm(w) = A^0_\pm + \sum_{\ell=1}^L Z^\ell_\pm \ln(w - p_\ell), \quad \overline{Z^\ell_\pm} = -Z^\ell_\mp, \quad \sum_{\ell=1}^L Z^\ell_\pm = 0$$

Vanishing of $G$ on $\partial \Sigma$ implies $L$ jump conditions (one for each pole) with $Z^{[\ell,\ell']} = Z^\ell_+ Z^-_\ell - Z^\ell_- Z^\ell_+$

$$A^0 Z^-_k + \overline{A}^0 Z^+_k + \sum_{\ell \neq k} Z^{[\ell,k]} \ln |p_\ell - p_k| = 0, \quad k = 1, 2, \ldots L$$

Number of moduli of our solutions: $2L-2$ free real parameters, they can be chosen to be $L - 1$ complex residues $Z^+_p$ correspond to $(p, q)$ 5-brane charges.
Behavior of solution near poles

- Solution is singular near poles at $x = p_m$. Expanding near the $m$-th pole $w = p_m + r e^{i\theta}$

- Near pole solution matches exactly the near brane solution for a $(p, q)$ 5-brane solution with the following identification: 

  $p = \frac{8}{3} \text{Re}(Z_m^+), \quad q = -\frac{8}{3} \text{Im}(Z_m^+)$

- $2(L - 1)$ moduli: $(p, q)$ five brane charges.
5-brane intersection

- The poles are remnants of semi-infinite fivebranes of a 5-brane intersection

\[ L \geq 3 \] - Minimum 3 \((p,q)\) 5-brane intersection.

\[ L - 1 \] \((p,q)\) 5-brane charges completely specify intersection and sugra solution.

\[ Z^I_+ \] cannot all be real or imaginary: Charges cannot all be parallel (no brane web with only D5 or NS5 branes)
4 pole example

- Solution with four poles. One can choose $Z_1^1 = -Z_3^3$ real, $Z_2^2 = -Z_4^4$ imaginary.
- Metric functions for $S^2$ and $AdS_6$
4 pole example

- metric factor for $\Sigma$ and dilaton

- Solution corresponds to an junction of $N$ D5 and $M$ NS 5-branes
4 pole example

- Brane web corresponding to this solutions studied before Aharony et al hep-th/9710116.

- When external five branes are terminated on seven branes the corresponding quiver theory is

\[ N - SU(N) \times \cdots \times SU(N) - N \]

\[ SU(N)^{M-1} \]

- Holographic calculations should provide evidence for this conjectured relation
Inclusion of 7 branes

Solution has single valued fields, can introduce $SL(2, \mathbb{Z})$ monodromy. Corresponds to inclusion of 7 branes.

- In near horizon limit, we can include (2) by modifying the supergravity solution

$$\partial_w A_\pm \rightarrow \partial_w A_\pm + f \times (\partial_w A_+ - \partial_w A_-)$$

- the function $f$ is given by

$$f = \sum_i \frac{n_i^2}{4\pi} \ln \left( \gamma_i \frac{w - w_i}{w - \bar{w}_i} \right)$$
Inclusion of 7 branes

- $w_i$ location of D7-branes with monodromy

$$f(w_i + e^{2\pi i}(w - w_i)) = f(w) + \frac{i}{2} n_i^2$$

- Corresponds to $SL(2, R)$ axion monodromy $\tau \rightarrow \tau + n_i^2$, i.e. $n_i^2$ D7 branes (at present we don’t know how to include noncommuting monodromies).

- Regularity conditions can be solved for the new solution with monodromy, constraining some of the new parameters.
On shell action and entanglement entropy

- Holographic entanglement entropy of a spherical region in the 5d SCFT: 8 dimensional RT surface wrapping $S^2$, $\Sigma$ and a 4dim minimal surface $\gamma_4$ in $AdS_6$

$$S_{EE} = \frac{1}{4G_N} \int_{\gamma_8} vol(\gamma_8) = \frac{1}{4G_N} vol(S^2) \mathcal{I} \text{Area}(\gamma_4)$$

- $\mathcal{I}$ is integral over $\Sigma$

$$\mathcal{I} = 4 \int_\Sigma d^2w f_6^2 f_2^2 \rho^2 = \frac{8}{3} \int_\Sigma d^2w \kappa^2 G$$

- Area of $\gamma_4$ divergent

$$\text{Area}(\gamma_4) = \text{Vol}(S_3)\left(\frac{r_0^3}{3\epsilon^3} - \frac{r_0}{\epsilon} + \frac{2}{3} + o(\epsilon)\right)$$

but finite piece is universal and corresponds to finite part of EE.
Applications and checks of duality

On shell action and entanglement entropy

- type IIB action with $C_4 = 0$ can be written as a total derivative \cite{Okuda,Trancanelli:2008}\n
\[
S_{IIB} = \frac{1}{64\pi G_N} \int d \left( \frac{1}{2} f^2 (1 + |B|^2) \tilde{C}_2 \wedge \ast dC_2 - f^2 \tilde{B} C_2 \wedge \ast dC_2 + c.c. \right)
\]

where

\[
B = \frac{1 + i\tau}{1 - i\tau}, \quad f = 1/\sqrt{1 - |B|^2}, \quad F = dC^1 + idC^2
\] (1)

- In \cite{Casini:2017} we evaluated both $S_{EE}$ and $S_{IIB}$ and showed that the finite regularized pieces agree \cite{Casini, Huerta, Myers:2011}\n
- No contribution to $S_{EE}$ from region close to poles.

- For the four pole example of above we find that $S_{IIB} \sim \zeta(3) N^2 M^2$

- KK reduction of solution gives operator spectrum of 5d SCFT
- Very hard for warped solution, complicated dependence on $\Sigma$, and mutual coupling of supergravity fields.
- Bachas and Estes arXiv:1103.2800 showed that transverse traceless spin 2 excitations on $AdS_6$ decouple. For type IIA $AdS_6$ solution Passias and Richmond, arxiv:1804.09728
- Perturbation of metric

$$ds^2 = f_{\epsilon}^2 \left( ds_{AdS_6}^2 + h_{\mu\nu} dx^\mu dx^\nu \right) + f_2^2 ds_{S^2}^2 + 4 \rho^2 |dw|^2,$$

with

$$h_{\mu\nu}(x, y) = h^{[tt]}_{\mu\nu}(x) \psi(y), \quad \Box_{AdS_6} h^{[tt]}_{\mu\nu} = m^2 h^{[tt]}_{\mu\nu}.$$
Massive spin two fluctuations

- Equation for $\psi$ on $S^2 \times \Sigma$ becomes

$$\frac{1}{f_6^4 f_2^2 \rho^2} \partial_a (f_6^6 f_2^2 \eta^{ab} \partial_b \psi) + \frac{f_6^2}{f_2^2} \nabla^2_{S^2} \psi + m^2 \psi = 0 ,$$

- expand $\psi$ in spherical harmonics on $S^2$ $\psi(y) = \phi_\ell(w, \bar{w}) Y_{\ell m}(S^2)$

$$6 \partial_a (G^2 \eta^{ab} \partial_b \phi_\ell) - \ell(\ell + 1) (9 \kappa^2 G + 6 |\partial G|^2) \phi_\ell + m^2 \kappa^2 G \phi_\ell = 0 .$$

- looks horrible but there are two simple solutions simple solution

$$\phi_\ell = G^\ell , \quad m^2 = 3 \ell (3 \ell + 5)$$

$$\phi_\ell = G^\ell (A_+ - \bar{A}_-) , \quad m^2 = 3 \ell (3 \ell + 6)$$

- This works because $\kappa$ and $G$ satisfy $\partial w \partial \bar{w} G = - \kappa^2$ and $A_\pm$ are holomorphic
Massive spin two fluctuations

- Using $m^2 = \Delta(\Delta - 5)$ we see that this solution is dual to a spin two operators of dimension

$$\Delta_{B_2} = 5 + 3\ell, \quad \Delta_{A_4} = 6 + 3\ell$$

- The spin 2 operators are $Q^4$ descendants in a short multiplets denoted $B_2$ and $A_4$ in the notation of Cordova, Dumitrescu and Intriligator [arXiv:1612.00809]

- The dimension of the scalar primaries in the two multiplets are

$$\Delta_{B_2} = 3 + 3\ell, \quad \Delta_{A_4} = 4 + 3\ell$$

 Universally present for all IIB solutions (no matter how complicated)
Other checks of duality

- Dual field theory for IIB supergravity solutions with $N (1, 0), (0, 1)$ and $(1, 1)$ branes: $T_N$ quiver theory.
- Five sphere partition function can be calculated using localization and extrapolated to large $N$.
- Operators in field theory with $o(N)$ scaling dimensions can be matched to string (junctions) in the IIB background. Bergman, Rodriguez-Gomez, Uhlemann: arXiv:1806.07898.
Conclusions

- Constructed new type IIB supergravity solutions which are dual to 5d SCFTs
- warped product of $AdS_6 \times S^2$ over Riemann surface with boundary
- Preserve 16 susy $F(4)$ superalgebra.
- Relation to $(p,q)$ 5-brane webs (including 7 branes).
- Some holographic observables have been calculated: entanglement entropy, free energy, spectrum of spin 2 excitations.
- Matches give confidence in duality and identification of field theory.
- Many things still need to be done: operator spectrum, other holographic observables, correlators, probe branes, any role for singularities.