Central Charge of Self-Dual Strings from Holographic Entanglement Entropy

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THE ROYAL SOCIETY

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Work in progress with

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NYC College of Technology

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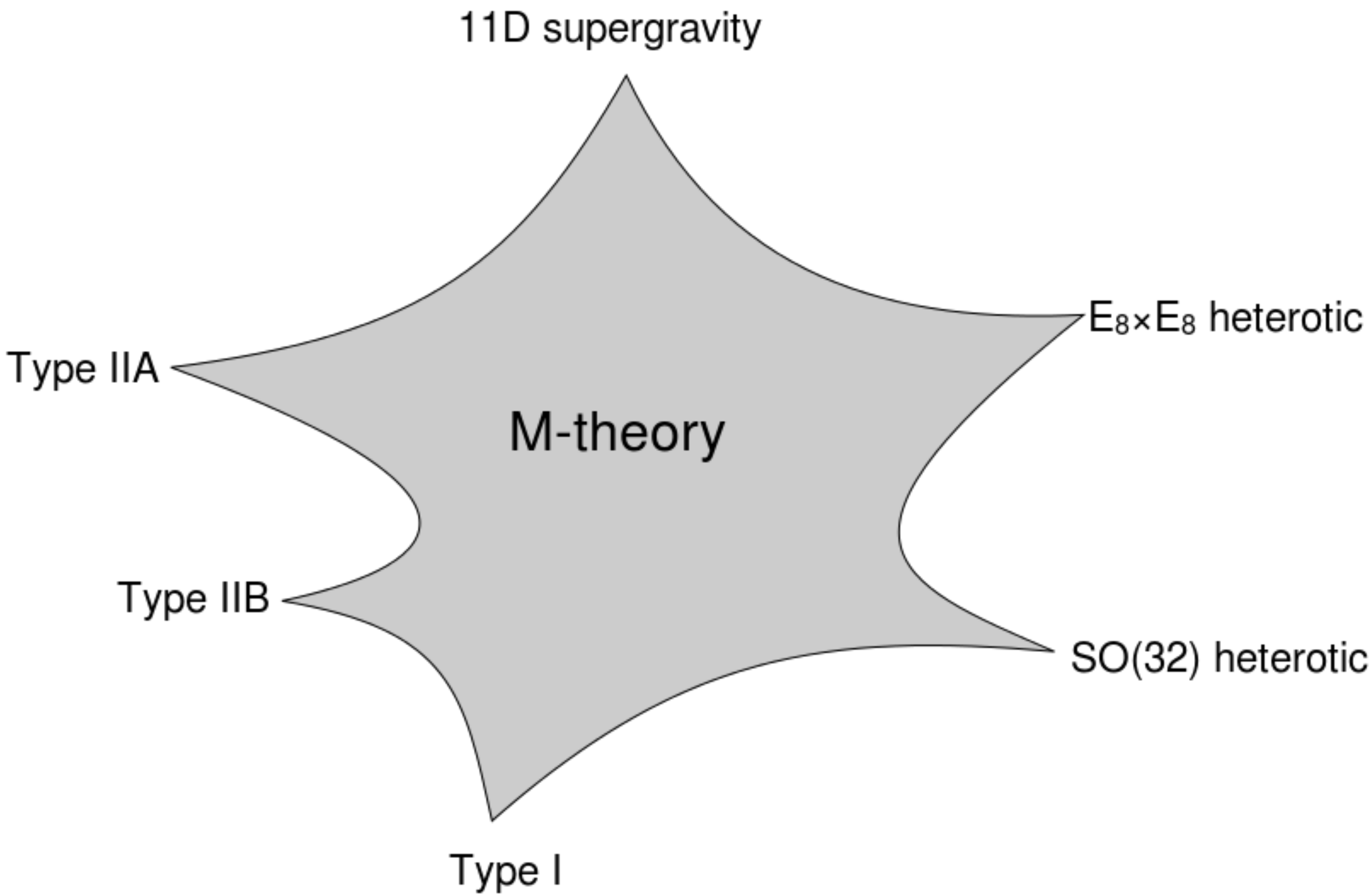
Southampton U.

Ronnie Rodgers
Southampton U.

Outline:

- M5-Branes and Self-Dual Strings
- Entanglement Entropy
- The Holographic Duals
- The Central Charge
- Summary and Outlook

M-theory


11D supergravity
$E_8 \times E_8$ heterotic
Type IIA
M-theory
Type IIB
SO(32) heterotic
Type I

M-theory

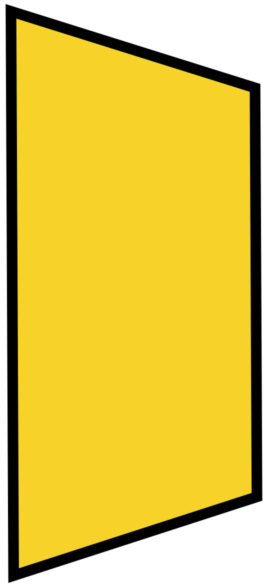
$$S = \frac{1}{16\pi G}\int d^{11}x\sqrt{-g}\left[R - \frac{1}{2}|F_4|^2\right] - \frac{1}{16\pi G}\frac{1}{6}\int C_3 \wedge F_4 \wedge F_4$$

$$F_4 \equiv dC_3 \quad \Rightarrow \quad F_7 \equiv \star_{11d} F_4 \quad \Rightarrow \quad F_7 \equiv dC_6$$

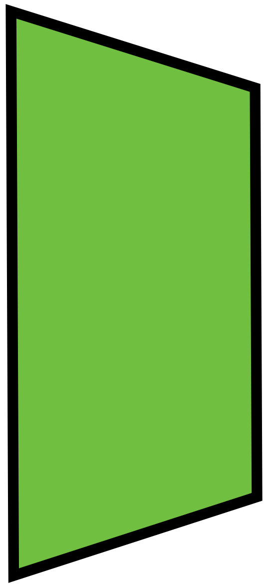
$$C_3 \quad \Rightarrow \quad M2$$

$$C_6 \quad \Rightarrow \quad M5$$

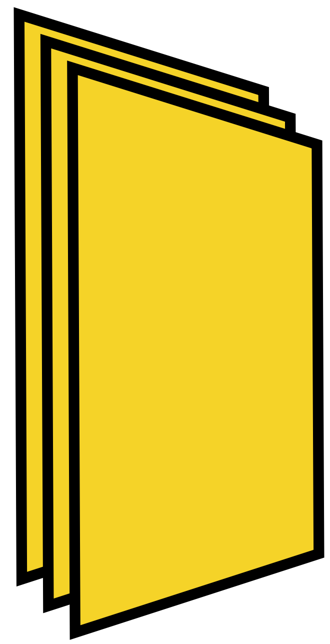
M2-brane



M5-brane



M-theory



	0	1	2	3	4	5	6	7	8	9	10
M2	•	•	•	–	–	–	–	–	–	–	–

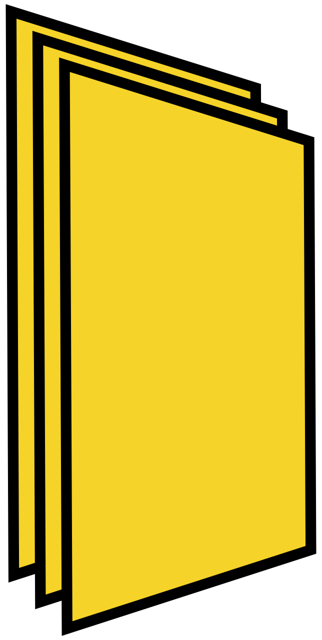
N coincident M2-branes' worldvolume theory

Aharony, Bergman, Jafferis, Maldacena 0806.1218
ABJM Theory

Bagger and Lambert hep-th/0611108, 0711.0955, 0712.3738

Gustavsson 0709.1260

M-theory



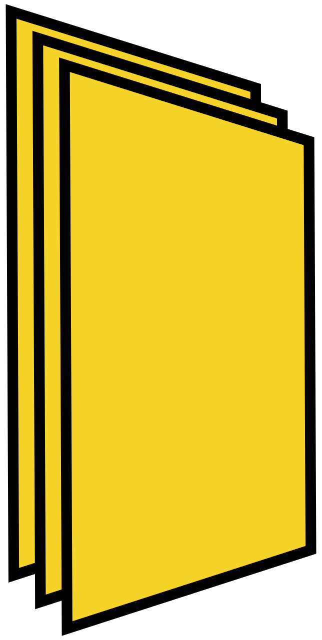
	0	1	2	3	4	5	6	7	8	9	10
M2	•	•	•	–	–	–	–	–	–	–	–

N coincident M2-branes' worldvolume theory

$d = 3$ $\mathcal{N} = 8$ superconformal Chern-Simons matter theory

Gauge group $U(N)_1 \times U(N)_{-1}$

M-theory



	0	1	2	3	4	5	6	7	8	9	10
M2	•	•	•	–	–	–	–	–	–	–	–

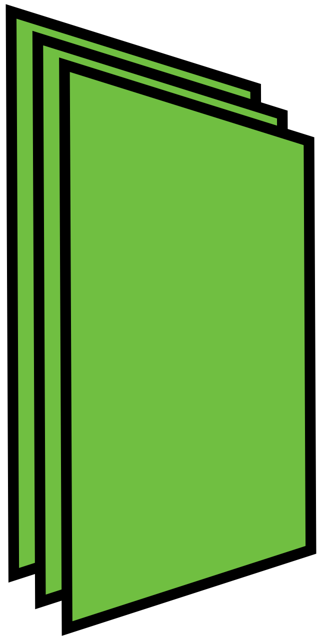
N coincident M2-branes' worldvolume theory

Maldacena limit

$$N \to \infty$$

$d = 11$ SUGRA on $AdS_4 \times S^7$

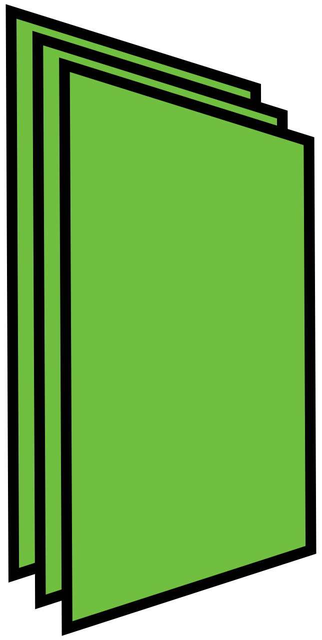
M-theory



	0	1	2	3	4	5	6	7	8	9	10
M5	●	●	●	●	●	●	–	–	–	–	–

M coincident M5-branes' worldvolume theory

M-theory

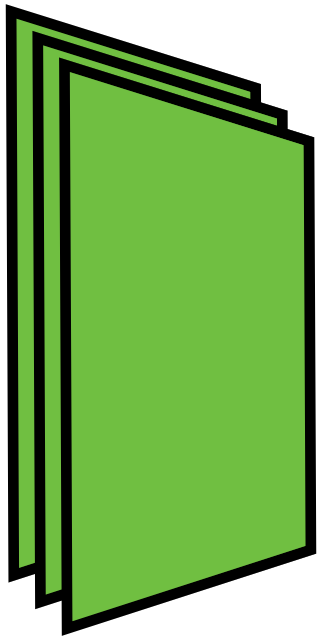


	0	1	2	3	4	5	6	7	8	9	10
M5	•	•	•	•	•	•	–	–	–	–	–

M coincident M5-branes' worldvolume theory

???????????

M-theory



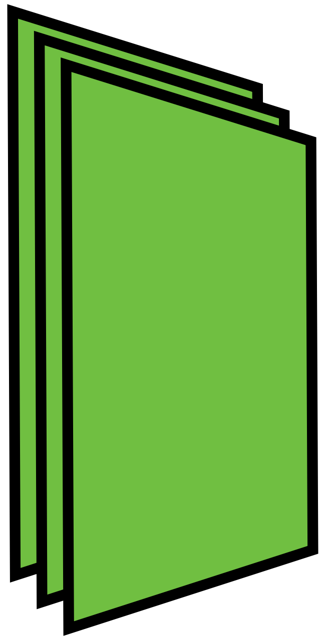
	0	1	2	3	4	5	6	7	8	9	10
M5	●	●	●	●	●	●	–	–	–	–	–

M coincident M5-branes' worldvolume theory

$d = 6$ $\mathcal{N} = (2,0)$ superconformal symmetry

Gauge group $U(M)$

M-theory



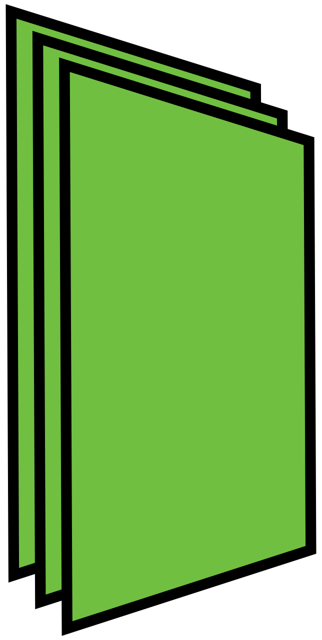
	0	1	2	3	4	5	6	7	8	9	10
M5	•	•	•	•	•	•	–	–	–	–	–

M coincident M5-branes' worldvolume theory

$d = 6$ $\mathcal{N} = (2, 0)$ superconformal symmetry

tensor multiplet
5 real scalar fields + fermions + self-dual two-form

M-theory



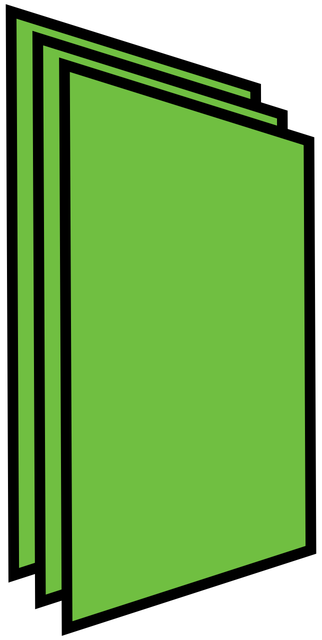
	0	1	2	3	4	5	6	7	8	9	10
M5	•	•	•	•	•	•	–	–	–	–	–

M coincident M5-branes' worldvolume theory

$$A_2 \quad \Rightarrow \quad F_3 \equiv dA_3$$

$$F_3 = \star_{6d} F_3$$

M-theory



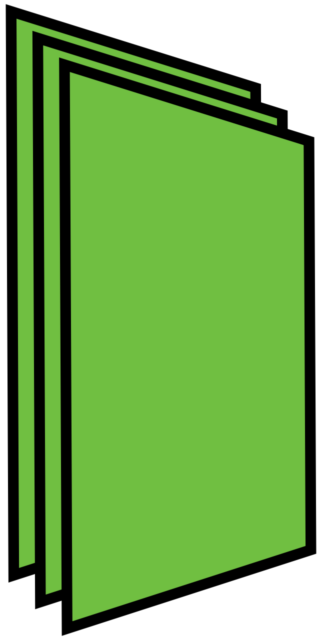
	0	1	2	3	4	5	6	7	8	9	10
M5	●	●	●	●	●	●	—	—	—	—	—

M coincident M5-branes' worldvolume theory

Difficult (impossible?) to write a
(local, Lorentz-invariant, superconformal, etc.) Lagrangian

"non-Lagrangian theory (?)"

M-theory



	0	1	2	3	4	5	6	7	8	9	10
M5	•	•	•	•	•	•	–	–	–	–	–

M coincident M5-branes' worldvolume theory

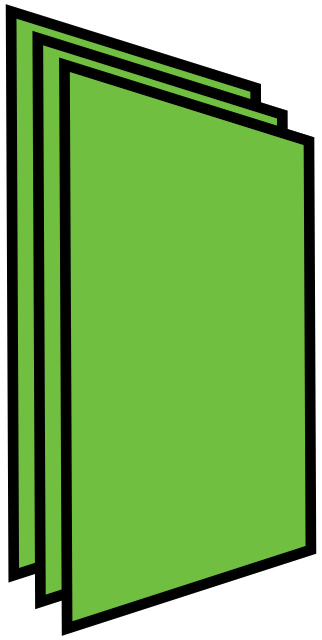
superconformal quantum field theories can only exist in $d \leq 6$

no free parameters besides M

"isolated fixed point"

derive all other QFTs from it?

M-theory



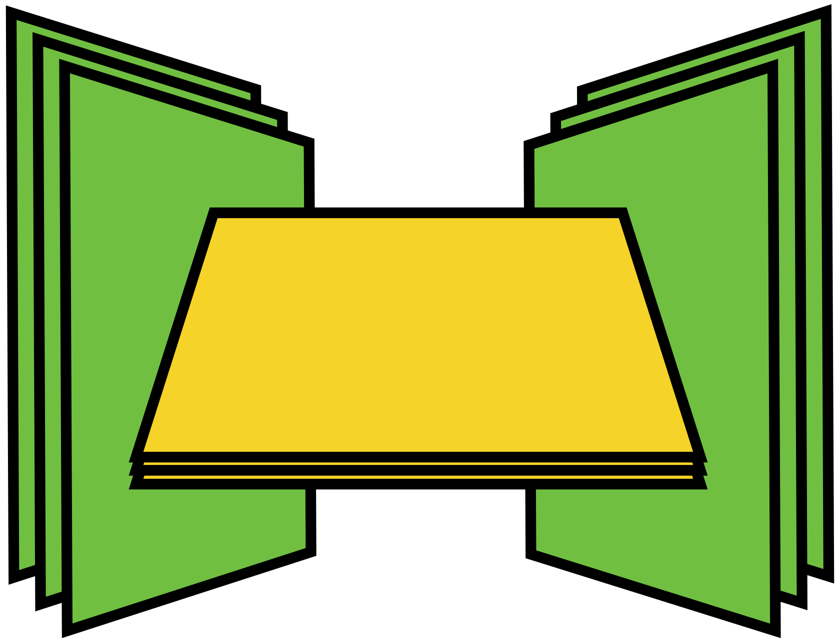
	0	1	2	3	4	5	6	7	8	9	10
M5	•	•	•	•	•	•	–	–	–	–	–

M coincident M5-branes' worldvolume theory

Maldacena limit
$M \to \infty$

$d = 11$ SUGRA on $AdS_7 \times S^4$

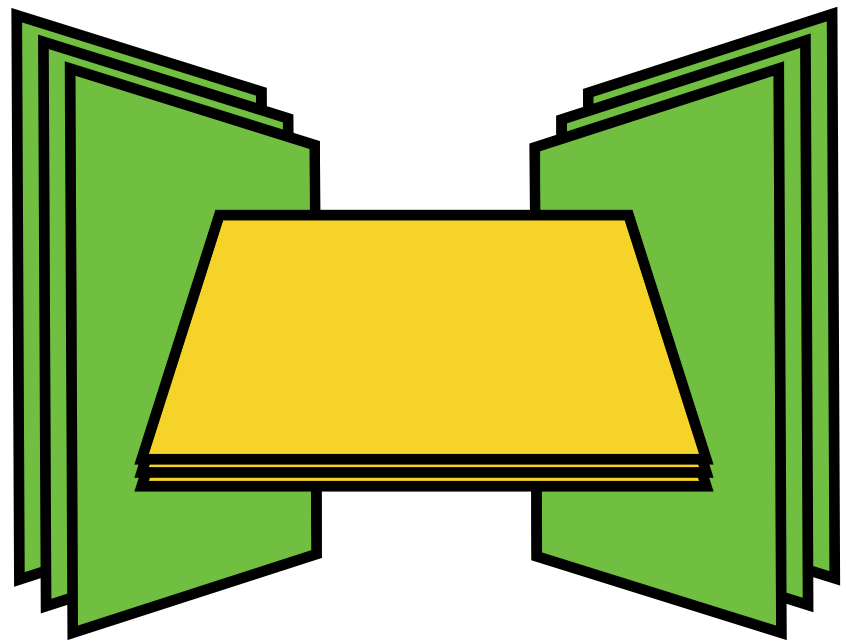
M-theory



	0	1	2	3	4	5	6	7	8	9	10
M5	●	●	●	●	●	●	—	—	—	—	—
M2	●	●	—	—	—	—	●	—	—	—	—

M2-branes can end on M5-branes

M-theory



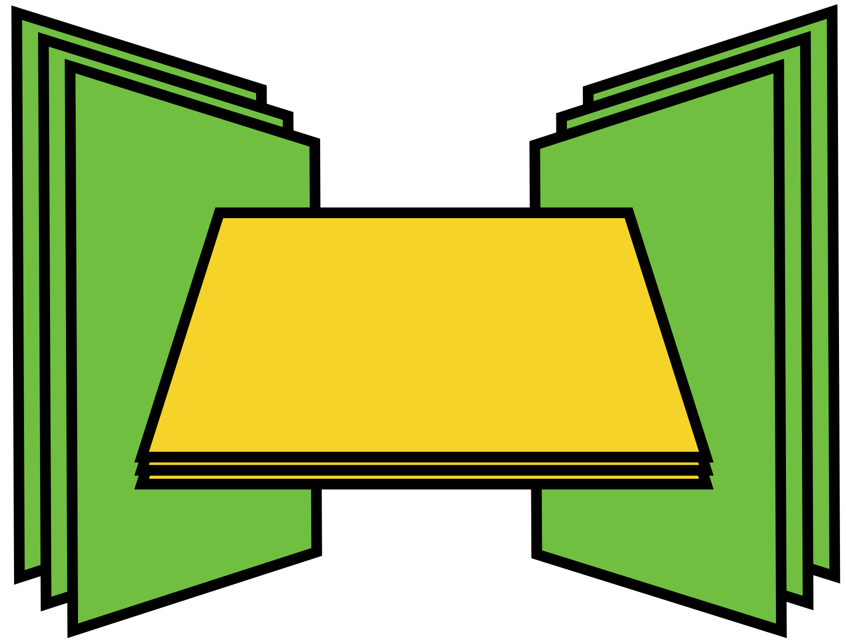
	0	1	2	3	4	5	6	7	8	9	10
M5	●	●	●	●	●	●	–	–	–	–	–
M2	●	●	–	–	–	–	●	–	–	–	–

M5-brane POV

$d = 2$ soliton $\Rightarrow$ string

charged under A_2

M-theory



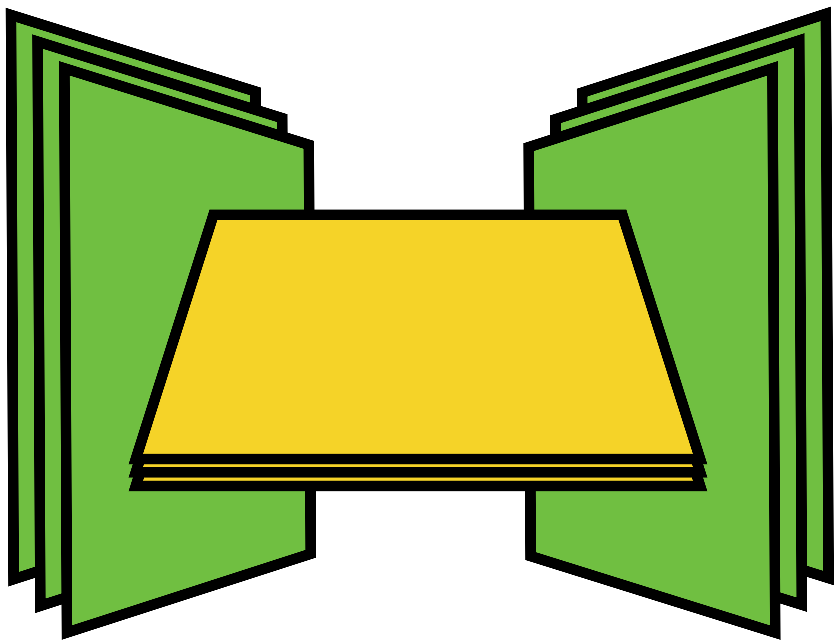
	0	1	2	3	4	5	6	7	8	9	10
M5	●	●	●	●	●	●	–	–	–	–	–
M2	●	●	–	–	–	–	●	–	–	–	–

M5-brane POV

$$\int_{S^3} \star_{6d} F_3 = \int_{S^3} F_3$$

"self-dual string"

M-theory



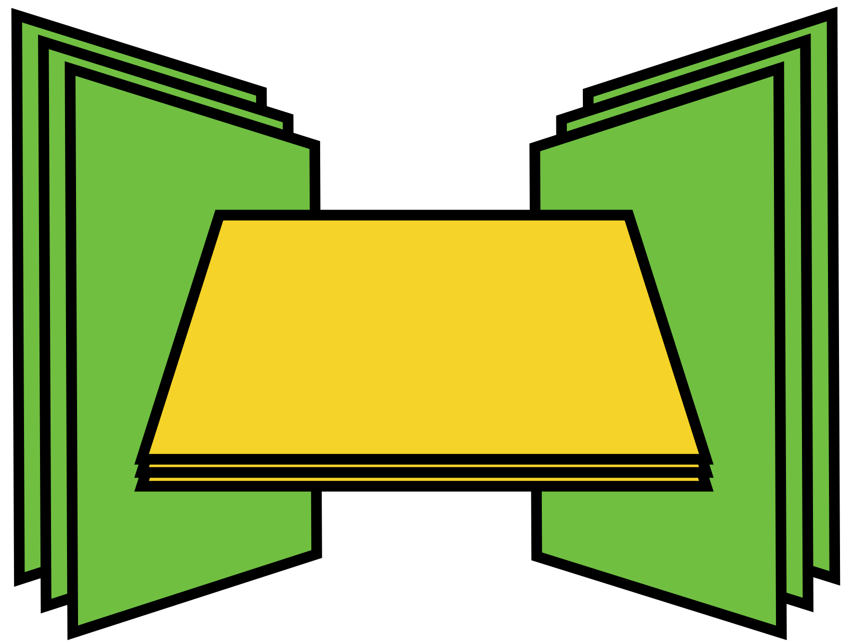
	0	1	2	3	4	5	6	7	8	9	10
M5	●	●	●	●	●	●	—	—	—	—	—
M2	●	●	—	—	—	—	●	—	—	—	—

M5-brane POV

$$SO(6,2) \times SO(5)_R \to SO(2,2) \times SO(4)_R \times SO(4)_R$$

$$d = 2 \ \text{ large } \ \mathcal{N} = (4,4) \text{ superconformal symmetry}$$

M-theory



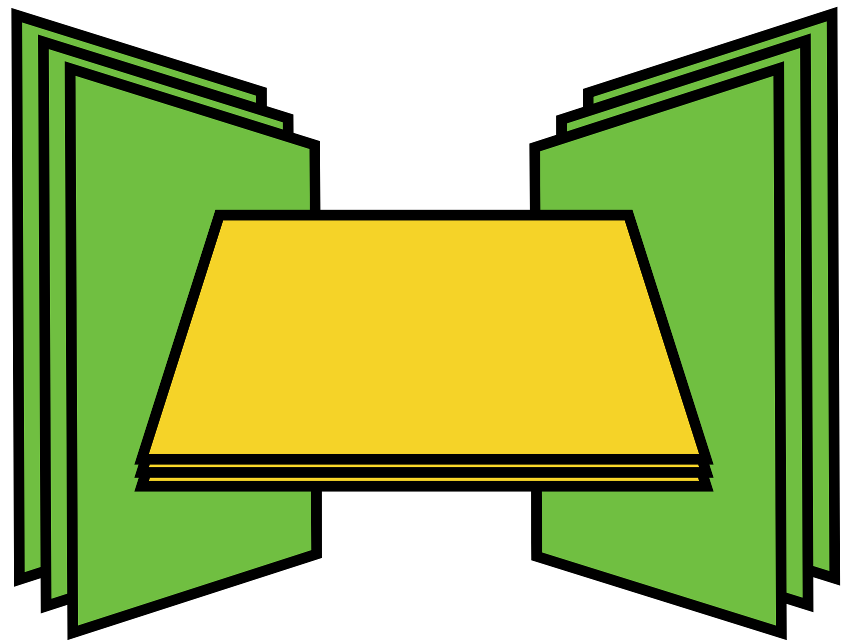
	0	1	2	3	4	5	6	7	8	9	10
M5	●	●	●	●	●	●	–	–	–	–	–
M2	●	●	–	–	–	–	●	–	–	–	–

M2-brane POV

ABJM theory with boundaries
+ superconformal boundary terms
+ massless SUSY multiplets at the boundaries

Niarchos 1509.07676

M-theory



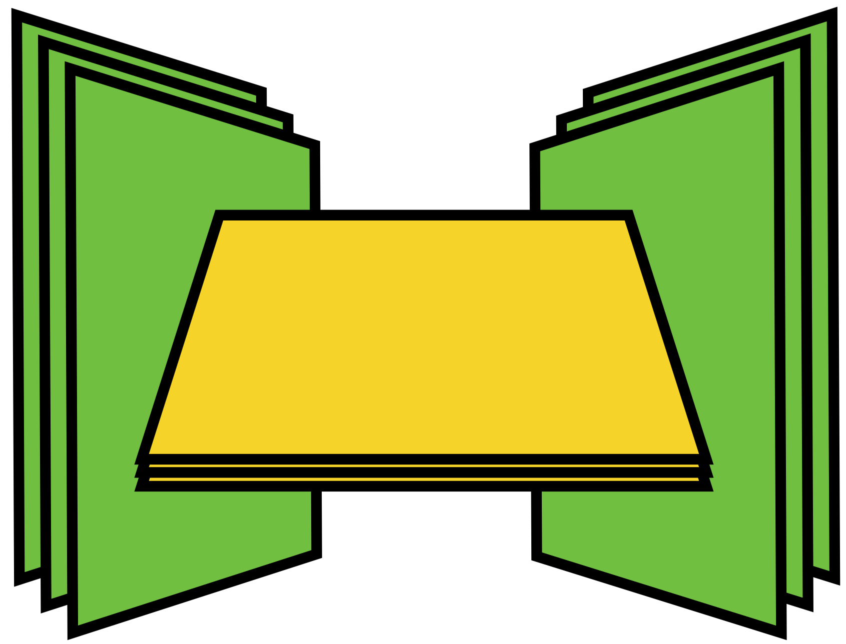
	0	1	2	3	4	5	6	7	8	9	10
M5	●	●	●	●	●	●	–	–	–	–	–
M2	●	●	–	–	–	–	●	–	–	–	–

M2-brane POV

$$SO(3,2) \times SO(8)_R \to SO(2,2) \times SO(4)_R \times SO(4)_R$$

$d = 2$ large $\mathcal{N} = (4,4)$ superconformal symmetry

M-theory



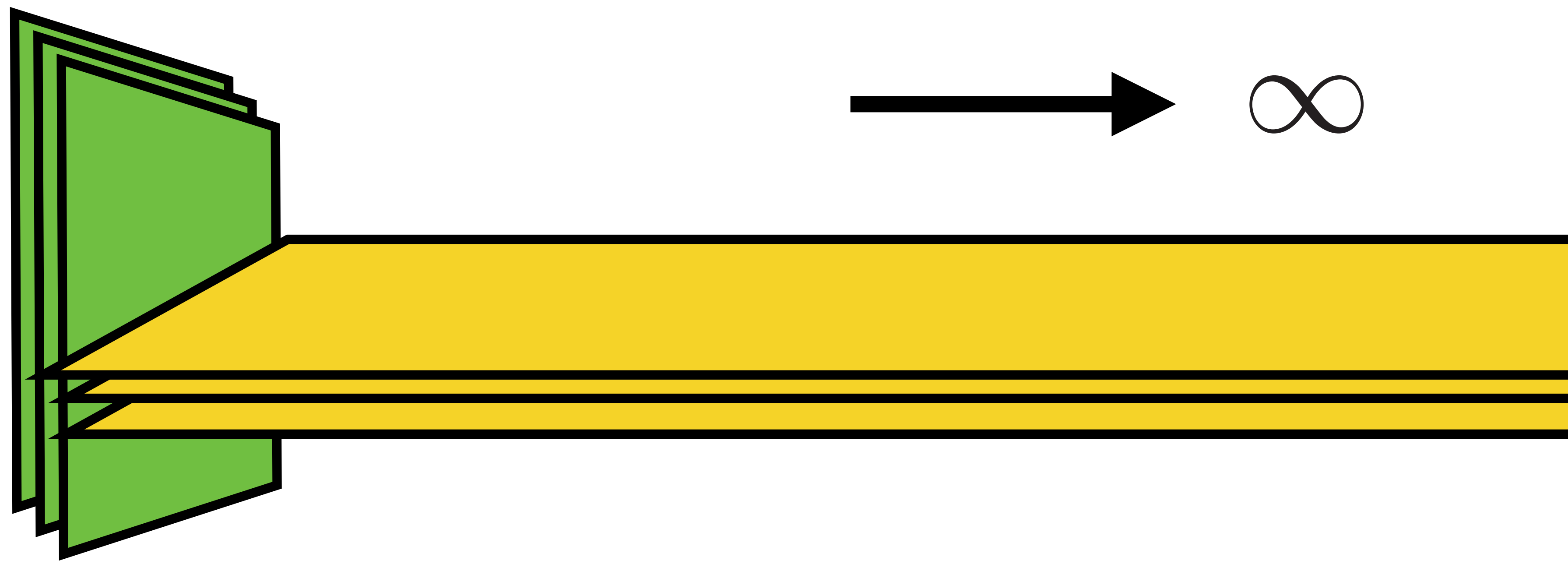
	0	1	2	3	4	5	6	7	8	9	10
M5	•	•	•	•	•	•	—	—	—	—	—
M2	•	•	—	—	—	—	•	—	—	—	—

M5-brane POV

Self-dual String Tension

$$T_{\text{string}} \propto T_{\text{M2}} |x_6|$$

M-theory



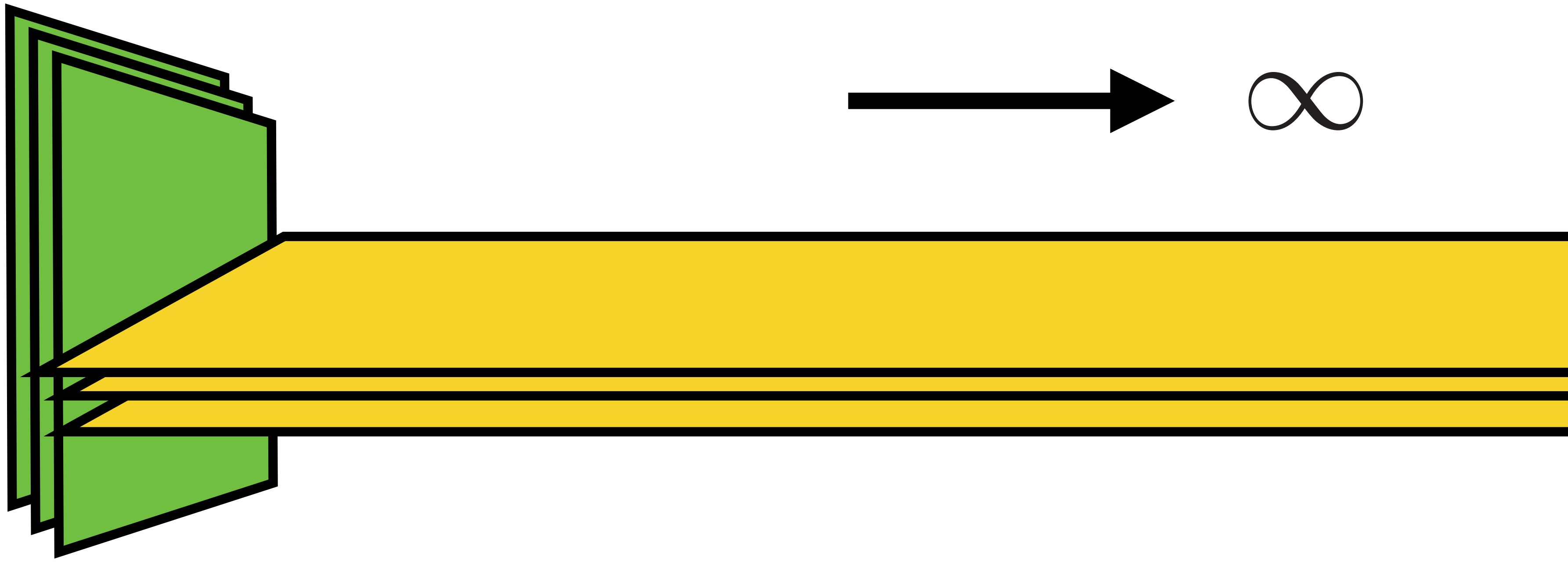
	0	1	2	3	4	5	6	7	8	9	10
M5	•	•	•	•	•	•	–	–	–	–	–
M2	•	•	–	–	–	–	•	–	–	–	–

M coincident M5-branes' worldvolume theory

$$|x_6| \to \infty \quad \Rightarrow \quad T_{\text{string}} \to \infty$$

"Wilson surface"

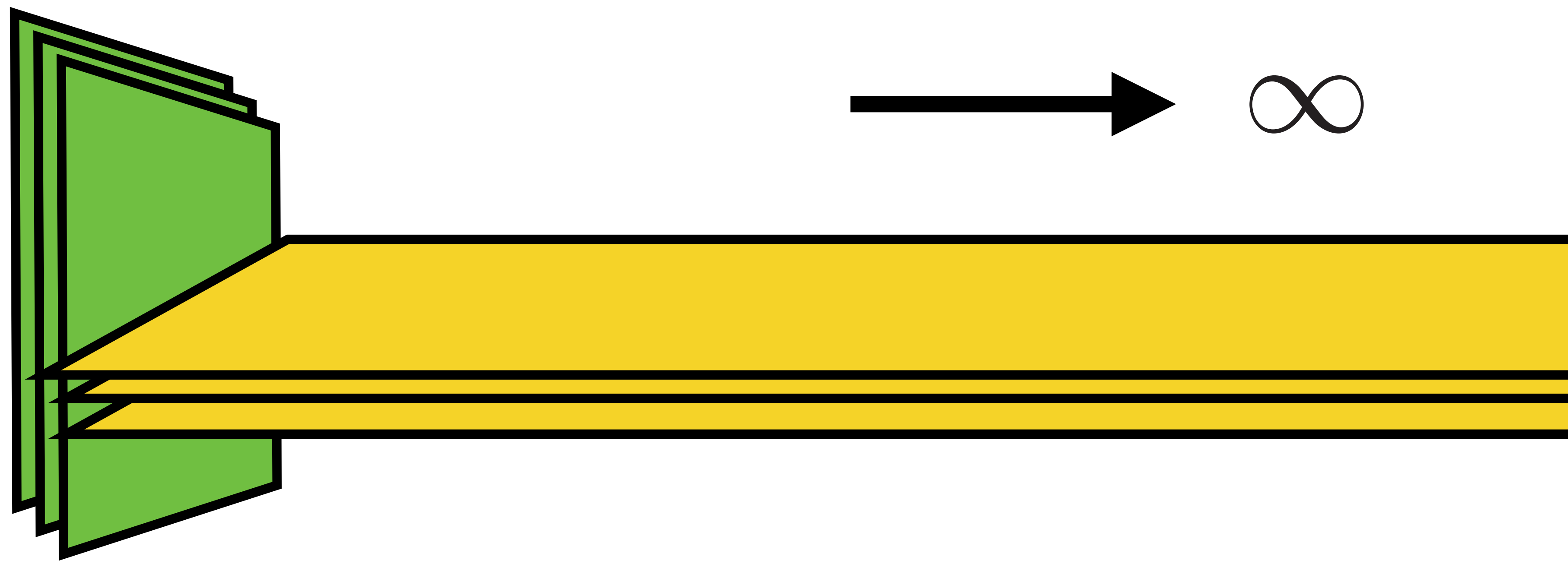
M-theory



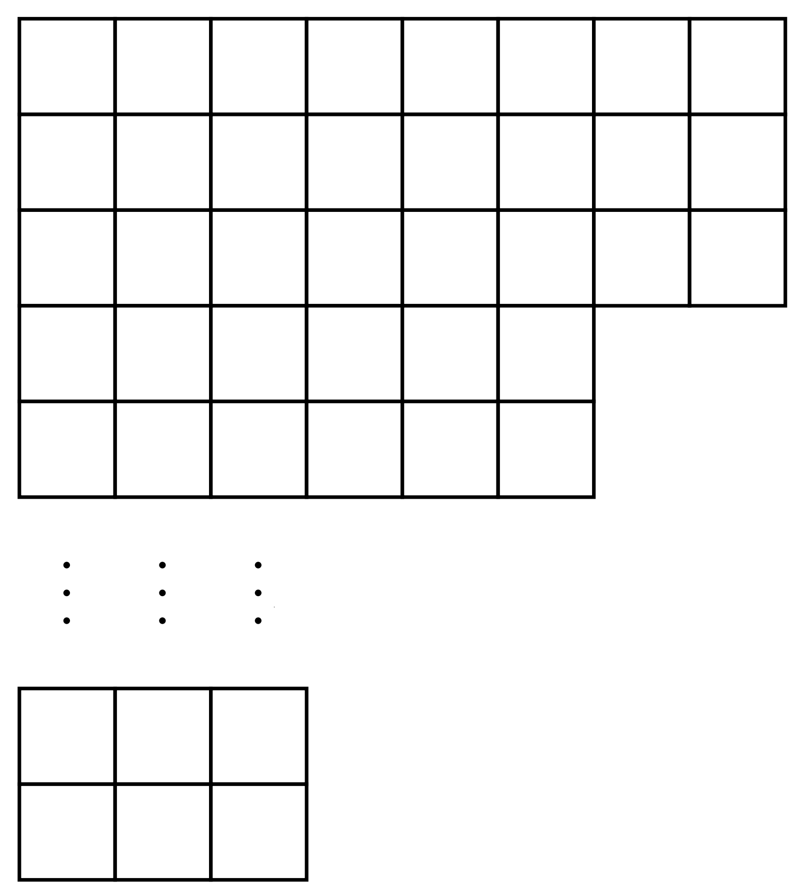
Wilson Surface

$$\langle \mathrm{W}_{\mathcal{R}} \rangle = \mathrm{tr}_{\mathcal{R}} \exp\left[i \int A_2 + \ldots \right]$$

M-theory



$\mathcal{R} =$



Question

What is the central charge of the self-dual string?

Counts massless degrees of freedom on the self-dual string

As a function of

$$M = \# \text{ of M5-branes}$$

$$N = \# \text{ of M2-branes}$$

$$\mathcal{R} = \# \text{ representation of } U(M)$$

Berman and Harvey hep-th/0408198

Niarchos and Siampos 1206.2935

Question

What is the central charge of the self-dual string?

$d = 11$ SUGRA solutions describing Wilson surfaces

D'Hoker, Estes, Gutperle, Krym 0806.0605, 0810.4647
Estes, Feldman, Krym 1209.1845
Bachas, D'Hoker, Estes, Krym 1312.5477

isometry $SO(2,2) \times SO(4)_R \times SO(4)_R$

asymptotically locally $AdS_4 \times S^7$ or $AdS_7 \times S^4$

M2-brane POV

M5-brane POV

GOAL

Compute the central charge of Wilson surfaces

$d = 11$ SUGRA solutions describing Wilson surfaces

D'Hoker, Estes, Gutperle, Krym 0806.0605, 0810.4647
Estes, Feldman, Krym 1209.1845
Bachas, D'Hoker, Estes, Krym 1312.5477

isometry $SO(2,2) \times SO(4)_R \times SO(4)_R$

asymptotically locally $AdS_4 \times S^7$ or $AdS_7 \times S^4$

↙ M2-brane POV

↘ M5-brane POV

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- M5-Branes and Self-Dual Strings
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Central Charge

$d = 2$ CFT

Virasoro algebra $\Rightarrow$ central charge c

Trace Anomaly
Duff hep-th/9308075

$$T_\mu{}^\mu = \frac{c}{24\pi} R$$

Entanglement Entropy
Holzhey, Larsen, Wilczek hep-th/9403108
Calabrese + Cardy hep-th/0405152

$$S_{\mathrm{EE}} = \frac{c}{3} \ln(\ell/\varepsilon) + \dots$$

stress tensor 2-pt. function

$$\langle T_{zz}(z,\bar{z}) T_{zz}(0,0) \rangle = \frac{c/2}{z^4}$$

Thermodynamic entropy
Cardy NPB 270 (186) 1986

$$S_{\mathrm{thermo}} = \frac{\pi}{3} c L T + \dots$$

Central Charge

$d = 2$ conformal defect/boundary in a $d > 2$ CFT

Virasoro algebra $\rightarrow$ central charge c

Trace Anomaly
Duff hep-th/9308075

??????

see C. Herzog's talk

Entanglement Entropy
Holzhey, Larsen, Wilczek hep-th/9403108
Calabrese + Cardy hep-th/0405152

??????

stress tensor 2-pt. function

??????

Thermodynamic entropy
Cardy NPB 270 (186) 1986

??????

Central Charge

$d = 2$ conformal defect/boundary in a $d > 2$ CFT

~~Virasoro algebra $\longrightarrow$ central charge c~~

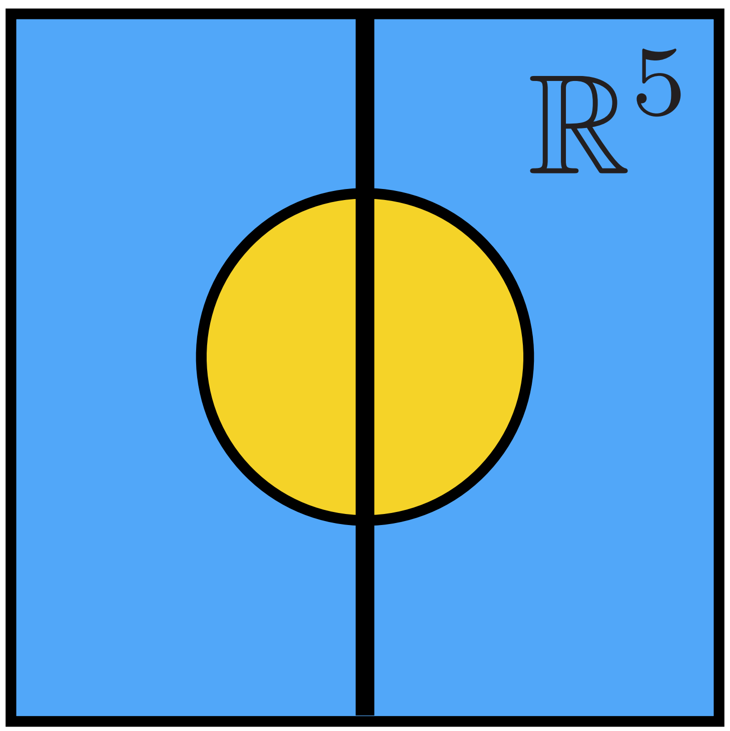
Trace Anomaly Duff hep-th/9308075	?????? see C. Herzog's talk
Entanglement Entropy Holzhey, Larsen, Wilczek hep-th/9403108 Calabrese + Cardy hep-th/0405152	??????
stress tensor 2-pt. function	??????
Thermodynamic entropy Cardy NPB 270 (186) 1986	??????

Central Charge

Entanglement Entropy

M5-brane POV

sphere of radius ℓ



$$S_{\mathrm{EE}} = S_{\mathrm{EE}}^{d=6} + S_{\mathrm{EE}}^{d=2}$$

$$S_{\mathrm{EE}}^{d=6} = c_4 \frac{A}{\varepsilon^4} + \frac{c_2}{\varepsilon^2} + \#a \ln(\ell/\varepsilon) + c_0 + \ldots$$

$$S_{\mathrm{EE}}^{d=2} = c_L \ln(\ell/\varepsilon) + c_0' + \ldots$$

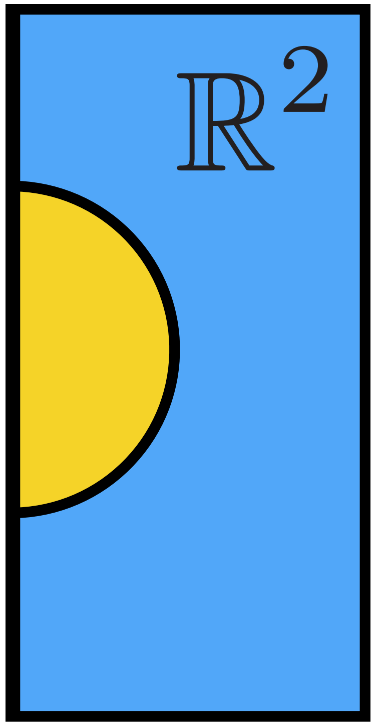
$$c \equiv 3\,\ell \frac{d}{d\ell}\left[S_{\mathrm{EE}} - S_{\mathrm{EE}}^{d=6}\right]$$

Central Charge

Entanglement Entropy

M2-brane POV

semi-circle of radius ℓ



$$S_{\mathrm{EE}} = \frac{1}{2} S_{\mathrm{EE}}^{d=3} + S_{\mathrm{EE}}^{d=2}$$

$$S_{\mathrm{EE}}^{d=3} = c_1 \, \frac{\ell}{\varepsilon} + c_0 + \ldots$$

$$S_{\mathrm{EE}}^{d=2} = c_L \, \ln\left(\ell/\varepsilon\right) + c_0' + \ldots$$

$$c \equiv 3\,\ell \frac{d}{d\ell} \left[S_{\mathrm{EE}} - \frac{1}{2} S_{\mathrm{EE}}^{d=3} \right]$$

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Holographic Duals

D'Hoker, Estes, Gutperle, Krym 0806.0605, 0810.4647
Estes, Feldman, Krym 1209.1845
Bachas, D'Hoker, Estes, Krym 1312.5477

M5-brane POV

Wilson surface

$$SO(6,2) \times SO(5)_R \to SO(2,2) \times SO(4)_R \times SO(4)_R$$

$$AdS_7 \times S^4 \to AdS_3 \times S^3 \times S^3$$

Holographic Duals

D'Hoker, Estes, Gutperle, Krym 0806.0605, 0810.4647
Estes, Feldman, Krym 1209.1845
Bachas, D'Hoker, Estes, Krym 1312.5477

M2-brane POV

ABJM theory with a boundary

$$SO(3,2) \times SO(8)_R \to SO(2,2) \times SO(4)_R \times SO(4)_R$$

$$AdS_4 \times S^7 \to AdS_3 \times S^3 \times S^3$$

Holographic Duals

D'Hoker, Estes, Gutperle, Krym 0806.0605, 0810.4647
Estes, Feldman, Krym 1209.1845
Bachas, D'Hoker, Estes, Krym 1312.5477

$$ds^2 = f_1^2(z,\bar{z})\, ds^2_{AdS_3} + f_2^2(z,\bar{z})\, ds^2_{S^3} + f_3^2(z,\bar{z}) ds^2_{S^3} + \rho^2(z,\bar{z}) dz\, d\bar{z}$$

$AdS_3 \times S^3 \times S^3$ fibered over a Riemann surface

asymptotically $AdS_7 \times S^4$ $\Rightarrow$ M5-brane POV; Wilson surface

asymptotically $AdS_4 \times S^7$ $\Rightarrow$ M2-brane POV; ABJM theory with a boundary

singularities indicate M2/M5-branes

Holographic Duals

Entanglement Entropy

Ryu and Takayanagi hep-th/0603001, 0605073

$$S_{\mathrm{EE}} = \frac{\mathrm{Area}}{4G}$$

Jensen and O'Bannon 1309.4523

$$S_{\mathrm{EE}} = \frac{\left(2\pi^2\right)^2}{4G} \int dz\, d\bar{z}\, \left(\rho^2 f_1 f_2^3 f_3^3\right) \int_{\varepsilon}^{\ell} \frac{\ell\, dy}{y\sqrt{\ell^2 - y^2}}$$

$$c \equiv 3\,\ell \frac{d}{d\ell}\left[S_{\mathrm{EE}} - S_{\mathrm{EE}}^{d=6}\right]$$

Gentle, Gutperle, Marasinou 1506.00052

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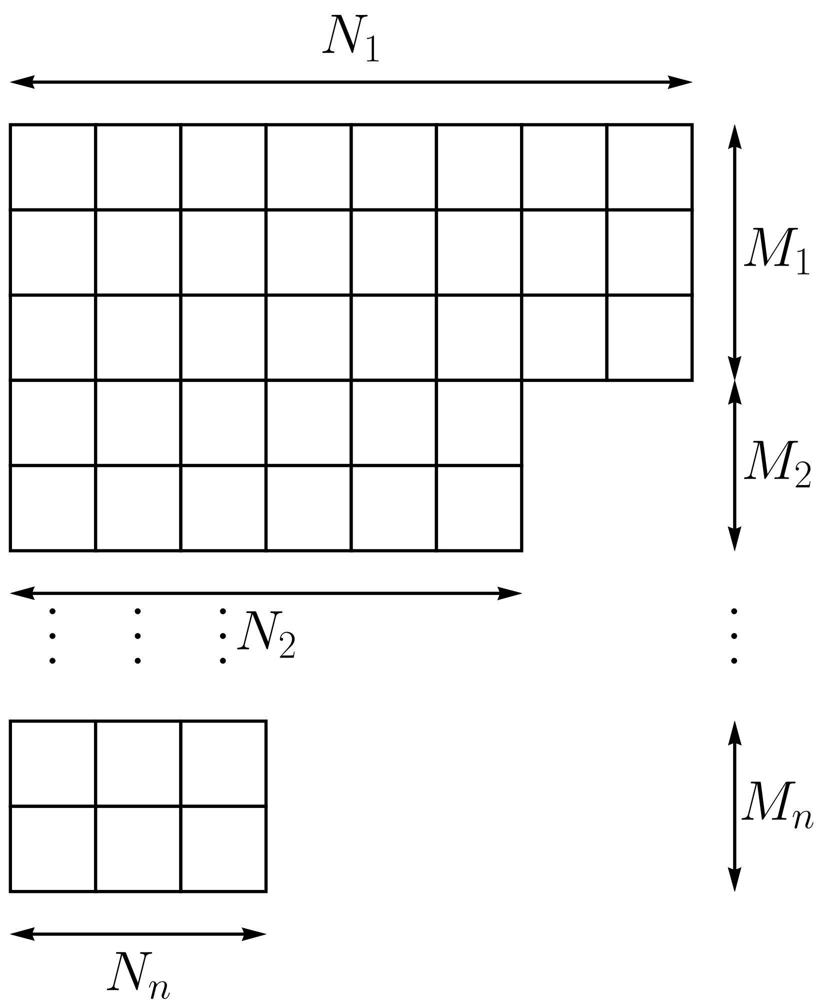
Central Charge

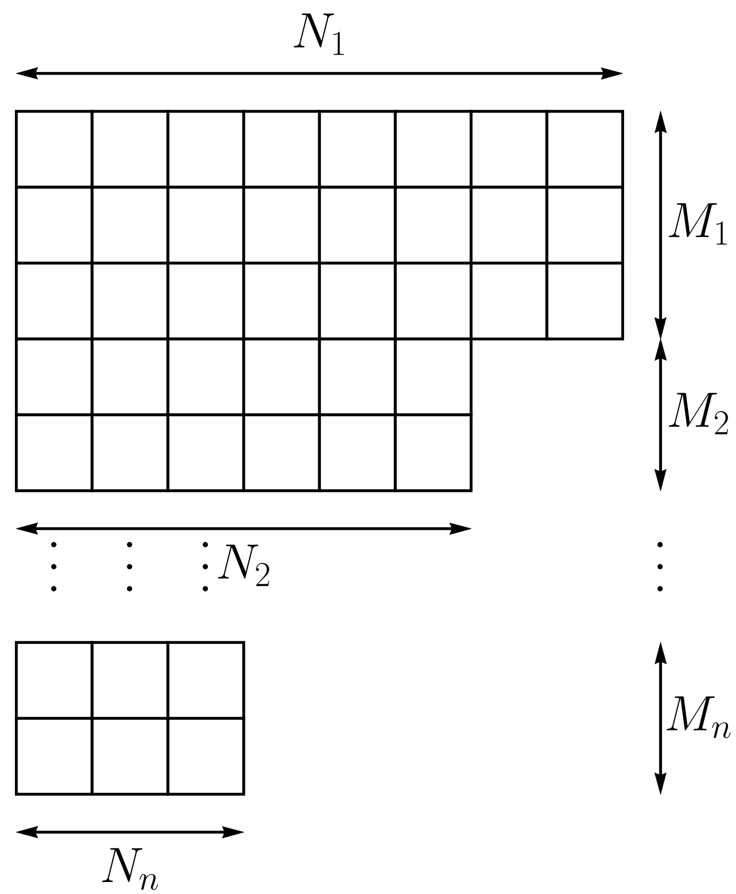


$M_a = \#$ of M5-branes with N_a M2-branes ending on them

$$M = \# \text{ of M5-branes} = \sum_{a=1}^{n+1} M_a$$

$$N = \# \text{ of M2-branes} = \sum_{a=1}^{n+1} M_a N_a$$

$$N_{n+1} = 0$$



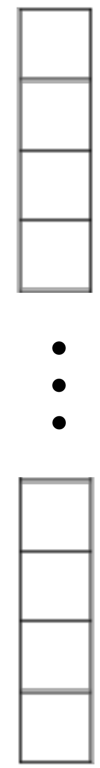
M5-brane POV

$$c_{M5} = \frac{3}{5}\left[8MN + \frac{N^2}{M} + \sum_{a=1}^{n}\left(8M_a^2 N_a - M_a N_a^2 - 16\sum_{b=1}^{a} N_a M_a M_b\right)\right]$$

M2-brane POV

$$c_{M2} = 3\sum_{a=1}^{n}\left[-M_a^2 N_a + \frac{1}{2}M_a N_a^2 + 2\sum_{b=1}^{a} N_a M_a M_b\right]$$

Antisymmetric



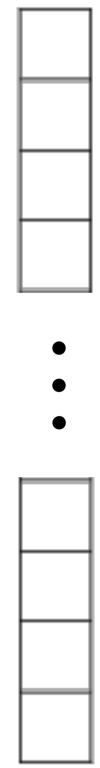
$$N_1 = 1$$

$$M_1 = N$$

M5-brane POV

$$c_{M5} = \frac{3}{5}\left[8MN + \frac{N^2}{M} - 8N^2 - N\right]$$

Antisymmetric



$$N_1 = 1$$

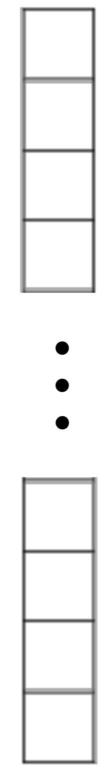
$$M_1 = N$$

M5-brane POV

$$c_{M5} = \frac{3}{5}\left[8MN + \frac{N^2}{M} - 8N^2 - N\right]$$

$$\text{if } M = N \text{ then } c_{M5} = 0$$

Antisymmetric



$$N_1 = 1$$

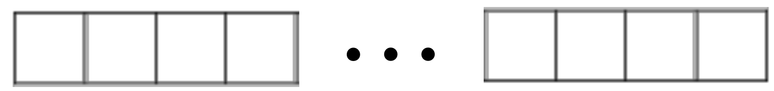
$$M_1 = N$$

M5-brane POV

$$c_{M5} = \frac{3}{5}\left[8MN + \frac{N^2}{M} - 8N^2 - N\right]$$

no M^3 or $N^{3/2}$

Symmetric



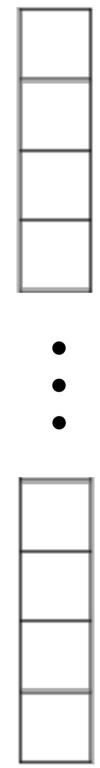
$$N_1 = N$$

$$M_1 = 1$$

M5-brane POV

$$c_{M5} = \frac{3}{5}\left[8MN + \frac{N^2}{M} - N^2 - 8N\right]$$

Antisymmetric



$$N_1 = 1$$

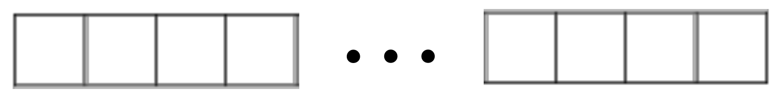
$$M_1 = N$$

M2-brane POV

$$c_{M2} = 3\left[N^2 + \frac{1}{2}N\right]$$

no M

Symmetric



$$N_1 = N$$

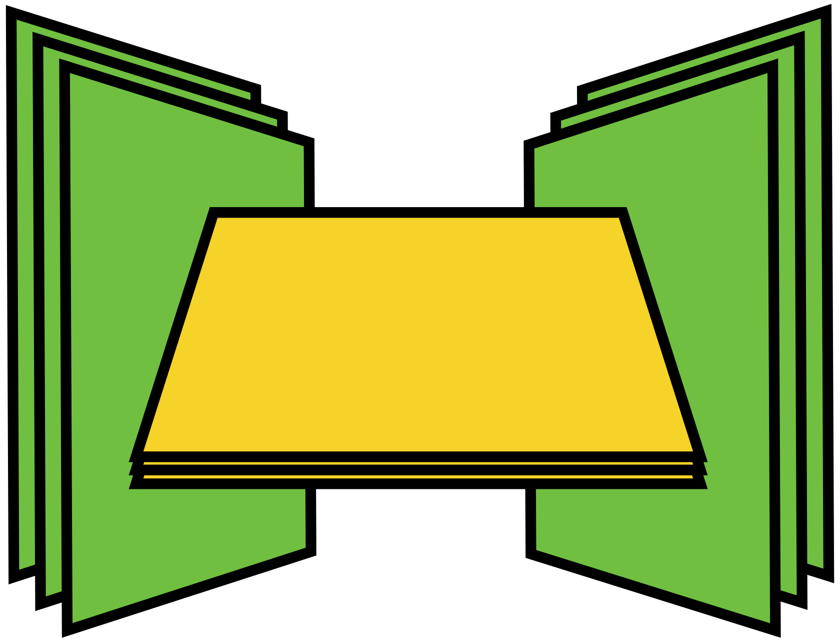
$$M_1 = 1$$

M2-brane POV

$$c_{M2} = 3\left[\frac{1}{2}N^2 + N\right]$$

Central Charge

Berman and Harvey hep-th/0408198
anomaly inflow

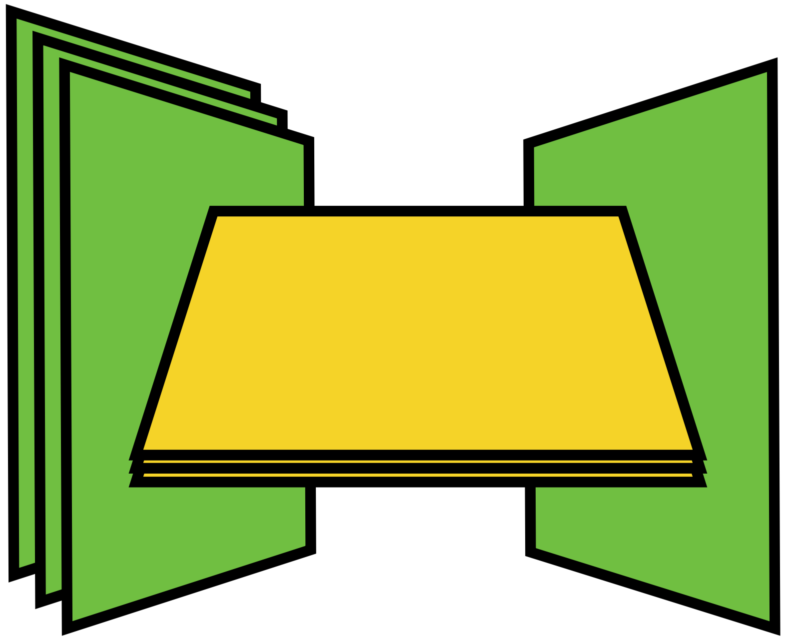


$$c_{\rm BH} = \frac{1}{2} N \alpha$$

α depends on the point in the Coulomb branch

Central Charge

Berman and Harvey hep-th/0408198
anomaly inflow

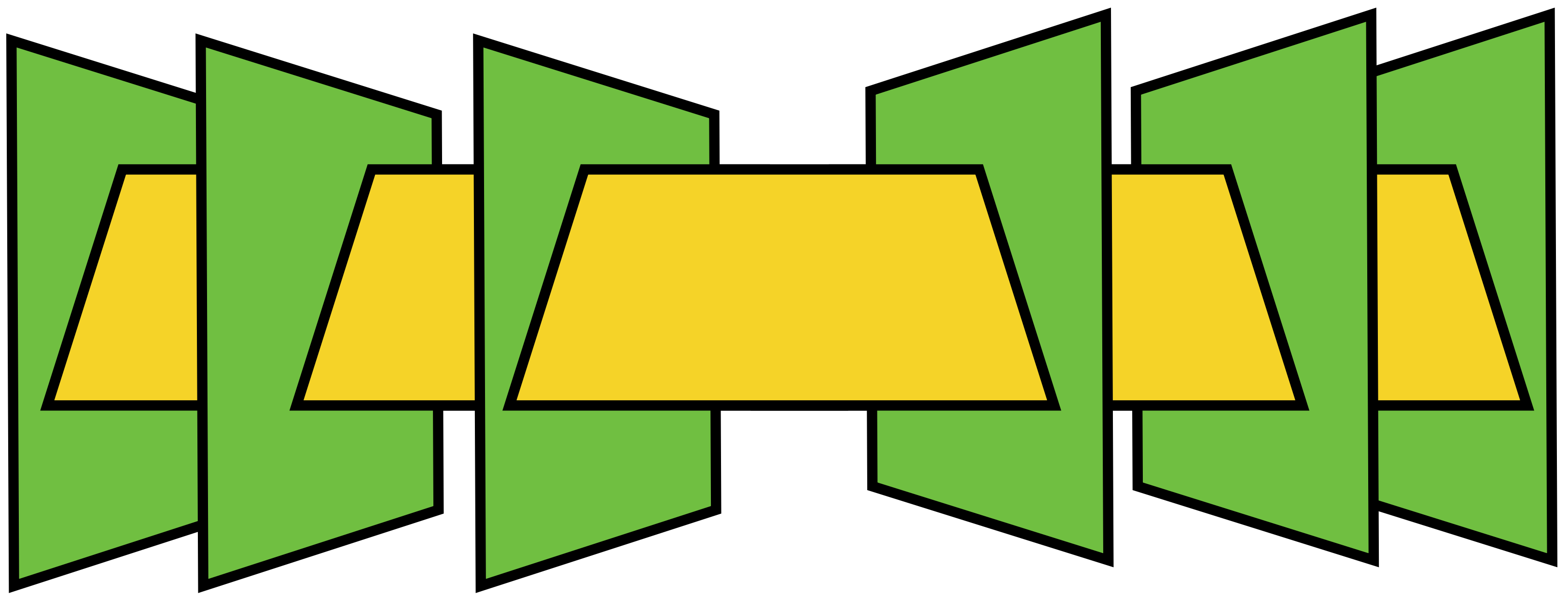


$$U(M+1) \to U(M) \times U(1)$$

$$c_{\mathrm{BH}} = \frac{1}{4} N M$$

Central Charge

Berman and Harvey hep-th/0408198
anomaly inflow



$$SU(M) \to U(1)^M$$

$$c_{\mathrm{BH}} = \frac{1}{8} N \left[M^2 + M - 1 \right]$$

Central Charge

Niarchos and Siampos 1206.2935
blackfolds

$$S_{\rm thermo} = \frac{\pi}{3} c L T + \dots$$

$$c_{\rm NS} = \frac{8}{45} \frac{\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{1}{6}\right)}{\sqrt{\pi}} \frac{N^2}{M} \approx 1.5 \frac{N^2}{M}$$

Central Charge

M5-brane POV

$$c_{M5} = \frac{3}{5}\left[8MN + \frac{N^2}{M} + \sum_{a=1}^{n}\left(8M_a^2 N_a - M_a N_a^2 - 16\sum_{b=1}^{a} N_a M_a M_b\right)\right]$$

$$\lambda \equiv \text{highest weight}$$

$$\rho \equiv \text{Weyl vector}$$

$$c_{M5} = \frac{3}{5}\left[16\left(\lambda, \rho\right) - \left(\lambda, \lambda\right)\right]$$

Central Charge

M5-brane POV

$d = 2$ CFT with W_M symmetry
(W-symmetry of type $\mathfrak{g} = A_{M-1}$)

$$c = (M-1) + 12\, Q^2\, (\rho, \rho)$$
$$Q = \text{“background charge”}$$

chiral primaries of dimension

$$\Delta_\lambda = \frac{1}{2}\left[2Q(\lambda, \rho) - (\lambda, \lambda)\right]$$

AGT correspondence? Alday, Gaiotto, Tachikawa 0906.3219

Outline:

- M5-Branes and Self-Dual Strings
- Entanglement Entropy
- The Holographic Duals
- The Central Charge
- Summary and Outlook

Summary

The M5-brane theory is fundamental to M-theory and QFT.

We can probe the M5-brane theory using Wilson surfaces.

SUGRA solutions exists describing Wilson surfaces holographically.

Using these, we computed the central charge of Wilson surfaces.

Outlook

Other comparisons? elliptic genus, BPS index, ...?

Field theory explanation of results? AGT?

Other calculations using these SUGRA solutions?

Relate to other definitions of the central charge?

Thank You.