Bulk and Witten Rules in Bi-Local Holography

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Outline

Vectorial AdS/CFT Duality [Klebanov & Polyakov '02⋯]

Holography from Collective Fields[S Das ,AJ]

Re-construction of Bulk AdS (HS) Gravity

Canonical map

Spinning particle picture

PRESENT TALK : BULK EQUATIONS and WITTEN DIAGRAMS

[R de Mello Koch , Kenta Suzuki, JungGi Yoon & AJ

arXiv: 18xxx….. (hep-th)]

Models

- d=1 : SYK Model/ NAdS_2 (?)
  Tensor models
- d=2, :AdS_3 HS / Minimal Models
- d=3 :O(N)Vector/HS Gravity (?)

$$\mathcal{L} = (\partial\vec{\phi})\cdot(\partial\vec{\phi}) + \frac{\lambda}{2N}(\vec{\phi}\cdot\vec{\phi})^2$$

VASILIEV: HIGHER SPIN THEORY

- d=4,5 : Fishnet Matrix Models [Kazakov et al'17] (?)

Bi-local Construction

$$\Psi(x_1, x_2) = \frac{1}{N} \sum_{i=1}^{N} \phi_i(x_1)\phi_i(x_2)$$

- Bi-local Space (of CFT_d) : AdS_{d+1} + S^{d-1}Spin)

$$\Psi(x_1^\mu, x_2^\mu) \quad \Leftrightarrow \quad H(x^\mu, z; S)$$

$$d + d \quad = \quad (d+1) + (d-1)$$

One to One Operator Identification

HS Equations

- Tensor Fields (Fronsdal)

$$-(\Box - m^2)h_{\mu_1\cdots\mu_s} + s\nabla_{(\mu_1}\nabla^{\nu}h_{\mu_2\cdots\mu_s)\nu} - \frac{s(s-1)}{2(d+2s-3)}g_{(\mu_1\mu_2}\nabla^{\nu_1}\nabla^{\nu_2}h_{\mu_3\cdots\mu_s)\nu_1\nu_2} = 0$$

- In AdS_(d+1) space-time

- Traceless ,Symmetric

Physical Space for HS Theory

- de Donder gauge conditions:

$$\eta^{\hat{\mu}_1\hat{\mu}_2} H_{\hat{\mu}_1\hat{\mu}_2\cdots\hat{\mu}_s} = 0$$

$$\left(\partial_z - \frac{1}{z}\right) H_{z\hat{\mu}_2\cdots\hat{\mu}_x} + \partial_\mu H_{\mu\hat{\mu}_2\cdots\hat{\mu}_s} = 0$$

- Solving the gauge Conditions

- Reduced HS field

$$H(x^i, z, \theta) = \sum_{s=1}^{\infty} e^{\pm is\theta} H_{(\pm s)}(x^i, z)$$

Identification: Momentum Space

$$\widetilde{\Psi}(k_1^i, k_2^i) \;=\; \widetilde{H}(p^i, p_z, \theta)$$

- Fourier transform(s)

$$\widetilde{\Phi}(k_1^i, k_2^i) \;\equiv\; \int dx_1^i dx_2^i \, e^{ix_1\cdot k_1} e^{ix_2\cdot k_2}\, \overline{\Phi}(x_1^i, x_2^i)\,,$$

$$\widetilde{H}(p^i, p^z, \Theta) \;\equiv\; \int dx^i dz dS \, e^{ip\cdot x} e^{ip^z z} e^{i\Theta S}\, H(x^i, z, S)$$

Change of momenta

(d=2)

$$\vec{p} = \vec{k}_1 + \vec{k}_2$$

$$p_z = 2\sqrt{k_1^+ k_1^-} - 2\sqrt{k_2^+ k_2^-}$$

$$\theta = \frac{1}{2}\left(\arcsin\frac{k^+}{p^+} - \arcsin\frac{k^-}{p^-}\right)$$

Realization of Conformal Symmetry SO(2,2

- HS AdS(3)

$$L_-^{\mathrm{AdS_3\times S^1}} = -\,ip^+$$

$$L_0^{\mathrm{AdS_3\times S^1}} = p^+x^- - \frac{1}{2}zp^z$$

$$L_+^{\mathrm{AdS_3\times S^1}} = -\,ip^+(x^-)^2 + ix^-zp^z - iz^2p^- + \frac{\sqrt{4p^+p^- - (p^z)^2}}{2p^+}izp^\theta - i\frac{(p^\theta)^2}{4p^+}$$

Bi-Local space

$$L_-^{\mathrm{bi}} = -i(k_1^+ + k_2^+)$$

$$L_0^{\mathrm{bi}} = k_1^+\frac{\partial}{\partial k_1^+} + k_2^+\frac{\partial}{\partial k_2^+}$$

$$L_+^{\mathrm{bi}} = i\left[k_1^+\left(\frac{\partial}{\partial k_1^+}\right)^2 + k_2^+\left(\frac{\partial}{\partial k_2^+}\right)^2\right]$$

Nonlinear Construction

- Collective Action:

$$S_{\rm col} = \frac{N}{2}\int d^dx\left[\left(\nabla_x^2\Psi(x,x')\right)_{x'=x} + \frac{\lambda}{2}\Psi^2(x,x)\right] - \frac{N}{2}\,{\rm Tr}\log\Psi$$

- Expand :Large N background

$$\Psi(x_1,x_2) = \Psi_0(x_1,x_2) + \frac{1}{\sqrt{N}}\overline{\Psi}(x_1,x_2)$$

$$\Phi_0(x_1,x_2) \equiv x_1 \bullet\!\!\longrightarrow\!\!\bullet\, x_2$$

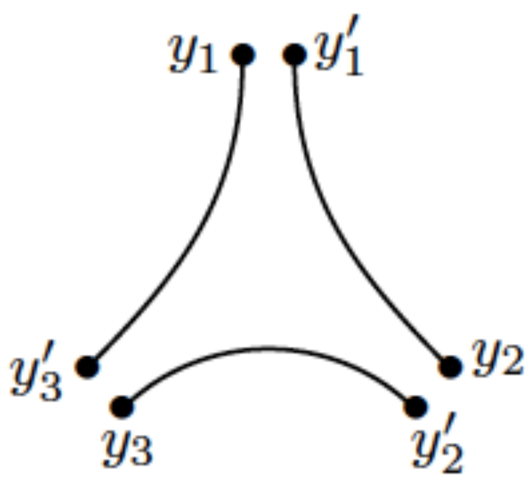
Complete sequence of Vertices: Expansion in 1/N=G

Interaction Vertices

All orders :

$$\sum_{n=3}^{\infty} \frac{(-1)^n}{2n\, N^{\frac{n}{2}-1}} \operatorname{Tr}\left(\Phi_0^{-1} \star \overline{\Phi} \star \cdots \star \Phi_0^{-1} \star \overline{\Phi}\right)$$

Star Product= Matrix product (bi-local space)



The Propagator

Collective Laplacian: in the quadratic term

$$\widehat{\mathcal{L}}_{\rm bi} = |x_{12}|^2|x_{34}|^2\Big[\Phi_0^{-1}(x_1,x_3)\Phi_0^{-1}(x_2,x_4) + \lambda\,\delta^d(x_{12})\delta^d(x_{13})\delta^d(x_{14})\Big]\,.$$

- Simplest(free) case

$$\begin{aligned}\widehat{\mathcal{L}}_{\rm bi} &\approx C_4 + \frac{1}{4}C_2^2 \\ &\approx \frac{1}{4}\,|x_1 - x_2|^4\,\frac{\partial}{\partial x_1}\cdot\frac{\partial}{\partial x_1}\frac{\partial}{\partial x_2}\cdot\frac{\partial}{\partial x_2}\end{aligned}$$

d=2 Example

- Eigenfunctions :

$$\int d^2x_3 d^2x_4\, \mathcal{K}(x_1, x_2; x_3, x_4)\psi_{h,s}(x_3, x_4; p) \;=\; 4\lambda_{h,s}\psi_{h,s}(x_1, x_2; p)$$

given by Bessel functions:

$$\psi_{h,s}(\vec{x}_1, \vec{x}_2; \vec{p}) \;=\; |\eta\bar{\eta}|^{\frac{1}{2}}\, e^{-i\vec{p}\cdot\vec{x}}\, K_{\frac{h+s-1}{2}}(ip\eta)\, K_{\frac{h-s-1}{2}}(i\bar{p}\bar{\eta})$$

With eigenvalues:

$$\lambda(h, s) \;=\; \frac{1}{4}(h+s)(h+s-2)(h-s)(h-s-2)$$

Momentum Space

- Fourier Transform of the eigen-functions (d=2)

$$\psi_{h,s}(\vec{k}_1, \vec{k}_2; \vec{p}) \; = \; \delta(\vec{k}_1 + \vec{k}_2 - \vec{p}) F(k^+) F(k^-)$$

$$F(k^+)F(k^-) \; = \; \frac{e^{i\frac{s}{2}(\arcsin\frac{k^+}{p^+} - \arcsin\frac{k^-}{p^-})} \; e^{i\frac{h}{2}(\arcsin\frac{k^+}{p^+} + \arcsin\frac{k^-}{p^-})}}{\sqrt{(k^+)^2 - (p^+)^2}\,\sqrt{(k^-)^2 - (p^-)^2}}$$

* Momentum space CFT: Skenderis et al

Cubic Vertex

- The bi-local cubic vertex/d=2 :

$$V^{+}_{(3)}(k_1, k_1'; k_2, k_2'; k_3, k_3') \propto \delta(k_1^+ + k_2'^+)\delta(k_2^+ + k_3'^+)\delta(k_3^+ + k_1'^+)$$

Transforms into: AdS form

$$\begin{aligned} & V^{+}_{(3)}(p_1, \phi_1, \theta_1; p_2, \phi_2, \theta_2; p_3, \phi_3, \theta_3) \\ \propto\ & \delta(p_1^+ + p_2^+ + p_3^+)\, \delta\Big(p_2^+ \big(1 + \sin(\phi_2 + \theta_2)\big) + p_3^+ \big(1 - \sin(\phi_3 + \theta_3)\big)\Big) \\ & \times\ \delta\Big(p_1^+ \big(1 - \sin(\phi_1 + \theta_1)\big) + p_3^+ \big(1 + \sin(\phi_3 + \theta_3)\big)\Big) \end{aligned}$$

Laplacian: $|x_1 - x_2|^4 \frac{\partial}{\partial x_1} \cdot \frac{\partial}{\partial x_1} \frac{\partial}{\partial x_2} \cdot \frac{\partial}{\partial x_2}$

- AdS form ;

$$S_{(2)} = \frac{1}{4} \int d^3x d\theta \sqrt{g}\, H \Big(\Box_{\mathrm{AdS}_3} + (\partial_\theta + i)^2 \Big) \Big(\Box_{\mathrm{AdS}_3} + (\partial_\theta - i)^2 \Big) H$$

A sequence of (physical) : h=s+d-2 (HS states

And : h=s+d (unphysical :higher twist states)

Decoupling of the second branch will be important when evaluating the exchange diagrams.

AdS Space (for the Fishnet Model)

- The 'Fishnet' Mode

$$\mathrm{Tr}\left[\phi(x_1)\phi(x_2)\right]$$

Summing 'fishnet' diagrams and solving for the eigenvalue spectrum gives

$$\lambda(h,s) = \frac{1}{16}(h+s-2)(h+s)(h-s-2)(h-s-4) - \xi^4;$$

.

Which translates into bi-local theory as:

Fishnet Model(ctnd)

- The Collective Laplacian :

$$\widehat{\mathcal{L}}_{\rm bi} = \frac{1}{4}(x_1 - x_2)^2\,\partial_1^2\,\partial_2^2 - \xi^4$$

Kazakov et al, have not given ξ^4 AdS_5 interpretation, but through the bi-local Map we see ;

$$\Big(\Box_{\rm AdS} - m_1^2(s)\Big)\Big(\Box_{\rm AdS} - m_2^2(s)\Big) + \xi^4$$

: a deformation of the 'free' bilocal theory.

The SYK Model/ADS_2

- The Collective Laplacian /

$$\Phi_0^{-1}(x_1, x_3)\Phi_0^{-1}(x_2, x_4) + \lambda\, \delta^d(x_{12})\delta^d(x_{13}$$

Translates into AdS_2

$$S \propto \Psi(t,z) \prod_{m=1}^{\infty} \left(\Box_{\mathrm{AdS}_2} - M_m^2\right) \Phi(t,z)\,,$$

With an infinite sequence of masses:

$$M_m^2 = p_m^2 - \frac{1}{4}\,. \qquad\qquad -\frac{2}{3}\,p_m = \tan\left(\frac{\pi p_m}{2}\right)\,.$$

BI-LOCAL WITTEN DIAGRAMS

- Background expansion

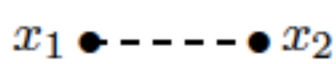
$$S[\eta] = \frac{N}{4} \int \prod_{a=1}^{4} d^d x_a \, \eta(x_1, x_2) \mathcal{K}(x_1, x_2; x_3, x_4) \eta(x_3, x_4)$$

$$- \frac{N}{6} \mathrm{Tr}\Big(\Psi_0^{-1} \star \eta \star \Psi_0^{-1} \star \eta \star \Psi_0^{-1} \star \eta \Big)$$

$$+ \frac{N}{8} \mathrm{Tr}\Big(\Psi_0^{-1} \star \eta \star \Psi_0^{-1} \star \eta \star \Psi_0^{-1} \star \eta \star \Psi_0^{-1} \star \eta \Big) + \cdots$$

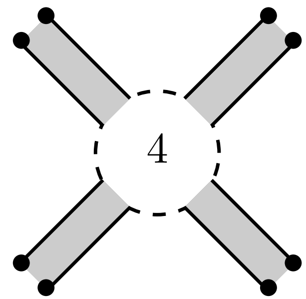
where

$\Phi_0^{-1}(x_1, x_2) \equiv$



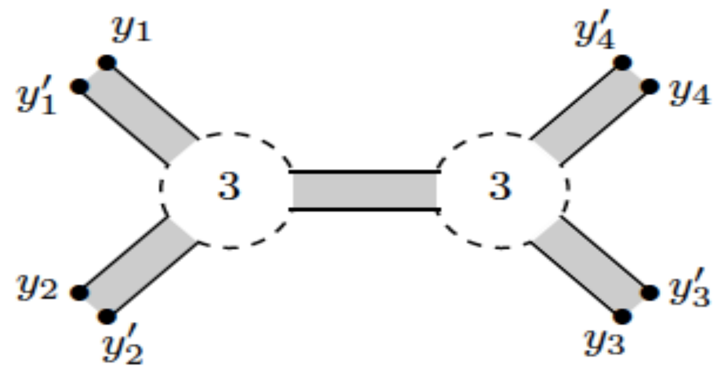
BI-LOCAL DIAGRAMS

- the contact diagram:



- The s -channel exchange diagram:

$$\left\langle \eta'(y_1, y_1')\eta'(y_2, y_2')\eta'(y_3, y_3')\eta'(y_4, y_4') \right\rangle_s =$$



Plus t and u channel diagrams.

Current Correlation Functions

- A 4-pt function of bi-locals:

$$\Big\langle \eta(x_1, x_1')\eta(x_2, x_2')\eta(x_3, x_3')\eta(x_4, x_4') \Big\rangle$$

generates general 4-pt functions of higher spin-currents

$$\Big\langle J_{s_1}(x_1, \hat{z}_1) J_{s_2}(x_2, \hat{z}_2) \cdots J_{s_n}(x_n, \hat{z}_n) \Big\rangle$$

by action of

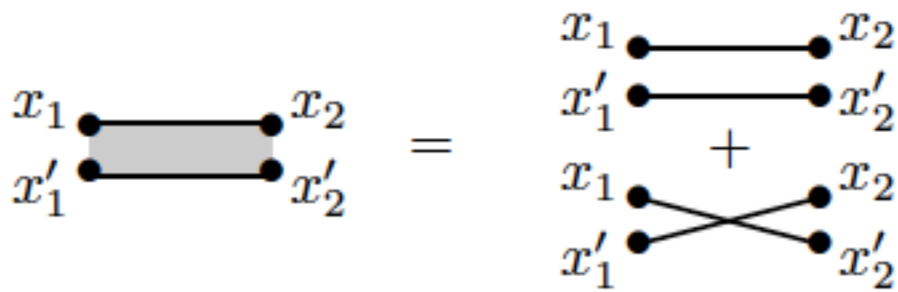
$$\widehat{D}_s(x, x') \equiv \sum_{k=0}^{\infty} \frac{(-1)^k}{k!(s-k)!} (\hat{z} \cdot \partial_x)^{s-k} (\hat{z} \cdot \partial_{x'})^k$$

on each leg/

EVALUATION:two roads

- The bi-local basis

$$D_{(\overline{\Phi})}(x_1, x_1'; x_2, x_2') \equiv$$



- Or the cpw basis

$$\mathcal{D}(x_1, x_2; x_3, x_4) \propto \sum_{h,s,\vec{p}} \frac{\psi^*_{h,s}(x_1, x_2; p)\psi_{h,s}(x_3, x_4; p)}{|x_1 - x_2||x_3 - x_4|\,\lambda_{h,s}}$$

Propagator(ctnd)

- Orthogonality condition of the Bessel function fixes normalization as

$$\int d^2x_1 d^2x_2\, |x_{12}|^{-4} \psi^*_{h,s}(x_1, x_2; p)\psi_{h',s'}(x_1, x_2; p')$$
$$= 16\pi^6\, \delta^2(p - p')\, \frac{\delta(h - h')\delta(s - s')}{\nu\bar{\nu}\sin(\pi\nu)\sin(\pi\bar{\nu})}$$

where

$$\nu \equiv \frac{h + s - 1}{2}, \qquad \bar{\nu} \equiv \frac{h - s - 1}{2}$$

- This bi-local propagator:

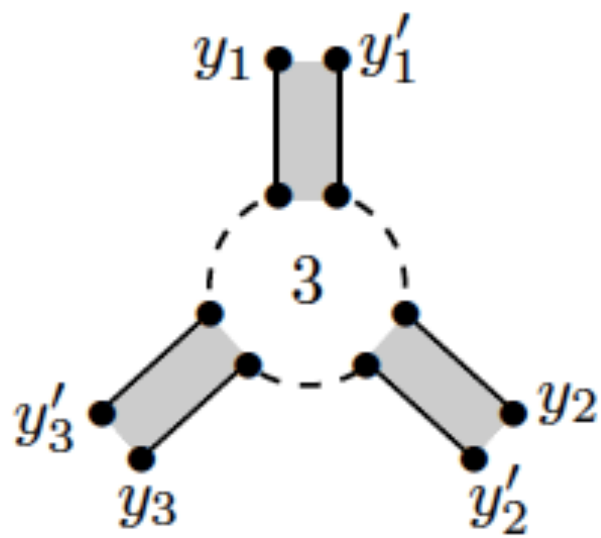
$$\mathcal{D}(x_1, x_2; x_3, x_4) = \sum_{h,s,\vec{p}} \frac{\nu\bar{\nu}\sin(\pi\nu)\sin(\pi\bar{\nu})}{36\pi^6\lambda_{h,s}} \frac{\psi^*_{h,s}(x_1, x_2; p)\psi_{h,s}(x_3, x_4; p)}{|x_1 - x_2|^2|x_3 - x_4|^2}$$

Bi-local Diagrams

- 3-point function

$$\left\langle \overline{\Phi}(y_1, y_1')\overline{\Phi}(y_2, y_2')\overline{\Phi}(y_3, y_3') \right\rangle$$

- $=$



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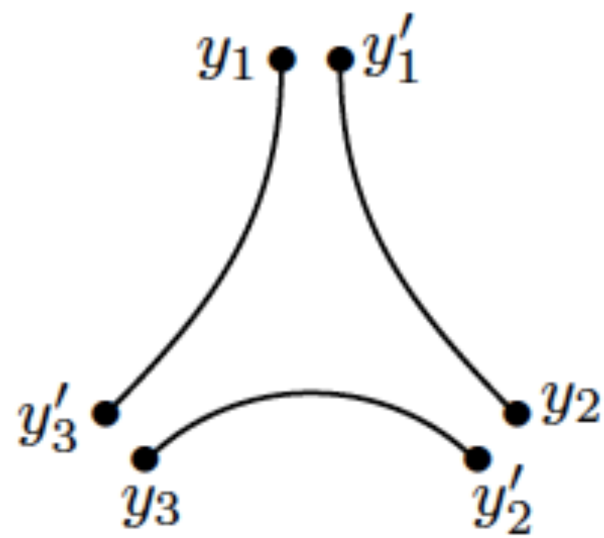
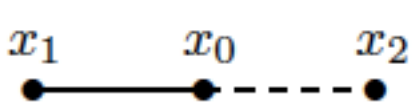


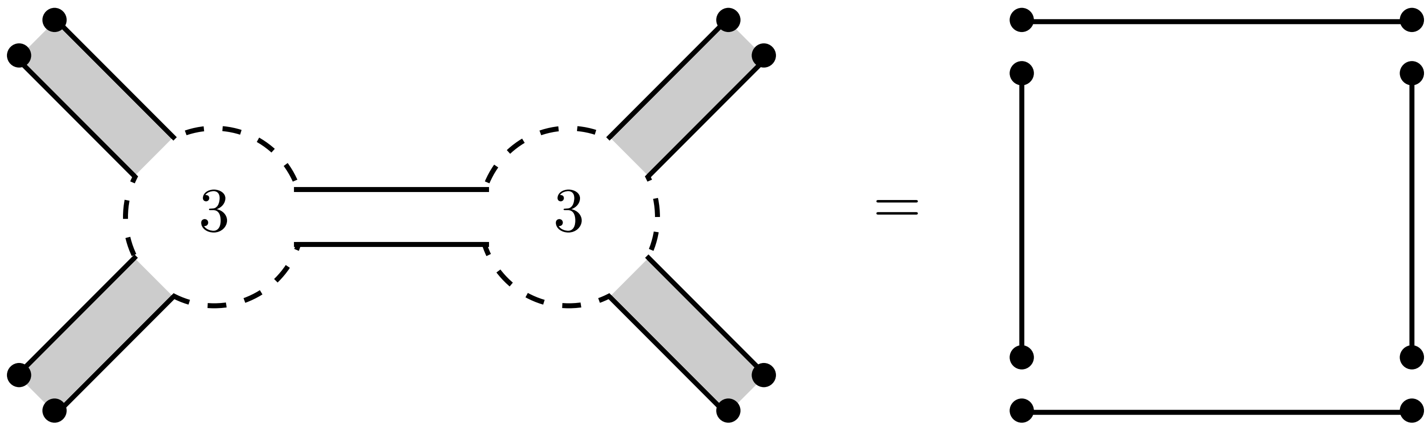
By



$= \delta^d(x_1 - x_2)$

S-channel diagram

- Diagrammatically this implies



Cancelation :at the vertex and each line of the bi-local propagator (free) theory.

S-channel (CPW version)

- S-channel :in terms of cpw's

$$\left\langle \eta(x_1,x_1')\eta(x_2,x_2')\eta(x_3,x_3')\eta(x_4,x_4')\right\rangle_s$$

$$\int_{\frac{d}{2}-i\infty}^{\frac{d}{2}+i\infty}\frac{dh}{2\pi i}\sum_{s=0}^{\infty}\frac{\rho(h,s)}{\lambda_{h,s}}\int d^dy\ A(x_1,x_1';x_2,x_2';y,h,s)A(y,h^*,s;x_3,x_3';x_4,x$$

A is the 3-pt function, expressed as

$$A(x_1,x_1';x_2,x_2';y,h,s) = g_3\,\Phi_0(x_1',x_2)\,\psi_{h,s}(x_1,x_2';y) + (3\text{ permutations of external points})$$

Conformal Block Expansion

- Obtained [Dolan & Osborn] after intermediate integration

$$\int d^2y \; \psi_{h,s}(y_1, y_2; y)\psi_{h^*,s}(y_3, y_4; y)$$

$$= \frac{\pi}{|y_{12}|^{2\Delta}|y_{34}|^{2\Delta}} \frac{\Gamma(1-\frac{h+s}{2})\Gamma(\frac{h+s}{2})}{\Gamma(\frac{h-s}{2})\Gamma(1-\frac{h-s}{2})}$$

$$\times \left[\frac{1}{K_{\frac{h^*+s}{2},\frac{h^*-s}{2}}} \frac{\Gamma^2(\frac{h-s}{2})}{\Gamma^2(\frac{h^*+s}{2})} F_{\frac{h+s}{2},\frac{h-s}{2}}(\eta,\bar{\eta}) + \frac{1}{K_{\frac{h+s}{2},\frac{h-s}{2}}} \frac{\Gamma^2(\frac{h^*-s}{2})}{\Gamma^2(\frac{h+s}{2})} F_{\frac{h^*+s}{2},\frac{h^*-s}{2}}(\eta,\bar{\eta}) \right]$$

with

$$F_{\beta,\bar{\beta}}(x,\bar{x}) \equiv x^{\beta} \, {}_2F_1(\beta,\beta;2\beta;x) \, \bar{x}^{\bar{\beta}} \, {}_2F_1(\bar{\beta},\bar{\beta};2\bar{\beta};\bar{x})$$

expressed in terms of cross-ratios

$$\eta \equiv \frac{y_{12}y_{34}}{y_{13}y_{24}}, \qquad \bar{\eta} \equiv \frac{\bar{y}_{12}\bar{y}_{34}}{\bar{y}_{13}\bar{y}_{24}}$$

S-channel Contribution

- Therefore, the s-channel contribution is given by the sum(integral) over conformal blocks

$$\sum_{s,\nu} \frac{2^{\frac{5}{2}+s}}{(2\pi)^3} \frac{\rho_s(i\nu)}{\lambda_{\Delta,s}} K_{i\nu,s} \widetilde{G}_{\frac{3}{2}+i\nu,s}(u,v)$$

- Here $$\frac{4}{\lambda_{i\nu,s}} = \frac{16}{\left[(i\nu)^2 - (s-\frac{1}{2})^2\right]\left[(i\nu)^2 - (s+\frac{3}{2})^2\right]}$$

- We have poles at the physical

$$i\nu = s - \frac{1}{2}$$

And cancellation of additional poles at(unphysical) values :

Evaluation(s-diagram)

- Cancelation, due to a nontrivial identity:

$$\operatorname*{Res}_{i\nu=s+2n+\frac{5}{2}} \frac{1}{|x_{12}|^2|x_{34}|^2}\frac{16}{N}\sum_{s,\nu}\frac{2^{\frac{5}{2}+s}}{(2\pi)^3}\frac{\rho_s(i\nu)}{\lambda_{\Delta,s}}K_{i\nu,s}\widetilde{G}_{\frac{3}{2}+i\nu,s}(u,v)$$

$$+\operatorname*{Res}_{i\nu=s+\frac{1}{2}} \frac{1}{|x_{12}|^2|x_{34}|^2}\frac{16}{N}\sum_{s,\nu}\frac{2^{\frac{5}{2}+s+2n+2}}{(2\pi)^3}\frac{\rho_{s+2n+2}(i\nu)}{\lambda_{\frac{3}{2}+i\nu,s+2n+2}}K_{i\nu,s}\widetilde{G}_{\frac{3}{2}+i\nu,s+2n+2}(u,v)=0$$

- The only contribution that remains is at $i\nu = s - \frac{1}{2}$

$$\frac{1}{(y_{12}^2 y_{34}^2)^{2\Delta}} \sum_{s\geq 0,\,\text{even}} c_s^2\, G_{s+d-2,s}(u,v)$$

$$c_s^2 = \frac{2^{s+1}}{s!}\,\frac{(\Delta)_s^2}{(2\Delta+s-1)_s}\,.$$

S-channel Diagram

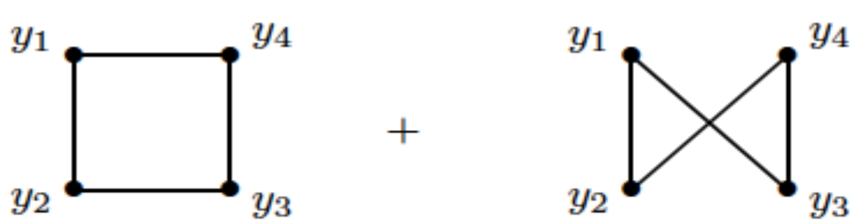
- Therefore, the s-channel diagram in(CPW) basis

$$\frac{1}{(y_{12}^2 y_{34}^2)^{2\Delta}} \sum_{s\geq 0,\,\text{even}} c_s^2\, G_{s+d-2,s}(u,v)$$

$$c_s^2 = \frac{2^{s+1}}{s!} \frac{(\Delta)_s^2}{(2\Delta+s-1)_s}\,.$$

- Equal to

$$\frac{1}{(y_{12}^2 y_{34}^2)^{2\Delta}} \left[\left(\frac{u}{v}\right)^{\Delta} + u^{\Delta} \right] \equiv A + B$$



Comparison

Bekaert, Erdmenger, Ponomarev and Sleight

arXiv:1508.04292

:evaluation of 4-point Witten diagrams (s=0) in AdS

- In the BEPS diagram :both single trace and double-trace participate
- In the BiLocal theory only single traces appear , representing the fields of the theory.

Question of Locality

[Sleight & Taronna '17] [Ponomarev '17] :

No-Go statement :quartic couplings of higher spins theory contain $1/\Box_{\mathrm{AdS}}$ type Non-locality

- In contrast, bi-local theory features

$$V_3 = \prod_{i=1}^{3} \delta_i(\cdots)\,, \qquad V_4 = \prod_{i=1}^{4} \delta_i(\cdots)$$

- Stuckelberg fields/

Outlook

- BiLocal Collective Theory : Field Theory of Higher Spins
- Loops : dynamical mass generation
- Classical , non-perturbative solutions(of HS gravity) (wormholes in SYK)

* THANK YOU !