Holographic relations at finite radius

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Marika Taylor Holographic relations at finite radius

• The original example of holography in string theory is the famous AdS/CFT conjecture of Maldacena:

- String theory on a background with (d + 1)-dimensional Anti-de Sitter asymptotics is dual to a d-dimensional conformal field theory.

 Many examples of gauge/gravity dualities involving various spacetime asymptotics.





- Original argument for holography: maximum entropy associated with a given spacetime volume scales as the surface area in Planck units.
- Follows from black holes being the most entropic objects for a given mass.
- No dependence on asymptotics!



- Consider a timelike
 hypersurface Σ_c, in a spacetime with generic asymptotics.
- Can we define a QFT on Σ_c , holographically dual to the interior of the spacetime?



- M.T. "TT deformations in general dimensions", 1805.10287.
- Old work: Compère, McFadden, Skenderis and M.T., 2011-2012.
- Related work: Hartman, Kruthoff, Shaghoulian and Tajdini, 1807.11401



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- **Top-down** models postulate a complete relationship between string theory in a given background and a specific QFT e.g. $AdS_5 \times S^5$ and $\mathcal{N} = 4$ SYM.
- In **bottom-up** models, we instead engineer the gravity theory to capture defining features of the QFT.



- Consider an RG flow to a UV fixed point, driven by a single operator O.
- The minimal ingredients required to describe this holographically are:

$$S = \int d^{d+1}x \sqrt{-g} \left(R - \frac{1}{2} (\partial \phi)^2 + V(\phi) \right)$$

where ϕ is the bulk scalar dual to O and the potential is such that the action admits AdS_{d+1} extrema.



Holographic dictionary

 More precisely, one can extract from asymptotic expansions near the conformal boundary ρ = 0:

$$ds^2 = \frac{d\rho^2}{\rho^2} + \frac{1}{\rho^2} \left(g_{(0)ij} + \rho^2 g_{(2)ij} \cdots + \rho^d g_{(d)ij} \cdots\right) dx^i dx^j$$

and

$$\phi = \rho^{d-\Delta}(\phi_{(d-\Delta)} + \cdots) + \rho^{\Delta}(\phi_{(\Delta)} + \cdots)$$

the dilatation Ward identity for $\langle T_{ij} \rangle \sim g_{(d)ij}$ and $\langle O \rangle \sim \phi_{(\Delta)}$ $\langle T_i^i \rangle + \phi_{(d-\Delta)} \langle O \rangle \sim 0$



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Use radial foliation near the conformal boundary

$$ds^2 = dr^2 + \gamma_{ij}(r, x) dx^i dx^j$$

where for AAdS $\gamma_{ij}(r, x) \sim e^{2r} g_{(0)ij} + \cdots$ as $r \to \infty$.

 The conjugate momentum to γ is the Brown-York quasi-local stress tensor

$$\mathcal{T}_{ij} = (\mathbf{K}_{ij} - \mathbf{K}\gamma_{ij})$$

where the extrinsic curvature $K_{ij} = \frac{1}{2} \partial_r \gamma_{ij}$.



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Holographic renormalization

- \mathcal{T}_{ij} is not finite as $r \to \infty$.
- Boundary counterterms added to the Einstein-Hilbert action

$$S_{\mathrm{ct}} = -\int d^d x \sqrt{-h} \left((d-1) + \cdots \right)$$

render the onshell action finite and give additional contributions to the quasi-local stress tensor:

$$T_{ij} = (K_{ij} - K\gamma_{ij} + (d-1)\gamma_{ij} + \cdots)$$

(Balasubramanian and Kraus; de Haro, Skenderis and Solodukhin)



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• T_{ij} does have a finite limit as $r \to \infty$:

$$\mathcal{L}_{r \to \infty} \left(T_{ij}
ight) = \langle T_{ij}
angle \sim g_{(d)ij}.$$

 The renormalized stress tensor satisfies the expected CFT identities e.g. for d = 2

$$\langle T_i^i
angle = rac{c}{6} \mathcal{R}(g_{(0)})$$



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Finite radius hypersurface

- Natural to ask about duality for finite radius hypersurface.
- From QFT perspective: radial evolution is RG flow.
- In presence of horizons, one obtains a fluid/gravity relation.



(Minwalla et al; Polchinski et al; Strominger et al; Compère, McFadden, Skenderis and Taylor;) **STAG**

- In the radial Hamiltonian decomposition, one can write the Einstein equations in Gauss-Codazzi form.
- In particular, for AdS gravity

$$K^2 - K^{ij}K_{ij} = \mathcal{R}(\gamma) + d(d-1)$$

which implies that, for flat hypersurfaces at finite radius,

$$T_i^i = -4\pi G\left(T_{ij}T^{ij} - \frac{1}{(d-1)}(T_i^i)^2\right)$$



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• We view this relation as a dilatation Ward identity:

$$T_i^i = -\lambda \mathcal{T}$$

where

$$\mathcal{T} = \left(T_{ij}T^{ij} - \frac{1}{(d-1)}(T_i^j)^2\right)$$

• In d = 2, T is the $T\overline{T}$ operator explored by Zamoldchikov.

 Holographic relation in d = 2 proposed by (McGough et al; Kraus et al).



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$T\bar{T}$ operator in 2d

Zamoldchikov showed that this operator has a remarkable
 OPE structure as x → y:

$$T\overline{T}(x,y) = T(y) + \sum_{\alpha} A_{\alpha}(x-y) \nabla_{y} \mathcal{O}_{\alpha}(x)$$

i.e. we can identify the operator as local, modulo derivatives of other local operators.

 Smirnov and Zamoldchikov also explored the behaviour of a CFT under deformations by T i.e.

$$S_{\rm CFT}
ightarrow S_{
m CFT} + \lambda \int d^2 x \ {\cal T}.$$

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- Consider the (Euclidean) theory on a cylinder of radius *R*.
- In a stationary state such that

$$\langle T_{ au au}
angle = -rac{E}{R}$$

the defining relation for the family of QFTs implies that

$$\frac{\partial E}{\partial \lambda} + 2E \frac{\partial E}{\partial R} = 0$$



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• This can be re-expressed in terms of dimensionless quantities (ϵ, α) using

$$\alpha = \frac{\lambda}{R^2} \qquad E = \frac{1}{R}\epsilon$$

with

$$\partial_{\alpha}\epsilon = \mathbf{2}\epsilon \left(\epsilon + \mathbf{2}\alpha\partial_{\alpha}\epsilon\right)$$

• This is the defining ODE for the **energy spectrum** $\epsilon(\alpha)$.



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In general dimensions:

$$\mathcal{T} = \left(T_{ij}T^{ij} - \frac{1}{(d-1)}(T_i^j)^2\right)$$

- Definite of composite operator more subtle; renormalization required as operators approach each other.
- Details of operator definition not required for energy spectrum, but would be needed for correlation functions, entanglement entropy etc.



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Energy spectrum

• Consider the (Euclidean) theory

$$S_{\text{CFT}} o S_{\text{CFT}} + \lambda \int d^{D+1}x \ \mathcal{T}.$$

on a cylinder of spatial volume R^{D} . With

$$\alpha = \frac{\lambda}{R^d} \qquad E = \frac{1}{R}\epsilon$$

dimensionless energy $\epsilon(\alpha)$ satisfies

$$\partial_{\alpha}\epsilon = \left(1 + \frac{1}{D}\right)\left(\epsilon + 2\alpha\epsilon\partial_{\alpha}\epsilon\right)$$

with $\epsilon(0)$ the CFT energy.

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• The conjectured holographic theory dual for finite radius is

$$S_{\text{CFT}} o S_{\text{CFT}} + \lambda \int d^{D+1}x \ \mathcal{T}.$$

- Identifying the quasi-local stress tensor as the dual stress tensor,
 Ward identity matches by construction.
- Can we also reproduce energy spectrum in gravity?





Consider a static black brane in (D+2) dimensions

$$ds^{2} = (\rho^{2} - \frac{\mu}{\rho^{D-1}})d\tau^{2} + \frac{d\rho^{2}}{(\rho^{2} - \frac{\mu}{\rho^{D-1}})} + \rho^{2}dx^{a}dx_{a}$$

We can then read off from the quasi local stress tensor the dimensionless energy:

$$\epsilon = \frac{D\rho^d}{2\lambda} \left(1 - \left(1 - \frac{\lambda M}{\rho^d} \right)^{\frac{1}{2}} \right)$$

where $\mu = 4\pi GM$.

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Black brane solutions

• In terms of dimensionless coupling $\alpha = \lambda / \rho^d$,

$$\epsilon = \frac{D}{2\alpha} \left(1 - (1 - \alpha M)^{\frac{1}{2}} \right)$$

Note that the CFT energy is

$$\epsilon(\mathbf{0}) = \frac{D}{4}M$$

and $\epsilon(\alpha)$ indeed satisfies:

$$\partial_{\alpha}\epsilon = \left(1 + \frac{1}{D}\right)(\epsilon + 2\alpha\epsilon\partial_{\alpha}\epsilon)$$

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- Trivial to generalize to boosted (spinning) branes.
- Addition of extra **bulk fields** (gauge fields, scalars etc) modifies CFT deformation e.g.

$$T_i^i = -\lambda \left(T^{ij} T_{ij} - \frac{1}{D} (T_i^i)^2 + 2 \mathcal{J}^i \mathcal{J}_i \right)$$

Also noticed in d = 2 by (Bzowski and Guica; Kraus et al).



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Conclusions and outlook

 The conjectured holographic theory dual for finite radius AdS is

$$S_{\rm CFT} o S_{
m CFT} + \lambda \int d^{D+1}x \ {\cal T}.$$

with

$$\mathcal{T} = \left(T^{ij}T_{ij} - \frac{1}{D}(T^{i}_{i})^{2}\right)$$

 Natural generalization of d = 2 proposal.



- Passes preliminary checks: Ward identity, energy relations.
- More detailed checks require renormalized definition of composite operator T.
- Proposal can easily be extended beyond AdS asymptotics (but UV behaviour is required to fix integration constants).



Gauge/Gravity Duality 2018: Summary and Outlook

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Summary



- Nunez: "The talks fall into two categories: using gravity to learn about strongly interacting theories and using our knowledge of quantum field theory to explore string theory/quantum gravity."
- 21 years after Maldacena's paper, a continual stream of new implications and applications.



- Goal since 1997: classify all AdS/CFT dualities visible in supergravity and find exact supergravity solutions for renormalization group flows.
- We still have very few exact solutions for RG flows exhibiting features such as confinement -Klebanov-Strassler is over used!



- Construction of supergravity solutions dual to 5d SCFTs (Gutperle).
- Explicit type II solutions describing 4d $\mathcal{N} = 1^*$ flows (Skenderis).
- Geometries obtained from (non-Abelian) T dualities, using knowledge of dual CFTs (Nastase, Nunez).
- Susy Q and boomerang RG flows (Gauntlett).



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Exploits G structures and generalized geometry techniques (see also double field theory, Park).

- Reconstruction of other susy RG flows?
- Extract information from new geometries: spectra, correlation functions etc.
- Numerical relativity techniques for constructing holographic geometries? (Ren)

Huge amount of unexploited information in 10d geometries -Kaluza-Klein holography! (see Porfyriadis)



Integrability techniques established $\mathcal{N} = 4$ SYM duality at planar level.

- Beyond conformality: integrability for defect conformal field theories (Kristjansen).
- Hints of unexploited integrable structure/exact solvability. (Bekaert, Ishii, Kraus)



- Initially, particle physics viewpoint: holographic descriptions of CFTs and relativistic RG flows.
- From 2008 AdS/CMT: why stick to relativistic theories?!



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- Non-relativistic gravity/field theory dualities (Obers)
- Modelling of momentum dissipation (Gauntlett and AdS/CMT talks)
- Boundary/defect CFT (Herzog, Kristjansen, O'Bannon,...)
- P-adic theories (Gubser)



Embedding of new dualities in holography



- Embedding of 3d Chern-Simons theories, using probe branes etc (Karch).
- Much to compute, compiling evidence for 3d CS dualities!



String landscape and swampland



- Ooguri and Vafa: Are many effective field theories doomed to be in the swampland?
- Holography: a huge landscape of (consistent) dualities, looking for new applications!



- Particle physics: hadrons, quark-gluon plasma
- Condensed matter physics
- Fluids: transport and turbulence
- New perspectives on quantum information



Particle physics: mesons



Phenomenological holographic models (Evans); necessity of including string effects (Sonnenschein).

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Other aspects of QCD



Deep inelastic scattering and pomerons (Costa); phases of QCD (Ewerz, Henriksson, Jarvinen) STAG

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Phases of matter



Holographic models for superconductors, intertwined order, cuprates, bad metals and semimetals, Mott insulators, charge density waves....(Amoretti, Arean, Cremonini, Krikun, Schalm, Sin, Zaanen, Zingg,..)

Quantum chaos



- New insights into quantum chaos from SYK. (Altland, Rozali)
- Hydrodynamic origin of quantum chaos in holographic theories. (Davison)

Note that SYK is not dual to an Einstein gravity theory: how are maximal chaos and gravity description entwined?



- String theory provides a landscape of strongly interacting phases.
- What features would be shared by small *N* (non-susy) systems?
- Can we sharpen intuitive arguments?



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Fluid/gravity relations



- New insights into thermalization, hydrodynamics, turbulence etc from holography. (Buchel, Craps, Ishii, Kaminski, Landsteiner, Sonner)
- Physical implications of anomalies in e.g. Weyl semi-metals. (Landsteiner et al)
- Elegant new formulations of hydrodynamics and fluid action principles.



Quantum information

Holographic entanglement entropy



Complexity (action of WdW patch)



(Holographic) entanglement entropy



- Generalise shape of entangling region and apply to new theories (Leigh, Tonni)
- Spacetime reconstruction and properties of HEE (Galli, Pedraza, Romero-Bermudez, Rota, Sarkar)
- Exploit entanglement to understand M5-brane/self dual strings. (O'Bannon)

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- One of the most popular themes of the conference.
- Defining complexity. (Heller, Jefferson, K-Y. Kim)
- Exploring (holographic) complexity. (Ageev, Bagrov, Flory, Roy, Thorlacius)



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Ambiguities? Derivation from defining relation of holography?



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What is complexity in a QFT?



(K-Y Kim)

- Complexity is defined in QI textbooks for finite numbers of qubits.
- Challenging to define complexity in an interacting quantum field theory (Heller, Jefferson, K-Y Kim).
- How are reference state and basis gates encoded in bulk?

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Emergence of (quantum) gravity from quantum field theory; explaining properties of black holes; singularity resolution.



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Spacetime reconstruction



SPACETIME RECONSTRUCTION

Reconstruction using entanglement.



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Explicit reconstruction of near boundary metric from QFT data (one point function of stress tensor etc).



- Generalisations of Fefferman-Graham spacetime reconstructions. (Korovin, M.T.)
- SYK suggests relevance of **radon transforms** (Das, Jevicki), which arose previously (de Boer et al, Ooguri et al.)
- How are approaches connected?



Higher spin theories

- Holography beyond Einstein gravity: tensionless string limit.
- Three-dimensional higher spin theories are an educational playground, capturing aspects of black holes (Castro).
- Many technical challenges, particularly for d > 3: characters (Bekaert); interactions and correlation functions (Mkrtchyan, Ponomarev, Skvortsov) and loops (Basile).

Long-standing issue of non-trivial solutions (black holes etc) in $d \ge 4$.



Ubiquitous in thermalisation, hydrodynamics, holographic phases.



- Black hole microstate counting. (Larsen)
- Low-dimensional black holes as controllable models (Castro, Thorlacius)
- Constructions of new explicit solutions. (e.g. Sybsema)

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- What does holography tell us about the experience of an observer falling into a black hole?
- How is information recovered from the black hole?
- Is soft hair (Semenoff) relevant to black hole physics?



Conclusions



- Many exciting directions of research for the next edition of Gauge/Gravity Duality!
- Thanks to Johanna, René and their group for hosting us!



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