Quantum Higher Spin Gravity

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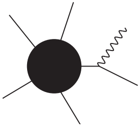
Main Messages

- Higher spin gravities (HSGRA), hypothetical theories with graviton and massless higher spin fields, have been studied for many years, but until recently there has not been a single example worked out in great detail (action, quantization, ...)
- There are many no-go theorems in flat space and some of them admit a straightforward extensions to AdS/CFT, where it is tempting to interpret them as $\frac{1}{\sqrt{2}}(|\text{yes-go}\rangle + |\text{no-go}\rangle)$, $AdS \sim$ flat
- We construct an example of a complete HSGRA, quantize it and discuss how it complies with the no-go's (Ponomarev, E.S.; E.S., Tung Tran, Mirian Tsulaia; see also talk by Ponomarev)
- In AdS this HSGRA is dual to a limit of Chern-Simons matter theories ($3d$ bosonization duality) — a complete model of AdS/CFT duality

Flat Space: HSGRA cannot exist

It has been long known that massless particles with $s > 2$ are somewhat special (do not want to exist). One of most powerful no-go theorems against HSGRA is the Weinberg low energy theorem:



- $s = 1$ we get charge conservation $\sum q_i$=0
- $s = 2$ we get equivalence principle $\sum g_i\, p^i_\mu$=0
- $s > 2$ we get too many conservation laws

$$\sum_i g_i\, p^i_{\mu_1} ... p^i_{\mu_{s-1}} = 0$$

May be massless higher spin fields confine? or do not exist?

Flat Space: HSGRA cannot exist

Coleman-Mandula theorem constrains the symmetries of nontrivial S-matrix to be a direct product of Poincare and inner symmetries.

$$\text{argument :} \qquad Q_{\mu_1\ldots\mu_{s-1}} \sim \sum_i p^i_{\mu_1}\ldots p^i_{\mu_{s-1}} \sim 0$$

so that we again get too many conservation laws. Exceptions: SUSY and 2d.

Deser and Aragone: If we use Fronsdal fields

$$\delta\Phi_{\mu_1\ldots\mu_s} = \partial_{\mu_1}\xi_{\mu_2\ldots\mu_s} + \text{permutations}$$

then the standard spell $\partial \to \nabla$ in the two-derivative $\int(\partial\Phi)^2$-type action does not work: $[\nabla,\nabla]$ will bring the four-index Riemann tensor.

This is avoided by low spins, $s=0,\frac{1}{2},1$, and results only in the Ricci-part for $s=\frac{3}{2},2$.

Flat Space: HSGRA cannot exist

As a summary we can use the quote from a textbook "Quantum Field Theory and the Standard Model" by Matthew D. Schwartz

8.7.3 Spin greater than 2

One can continue this procedure for integer spin greater than 2. There exist spin-3 particles in nature, for example the ω_3 with mass of 1670 MeV, as well as spin 4, spin 5, etc. These particles are all massive. One can construct free Lagrangians for them using the same trick. An interesting and profound result is that it is impossible to have an *interacting* theory of massless particles with spin greater than 2. The required gauge invariance would be so restrictive that nothing could satisfy it. We will prove this in the next chapter. Constructing the kinetic term for a spin-3 particle is done in Problem 8.8.

Flat Space: HSGRA may exist

The no-go theorems constrain the physics at infinity by saying that $S = 1$ once at least one massless higher spin particle is present

However, they have little to say about possible local interactions

Long ago some local cubic interactions were found by Brink, Bengtsson^2, Linden using the light-cone approach. **How these local effects comply with the global restrictions?**

Let's now move to AdS HSGRA and see what is the difference

The most basic higher-spin AdS/CFT duality conjecture Klebanov, Polyakov; Sezgin, Sundell; Leigh, Petkou says that

- free vector model (fancy name for free scalars) should be dual to a higher-spin theory whose spectrum contains totally-symmetric massless fields
- critical vector model (Wilson-Fisher) should be dual to the same theory for $\Delta = 2$ boundary conditions on $\Phi(x)$. **This duality is kinematically related to the first one** (Hartman, Rastelli; Giombi, Yin; Bekaert, Joung, Mourad).

$$
\begin{array}{lcl}
J_{a_1 \ldots a_s} = \phi \partial_{a_1} \ldots \partial_{a_s} \phi & \leftrightarrow & \delta\Phi_{\mu_1 \ldots \mu_s}(x) = \nabla_{\mu_1} \xi_{\mu_2 \ldots \mu_s} \\
\langle J \ldots J \rangle \neq 0 & \leftrightarrow & \text{interactions}
\end{array}
$$

HSGRA in AdS

HS Current Conservation implies Free CFT, i.e. given a CFT with stress-tensor J_2 and at least one higher-spin current J_s, one can prove Maldacena, Zhiboedov; Boulanger, Ponomarev, E.S., Taronna; Alba, Diab, Stanev that

- there are infinitely many higher-spin currents and spin is unbounded;
- correlation function corresponds to free CFT (which CFT, depends on the spectrum)

This essentially proves the duality no matter how the bulk theory is realized. Loops still need to be shown to vanish (be proportional to the tree result)

This is a generalization of the Coleman-Mandula theorem to AdS/CFT: higher spin symmetries imply free CFT, i.e. $S = 1$. See also $S = 1$ in O(N)/HS duality, (de Mello Koch, Jevicki, Jin, Rodrigues, Ye)

AdS versus Flat

With a 50 years delay we see that asymptotic higher spin symmetries

$$\delta\Phi_{\mu_1\ldots\mu_s}(x) = \nabla_{\mu_1}\xi_{\mu_2\ldots\mu_s}$$

always completely fix (holographic) S-matrix to be

$$\text{HSGRA S-matrix} = \begin{cases} 1, & \text{flat space} \\ \text{free CFT}, & \text{asymptotic AdS} \\ ???, & \text{some other space} \end{cases}$$

There is not much difference between flat and AdS space: **S-matrix is already known and can be used to reconstruct the theory**, the theories should exhibit some sort of non-locality starting from the quartic order (Bekaert, Erdmenger, Ponomarev, Sleight; Taronna, Sleight; Ponomarev; Roiban, Tseytlin; Ponomarev, E.S.).

Let's move back to flat space since AdS complicates things without bringing anything significantly new as far as problems of constructing HSGRA are concerned. Once they are solved we will be back to AdS.

Unless one gives S-matrix right away, the light-cone approach seems to be the most fundamental approach to local dynamics: no extra assumptions, just study the interactions of a given set of particles, unitarity, but: quantum computations are harder, most of the covariant structures, e.g. diffeomorphisms, get lost.

HSGRA in Flat Space

The idea of the light-cone approach is that QFT is about writing explicitly Poincare generators P^A and J^{AB}

$$
\begin{aligned}
[P^A, P^B] &= 0 \\
[J^{AB}, P^C] &= P^A \eta^{BC} - P^B \eta^{AC} \\
[J^{AB}, J^{CD}] &= J^{AD}\eta^{BC} - J^{BD}\eta^{AC} - J^{AC}\eta^{BD} + J^{BC}\eta^{AD}
\end{aligned}
$$

in terms of fields ($p = (p^-, p^+, p_\perp)$)

$$
H \equiv P^- = \int d^3p \, \Phi_{-p} \, \frac{p_\perp^2}{2p^+} \, \Phi_p + \mathcal{O}(\Phi^3)
$$

HSGRA in Flat Space

Most of the generators stay free and one has to solve for

$$[H, J^{a-}] = 0$$

or perturbatively

$$[H_2, \delta J^{a-}] = [J_2^{a-}, \delta H]$$

which looks like one equation for two functions:

$$\delta J^{a-} \sim \frac{[J_2^{a-}, \delta H]}{\sum_i \frac{(p_\perp^i)^2}{2p^+}}$$

Unless locality is imposed, any δH looks like an ok formal deformation and gives some δJ! All theories can be bootstrapped in this way, in principle, e.g. Yang-Mills, Gravity.

HSGRA in Flat Space

In $4d$ a massless spin-$|\lambda|$ field equals two scalars, $\Phi^{\pm\lambda}$.

Brink, Bengtsson^2, Linden; Metsaev showed that there exists δH:

$$\delta H \sim C^{\lambda_1,\lambda_2,\lambda_3} \int V^{\lambda_1,\lambda_2,\lambda_3} \; \Phi^{\lambda_1}\Phi^{\lambda_2}\Phi^{\lambda_3} + c.c.$$

where the vertex has a clear spinor-helicity interpretation

$$V^{\lambda_1,\lambda_2,\lambda_3} = \frac{\bar{\mathbb{P}}_{12}^{\lambda_1+\lambda_2+\lambda_3}}{\beta_1^{\lambda_1}\beta_2^{\lambda_2}\beta_3^{\lambda_3}} \sim [\mathbf{12}]^{\boldsymbol{\lambda_1+\lambda_2-\lambda_3}}[\mathbf{23}]^{\boldsymbol{\lambda_2+\lambda_3-\lambda_1}}[\mathbf{13}]^{\boldsymbol{\lambda_1+\lambda_3-\lambda_2}}$$

where $\beta \equiv p^+$ and $\mathbb{P}_{12} = p_1\beta_2 - p_2\beta_1$ and similarly for c.c.

Coupling constants $C^{\lambda_1,\lambda_2,\lambda_3}$ and $\bar{C}^{\lambda_1,\lambda_2,\lambda_3}$ are any numbers so far.

HSGRA in Flat Space

Now, $(+s, -s, 2)$ gives a two-derivative coupling to gravity

$$V^{\lambda_1,\lambda_2,\lambda_3} = \frac{\bar{\mathbb{P}}_{12}^{\lambda_1+\lambda_2+\lambda_3}}{\beta_1^{\lambda_1}\beta_2^{\lambda_2}\beta_3^{\lambda_3}} + c.c.$$

so we can avoid the Deser-Aragone argument. Also, there are no higher-spin gauge/global symmetries, so the Coleman-Mandula theorem is avoided.

$C^{\lambda_1,\lambda_2,\lambda_3}$ and $\bar{C}^{\lambda_1,\lambda_2,\lambda_3}$ are any numbers so far.

But the existence of cubic vertices does not yet entail existence of any theory (Example: for YM, cubic vertices exist for any anti-symmetric f_{ijk} and it is the quartic closure of the Poincare algebra that imposes Jacobi identity)

We need to go to the quartic order and higher!

One can rediscover the equivalence principle by trying to couple, say scalar to gravity ($C^{0,0,2} = C^{2,2,-2}$):

$$H_3 = \Phi^2\Phi^2\Phi^{-2}\bar{\mathbb{P}}^2 C^{2,2,-2} + \Phi^0\Phi^0\Phi^2\bar{\mathbb{P}}^2 C^{0,0,2}$$

Analogously, one can see that the equivalence principle extends to all spins

$$s - s - 2: \qquad\qquad C^{s,-s,2} = C^{2,2,-2} = g\, l_{pl}$$

It was shown by Metsaev that the necessary condition for the quartic closure is

$$C^{\lambda_1,\lambda_2,\lambda_3} = \frac{g(l_{pl})^{\lambda_1+\lambda_2+\lambda_3}}{\Gamma[\lambda_1+\lambda_2+\lambda_3]}$$

and the same for $\bar{C}$ if we want a parity even theory.

Chiral Higher Spin Gravity

Complete chiral HSGRA is obtained by setting $\bar{C} = 0$ (Ponomarev, E.S.):

$$S = \sum_{\lambda} \int \Phi^{-\lambda}\, p^2\, \Phi^{\lambda} + \sum_{\lambda_i} C^{\lambda_1,\lambda_2,\lambda_3} \int V^{\lambda_1,\lambda_2,\lambda_3} \Phi^{\lambda_1} \Phi^{\lambda_2} \Phi^{\lambda_3}$$

where the couplings discriminate negative helicities

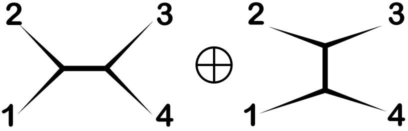
$$C^{\lambda_1,\lambda_2,\lambda_3} = \frac{g(l_{pl})^{\lambda_1+\lambda_2+\lambda_3}}{\Gamma[\lambda_1+\lambda_2+\lambda_3]}$$

One can also add color (Metsaev) leading to higher-spin glue. Interestingly, color should be added in the Chan-Paton way. The theory is nontrivial and contains parts of YM and EH actions.

Once we have a complete theory, it is interesting to quantize gravity and see how it complies with the no-go's

First, let's have a look at trees. Using higher-spin glue allows us to look at color-ordered amplitudes only.

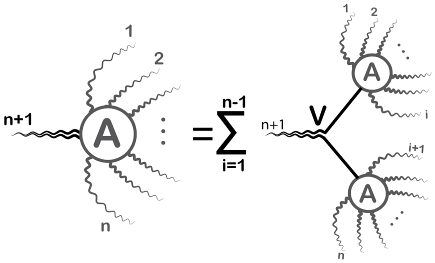
The four-point amplitude



$$\sim \frac{(\bar{\mathbb{P}}_{12}+\bar{\mathbb{P}}_{34})^{\Lambda_4-2}\beta_2\,\boldsymbol{p}_4^2}{\Gamma(\Lambda_4-1)\beta_4\mathbb{P}_{12}\mathbb{P}_{23}}$$

vanishes on-shell

Now we can apply a useful identity (Berends, Giele)



which gives

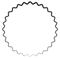
$$A_n \sim \frac{1}{\Gamma(\Lambda_n-(n-3))\prod_{i=1}^n \beta_i^{\lambda_i-1}} \frac{\alpha_n^{\Lambda_n-(n-2)}\beta_2 ... \beta_{n-2}\,\boldsymbol{p}_n^2}{\beta_n \mathbb{P}_{12} ... \mathbb{P}_{n-2,n-1}}$$

$\alpha_n = \sum_{i<j}^{n-2} \bar{\mathbb{P}}_{ij} + \bar{\mathbb{P}}_{n-1,n}$

At least at the tree-level we do not see any signs of higher spin interactions in S-matrix (at infinity) due to the coupling conspiracy. This is in agreement with the no-go's

Chiral Higher Spin Gravity

The simplest loop corrections are vacuum diagrams:

:
$$Z_{\text{1-loop}} = \frac{1}{(|-\partial^2|_0)^{1/2}} \prod_{s>0} \frac{(|-\partial^2|_{s-1})^{1/2}}{(|-\partial^2|_s)^{1/2}}\,,$$

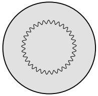
and should count the total number of degrees of freedom $Z_{\text{1-loop}} = (z_0)^{\nu_0/2}$. It was argued (Tseytlin, Beccaria) that it should be understood as

$$\nu_0 = \sum_\lambda 1 = 1 + 2\sum_{s=1}^{\infty} 1 = 1 + 2\zeta(0) = 0\,,$$

Much more nontrivial examples of one-loop det's in AdS show that the above prescription is correct.

AdS Detour

Vacuum bubbles in AdS HSGRA contain a lot of non-trivial information:



$$F^1 \sim \sum_s \log\det[\Box + m_s^2]$$

Those are related to F, c, a theorems, Casimir energy etc.

Can be computed via ζ-function! Giombi, Klebanov, Safdi, Tseytlin, Beccaria, Joung, Lal, Bekaert, Basile, Boulanger, Gunaydin, E.S, Tung, ...

$$F = -\zeta(0)\log\Lambda l - \frac{1}{2}\zeta'(0)$$

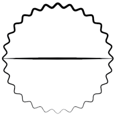
For example,

$$\zeta(0) = \frac{1}{360} + \sum_s \left(\frac{1}{180} - \frac{s^2}{24} + \frac{5s^4}{24}\right) = 0 \quad \text{vs.} \quad \sum_s (-)^{2s} d(s) s^p = 0$$

$a_\phi = \frac{1}{90}, -\frac{1}{756}, \ldots$ or $F_\phi^3 = \frac{1}{16}\left(2\log 2 - \frac{3\zeta(3)}{\pi^2}\right)$, which are hard to fake

There is a difference between vacuum one-loop and higher loops.

Higher vacuum loops vanish due to the coupling conspiracy: sum over all helicities must be zero, but in order for a vertex to contribute the sum must be positive. For example,

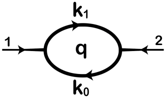


$$= 0$$

since both $(\lambda_1 + \lambda_2 + \lambda_3)$ and $-(\lambda_1 + \lambda_2 + \lambda_3)$ cannot be positive and the coupling contains $1/\Gamma[\lambda_1 + \lambda_2 + \lambda_3]$.

Chiral Higher Spin Gravity

The legged diagrams are supposed to be the most difficult ones. Vanishing of tree amplitudes should improve the behaviour of loop diagrams.



$$= \frac{\nu_0 (l_p)^{\Lambda_2 - 2}}{\Gamma[\Lambda_2 - 1]} \int \frac{d^4 q}{(2\pi)^4} \frac{\bar{\mathbb{P}}^2_{k_0 - q, p} \delta_{\Lambda_2, 2}}{(q - k_0)^2 (q - k_1)^2} \,,$$

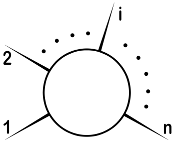
where most importantly we have an overall factor

$$\nu_0 = \sum_{\lambda} 1 = 0$$

which is known to vanish. Anyway, the integral can be regularized and shown to be finite.

Chiral Higher Spin Gravity

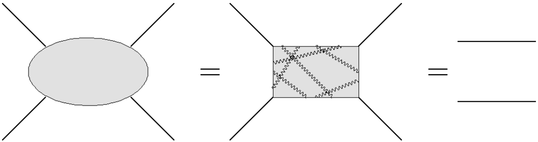
General loop diagram can be decomposed into elementary sunrise diagrams



$$= \frac{\nu_0 (l_p)^{\Lambda_2 - 2}}{\Gamma[\Lambda_2 - 1]} \int \frac{d^4 q}{(2\pi)^4} \frac{f(\bar{\mathbb{P}})}{(q - k_0)^2 (q - k_1)^2} \,,$$

Crucially, they all have an overall factor of $\nu_0 = 0$. The finite leftovers are rational amplitudes, which is similar to SDYM (Ponomarev).

Therefore, all loops vanish! We have **coupling conspiracy**



Flat space summary

- Really many no-go's
- Light-cone allows to avoid all of them in $4d$, at least formally
- Quantum Chiral HSGRA does exist
- The only way out seems to have **coupling conspiracy**: local interactions conspire to get $S = 1$
- Some stringy features are still present in the form of $\sum_\lambda 1 = 0$ and Chan-Paton factors.
- non-chiral HSGRA is unlikely to exist (recent: Roiban, Tseytlin; Taronna; Ponomarev, E.S.) in the usual sense: parity preserving interactions will face non-localities. One could try to achieve $S = 1$ with some sort of non-locality, c.f. AdS/CFT reconstruction.

Chiral HSGRA in AdS

In AdS the dependence of couplings on the spin $1/\Gamma[s_1+s_2+s_3]$ is the same (Bekaert, Erdmenger, Ponomarev, Sleight; E.S.; Sleight, Taronna)

Chern-Simons matter theories — $3d$ CFT's with at least two parameters N and $\lambda = N/k$, which exhibit three-dimensional bosonization duality and many others (Minwalla, Giombi, Yin, Aharony, Witten, Seiberg, Karch, Tong, ...) should have AdS_4 gravitational dual — HSGRA.

The Chiral HSGRA should corresponds to taking t'Hooft coupling $\lambda = \pm i\infty$. In particular the three-point functions should be the limiting case of (Maldacena, Zhiboedov)

$$\langle JJJ\rangle = \frac{\tilde{N}}{1+\tilde{\lambda}^2}\langle JJJ\rangle_{F.B.} + \frac{\tilde{N}\tilde{\lambda}}{1+\tilde{\lambda}^2}\langle JJJ\rangle_{Odd} + \frac{\tilde{N}\tilde{\lambda}^2}{1+\tilde{\lambda}^2}\langle JJJ\rangle_{F.F.}$$

We have a local gravitational dual of a well-defined CFT

Concluding Remarks

- Contrary to what has been previously thought there is not much difference between problems in flat space and AdS for HSGRA: some no-go directly generalize from flat space to AdS (where they are interpreted as yes-no-go, but the S-matrix is fixed by the symmetry)
- At least some of HSGRA seem to exist: chiral (proved!), conformal should have similar properties (Segal; Tseytlin; Bekaert, Joung, Mourad; Joung, Nakach, Tseytlin) and in three dimensions via Chern-Simons. The propagating examples reveal trivial S-matrix in flat space, but not in AdS.
- Chiral HSGRA is a complete toy model that displays some stringy features and shows how gravity can be quantizable thanks to higher spin fields. Its AdS uplift provides a local bulk dual of certain limit of Chern-Simons matter theories.

Thank you for your attention!