Hubbard model and Mott transition in holography

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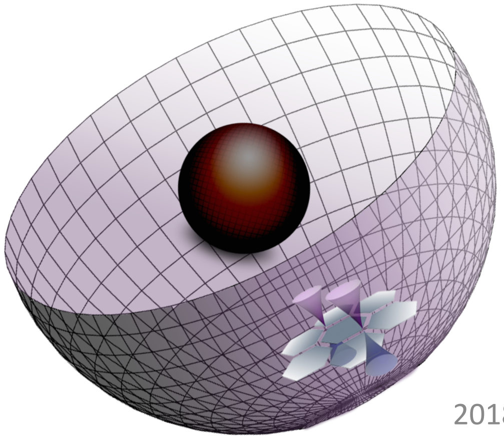
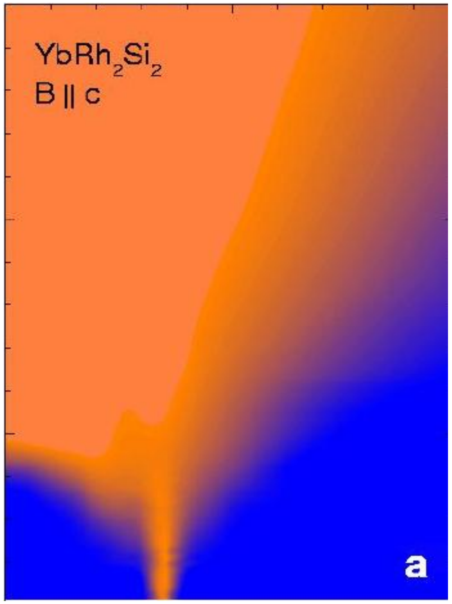
2018.08@Wurzburg

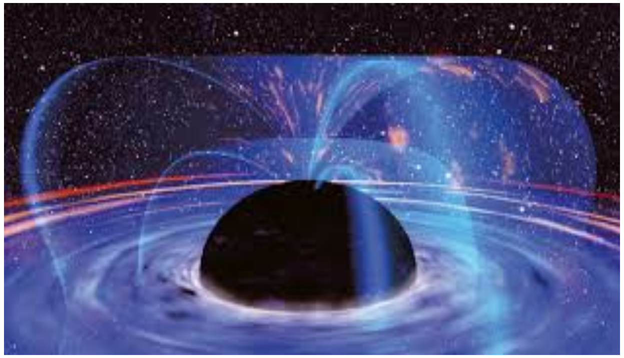


AdS/CMT: Similarity of QCDP and BH



QCP

$\omega = k^z$, $[s] = D - \theta$



Both have

i) Matching symmetry, parameters with Z, θ:

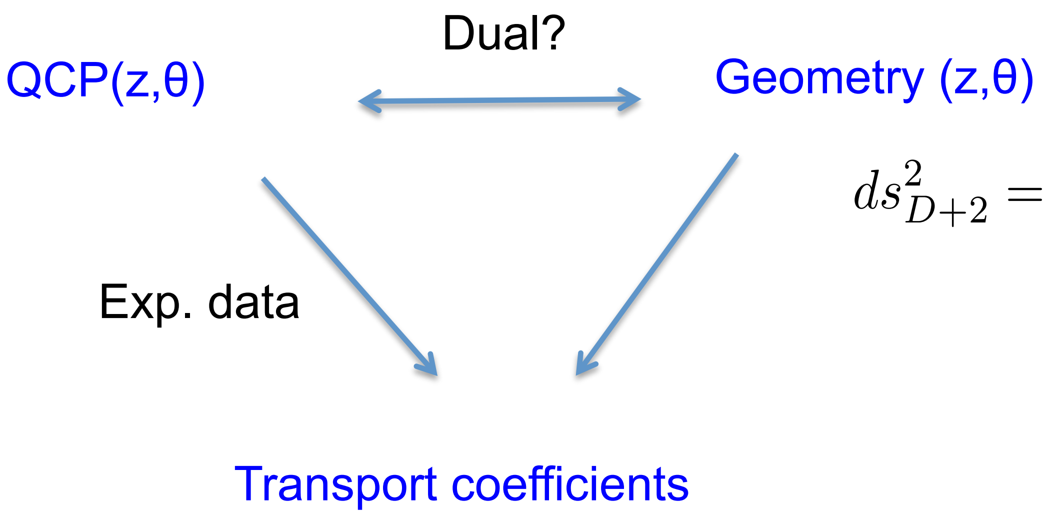
ii) Information loss → Universality

iii) Thermodynamics

iv) Transports

Ads/cmt=postulate the dictionary of ads/cft works here

General idea is to identify QCP and the ads BH.



$$ds^2_{D+2} = r^{\frac{2\theta}{D}}\left(\frac{-dt^2}{r^{2z}} + \frac{dr^2}{r^2} + \frac{1}{r^2}\sum_{i=1}^{D} dx_i^2\right)$$

Therefore, ….

So, we ask

i) How materials becomes Strongly interacting

ii) What materials can actually be described by holography ?

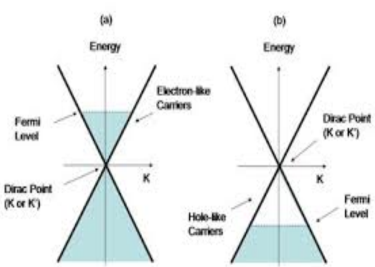
Two classes of SIS

Mechanism/Class : Examples :Phenomena

Slow Electron :Transition-metal Oxides :Mott transition

Small FS : Dirac materials :Anomalous Transport -

$$g^2 = \frac{e^2 \cdot c}{4\pi\epsilon\hbar c \cdot v_F} \sim 1$$

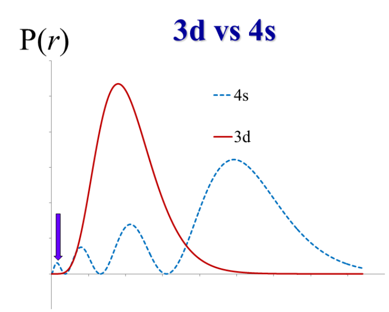


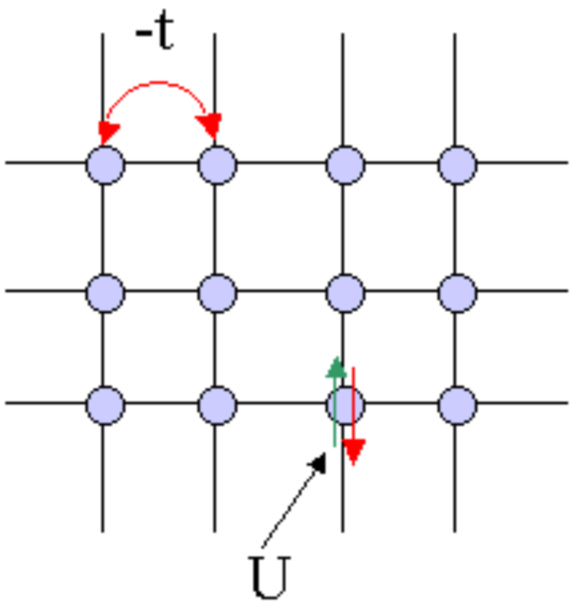
Class 2,

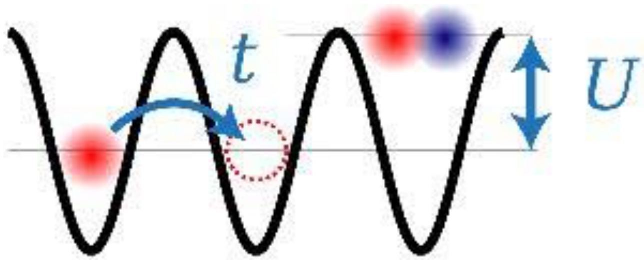
Small FS

Class 1,

$3d^{1-10}\,4s^{1-2}$







Coupling = U/t →two ways

$$H = -t \sum_{\langle i,j\rangle,\sigma} (c^{\dagger}_{i,\sigma} c_{j,\sigma} + c^{\dagger}_{j,\sigma} c_{i,\sigma}) + U \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow},$$

1st class : Transition metal Oxide

1. Mott transition is the first object to understand.

2. Spectral function– ARPES. ➔ Phases of the model

3. Compare with DMFT.

4. Compare with Experiment

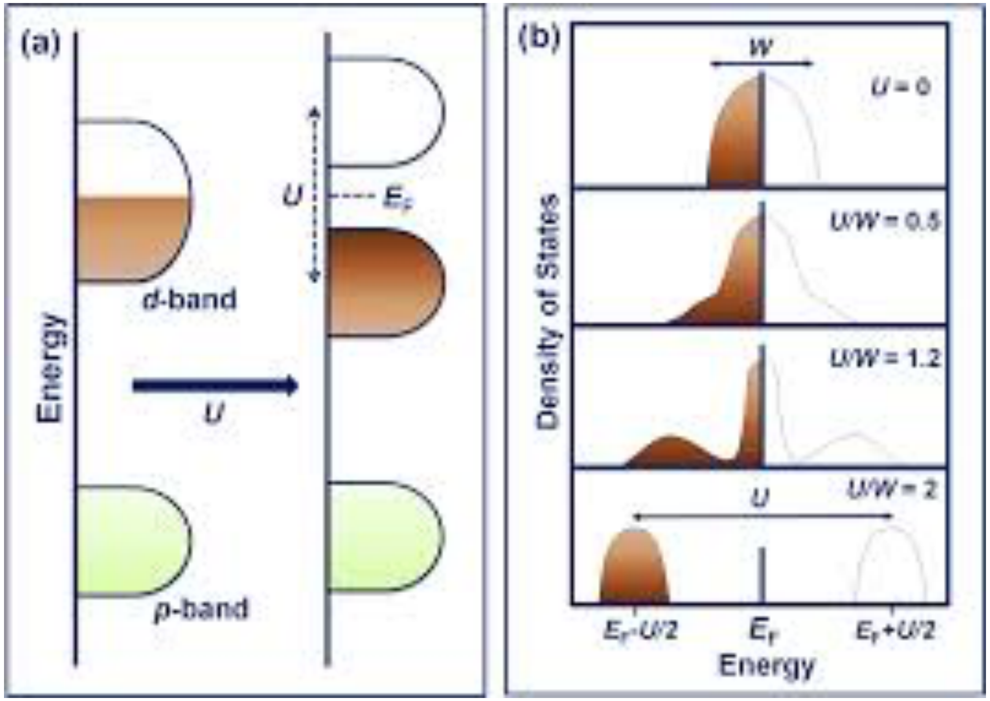
Based on

- **Mott transition with Holographic Spectral function**

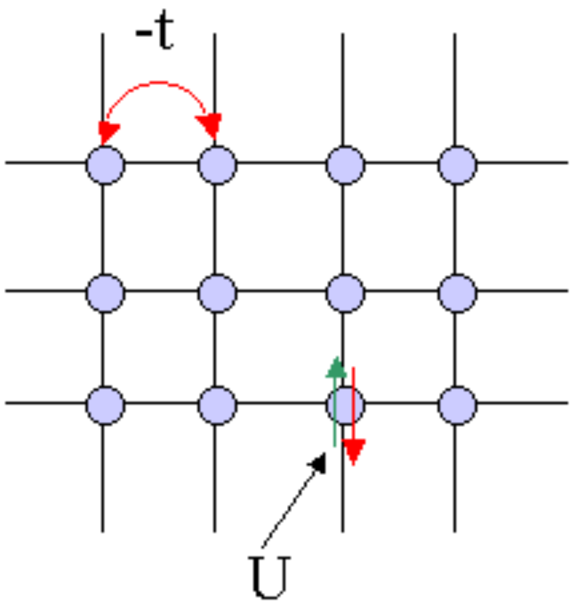
  Yunseok Seo, Geunho Song, Yong-Hui Qi +SJS **1803.01864**

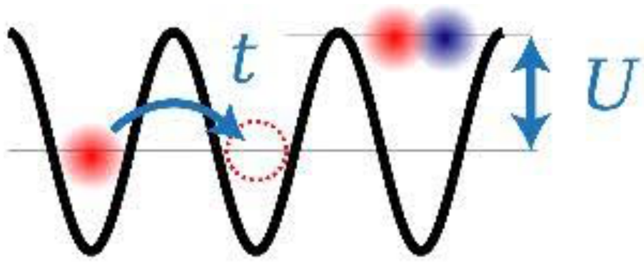
- **Classification of Mott Gap and Pseudo-gap in Holography.**
- **Instability and its information in holography.**

Mott Transition



Mott transition in terms of Hubbard model





Coupling = U/t

$$H = -t \sum_{\langle i,j\rangle,\sigma} (c^{\dagger}_{i,\sigma} c_{j,\sigma} + c^{\dagger}_{j,\sigma} c_{i,\sigma}) + U \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow},$$

Theme

1. Hubbard model in d>1 → Not solvable
2. finding its gravity dual is V. difficult. → title
3. Can we replace the Hubbard Model by a holographic-model?
4. Try a Holographic Fermion Model
   with free fermion like behavior and gap generation.

My original title: Toward holographic Hubbard model .....

Spectral function

$$S_D = \int d^4x\sqrt{-g}i\bar{\psi}\left(\Gamma^M\mathcal{D}_M - m - ip\,\Gamma^{MN}F_{MN}\right)\psi + S_{\rm bd},$$

$$\mathcal{D}_M = \partial_M + \frac{1}{4}\omega_{abM}\Gamma^{ab} - iqA_M.$$

$$S_{\rm bd} = \frac{\pm 1}{2}\int d^3x\sqrt{h}\bar{\psi}\psi = \frac{\pm 1}{2}\int d^3x\sqrt{h}(\bar{\psi}_-\psi_+ + \bar{\psi}_+\psi_-),$$

$h = -gg^{rr}$, $\psi_\pm$ are the spin-up and down

Calculating the spectral function is already standard.

SS. Lee, H. Liu, Iqbal, T. Faulkner, J. Mcgreevy, Vegh, Cubrovic, K. Schalm, J. Zaanen, ……

Phase Diagram

$$S_\psi = \int d^4x \sqrt{-g}\, i\bar{\psi}(\not{D} - m - ip\not{F})\psi + S_{bdy}$$

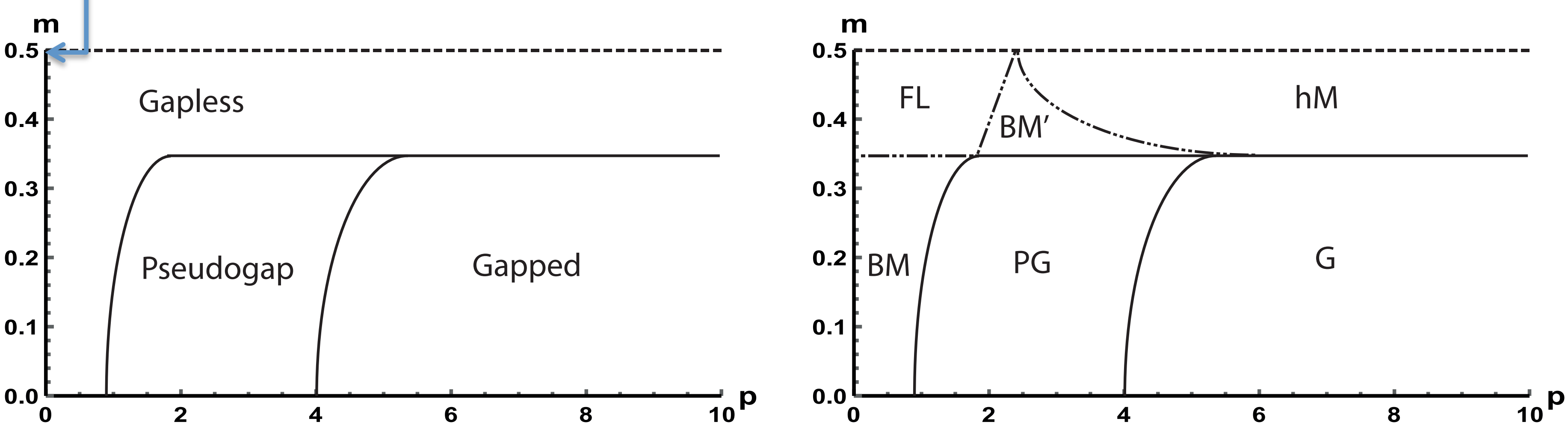
$$\Delta = d/2 - m$$

$$\Delta_{FF} = (d-1)/2$$

m=1/2 is Free fermionic.

Known to Gap generating Phillips et.al

Free fermion

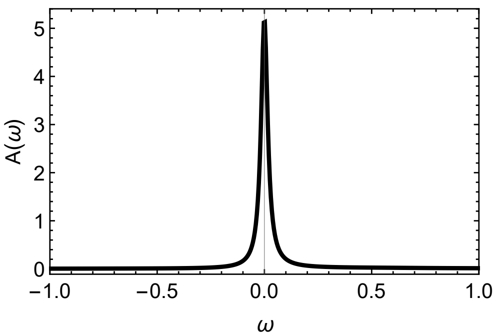


(a) Phase diagram

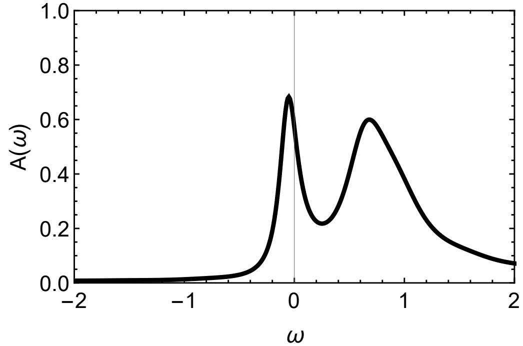
(b) Substructure in gapless phase

"Transition" is smooth everywhere.

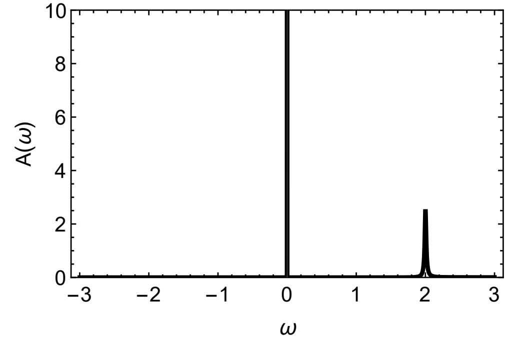
Naïve spectral function



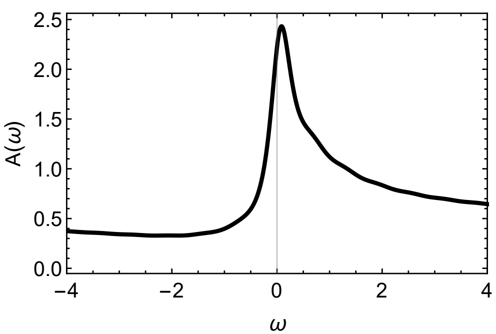
(a) Fermi liquid like (FL)



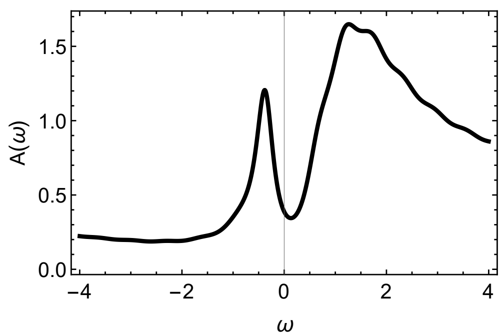
(b) bad metal prime (BM')



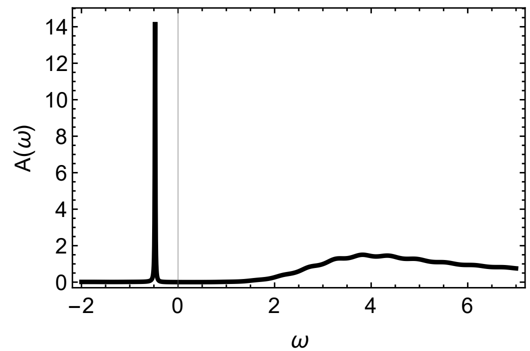
(c) half-metal (hM)



(d) bad metal (BM)

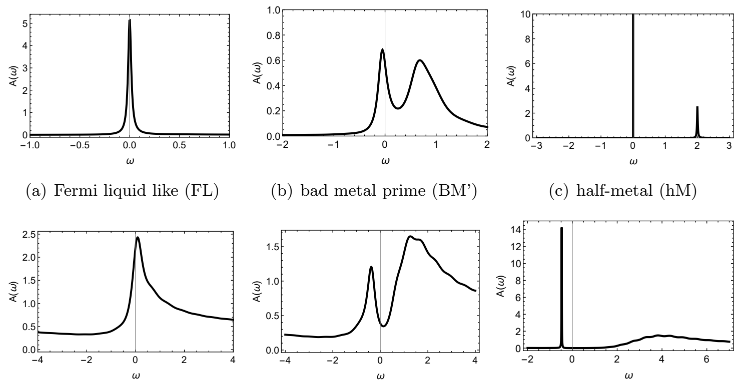


(e) pseudogap (PG)



(f) gapped (G)

Diagnosis



(a) Fermi liquid like (FL) (b) bad metal prime (BM') (c) half-metal (hM)

(d) bad metal (BM) (e) pseudogap (PG) (f) gapped (G)

1. Problem: Spectral function gives too much asymmetry.
   This is the evidence of Pauli principle is working partially.

2. Reason: Hole degree of freedom is not encoded.
   (Positive and Negative energy spectrum have the same charge)

3. Spectral function of hole = Spectral function of particle (q→-q)

How to add hole spectrum?

$$S_\psi = \int d^4x\sqrt{-g}i\bar{\psi}(\not{D} - m - ip\not{F})\psi + S_{bdy}$$

p → p Sign[q]

Minimal interaction contains qA_t

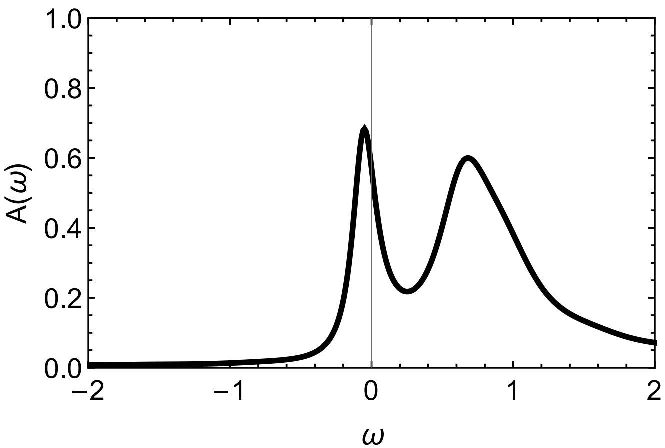
1. We change Lagrangian → L[q] +L[-q]

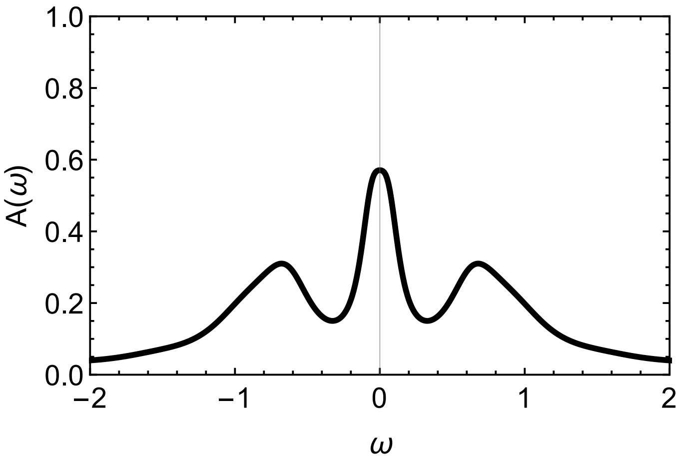
2. Consequence:
   equivalent to Spectral function is Symmetrized.
   due to the relation

   G[w, k, q] =-G*[-w,-k,-q]

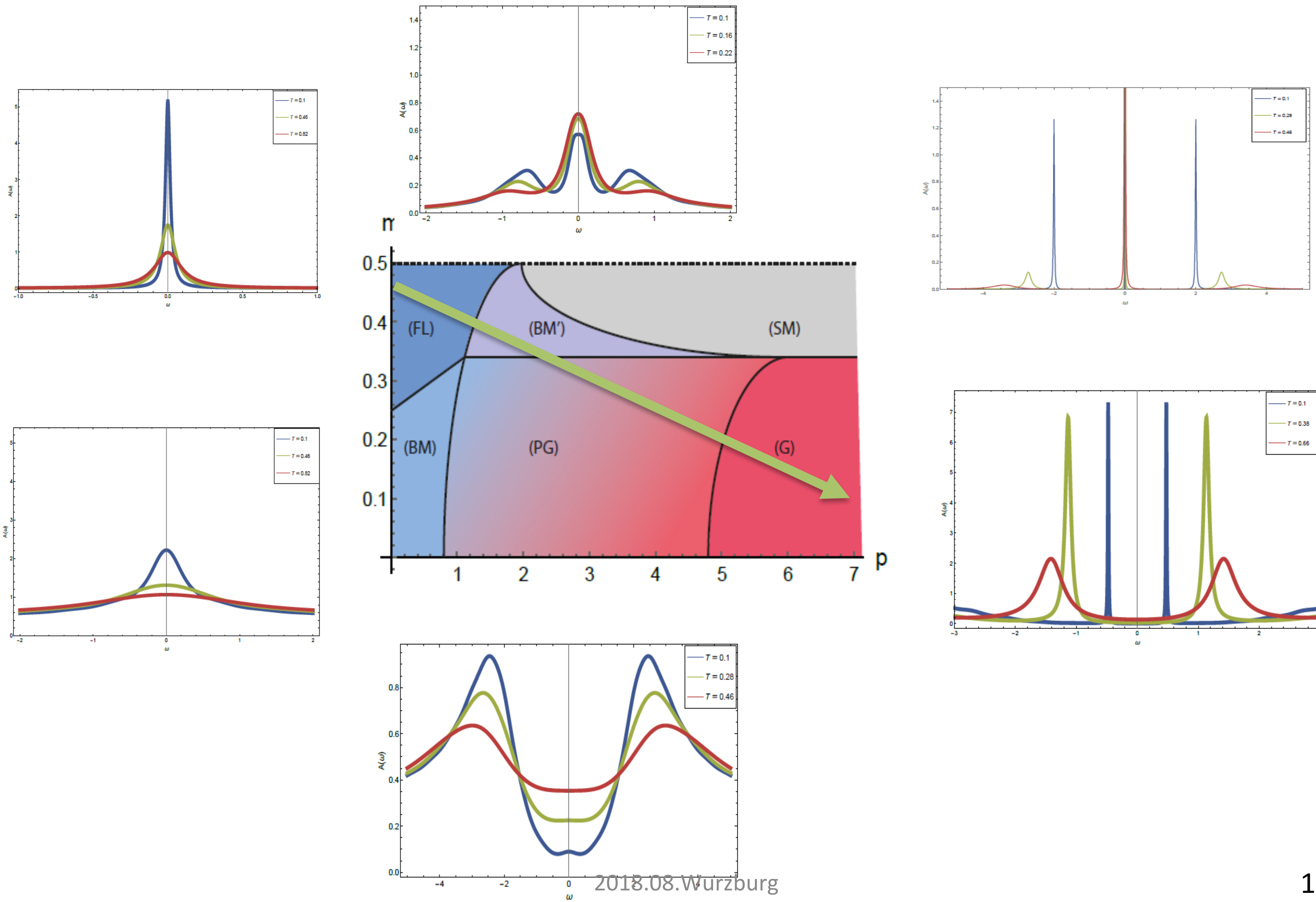
   A[w, k,q]+A[w,k,-q]=A[w, k,q]+A[-w,-k,q]

Consequece of adding hole spectrum

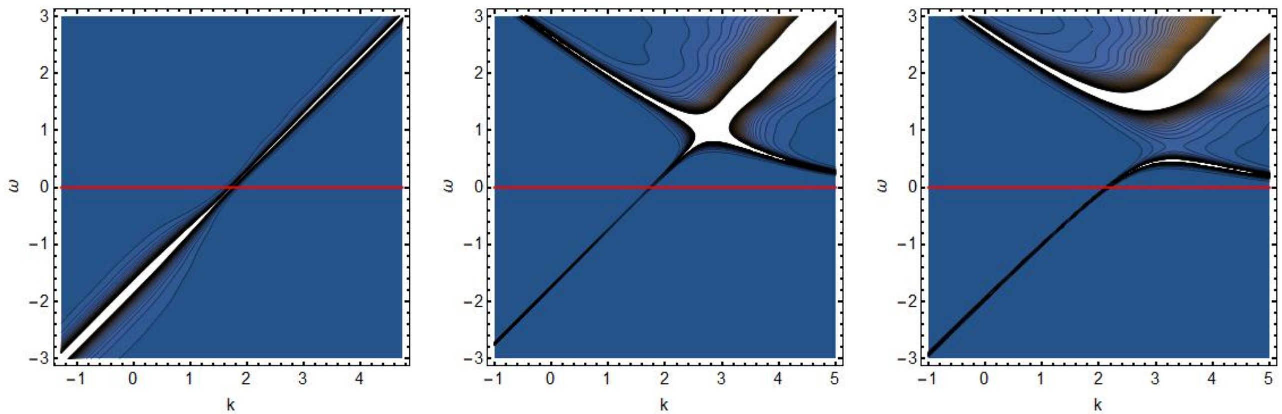




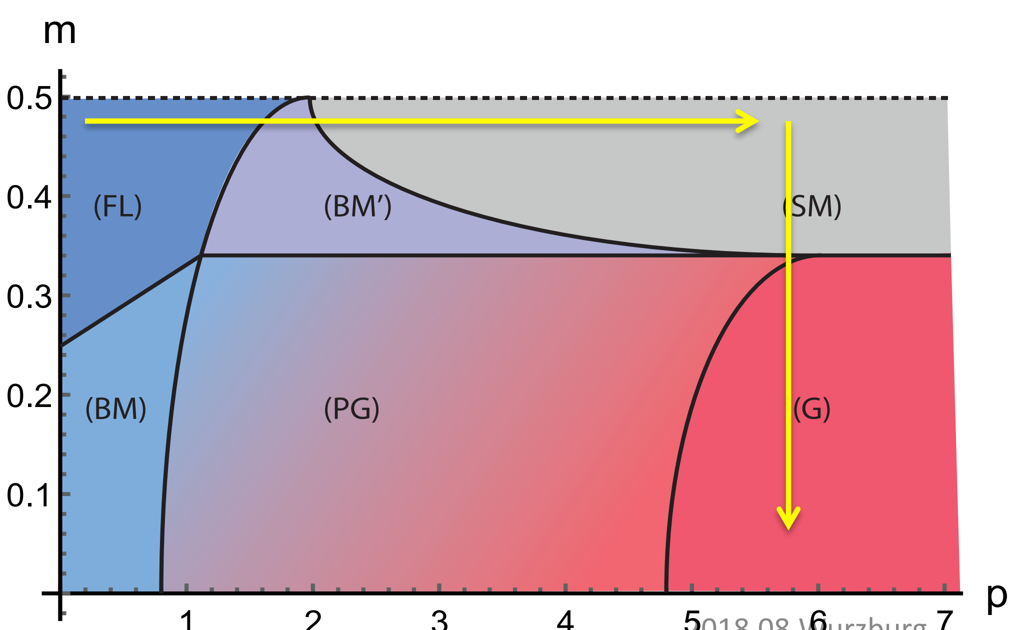
6 phases : with symmetrized spectral function

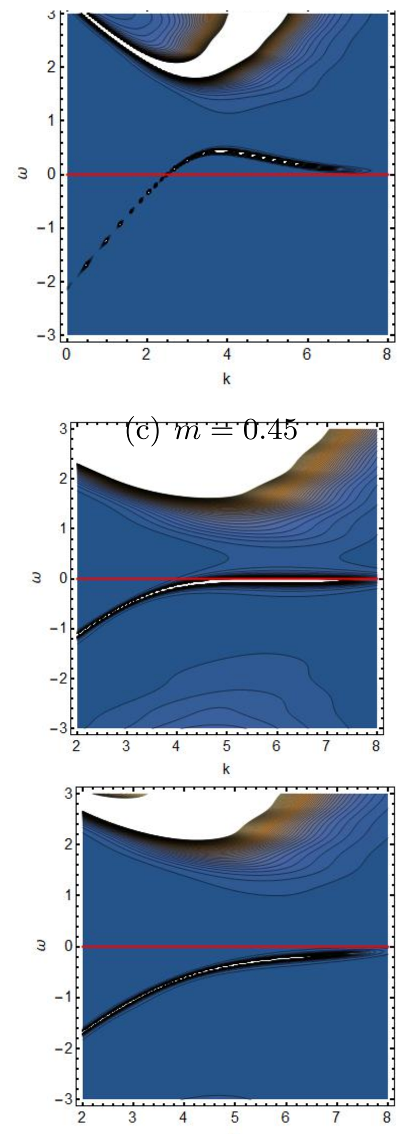


Understanding the Gap creation

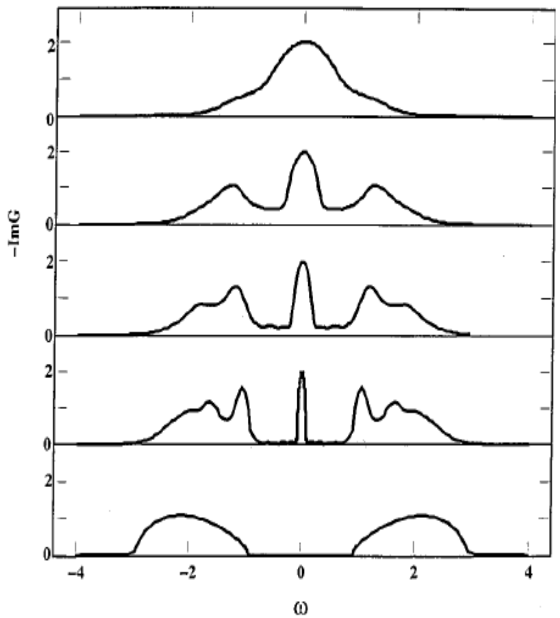


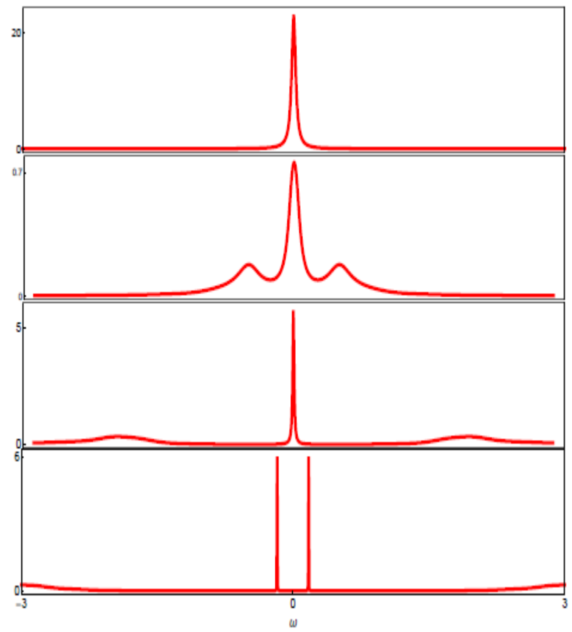
(a) $m = 0.5, p = 0$ (b) $m = 0.5, p = 5$ (c) $m = 0.47, p = 5$





Comparision with DMFT results

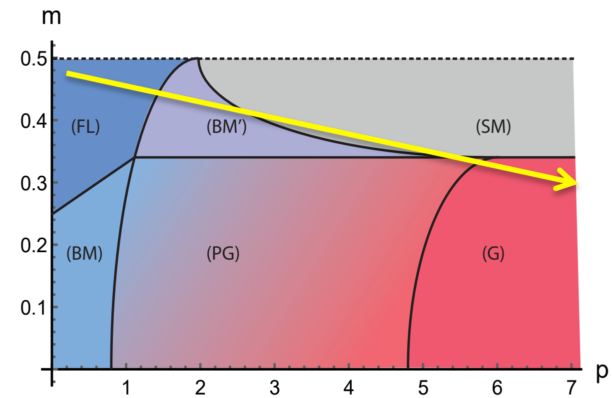




Single-site DMFT result

Holography with embedding

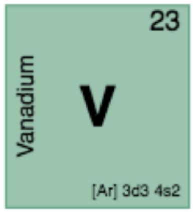
A. Georges, et.al Rev. Mod. Phys. 68 (Jan, 1996) 13–125.



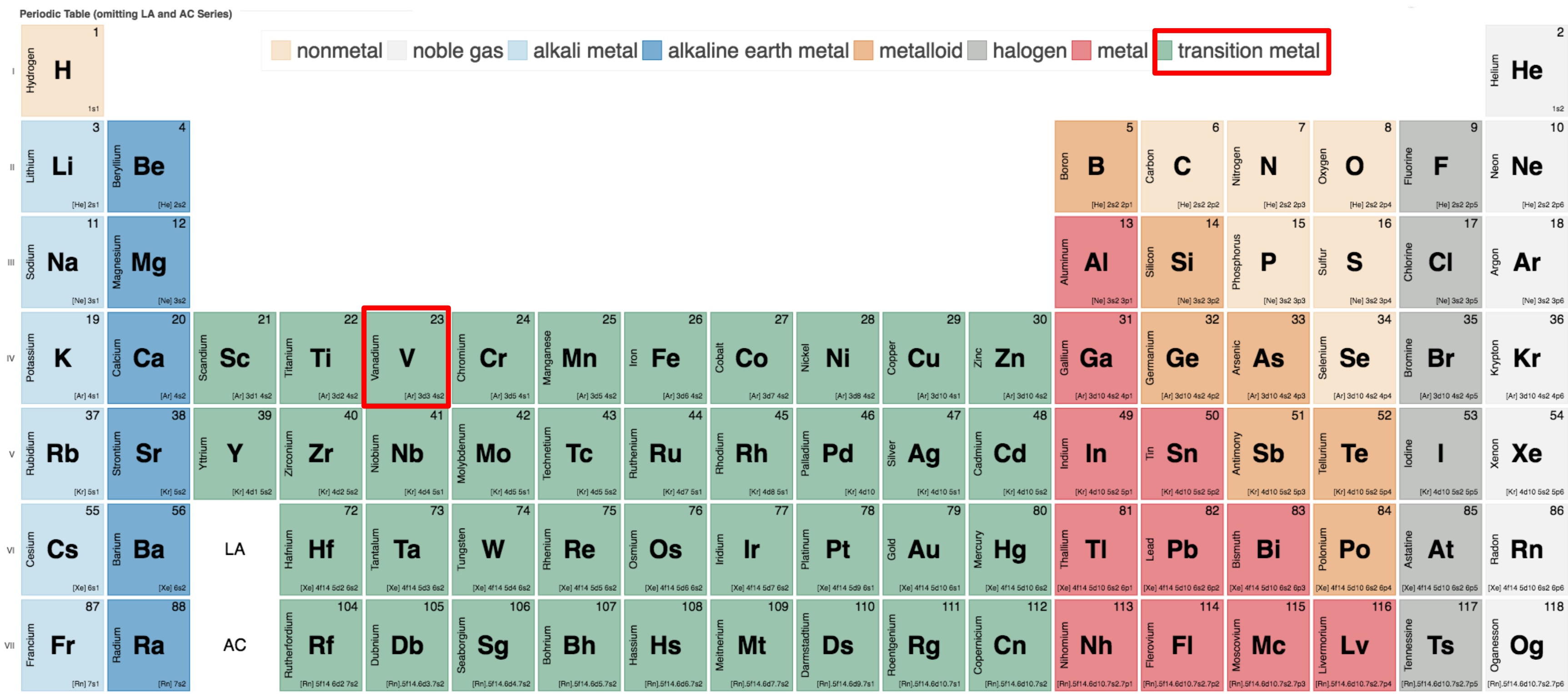
Depending on the path, evolution is different.

Data for Transition Metal Oxide

O^{2-}: anion

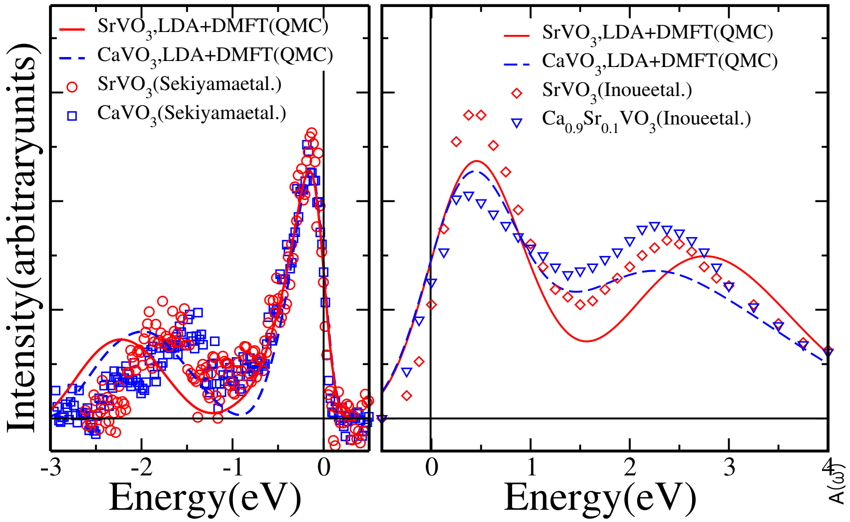


V^{3+}: V_2O_3, d^2
V^{4+}: VO_2, d^1

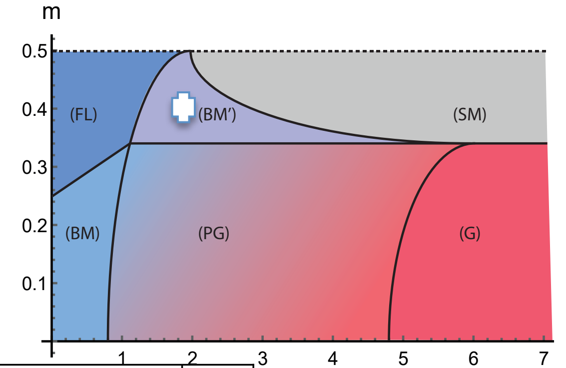


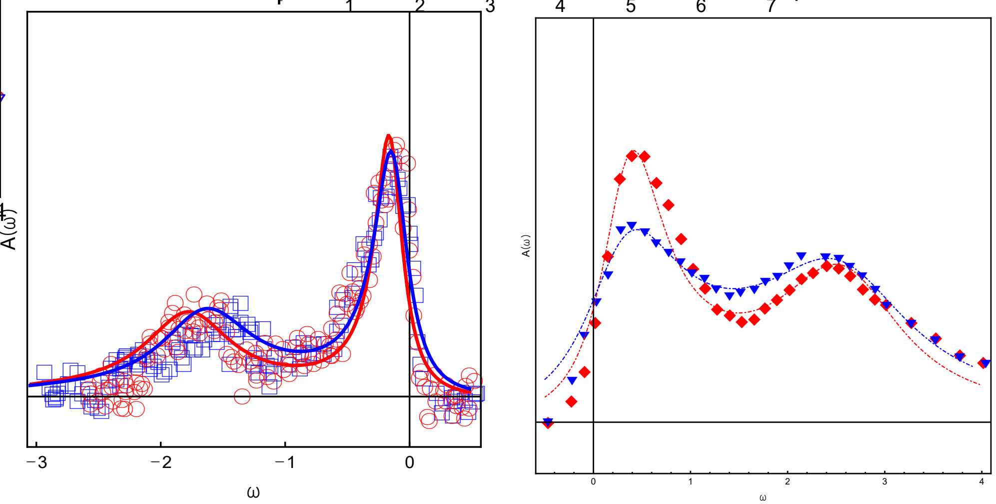
.ppt from Ara Go

Cf: DMFT vs Experiment // Holography vs Exp



DMFT school (2011) Dieter Vollhardt





(a)PES data vs theory (b)XAS data vs theory

FIG. 5. Experimenal data vs holographic theory: (a) PES data, (b) XAS data ; In both case (color red) is for $SrVO_3$ and (color blue) is for $CaVO_3$. The data for $SrVO_3$ is from [26], and that for $CaVO_3$ is from [25].

Future directions

- asymmetry, magnetism, backreaction.
- instability, d-wave condensation.
- other gap generation mechanism

$$S = \int \sqrt{-g} i\bar{\psi} \left(\not{D} - m - i g_v B_\mu \Gamma^\mu - i g_a B_\mu^{(5)} \Gamma^{5\mu} - i g_T M_{\mu\nu} \Gamma^{\mu\nu} \right) \psi \; - \int \sqrt{-g} i\bar{\psi} \left(g_0 M_0 + i g_5 M_5 \Gamma^5 \right) \psi,$$

α	0	1	2	3
M_s	$\cdot$	○	○	○

M_s	0	○	○	○

α	0	1	2	3
$B_t \gamma^t$	$\cdot$	●	●	●
$B_x \gamma^x$	●	○	○	○
$B_y \gamma^y$	○	○	○	○

Table 1: $g_v \neq 0$, ●: Gapless, ○: Gapped

α	0	1	2	3
$B_t^{(5)} \gamma^{5t}$	$\cdot$	●	●	●
$B_x^{(5)} \gamma^{5x}$	●′	○	○	○
$B_y^{(5)} \gamma^{5y}$	○	○	○	○

$$B_t = p \left[1 - \left(\frac{r_0}{r} \right)^\alpha \right]$$

$$B_i = p\, r^{-\alpha}$$

$$M_{\mu\nu} = p\, r^{-\alpha},$$

α	0	1	2	3
$M_{tr} \gamma^{tr}$	0	●	○	○
$M_{tx} \gamma^{tx}$	●	●	●	●
$M_{ty} \gamma^{ty}$	●	●	●	●
$M_{rx} \gamma^{rx}$	●	●	●	●
$M_{ry} \gamma^{ry}$	●	●	●	●
$M_{xy} \gamma^{xy}$	○	○	○	○

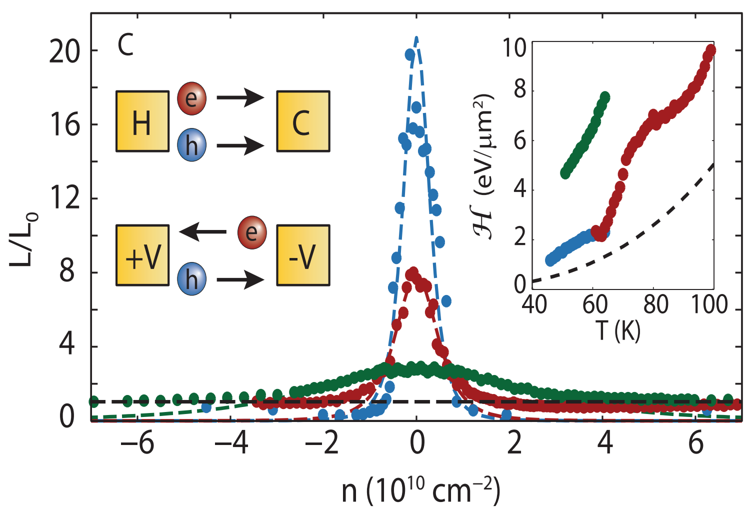
2nd class : Dirac Materials

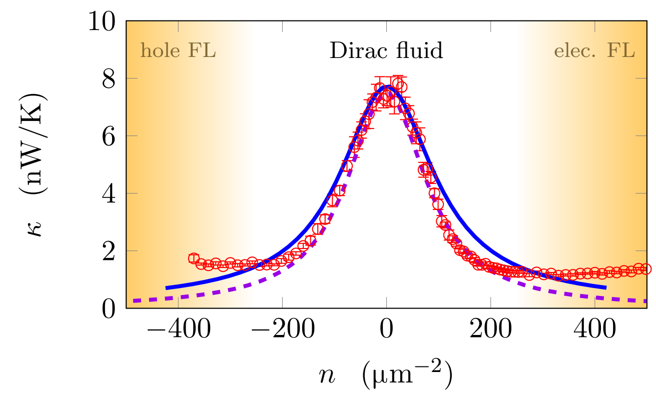
1. Graphene and anomalous transport

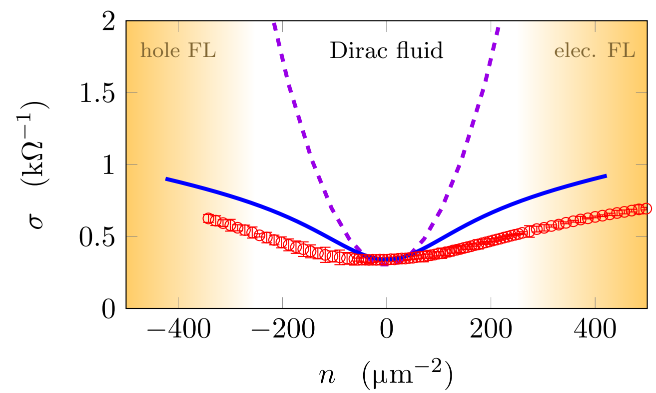
2. TI and magnetotransport

Observation of the Dirac fluid and the breakdown of the Wiedemann-Franz law in graphene

Jesse Crossno,[1,2] Jing K. Shi,[1] Ke Wang,[1] Xiaomeng Liu,[1] Achim Harzheim,[1]
Andrew Lucas,[1] Subir Sachdev,[1,3] Philip Kim,[1,2*] Takashi Taniguchi,[4] Kenji Watanabe,[4]
Thomas A. Ohki,[5] Kin Chung Fong[5*]







Science AAAS March 2016

arXiv:1510.01738 PRB

A Holographic model

Idea : neutral current → Enhance the heat conductivity

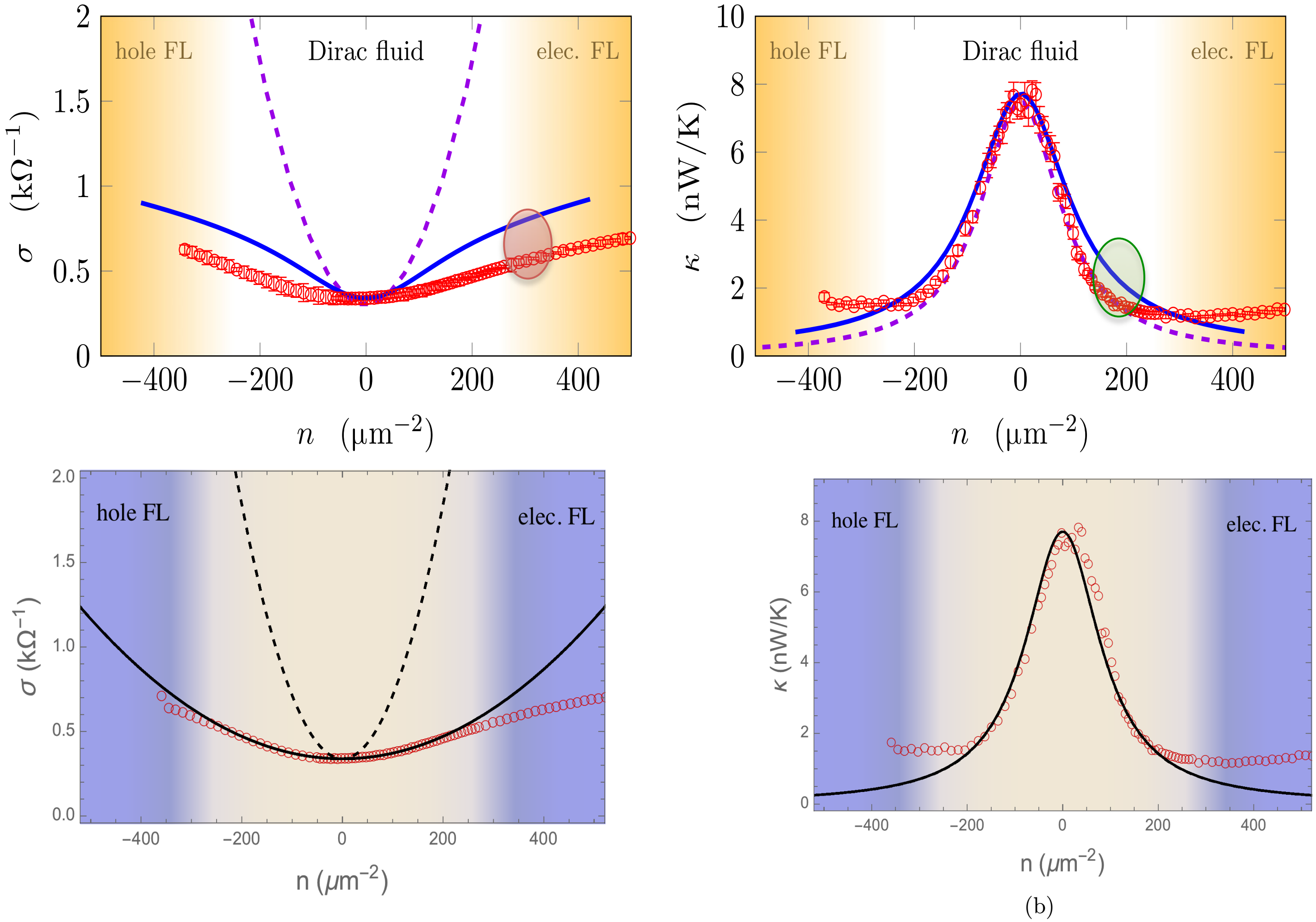
Two current model→ Two gauge field in AdS charged BH

$$S = \int d^4x\sqrt{-g}\left[R - \frac{1}{2}\left[(\partial\phi)^2 + \Phi_1(\phi)(\partial\chi_1)^2 + \Phi_2(\phi)(\partial\chi_2)^2\right] - V(\phi) - \frac{Z(\phi)}{4}F^2 - \frac{W(\phi)}{4}G^2\right]$$

$$\sigma = \sigma_0(1 + (Q/Q_0)^2), \qquad \kappa = \frac{\bar{\kappa}}{1 + (1 + g_n^2)(Q/Q_0)^2}$$

$$\sigma_0 = \frac{e^2}{\hbar}2Z_0, \quad \bar{\kappa} = \frac{4\pi k_B}{\hbar}\frac{sT}{k^2}, \quad Q_0^2 = \frac{\hbar\sigma_0}{4\pi k_B}sk^2.$$

Hydrodynamics vs quantum Holography in data fitting



(b)

4 parameters at 75K, $\sigma_0 = 0.338/k\Omega$, $\bar{\kappa} = 7.7nW/K$, $Q_0 = e\cdot 320/(\mu m)^2$,

Appeared in

PRL **118,** 036601 (2017) PHYSICAL REVIEW LETTERS week ending 20 JANUARY 2017

Holography of the Dirac Fluid in Graphene with Two Currents

Yunseok Seo,[1] Geunho Song,[1] Philip Kim,[2,3] Subir Sachdev,[2,4] and Sang-Jin Sin[1]
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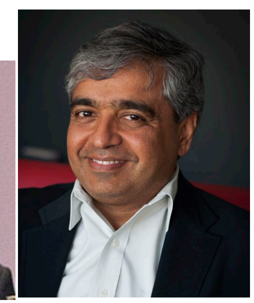


Prediction

For all Dirac material listed in (1405.5774), transport is similarly anomalous.

Material	Pseudospin	Energy scale (eV)	References
Graphene, Silicene, Germanene	Sublattice	1–3 eV	[5, 6, 17, 19, 36, 37]
Artificial Graphenes	Sublattice	10^{-8}–0.1 eV	[28, 29, 38–40]
Hexagonal layered heterostructures	Emergent	0.01–0.1 eV	[41–47]
Hofstadter butterfly systems	Energent	0.01 eV	[46]
Graphene-hBN heterostructures in high magnetic fields			
Band inversion interfaces	Spin-orbit ang. mom.	0.3 eV	[48–50]
SnTe/PbTe, CdTe/HgTe, PbTe			
2D Topological Insulators	Spin-orbit ang. mom.	< 0.1eV	[7, 8, 22, 24, 51, 52]
HgTe/CdTe, InAs/GaSb, Bi bilayer, ...			
3D Topological Insulators	Spin-orbit ang. mom.	$\lesssim 0.3$eV	[7, 8, 23, 52–55]
$Bi_{1-x}Sb_x$, Bi_2Se_3, strained HgTe, Heusler alloys, ...			
Topological crystalline insulators	orbital	$\lesssim 0.3$eV	[56–59]
SnTe, $Pb_{1-x}Sn_xSe$			
d-wave cuprate superconductors	Nambu pseudospin	$\lesssim 0.05$eV	[60, 61]
^3He	Nambu pseudospin	0.3 μeV	[2, 3]
3D Weyl and Dirac semimetals	Energy bands	Unclear	[32–34]
Cd_3As_2, Na_3Bi			

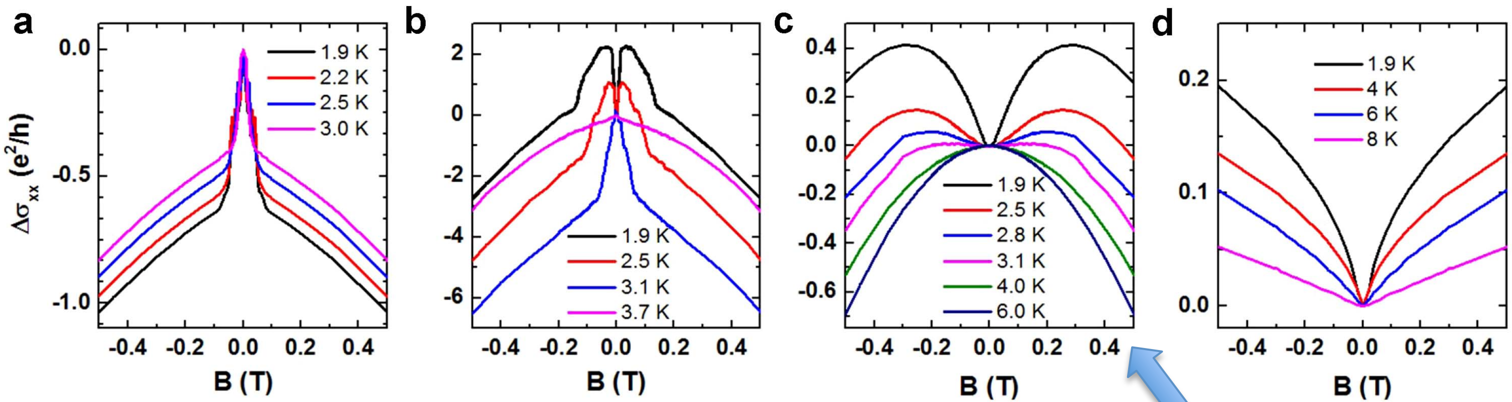
Table 1. Table of Dirac materials indicated by material family, pseudospin realization in the Dirac Hamiltonian, and the energy scale for which the Dirac spectrum is present without any other states.

Ex: Surface of TI

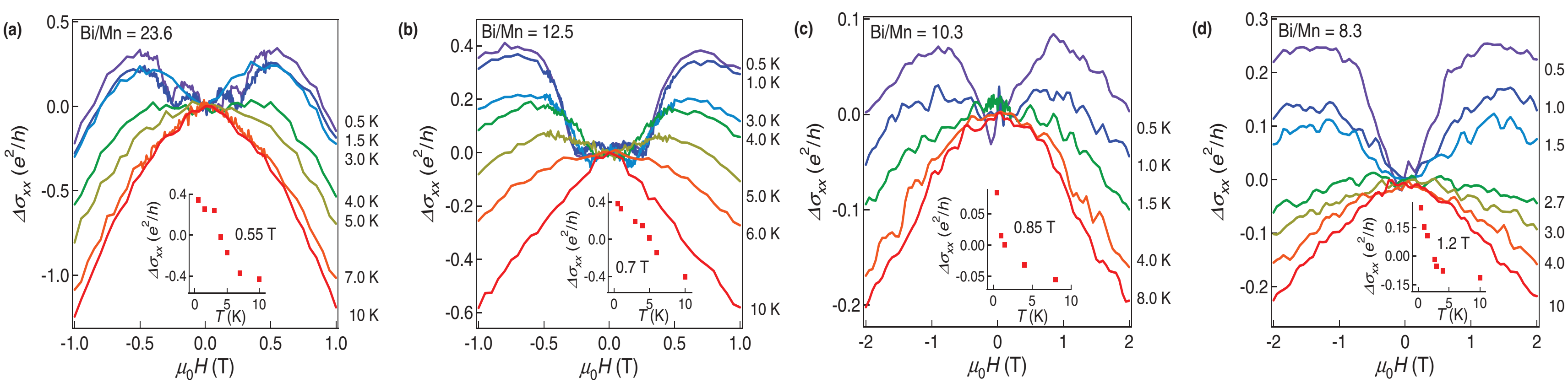
Similar, but differ by strong spin-orbit interaction

Surface of TI : WAL → WL transition

1. Bi_2Te_3 with Cr doping: Bao et.al, SREP02391

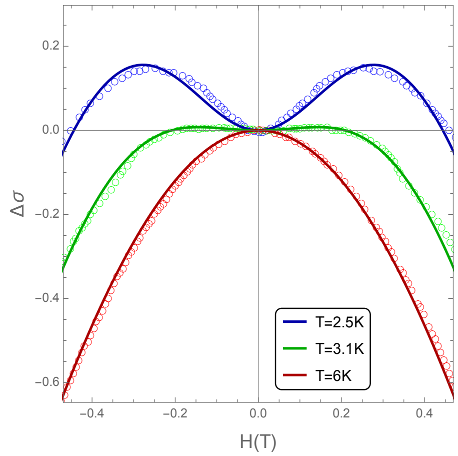


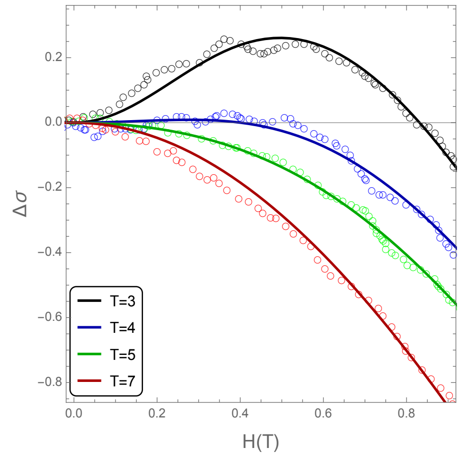
2. . Bi_2Se_3 with Mn doping : Zhang et.al, prB86,205127(2012)



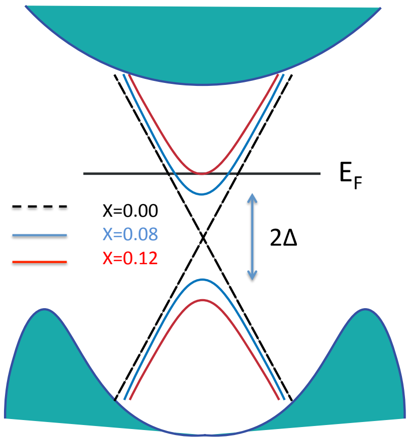
Result : Interplay of bulk and boundary

Universality : Both Cr doped Bi_2Te_3, Mn doped Bi_2Se_3
can be fit by the same theory with different parameters.





Reason why it works:
Small FS at intermediate doping.



results

Strong Correlation Effects on Surfaces of Topological Insulators via Holography

Yunseok Seo, Geunho Song and Sang-Jin Sin
Department of Physics, Hanyang University, Seoul 04763, Korea.

Phys.Rev. B96 rapid comm. (2017) no.4, 041104

Small Fermi Surfaces and Strong Correlation Effects in Dirac Materials with Holography

Y. Seo, G. Song, C. Park + SJS

JHEP 1710 (2017) 204

Conclusion

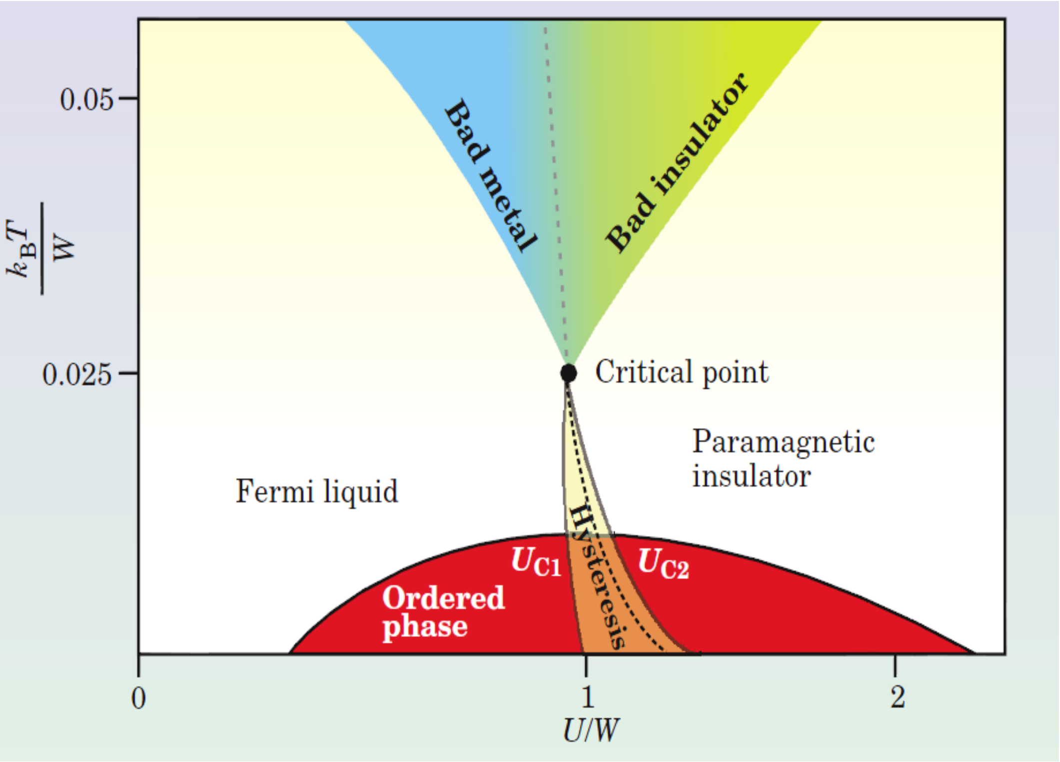
- New insight of gap creation by considering m as well as p
- Described how to encode the holes in holography.
- Hubbard model can be replaced by a holographic model
- Data for Dirac material and Transition metal can be fit.

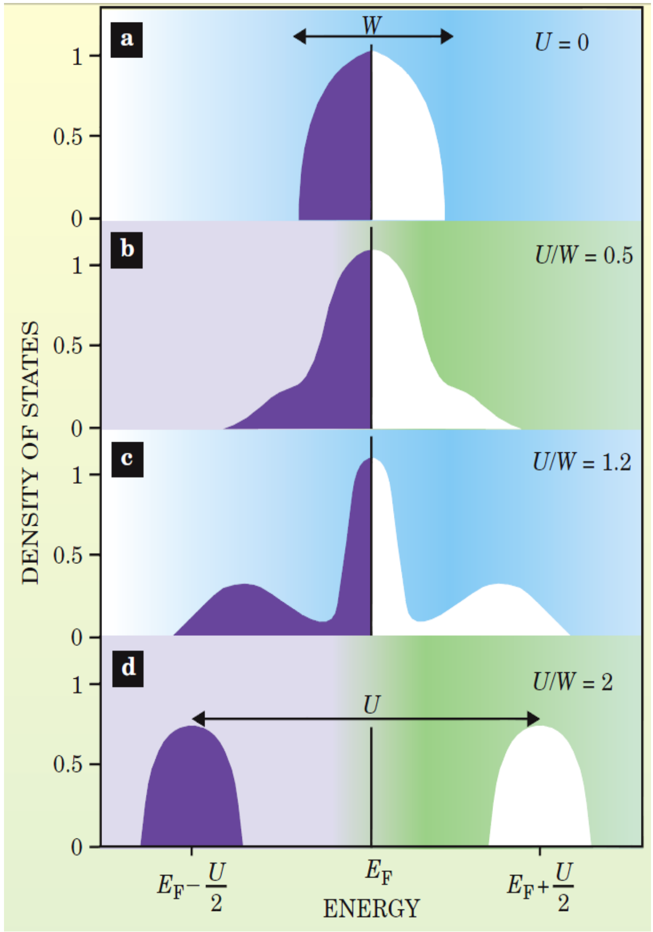
Thank you.

*

*

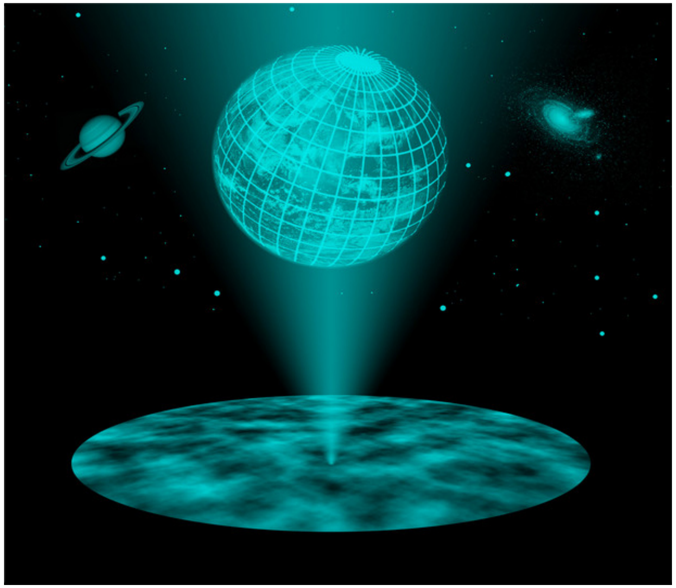
Mott Transition First order?



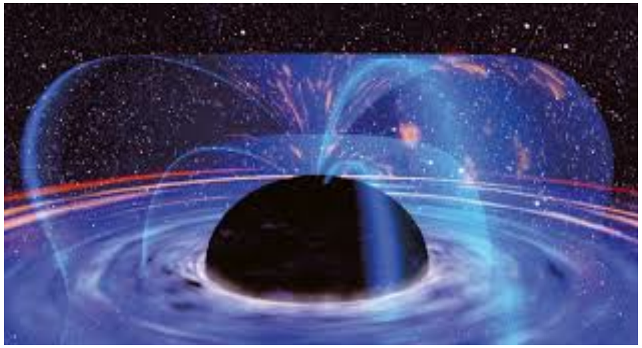


G Kotliar and D Vollhardt, Physics Today **57**, 53 (2004)

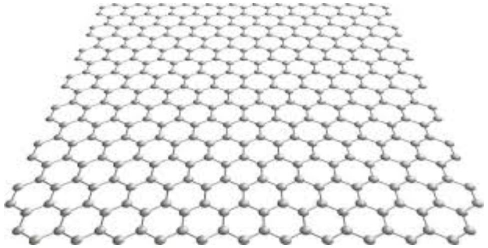
Method : quantum holography



3 dim.
Classical
BH



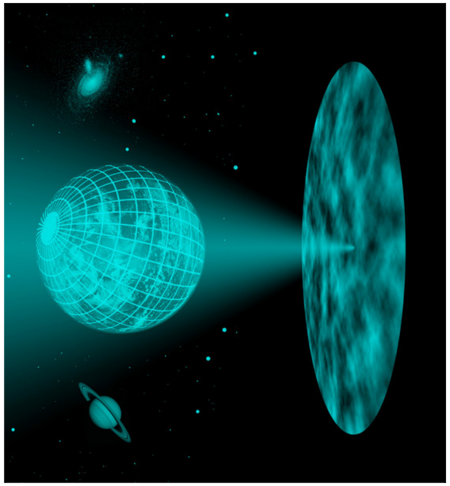
2 dim.
Quantum
Matter



2d SIS (near QCP) hologram = 3dim Black hole

→ quantum black hole

quantum holography



QCP : dynamical exponent

But

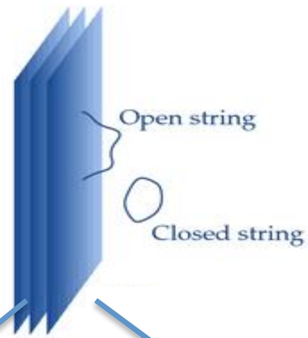
BH :

←→ equilibrium, fluid dynamic behavior

←→ transport(transport is input in traditional fluid dynamics)

method

Supersymmetric D-brane



Susy gauge theory

Susy AdS gravity

Theory of Quantum Matter ⟷ Classical Geometry

General case without SUSY

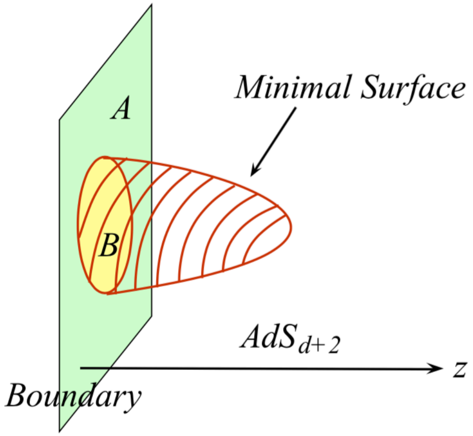
i. Find example outside string theory.

ii. Asume→ caculate → compare with exp..

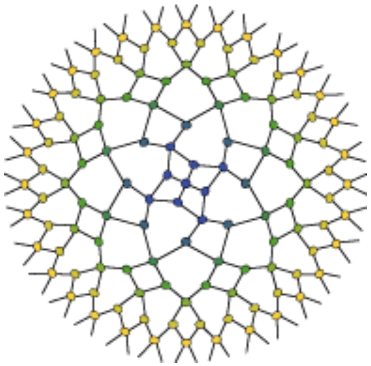
i) Evidence outside string theory

1. entanglement entropy calculation in 2d

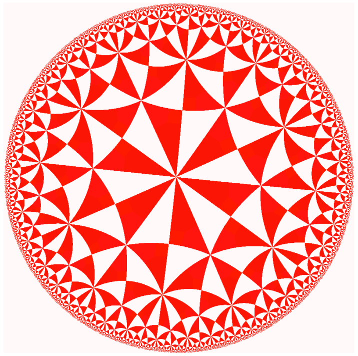
Ryu & Takayanagi (2006)



2. Tensor network : (Multiscale Entanglement Renormalization Ansatz)

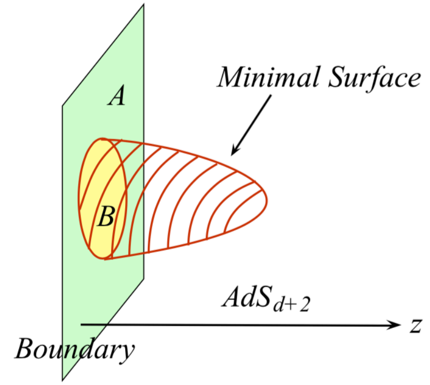


[Swingle]

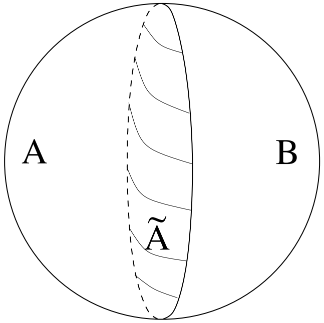


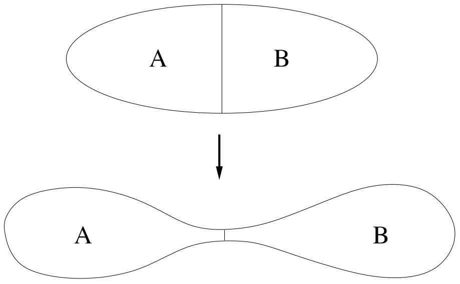
Comment : Entanglement and Holography

Ryu & Takayanagi (2006) by product.
Presence of dual space time=
presence of high entanglement



Raamsdonk : classical .
Space is sewn by entanglement.
Entanglement first law → Linearized gravity equation.



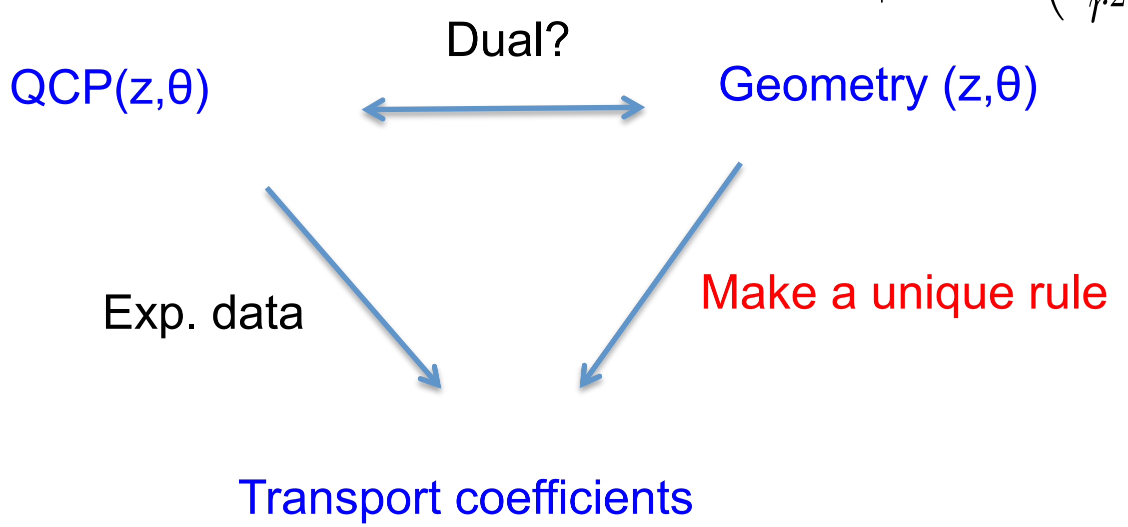


Complete Einstein equation from the generalized First Law of Entanglement
Eunseok Oh (Hanyang U.), I.Y. Park (Philander Smith Coll.), Sang-Jin Sin (Hanyang U.):
arXiv:1709.05752 [hep-th] | PDF

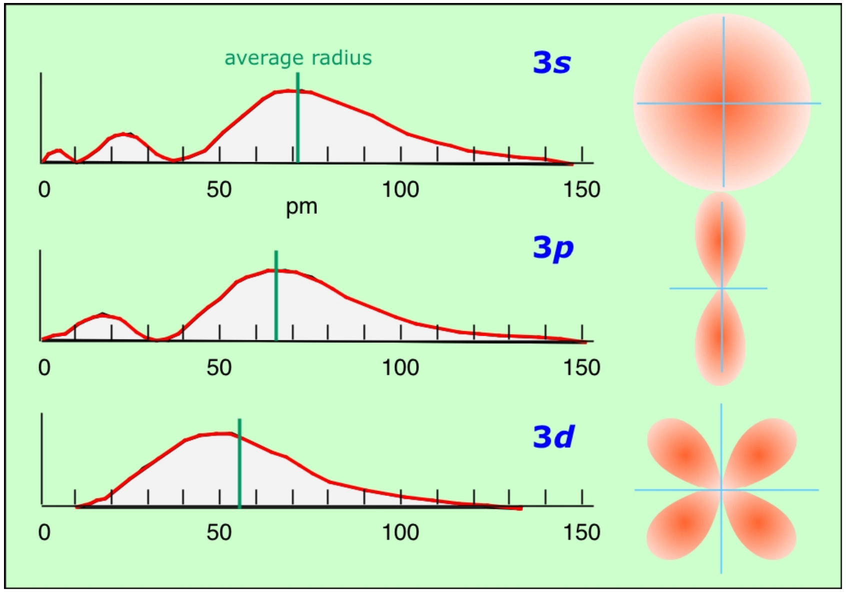
AdS/CFT : an exact duality where dictionary is given

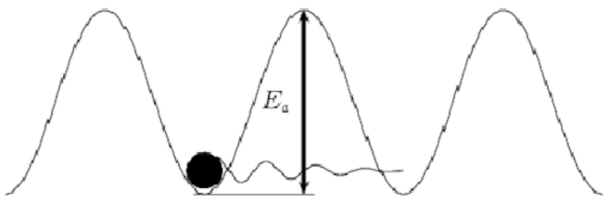
Hydrogen atom of Holographic Duality

$$ds^2_{D+2} = r^{\frac{2\theta}{D}} \left(\frac{-dt^2}{r^{2z}} + \frac{dr^2}{r^2} + \frac{1}{r^2} \sum_{i=1}^{D} dx_i^2 \right)$$

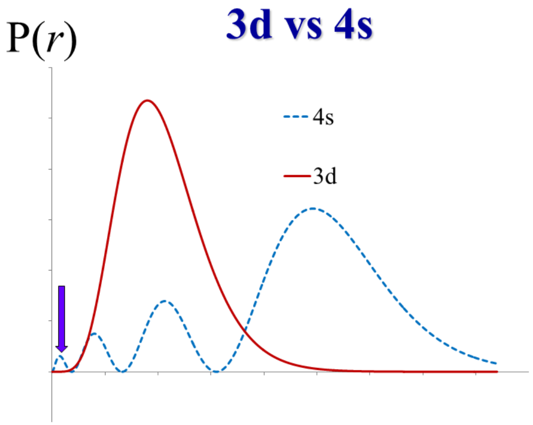


Why 3d? why Oxide?

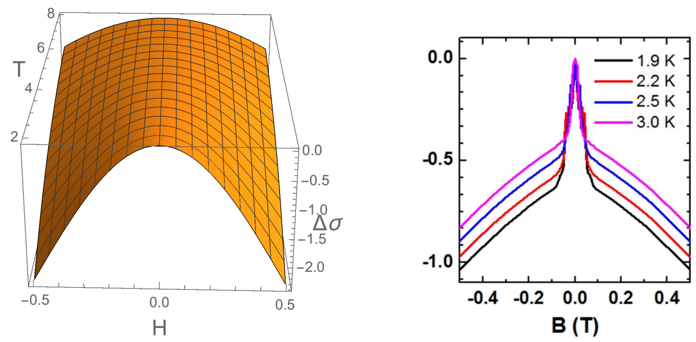




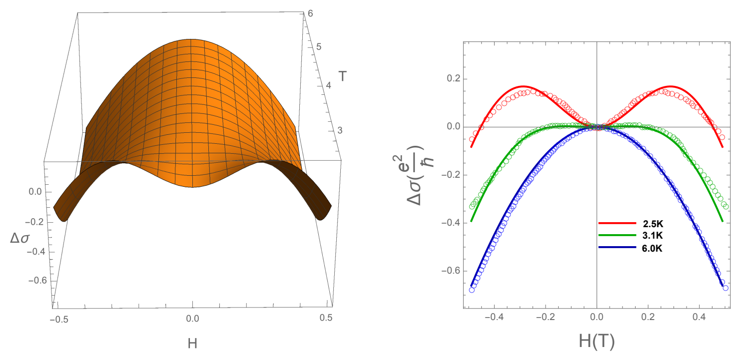
Transition metal oxide 3d $^{1-10}$ 4s^{1-} O



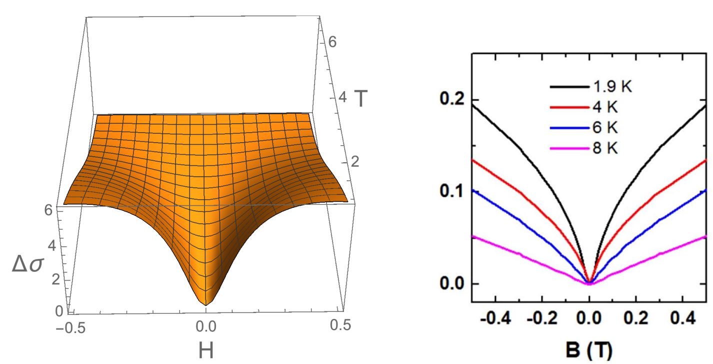
Surface states of TI [1703.07361]



(a) (b)

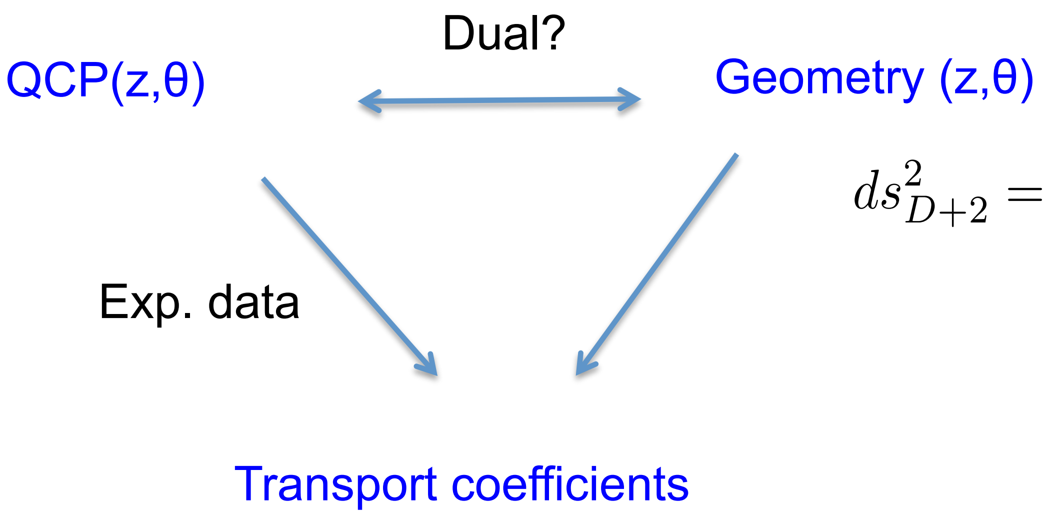


(c) (d)



(e) (f)

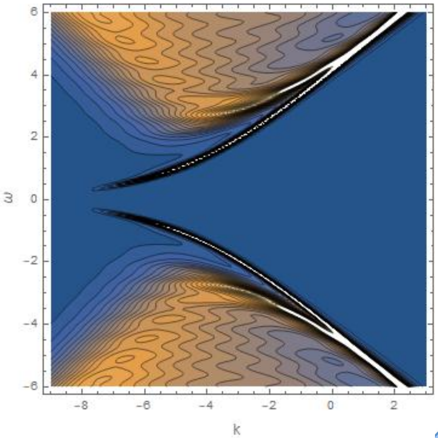
Postulate: the dictionary of ads/cft works for in general



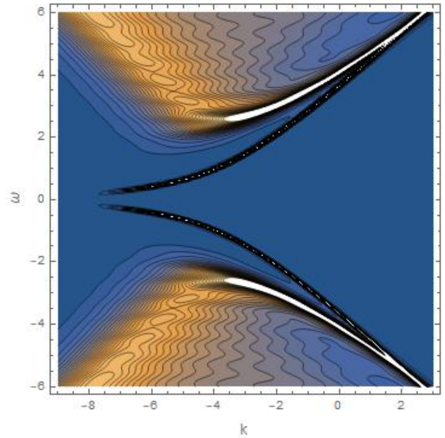
$$ds^2_{D+2} = r^{\frac{2\theta}{D}}\left(\frac{-dt^2}{r^{2z}} + \frac{dr^2}{r^2} + \frac{1}{r^2}\sum_{i=1}^{D} dx_i^2\right)$$

Analogy: wave eq. → Schrodinger eq. under force.

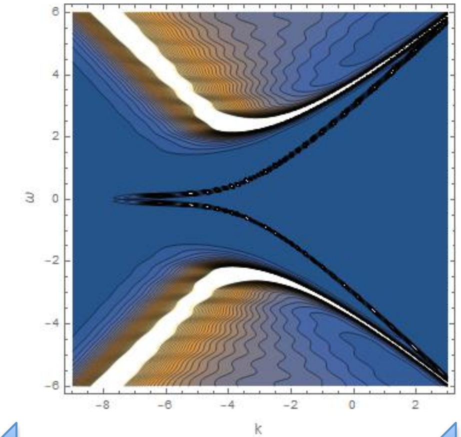
Role of m in Gap creation



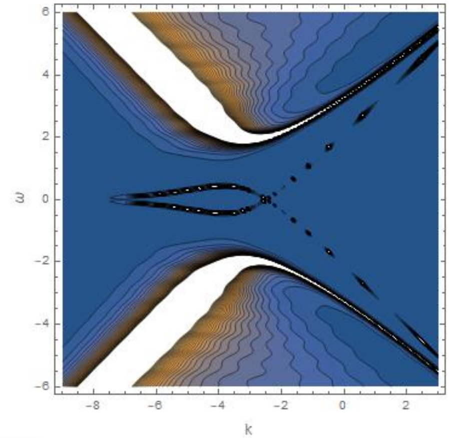
(i) $m = 0 \ p = 6$



(j) $m = 0.15 \ p = 6$

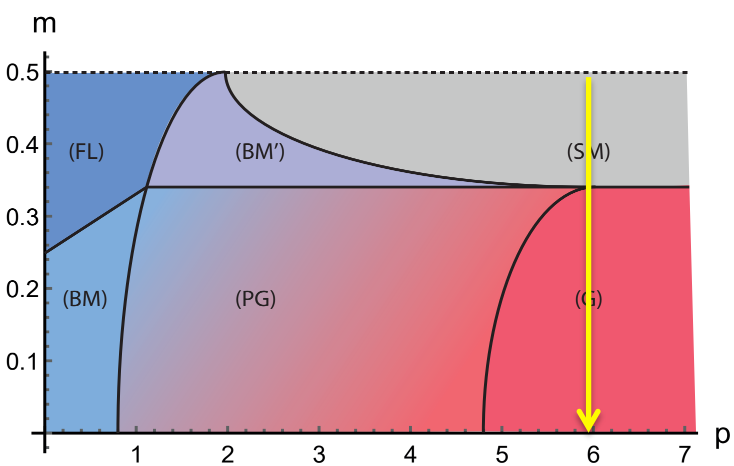


(k) $m = 0.3 \ p = 6$

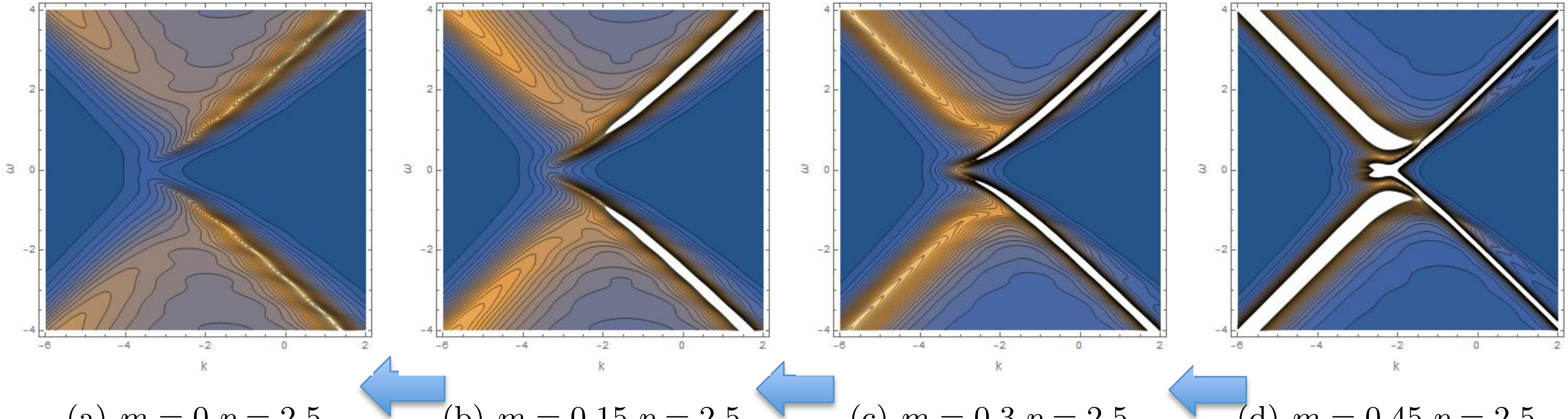


(l) $m = 0.45 \ p = 6$

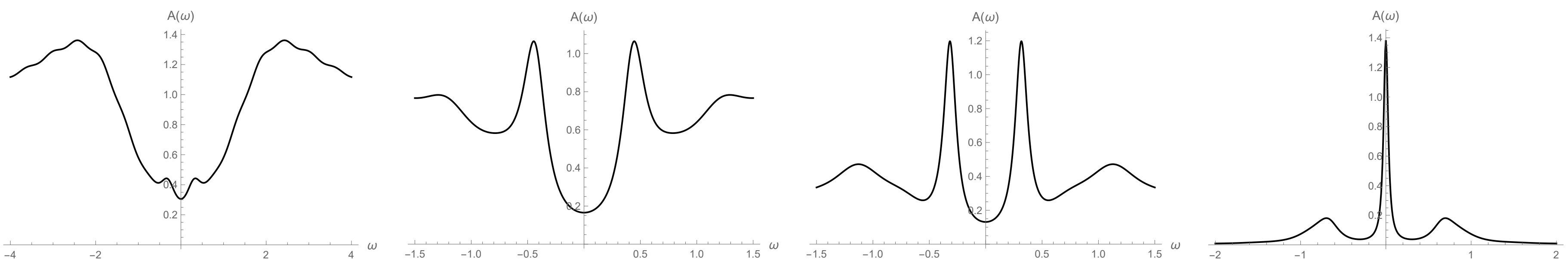
Now as we decreases m further, the lower one goes down upper one goes up → gap creation, i.e, As (½-m) increases, gap is created.



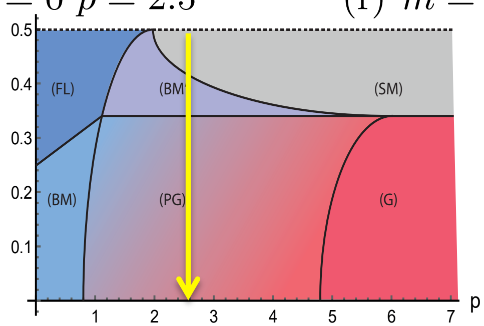
Role of m in psuedo-Gap generation



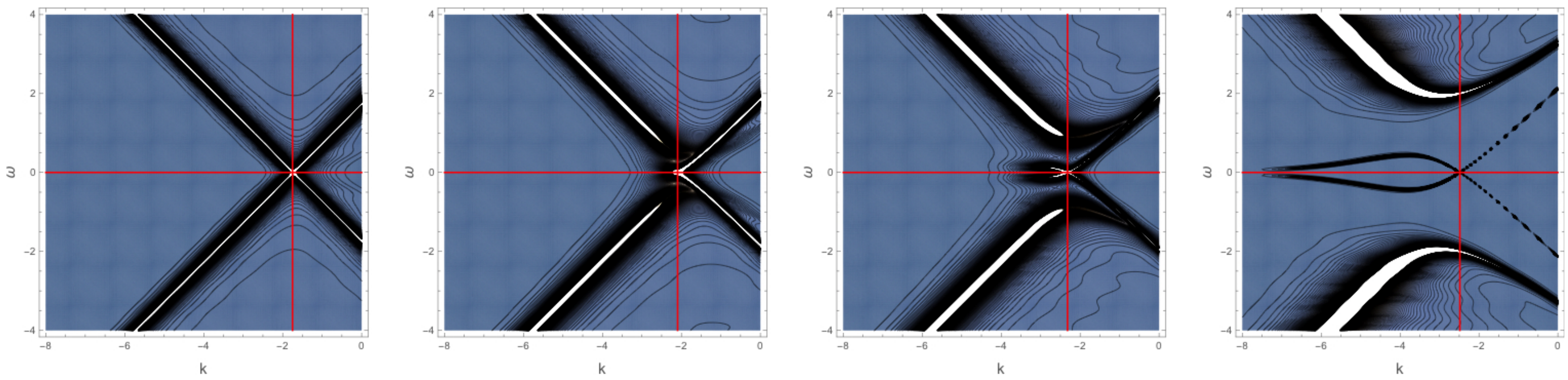
(a) $m = 0\ p = 2.5$ (b) $m = 0.15\ p = 2.5$ (c) $m = 0.3\ p = 2.5$ (d) $m = 0.45\ p = 2.5$



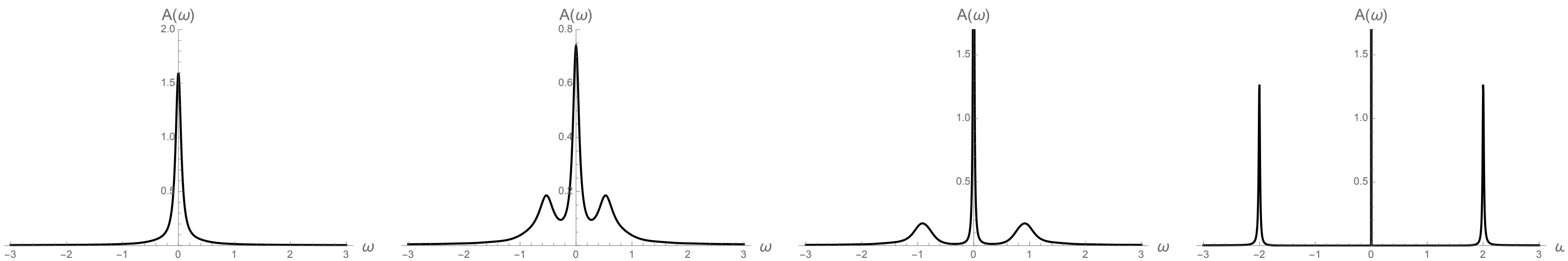
(e) $m = 0\ p = 2.5$ (f) $m = 0.15\ p = 2.5$ (g) $m = 0.3\ p = 2.5$ (h) $m = 0.45\ p = 2.5$



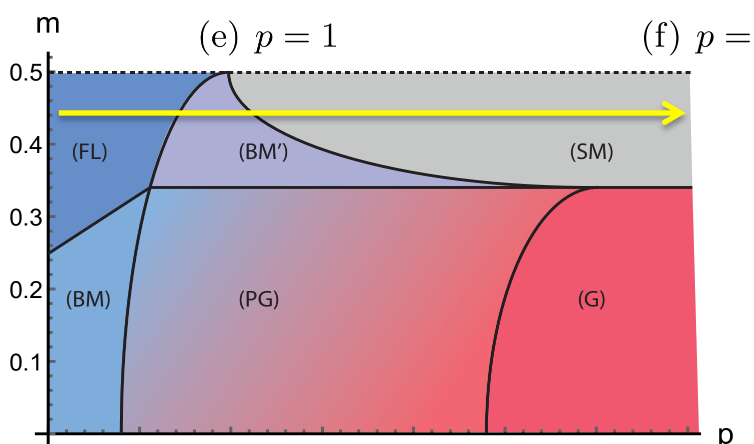
Role of p : creation of new band



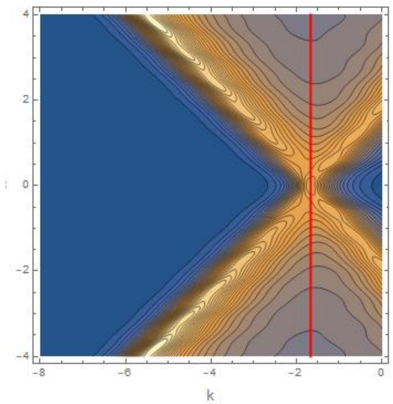
(a) $p = 1$ (b) $p = 2$ (c) $p = 3$ (d) $p = 6$



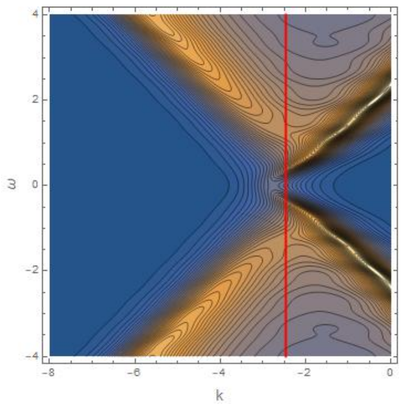
(e) $p = 1$ (f) $p = 2$ (g) $p = 3$ (h) $p = 6$



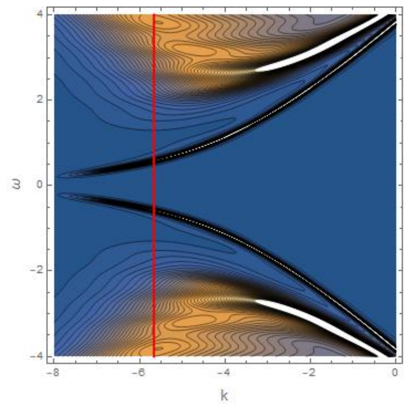
Role of p in PG and Gap creation



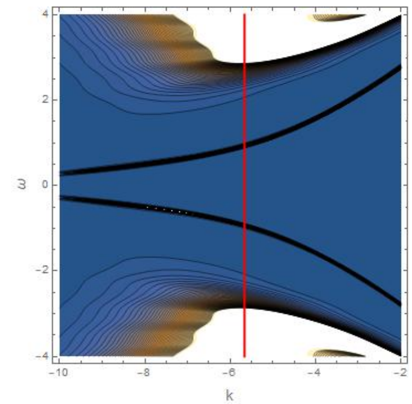
(a) $p = 1$



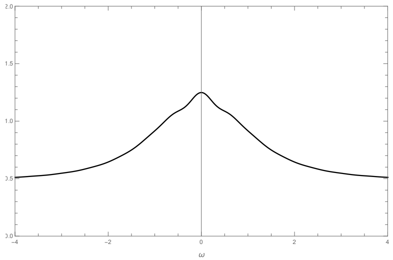
(b) $p = 2$



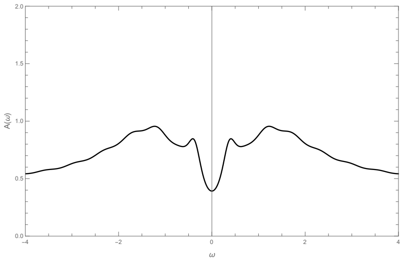
(c) $p = 6$



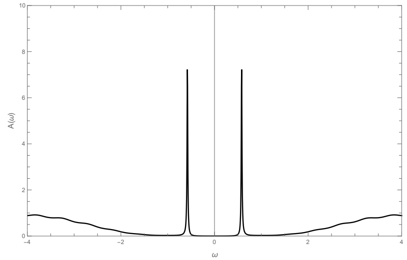
(d) $p = 8$



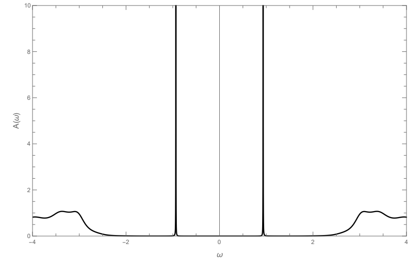
(e) $p = 1$



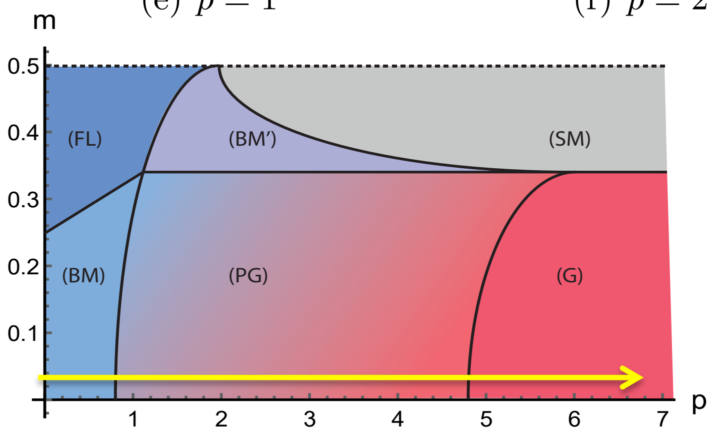
(f) $p = 2$



(g) $p = 6$



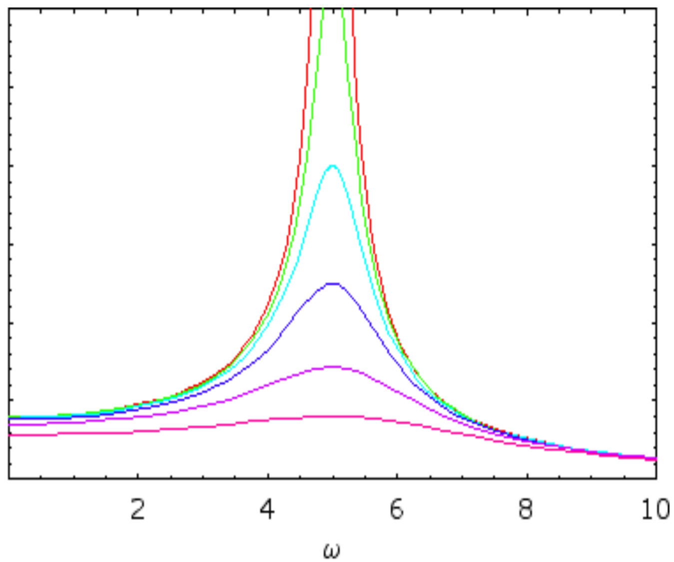
(h) $p = 8$

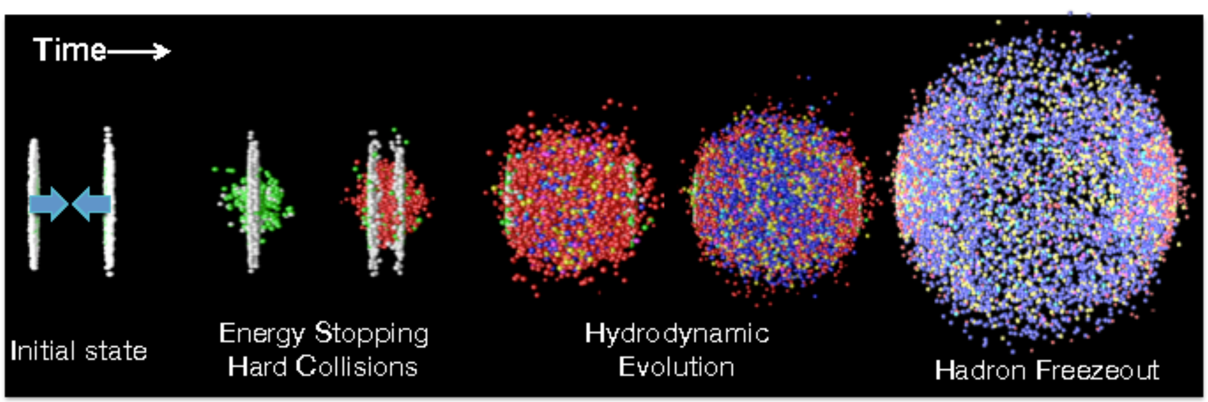


Effect of strong Interaction

1. Loss of particle → Loss of calculability, Loss of Fermi sea

2. Rapid Thermalization → enables Hydro-dynamics w/small viscosity
   ~ Gravity dual description

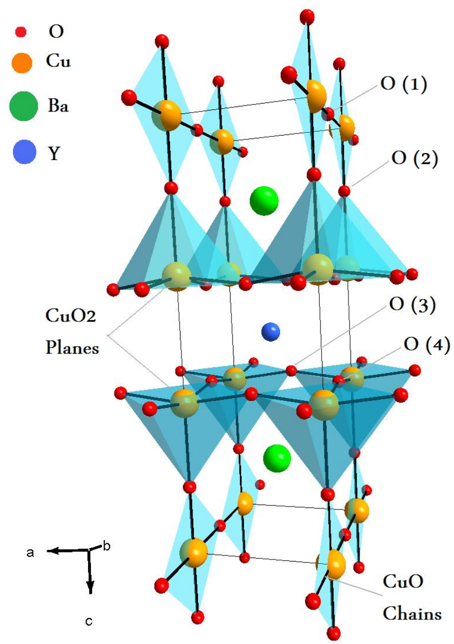




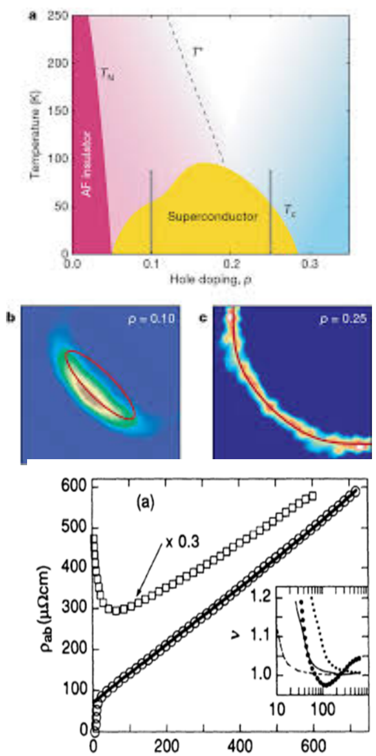
Condensed matter theory

= Structure (UV) → functionality (IR) Transport and Spectrum

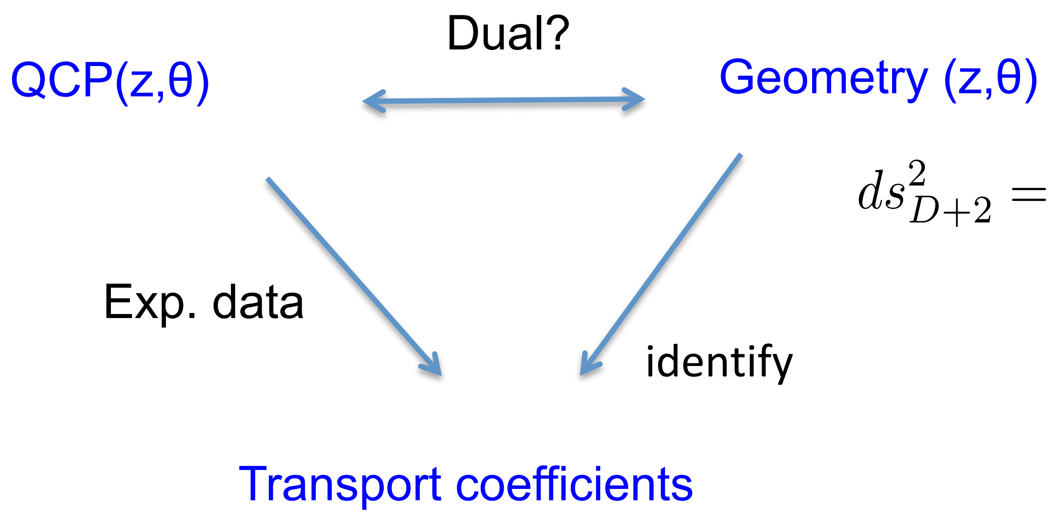
Not applicable to SIS.



keV → meV



General idea is to identify QCP and the ads BH.



$$ds^2_{D+2} = r^{\frac{2\theta}{D}}\left(\frac{-dt^2}{r^{2z}} + \frac{dr^2}{r^2} + \frac{1}{r^2}\sum_{i=1}^{D} dx_i^2\right)$$

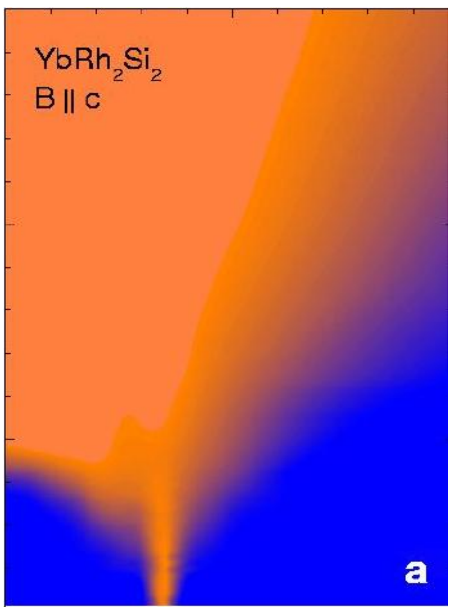
Therefore, ...

My favorate reason why AdS/CMT:

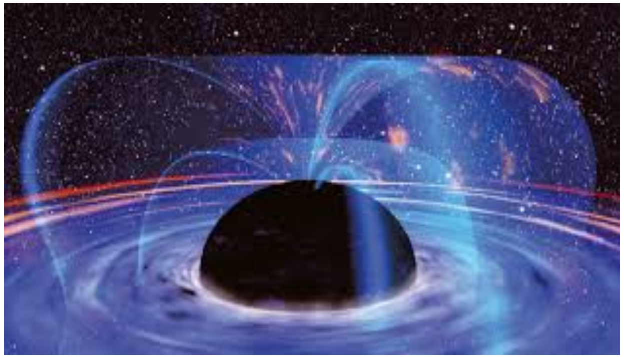
Similarity of QCDP and BH

absence of scale→absence of structural dependence
→ Universality

Classify QCP by dynamical exponent Z, θ: $\omega = k^z$, $[s] = D - \theta$

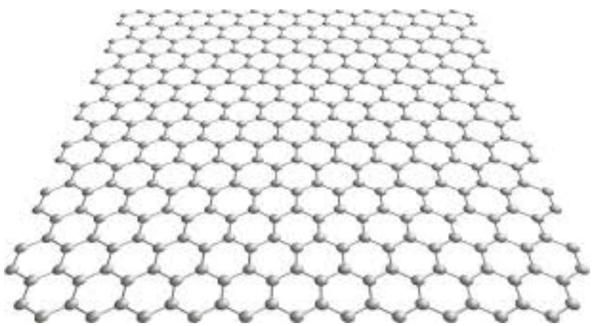


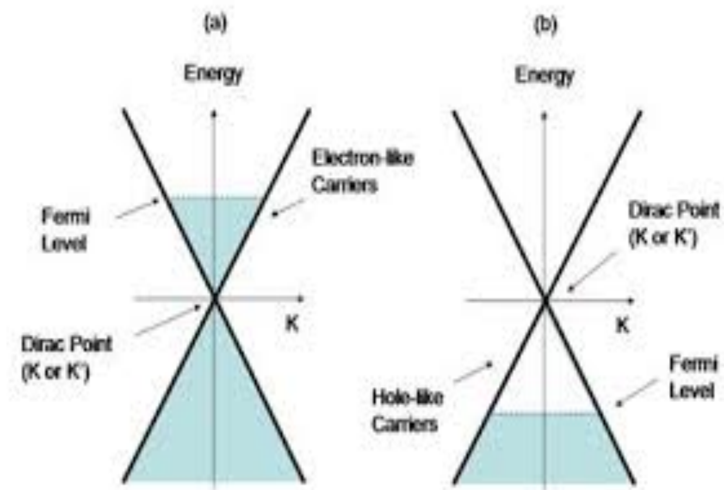
QCP



Both have
I0 Matching symmetry, parameters
ii) Information loss → Universality
iii) Thermodynamics
iv) Transports

Simplest QCP with z=1 : include graphene





Q: strong coupling? …. Yes, generically. Weak in metal due to screening

1. $g^2 = \dfrac{e^2 \cdot c}{4\pi\epsilon\hbar c \cdot v_F} \sim 1$

2. near Dirac Point → Tiny FS → No (insufficient) screening