Masterarbeit

Massive Neutrinos in Supersymmetric Models from Higher than $d = 5$ Effective Operators

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am **29. September 2010**

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Zusammenfassung

Durch die Messung des Flusses solarer Neutrinos auf der Erde sowie durch Experimente zu atmosphärischen und in Kernreaktoren erzeugten Neutrinos, wurde das Phänomen der Flavor-Oszillation von Neutrinos entdeckt. Diese lassen sich nur erklären, wenn Neutrinos unterschiedliche Flavor- und Massen-Eigenzustände besitzen. Daraus folgt auch, dass sie nichtentartete endliche Massen besitzen müssen, die jedoch sehr klein sind. Die geringe Größe dieser Massen lässt sich mit Hilfe des Seesaw-Mechanismus beschreiben. Erweiterungen des Seesaw-Mechanismus haben phänomenologische Eigenschaften, die – im Gegensatz zum gewöhnlichen Seesaw-Mechanismus – mit Experimenten an der TeV-Skala, wie etwa dem *Large Hadron Collider* am CERN, getestet werden können. Eine weitere gut motivierte Theorie jenseits des Standard Modells der Teilchenphysik ist die Supersymmetrie. In dieser Arbeit soll daher die Möglichkeit erweiterter Seesaw-Szenarien in supersymmetrischen Modellen untersucht werden. Nach einer Einführung in die Grundlagen der Neutrinophysik, effektiver Operatoren und der Supersymmetrie, werden daher die Bedingungen für derartige Modelle und ihre Eigenschaften diskutiert, sowie unterschiedliche Beispiele vorgestellt. Anhand eines Modells mit neutralen Fermion-Singlets und geladenen Fermion-Doublets als Mediatoren wird exemplarisch die Möglichkeit der Reproduktion der Flavor-Struktur der leichten Neutrinos gezeigt. Abschließend ist untersucht worden, welche phänomenologischen Eigenschaften dieses Modell im Hinblick auf einen experimentellen Nachweis hat.

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1 Introduction

Neutrinos have been considered massless for a long time. But when the flux of solar neutrinos arriving at Earth was measured in experiments, only a fraction of the predicted value was observed [1, 2]. Assuming that the models of the Sun are reliable, where did the missing neutrinos go? Other experiments concerning atmospheric neutrinos [3] and neutrinos from nuclear reactors [4] were confronted with similar issues.

An answer to this question is given by the phenomenon of flavor oscillations. If some of the electron neutrinos produced in solar fusion processes change into other flavors, not all of them can be detected at the Earth. More recent experiments, which are sensitive to all three flavors using neutral currents [5], have shown indeed, that the total neutrino flux is in agreement with solar models (for a review see, e.g., Ref. [6]). The requirement for neutrino oscillations is that their mass eigenstates are distinct from their flavor states. Furthermore they must have non zero masses which are non degenerate. This on the other hand raises the question, why the observed neutrino masses are so small. Since a description is difficult in the framework of the Standard Model, this points to new physics. A possible explanation is the so called seesaw mechanism, but it sets the new physics scale at the scale of grand unified theories (GUT) (10^{15} GeV), which is not testable with current experiments. One of various possibilities to lower this scale are extended seesaw scenarios, involving higher-dimensional effective operators [7, 8, 9, 10, 11].

Other indications for physics beyond the Standard Model, such as the hierarchy problem, which is the problem of the small Higgs mass, or the unification of gauge couplings, support the existence of a symmetry between fermions and bosons. This supersymmetry (SUSY) can solve those problems, but it cannot easily describe neutrino physics.

Therefore it is the objective of this thesis, to study the possible realization of extended seesaw scenarios in a supersymmetric framework. In Chapter 2 we will give an introduction to the basic aspects of neutrino physics and phenomenology. The seesaw mechanism and more complex theories about the generation of neutrino masses are best understood in the framework of effective theories. Hence their aspects, including higher-dimensional effective operators, are described in the following Chapter. As supersymmetry is important for our approach, its principles are outlined in Chapter 4. After this introductory part, we will discuss the possible realization of extended seesaw scenarios in SUSY. An example of a possible decomposition of an dimension seven effective operator will be studied thereafter. Finally, some phenomenological considerations in regard to this example are presented, which concern the theoretical verifyability of our model.

2 Neutrino Physics

Neutrino Physics is one of the currently most interesting topics in physics, since it points to physics beyond the SM (see e.g. Ref. [12] for a recent review). Here we will present an overview of its theoretical aspects.

2.1 Flavor oscillations

Neutrinos exist as three different flavor states $|\nu_\alpha\rangle$ ($\alpha = e, \mu, \tau$) and three distinct mass eigenstates $|\nu_i\rangle$ ($i = 1, 2, 3$). It is common to use Greek indices for flavor states and Latin ones for mass eigenstates. Hence we can define a unitary flavor mixing matrix U in a flavor diagonal basis, so that

$$|\nu_k\rangle = U_{k\alpha}|\nu_\alpha\rangle \qquad \text{and} \qquad |\nu_\alpha\rangle = U^{-1}_{\alpha k}|\nu_k\rangle\,, \tag{2.1}$$

if the lepton Yukawa coupling Y_e is diagonal. It is also known as Pontecorvo-Maki-Nakagawa-Sakata or PMNS matrix.

The time evolution of the mass eigenstates is given by

$$|\nu_k(t)\rangle = \exp(-\mathrm{i}E_k t)|\nu_k\rangle\,. \tag{2.2}$$

The vacuum transition Amplitude therefore reads

$$A_{\alpha\to\beta} = \langle\nu_\beta|\nu_\alpha(t)\rangle = \langle\nu_k|U_{k\beta}U^{-1}_{\alpha k}|\nu_k(t)\rangle = U^{-1}_{\alpha k}U_{k\beta}\exp(-\mathrm{i}E_k t)\,. \tag{2.3}$$

The transition probability is accordingly

$$P_{\alpha\beta} = |A_{\alpha\to\beta}|^2 = U^{-1}_{\alpha k}U_{k\beta}U^{-1}_{\alpha l}U_{l\beta}\exp\left[-\mathrm{i}(E_k - E_l)t\right]\,. \tag{2.4}$$

Because of their small mass, neutrinos can be considered ultra-relativistic ($E \gg m$). Therefore the approximation

$$E_k = \sqrt{\vec{p}^{\,2} + m_k^2} \approx E + \frac{m_k^2}{2E} \tag{2.5}$$

can be used, which means Eq. (2.4) becomes

$$P_{\alpha\beta} = \delta_{\alpha\beta} - 4\,\mathrm{Re}\,J^{\alpha\beta}_{kl}\sin^2\left(\frac{\Delta m^2_{kl}L}{2E}\right) + 2\,\mathrm{Im}\,J^{\alpha\beta}_{kl}\sin^2\left(\frac{\Delta m^2_{kl}L}{2E}\right)\,, \tag{2.6}$$

where $J^{\alpha\beta}_{kl} = U^{-1}_{\alpha k}U_{k\beta}U^{-1}_{\alpha l}U_{l\beta}$ and $\Delta m^2_{kl} = m_k^2 - m_j^2$. The last term is only non-zero if CP symmetry is violated. Note that the physical observables (at a given energy)

Parameter	Best-fit value
$\Delta m^2_{21}/10^{-5}\,\mathrm{eV}^2$	$7.65^{+0.23}_{-0.20}$
$\Delta m^2_{31}/10^{-3}\,\mathrm{eV}^2$	$2.40^{+0.12}_{-0.11}$
$\sin^2\theta_{12}$	$0.304^{+0.022}_{-0.016}$
$\sin^2\theta_{23}$	$0.50^{+0.07}_{-0.06}$
$\sin^2\theta_{13}$	$0.01^{+0.016}_{-0.011}$

Table 2.1: Experimental best-fit values for neutrino oscillation data. The errors are the 1σ interval. Taken from Ref. [13].

depend only on the *differences* of the squared masses. The current best-fit values are shown in Tab. 2.1. There are two possible hierarchies for the neutrino mass spectrum, the normal hierarchy, where $m_1 < m_2 < m_3$, and the inverted hierarchy, where $m_3 < m_1 < m_2$.

The mixing matrix can be parameterized as

$$U^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}\mathrm{e}^{\mathrm{i}\delta_{CP}} \\ 0 & 1 & 0 \\ -s_{13}\mathrm{e}^{-\mathrm{i}\delta_{CP}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2.7}$$

where $s_{ij} = \sin\theta_{ij}$, $c_{ij} = \cos\theta_{ij}$. δ_{CP} is a CP phase. The mixing angles are parameters. Their best-fit values are also listed in Tab. 2.1. θ_{13} must be very small or even zero. Future experiments will show, whether a non-zero value can be verified. The other two angles are much larger. We might even have maximal mixing between ν_μ and ν_τ, since $\theta_{23} = \pi/4$ is in good accordance with experimental limits. The 3σ interval for $\sin^2\theta_{12}$, however, is $0.25 - 0.37$ [13] so that maximal mixing is unlikely in this case.

In the case of tri-bimaximal mixing [14] we have $\sin^2\theta_{12} = 1/3$, $\sin^2\theta_{23} = 1/2$ and $\sin^2\theta_{13} = 0$, which is in accordance with the mentioned boundaries. The according mixing matrix reads

$$U^{-1} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \tag{2.8}$$

This mixing pattern might be generated by some fundamental symmetries. It will have to be tested in future experiments to what precision this flavor structure is realized in nature.

2.2 Seesaw mechanism

The principle of the seesaw mechanism can be understood by looking at the neutrino mass matrix.

One has to assume that besides the usual left handed (LH) neutrinos ν_L, there are right handed (RH) neutrinos ν_R, which are not strictly forbidden by the SM. Therefore one can construct a Dirac mass term for neutrinos,

$$\begin{aligned} \mathcal{L}^D_{\text{mass}} &= m_D \overline{\nu_R} \nu_L + \text{h.c.} \\ &= \frac{1}{2} \left(m_D \overline{\nu_R} \nu_L + m_D \overline{\nu^c_L} \nu^c_R \right) + \text{h.c.} \end{aligned} \tag{2.9}$$

Since neutrinos have zero electric charge, in general also Majorana mass terms are possible,

$$\mathcal{L}^L_{\text{mass}} = \frac{1}{2} m_L \overline{\nu^c_L} \nu_L + \text{h.c.}\,, \tag{2.10a}$$

$$\mathcal{L}^R_{\text{mass}} = \frac{1}{2} m_R \overline{\nu^c_R} \nu_R + \text{h.c.} = \frac{1}{2} m_R \overline{\nu_R} \nu^c_R + \text{h.c.} \tag{2.10b}$$

A left handed vector n_L can be defined, that contains the neutrino fields,

$$n_L = \begin{pmatrix} \nu_L \\ \nu^c_R \end{pmatrix}, \qquad \overline{n^c_L} = \begin{pmatrix} \overline{\nu^c_L} & \overline{\nu_R} \end{pmatrix}. \tag{2.11}$$

Now one can introduce a mass matrix M, so that

$$\mathcal{L}_{\text{mass}} = \mathcal{L}^D_{\text{mass}} + \mathcal{L}^L_{\text{mass}} + \mathcal{L}^R_{\text{mass}} = \frac{1}{2} \overline{n^c_L} M n_L\,, \tag{2.12}$$

where

$$M = \begin{pmatrix} m_L & m_D \\ m_D^T & m_R \end{pmatrix} \tag{2.13}$$

in the most general case. The positive mass eigenstates for this matrix are

$$m_{1,2} = \left| \frac{1}{2} \left(m_L + m_R \pm \sqrt{(m_L - m_R)^2 + 4 m_D^2} \right) \right|. \tag{2.14}$$

In the seesaw scenario the RH neutrino fields $\nu_R = N_R$ are assumed to be fields with a heavy mass, whereas m_D is of the electroweak scale. Therefore $m_D \ll m_R$. Since ν_L possesses non-zero isospin and hypercharge, the LH Majorana term is forbidden by the SM gauge symmetries, hence $m_L = 0$. This means in a fundamental theory that respects the SM symmetries one obtains the mass eigenstates

$$m_1 \approx \frac{m_D^2}{m_R}\,, \tag{2.15a}$$

$$m_2 \approx m_R\,. \tag{2.15b}$$

As a consequence, one has a neutrino at a mass scale $\Lambda_N = m_R$ of new physics and a very light neutrino, the mass of which is suppressed by m_D/Λ_N. To explain the low experimental upper limit for the neutrino mass, the new mass scale has to be close to the GUT scale.

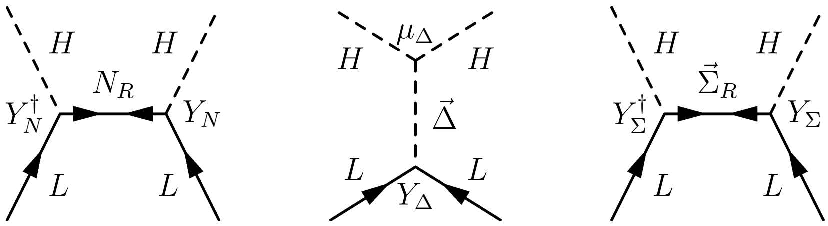


Figure 2.1: The three types of seesaw mechanism. H denotes the Higgs field, L the left handed lepton SU(2) doublet containing the SM neutrinos and N_R, Δ and $\vec{\Sigma}_R$ are the respective heavy mediators. The coupling constants Y are similar to the Yukawa couplings of the SM.

2.3 The three types of seesaw mechanism

There are only three different realizations of the seesaw mechanism at tree level [15], which are shown in Fig. 2.1. The existence of these three types is explained by the fact that two doublets can be decomposed into a triplet and a singlet ($\mathbf{2} \otimes \mathbf{2} = \mathbf{3} \oplus \mathbf{1}$). (This is an analogy to the addition of two spin 1/2 particles.) Hence the left handed leptons and the Higgs doublet of the SM can couple to a triplet and a singlet.

If the heavy particle is a scalar, a pair of Higgs doublets appear at the same vertex in the Feynman graph (Fig. 2.1). Since these are scalar boson fields, they have to be in a symmetric state, which the triplet is but not the singlet. Hence a coupling to a singlet scalar is prohibited. The same constraint can also be deduced from the lepton vertex. The combination $\nu_i e_j - e_i \nu_j$ would be an isospin singlet, but solely the triplet state $\nu_i \nu_j$ can generate an effective neutrino mass.

Type I

The first type of the seesaw mechanism couples the lepton and the Higgs fields via the exchange of a heavy virtual fermion N_R, which is a singlet under all SM gauge groups [16, 17, 18, 19].

Type II

By replacing the fermion SU(2) singlet by a scalar SU(2) triplet $\vec{\Delta}$, one obtains the second type of the seesaw mechanism. Furthermore the topology of the Feynman graph changes, because a vertex involving leptons always must have two fermion lines to obtain a Lagrangian of an integer dimension [20, 21].

Type III

The third type is nearly the same as the first one, except for the replacement of the fermion SU(2)-singlet by a fermion SU(2)-triplet [22, 23].

In the following we will discuss the structure of these three realizations of the seesaw mechanism (see also Ref. [24]).

2.3.1 Type I: Fermion singlet

In type I the left-handed lepton fields couple to the right handed heavy fermion singlet fields. They also have to couple to the Higgs field, which gives the neutrinos their mass after electroweak symmetry breaking. Furthermore there have to be terms that describe the dynamics of the heavy fields. Therefore the Lagrangian $\mathcal{L} = \mathcal{L}^{\mathrm{SM}} + \mathcal{L}^{N}$ reads:

$$\mathcal{L}^{N} = i\overline{N_R}\partial\!\!\!/ N_R - \left[\frac{1}{2}\overline{N_R}M_N {N_R}^c + \bar{L}_L\widetilde{H}Y_N^\dagger N_R + \text{h.c.}\right], \tag{2.16}$$

where $\widetilde{H} = i\tau_2 H^*$ and the notation $\psi_R \equiv P_R\psi \equiv (1-\gamma_5)\psi$ is used. Due to this additional interaction, the Yukawa couplings of the leptons,

$$\bar{L}_L H Y_e e_R + \bar{L}_L \widetilde{H} Y_N^\dagger N_R + \text{h.c.}\,, \tag{2.17}$$

are now formally equivalent to those of the quarks,

$$\bar{q}_L H Y_d d_R + \bar{q}_L \widetilde{H} Y_u^\dagger u_R + \text{h.c.} \tag{2.18}$$

The total weak hypercharge Y of the new terms has to be zero.[1] Since L_L has hypercharge $-1/2$ and H has $+1/2$, the hypercharge of the conjugated fields is $Y(\overline{L_L}) = +1/2$ and $Y(\widetilde{H}) = -1/2$. Therefore the heavy fields have $Y(N_R) = 0$.

To check the electromagnetic charge, one has to sum over the SU(2) components of the doublets,

$$\overline{L_L}\,\widetilde{H}N_R = \overline{\nu_L}H_0^* N_R - \overline{e_L}H_- N_R\,, \tag{2.19}$$

where $\widetilde{H} = \begin{pmatrix} H_0^* \\ -H_- \end{pmatrix} = i\tau_2 H^* = i\tau_2 \begin{pmatrix} H_+ \\ H_0 \end{pmatrix}^*$. Since $Q(N_R) = 0$ the total charge is zero.

2.3.2 Type II: Scalar Triplet

The Lagrangian for the type II seesaw contains the additional terms

$$\mathcal{L}_{\Delta}^{\text{coupling}} = \overline{\widetilde{L_L}}Y_\Delta(\vec{\tau}\cdot\vec{\Delta})L_L + \mu_\Delta \widetilde{H}^\dagger(\vec{\tau}\cdot\vec{\Delta})^\dagger H + \text{h.c.}\,, \tag{2.20}$$

where $\overline{\widetilde{L_L}} = \overline{L_L^c}i\tau_2 = \begin{pmatrix} -\overline{e_L^c} & \overline{\nu_L^c} \end{pmatrix}$.

Since L_L and also $\overline{\widetilde{L_L}}$ have hypercharge $-1/2$, the scalar $\vec{\Delta}$ must have hypercharge $+1$.

[1] The Definition $Q = T_3 + Y$ is used, where Q is the electromagnetic charge and T_3 is the third component of the weak isospin. One finds also often the definition $Q = T_3 + \frac{Y}{2}$ in literature.

The electromagnetic charge states can be obtained from the couplings to the leptons.

$$\overline{\widetilde{L_L}}(\vec{\tau}\cdot\vec{\Delta})L_L = \begin{pmatrix}-\overline{e_L^c} & \overline{\nu_L^c}\end{pmatrix}\begin{pmatrix}\Delta_3 & \Delta_1 - i\Delta_2 \\ \Delta_1 + i\Delta_2 & -\Delta_3\end{pmatrix}\begin{pmatrix}\nu_L \\ e_L\end{pmatrix}$$

$$= -\overline{e_L^c}\Delta_3\nu_L - \overline{e_L^c}(\Delta_1 - i\Delta_2)e_L + \overline{\nu_L^c}(\Delta_1 + i\Delta_2)\nu_L - \overline{\nu_L^c}\Delta_3 e_L \tag{2.21}$$

Since e_L and $\overline{e_L^c}$ have charge -1 and ν_L and $\overline{\nu_L^c}$ have no charge, the charged states read:

$$\Delta^{++} \equiv \frac{1}{\sqrt{2}}(\Delta_1 - i\Delta_2)\,, \quad \Delta^{+} \equiv \Delta_3\,, \quad \Delta^0 \equiv \frac{1}{\sqrt{2}}(\Delta_1 + i\Delta_2)\,. \tag{2.22}$$

The minimal Lagrangian is

$$\mathcal{L}_\Delta = \left(D_\mu\vec{\Delta}\right)^\dagger\left(D^\mu\vec{\Delta}\right) + \left(\overline{\widetilde{L_\mathrm{L}}}Y_\Delta(\vec{\tau}\cdot\vec{\Delta})L_\mathrm{L} + \mu_\Delta\widetilde{H}^\dagger(\vec{\tau}\cdot\vec{\Delta})^\dagger H + \text{h.c.}\right) - \Big[\vec{\Delta}^\dagger M_\Delta{}^2\vec{\Delta}$$
$$+\frac{1}{2}\lambda_2\left(\vec{\Delta}^\dagger\vec{\Delta}\right)^2 + \lambda_3\left(H^\dagger H\right)\left(\vec{\Delta}^\dagger\vec{\Delta}\right) + \frac{\lambda_4}{2}\left(\vec{\Delta}^\dagger T^i\vec{\Delta}\right)^2 + \lambda_5\left(\vec{\Delta}^\dagger T^i\vec{\Delta}\right)H^\dagger\tau^i H\Big]\,, \tag{2.23}$$

where

$$T_1 = \begin{pmatrix}0 & 0 & 0\\ 0 & 0 & -\mathrm{i}\\ 0 & \mathrm{i} & 0\end{pmatrix}\,, \quad T_2 = \begin{pmatrix}0 & 0 & \mathrm{i}\\ 0 & 0 & 0\\ -\mathrm{i} & 0 & 0\end{pmatrix}\,, \quad T_3 = \begin{pmatrix}0 & -\mathrm{i} & 0\\ \mathrm{i} & 0 & 0\\ 0 & 0 & 0\end{pmatrix}\,. \tag{2.24}$$

The quartic couplings, λ_2 and λ_4, can be neglected, since the heavy fields appear only as virtual particles at low energies and therefore quartic couplings have to be considered only at higher orders of perturbation. The remaining terms with λ_3 and λ_5 give corrections to the heavy scalar mass after the electroweak symmetry breaking.

2.3.3 Type III: Fermion triplet

The type III seesaw mechanism is obtained by replacing the fermion singlet of type I with a triplet, so that

$$\mathcal{L}_\Sigma = i\,\overline{\vec{\Sigma}}_R \not{D}\,\vec{\Sigma}_R - \left[\frac{1}{2}\overline{\vec{\Sigma}}_R M_\Sigma \vec{\Sigma}_R^c + \overline{\vec{\Sigma}}_R Y_\Sigma(\widetilde{H}^\dagger\vec{\tau}L_L) + \text{h.c.}\right]\,. \tag{2.25}$$

The heavy triplet $\vec{\Sigma}_R$ has hypercharge zero. The electromagnetic charge states are

$$\Sigma^\pm \equiv \frac{\Sigma_1 \mp i\Sigma_2}{\sqrt{2}}\,, \qquad \Sigma_0 \equiv \Sigma_3\,. \tag{2.26}$$

A more detailed description of the seesaw mechanism can be given in the framework of effective theories, which will be explained in the next chapter.

3 Effective Operators

Physics at high energy scales can also affect the phenomenology at much lower scales. In this case it is not necessary to have a detailed understanding of the fundamental theory, but instead one can use an effective theory as a good approximation.

3.1 Effective theories in general

Assume a fundamental theory containing heavy fields with masses at a high energy scale Λ. At energies below this mass scale, these particles cannot be directly observed. In scattering processes, however, they appear off-shell as virtual states.

Technically spoken, one can expand the propagator of the heavy field, since the mass is much larger than the kinetic energy. For fermions this means

$$\frac{1}{\partial\!\!\!/ - M} = -\frac{1}{M} - \frac{\partial\!\!\!/}{M^2} + \cdots , \tag{3.1}$$

Since Feynman graphs with several propagators of the heavy particles are possible, one obtains a tower of effective operators, which have to be added to the SM Lagrangian:

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda}\mathcal{L}^{d=5} + \frac{1}{\Lambda^2}\mathcal{L}^{d=6} + \frac{1}{\Lambda^3}\mathcal{L}^{d=7} + \cdots \tag{3.2}$$

For each propagator involved, one gets a suppression factor $\frac{1}{M} \sim \frac{1}{\Lambda}$.

Heavy fields at a mass scale M, like those in the seesaw mechanism, can be integrated out to obtain an effective theory, which is valid for energies $E \ll M$ and which only contains the low energy fields. A well known example is the Fermi interaction. It is an effective theory for the weak interaction without W- and Z-bosons. Therefore it is only viable at energies $E \ll m_Z \approx 10^2$ GeV.

For example[1] the weak decay process $c \longrightarrow s u \bar{d}$ has the tree-level amplitude

$$M = -\frac{g^2}{8} V_{cs}^* V_{ud} \bar{s}\gamma_\mu (1-\gamma_5) c \frac{g^{\mu\nu}}{k^2 - m_W^2} \bar{u}\gamma_\nu (1-\gamma_5) d \,, \tag{3.3}$$

[1] The example has been taken from Ref. [25].

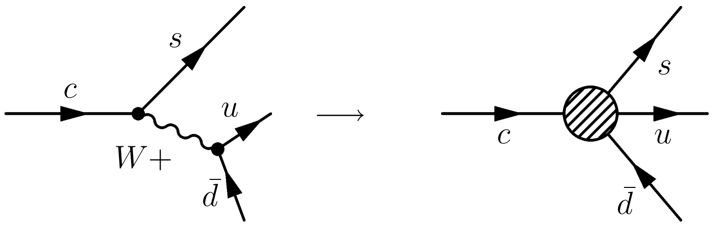


Figure 3.1: Fermi theory of weak interaction. The QFD Feynman graphs are replaced by effective four-vertices.

where V is the CKM matrix and g is the coupling constant of the weak interaction. After the expansion of the propagator it becomes

$$M_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} \bar{s}\gamma_\mu (1-\gamma_5) c g^{\mu\nu} \bar{u}\gamma_\nu (1-\gamma_5) d \,, \tag{3.4}$$

where the Fermi constant G_F is an effective coupling, obtained as $\frac{G_F}{\sqrt{2}} = \frac{g_2^2}{8m_W^2}$, and is suppressed by the weak scale $\Lambda_{\text{weak}} \sim m_W$. Fig. 3.1 shows how the weak interaction is replaced by an effective 4-vertex.

The expansion of the propagator gives us only an effective interaction. We want, however, to obtain the complete effective Lagrangian and not just single interactions. This will be discussed in the following sections.

3.2 Effective Action

The effective theory can be calculated in the path integral formalism[2]. In the simple case of a scalar field H path integrals have the form

$$\int \mathcal{D}H \, e^{iS[H]} = \int \mathcal{D}H \, e^{i\int d^4x \mathcal{L}[H(x)]} \,, \tag{3.5}$$

where $S[H]$ is the action depending on the configuration of H, and $\mathcal{D}H$ is the integration measure.

If an additional heavy scalar field ϕ appears, one obtains

$$\int \mathcal{D}H \mathcal{D}\phi \, e^{iS[\phi,H]} \,. \tag{3.6}$$

Since the heavy fields do not appear explicitly at low energies, one can separate the low energy (LE) and high energy (HE) sector and obtains

$$\int \mathcal{D}H \, e^{iS_{\text{LE}}[H]} \int \mathcal{D}\phi \, e^{iS_{\text{HE}}[\phi,H]} \,. \tag{3.7}$$

[2]For more details on path integrals and effective field theory see for example Ref. [26], Sec. 5 of Ref. [25] or Ref. [27].

This can be rewritten as

$$\int \mathcal{D}H\, e^{iS_{\text{eff}}[H]}\,, \tag{3.8}$$

where $e^{iS_{\text{eff}}[H]} = e^{iS_{\text{LE}}[H]} \int \mathcal{D}\phi\, e^{iS_{\text{HE}}[\phi,H]}$. With $S_{\text{eff}}[H] = \int d^4x \mathcal{L}_{\text{eff}}(H)$ one has now obtained an effective Lagrangian which depends only on the light field.

This procedure can now be adapted to more sophisticated scenarios [28]. In the type I seesaw an effective action S_{eff} is obtained by separating the terms involving the heavy fermion fields and their interactions S_N and the ones involving the SM interactions and fields S_{SM},

$$e^{iS_{\text{eff}}} \equiv \int \mathcal{D}N\mathcal{D}\overline{N}e^{iS} = e^{iS_{\text{SM}}} \int \mathcal{D}N\mathcal{D}\overline{N}e^{iS_N}\,, \tag{3.9}$$

where N is the heavy field and $\mathcal{D}N$ is the integration measure.

$S_N \equiv S_N[N]$ can be expanded around the stationary configuration N_0, so that

$$\begin{aligned} e^{iS_N^{\text{eff}}} &= \int \mathcal{D}N\mathcal{D}\overline{N}e^{iS_N[N]} \\ &= \int \mathcal{D}N\mathcal{D}\overline{N}e^{i(S_N[N_0]+\delta S_N[N_0]+\delta^2 S_N[N_0]+\cdots)} \\ &= e^{iS_N[N_0]} \int \mathcal{D}N\mathcal{D}\overline{N}e^{i\delta^2 S_N[N_0]+\cdots} \\ &\approx e^{iS_N[N_0]}\,. \end{aligned} \tag{3.10}$$

where

$$\delta S[N_0] \equiv \left.\frac{\delta S}{\delta N}\right|_{N_0} + \left.\frac{\delta S}{\delta \overline{N}}\right|_{\overline{N_0}}\,. \tag{3.11}$$

This order has to be zero due to the requirement of stationarity. The higher orders contain higher powers of the couplings of the fields. Since (in the full theory) the heavy fields are only virtual states at low energies, these terms do not appear at tree level. Hence they can be perturbatively neglected. Therefore the effective action reads

$$S_{\text{eff}} = \int d^4x\, \mathcal{L}_{\text{eff}} = S_{\text{SM}} + S_N[N_0] = \int d^4x\, (\mathcal{L}_{\text{SM}} + \mathcal{L}_N(N_0))\,. \tag{3.12}$$

The stationary fields are defined by the condition

$$\left.\frac{\delta \mathcal{L}}{\delta N_i}\right|_{N_{0i}} = 0 \qquad\qquad \left.\frac{\delta \mathcal{L}}{\delta \overline{N}_i}\right|_{\overline{N_{0i}}} = 0\,. \tag{3.13}$$

Inserting the stationary fields in $S_N[N_0]$ leads to the effective Lagrangian

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \mathcal{L}_N(N_0) = \mathcal{L}_{\text{SM}} + \delta\mathcal{L}^{d=5} + \delta\mathcal{L}^{d=6} + \cdots \tag{3.14}$$

For $d = 5$ there is only one possible operator, which is the WEINBERG operator [29]

$$\delta\mathcal{L}^{d=5} = \frac{1}{2} c^{d=5}_{\alpha\beta} (\overline{L^c_{L\alpha}} \widetilde{H}^*)(\widetilde{H}^\dagger L_{L\beta}) + \text{h.c.}\,, \tag{3.15}$$

where the coefficient matrix $c^{d=5}_{\alpha\beta}$ is of $\mathcal{O}(1/M)$. After electroweak symmetry breaking this term becomes a Majorana mass term of $\mathcal{O}(v^2/M)$ by inserting the Higgs VEV (vacuum expectation value). This term is at the scale of neutrino masses if M is close to the GUT scale, assuming that the Yukawa couplings are of $\mathcal{O}(1)$.

Eq. (3.13) can also be interpreted from another point of view: Since the mass terms are much larger than the kinetic terms of the heavy fields, the kinetic terms can be neglected. This procedure can be outlined as follows:

By integrating out heavy fields, integrals of the form

$$\int \mathcal{D}N\mathcal{D}\overline{N} e^{i\int d^4x\left(-\frac{1}{2}\overline{N}D^{-1}N + \overline{J}N\right)} \propto e^{\frac{1}{2}\overline{J}DJ} \tag{3.16}$$

appear, where N_i are the heavy fields, J contains the low energy fields to which the latter couple and D is the propagator,

$$D = \frac{1}{\not\partial - M} \qquad\qquad D^{-1} = \not\partial - M\,. \tag{3.17}$$

This means, that the expansion of the propagator

$$D = \frac{1}{\not\partial - M} \approx -\frac{1}{M} + \cdots \tag{3.18}$$

equals neglecting the kinetic terms of the Lagrangian

$$D^{-1} = (\not\partial - M) \approx -M + \cdots\,, \tag{3.19}$$

which means for $\delta S = 0$ we can use the equations of motion, which become

$$\partial_\mu \frac{\partial\mathcal{L}}{\partial(\partial_\mu N)} - \frac{\partial\mathcal{L}}{\partial N} = 0 \longrightarrow \frac{\partial\mathcal{L}}{\partial N} \approx 0\,. \tag{3.20}$$

3.3 Effective operators for the seesaw mechanism

We will now derive the effective operators for the three types of the seesaw mechanism by integrating out the heavy mediators [28, 24].

3.3.1 Type I seesaw

The Lagrangain for the heavy fermion singlets reads

$$\mathcal{L}_N = i\overline{N_R}\partial\!\!\!/ N_R - \overline{L_L}\widetilde{H}Y_N^\dagger N_R - \frac{1}{2}\overline{N_R}M_N N_R^c + \text{h.c.} \tag{3.21}$$

The covariant derivative D_μ can be replaced by ∂_μ since as SM singlets the heavy fermions do not interact with the gauge fields. The complex eigenvalues of the Majorana mass matrix can be expressed as $M_i = e^{\theta_i}|M_i| \equiv \eta_i|M_i|$, where η_i is a phase. Working in a real and diagonal basis of M one can define

$$\begin{aligned} N_i &\equiv e^{i\theta_i/2}N_{Ri} + e^{-i\theta_i/2}N_{Ri}^c \\ &= \sqrt{\eta_i}N_{Ri} + \sqrt{\eta_i^*}N_{Ri}^c \end{aligned} \tag{3.22}$$

so that $N_i = N_i^c$, which are the Majorana mass eigenstates. Eq. (3.21) can then be rewritten as

$$\begin{aligned} \mathcal{L}^N &= \frac{1}{2}\overline{N}_i\left(i\partial\!\!\!/ - M_i\right)N_i \\ &- \frac{1}{2}\left[\overline{L_L}\,\widetilde{H}\,Y_\nu\sqrt{\eta^*} + \overline{L_L{}^c}\,\widetilde{H}^*\,Y_\nu^*\,\sqrt{\eta}\right]_i N_i \\ &- \frac{1}{2}\overline{N_i}\left[\sqrt{\eta^*}\,Y_\nu^\intercal\,\widetilde{H}^\intercal\,L_L{}^c + \sqrt{\eta}\,Y_\nu^\dagger\,\widetilde{H}^\dagger\,L_L\right]_i . \end{aligned} \tag{3.23}$$

Using Eq. (3.13) one obtains the equations

$$\overline{N_{0i}}\,(-i\overleftarrow{\partial\!\!\!/} - M_i) - \left(\overline{L_L}\,\widetilde{H}\,Y_\nu\,\sqrt{\eta^*} + \overline{L_L{}^c}\,\widetilde{H}^*\,Y_\nu^*\,\sqrt{\eta}\right)_i = 0\,, \tag{3.24}$$

$$(i\overrightarrow{\partial\!\!\!/} - M_i)\,N_{0i} - \left(\sqrt{\eta}\,Y_\nu^\dagger\,\widetilde{H}^\dagger\,L_L + \sqrt{\eta^*}\,Y_\nu^\intercal\,\widetilde{H}^\intercal\,L_L{}^c\right)_i = 0\,. \tag{3.25}$$

Now we solve for N and eliminate it from Eq. (3.23):

$$\begin{aligned} S_N[N_0] &\approx -\frac{1}{2}\int d^4x \left(\overline{L_L}\,\widetilde{H}\,Y_\nu\,\sqrt{\eta^*} + \overline{L_L{}^c}\,\widetilde{H}^*\,Y_\nu^*\sqrt{\eta}\,\right)_i \left(\frac{\delta_{ij}}{i\overrightarrow{\partial\!\!\!/} - M_i}\right) \\ &\times \left(\sqrt{\eta}\,Y_\nu^\dagger\,\widetilde{H}^\dagger\,L_L + \sqrt{\eta^*}\,Y_\nu^\intercal\,\widetilde{H}^\intercal\,L_L{}^c\right)_j \\ &\equiv \int d^4x\;\delta\mathcal{L}\,. \end{aligned} \tag{3.26}$$

After expanding the propagator,

$$\frac{1}{i\overrightarrow{\partial\!\!\!/} - M} = -\frac{1}{M} - \frac{i\overrightarrow{\partial\!\!\!/}}{M^2} + \cdots\,, \tag{3.27}$$

the terms of $\delta\mathcal{L}$ can be combined to obtain the higher dimensional operators. This results in the effective $d = 5$ operator

$$\delta\mathcal{L}^{d=5} = \frac{1}{2}c_{\alpha\beta}^{d=5}\left(\overline{L_{L\alpha}^c}\widetilde{H}^*\right)\left(\widetilde{H}^\dagger\,L_{L\beta}\right) + \text{h.c.}\,, \tag{3.28}$$

where

$$c^{d=5} = Y_N^\mathsf{T} \frac{1}{M_N} Y_N \,. \tag{3.29}$$

After electroweak symmetry breaking, the neutrino mass is obtained by inserting the Higgs VEV in Eq. (3.28),

$$m_\nu \equiv -\frac{v^2}{2} c^{d=5} = -Y_N^\mathsf{T} \frac{v^2}{M_N} Y_N . \tag{3.30}$$

With the next order in the propagator expansion Eq. (3.27) the dimension 6 operator

$$\delta\mathcal{L}^{d=6} = c^{d=6}_{\alpha\beta} \left(\overline{L_{L\alpha}} \widetilde{H}\right) i\partial\!\!\!/ \left(\widetilde{H}^\dagger L_{L\beta}\right) \tag{3.31}$$

can be constructed, where

$$c^{d=6} = Y_N^\dagger \frac{1}{M_N^\dagger} \frac{1}{M_N} Y_N \,. \tag{3.32}$$

As we will show below, Eq. (3.31) leads to non-unitarity effects. This is also known as “minimal unitarity violation” [30].

After symmetry breaking this operator adds corrections to the kinetic terms of the neutrinos, so that

$$\mathcal{L}_\nu^{\text{kin}} = i\bar{\nu}_{L\alpha} \partial\!\!\!/ (\delta_{\alpha\beta} + \epsilon^N_{\alpha\beta}) \nu_{L\beta} \tag{3.33}$$

where

$$\epsilon^N \equiv \frac{v^2}{2} c^{d=6} \,. \tag{3.34}$$

By rescaling the neutrino fields

$$\nu_{L\alpha} \longrightarrow \nu'_{L\alpha} \equiv (\delta_{\alpha\beta} + \epsilon^N_{\alpha\beta})^{\frac{1}{2}} \nu_{L\beta} \tag{3.35}$$

the kinetic term can be rewritten as

$$\mathcal{L}_\nu^{\text{kin}} = i\bar{\nu}'_{L\alpha} \partial\!\!\!/ \delta_{\alpha\beta} \nu'_{L\beta} \tag{3.36}$$

But this redefinition also affects the interactions of the neutrinos, for example the charged current interaction reads now in the rescaled flavor basis

$$\mathcal{L}_{CC} = \frac{g}{\sqrt{2}} \overline{e_{L\alpha}} W\!\!\!/^- \left(\delta_{\alpha\beta} - \frac{1}{2}\epsilon^N_{\alpha\beta}\right) \nu'_{L\beta} + \text{h.c.} \,. \tag{3.37}$$

Since we have modified our flavor basis we also have to modify the U_{PMNS} matrix for rotating from the rescaled flavor states to the mass eigenstates of the neutrinos.

Because the rescaling is flavor-dependent, the mixing matrix has to be replaced by a non-unitary matrix N. After unphysical phases have been reabsorbed in the lepton field definition it reads

$$N = \left(1 - \frac{\epsilon^N}{2}\right) U_{\mathrm{PMNS}} \,. \tag{3.38}$$

So the weak currents in the mass basis read

$$J_\mu^{-\,CC} \equiv \overline{e_{L\alpha}}\, \gamma_\mu\, N_{\alpha i}\, \nu_i , \tag{3.39a}$$

$$J_\mu^{NC} \equiv \frac{1}{2} \overline{\nu}_i\, \gamma_\mu (N^\dagger\, N)_{ij}\, \nu_j . \tag{3.39b}$$

This in turn changes the observable parameters such as the Fermi constant measured from $\mu \to \nu_\mu \bar{\nu}_e e$ decays, which is connected to $G_F^{SM} = \sqrt{2} g^2/(8 M_W^2)$, the SM Fermi constant at tree level, by

$$G_F = G_F^{SM} \sqrt{(N N^\dagger)_{ee} (N N^\dagger)_{\mu\mu}} \,, \tag{3.40}$$

since $N N^\dagger = (1 - \epsilon^N)$ at $\mathcal{O}(1/M_N^2)$ and hence $\nu_e' = (1-\epsilon^N)_{ee}^{\frac{1}{2}}\, \nu_e = \sqrt{(N N^\dagger)_{ee}}\, \nu_e$ (accordingly for ν_μ').

3.3.2 Type II seesaw

The minimal Lagrangian for a heavy scalar triplet written with the triplet indices in the mass basis and neglecting the quartic couplings reads

$$\begin{aligned} \mathcal{L}_\Delta =& \Delta_a^\dagger (D^\mu)^2_{ab} \Delta_b + \left(\widetilde{\overline{L_{\mathrm{L}}}} Y_\Delta (\tau_a \Delta_a) L_{\mathrm{L}} + \mu_\Delta \widetilde{H}^\dagger (\Delta_a^\dagger \tau_a) H + \mathrm{h.c.} \right) \\ & - \left[\Delta_a^\dagger {M_\Delta}^2 \delta_{ab} \Delta_b + \lambda_3 \left(H^\dagger H\right) \left(\Delta_a^\dagger \delta_{ab} \Delta_b\right) + \lambda_5 \left(\Delta_a^\dagger T^i_{ab} \Delta_b\right) H^\dagger \tau^i H \right] \\ =& \Delta_a^\dagger \left[(D_\mu)^2 - \lambda_5 \overrightarrow{T} H^\dagger \overrightarrow{\tau} H - \left({M_\Delta}^2 + \lambda_3 (H^\dagger H)\right) \mathbb{1}_{\mathrm{isospin}} \right]_{ab} \Delta_b \\ & + \left[\left(\widetilde{\overline{L_{\mathrm{L}}}} Y_\Delta \tau_a L_{\mathrm{L}} + \mu_\Delta^* H^\dagger \tau_a \widetilde{H} \right) \Delta_a + \mathrm{h.c.} \right] , \end{aligned} \tag{3.41}$$

where $(D^\mu)^2_{ab} \equiv (\overleftarrow{D}^\mu)^\dagger \overrightarrow{D}_\mu \delta_{ab}$ and $\mathbb{1}_{\mathrm{isospin}}$ is the identity in the basis of the triplet states Δ_1, Δ_2 and Δ_3, so that $(\mathbb{1}_{\mathrm{isospin}})_{ab} = \delta_{ab}$.

Modifying the formalism from 3.2 for heavy scalar fields one obtains in analogy to Eq. (3.13)

$$\left.\frac{\delta \mathcal{L}}{\delta \Delta_a}\right|_{(\Delta_0)_a} = 0 \,, \qquad \left.\frac{\delta \mathcal{L}}{\delta (\Delta^\dagger)_a}\right|_{(\Delta_0^\dagger)_a} = 0 \,. \tag{3.42}$$

One can determine the solutions

$$(\Delta_0)_a = \left[(D_\mu)^2 - \lambda_5 \overrightarrow{T} H^\dagger \overrightarrow{\tau} H - \left({M_\Delta}^2 + \lambda_3 (H^\dagger H)\right) \mathbb{1}_{\mathrm{isospin}} \right]^{-1}_{ab} \left[\mu_\Delta \widetilde{H}^\dagger \tau_b H + \overline{L_{\mathrm{L}}} Y_\Delta^\dagger \tau_b \widetilde{L_{\mathrm{L}}} \right] \tag{3.43a}$$

$$(\Delta_0^\dagger)_a = \left[\mu_\Delta^* H^\dagger \tau_b \widetilde{H} + \widetilde{\overline{L_{\mathrm{L}}}} \tau_b Y_\Delta L_{\mathrm{L}} \right] \left[(D_\mu)^2 - \lambda_5 \overrightarrow{T} H^\dagger \overrightarrow{\tau} H - \left({M_\Delta}^2 + \lambda_3 (H^\dagger H)\right) \mathbb{1}_{\mathrm{isospin}} \right]^{-1}_{ba} . \tag{3.43b}$$

Inserting this in Eq. (3.41) one obtains

$$\mathcal{L}_{\Delta}^{\text{eff}} = \left[\mu_{\Delta}^{*} H^{\dagger}\tau_a \widetilde{H} + \overline{\widetilde{L_{\text{L}}}}\tau_a Y_{\Delta} L_{\text{L}}\right] \left[(D_\mu)^2 - \lambda_5 \overrightarrow{T} H^{\dagger} \overrightarrow{\tau} H - \left({M_{\Delta}}^2 + \lambda_3 (H^{\dagger}H)\right) \mathbb{1}_{\text{isospin}}\right]^{-1}_{ab} \times \left[\mu_{\Delta}\widetilde{H}^{\dagger}\tau_b H + \overline{L_{\text{L}}} Y_{\Delta}^{\dagger}\tau_b \widetilde{L_{\text{L}}}\right] . \tag{3.44}$$

The propagator can be expanded in inverse powers of M_{Δ}^2,

$$\left[(D_\mu)^2 - \lambda_5 \overrightarrow{T} H^{\dagger} \overrightarrow{\tau} H - \left({M_{\Delta}}^2 + \lambda_3 (H^{\dagger}H)\right)\right]^{-1} \approx -\frac{1}{M_{\Delta}^2} + \frac{(D_\mu)^2 - \lambda_5 \overrightarrow{T} H^{\dagger} \overrightarrow{\tau} H - \lambda_3 (H^{\dagger}H)}{M_{\Delta}^4} + \cdots \tag{3.45}$$

Expanding Eq. (3.44) to $\mathcal{O}(M_{\Delta}^{-2})$ one obtains besides a dimension four operator correcting the four Higgs coupling a dimension five operator,

$$\delta\mathcal{L}^{d=5} = \frac{1}{4} c_{\alpha\beta}^{d=5} \left(\overline{\widetilde{L_{L\alpha}}} \overrightarrow{\tau} L_{L\beta}\right) \left(\widetilde{H}^{\dagger} \overrightarrow{\tau} H\right) + \text{h.c.} , \tag{3.46}$$

where

$$c_{d=5} = 4Y_{\Delta}\frac{\mu_{\Delta}}{M_{\Delta}^2} . \tag{3.47}$$

Eq. (3.46) can be rewritten to match the form of Eq. (3.28) After inserting the Higgs VEV the neutrino mass reads

$$m_\nu = -2Y_{\Delta}v^2\frac{\mu_{\Delta}}{M_{\Delta}^2} . \tag{3.48}$$

Including the next order in the expansion of the propagator, one obtains an effective dimension six Lagrangian. It consists of several components in the type II case

$$\delta\mathcal{L}_{\Delta}^{d=6} = \delta\mathcal{L}_{4F} + \delta\mathcal{L}_{HD} + \delta\mathcal{L}_{6H} , \tag{3.49}$$

where

$$\delta\mathcal{L}_{4F} = \frac{1}{{M_{\Delta}}^2}\left(\overline{\widetilde{L_{\text{L}}}}\, Y_{\Delta}\, \overrightarrow{\tau} L_{\text{L}}\right)\left(\overline{L_{\text{L}}}\, \overrightarrow{\tau}\, Y_{\Delta}^{\dagger}\, \widetilde{L_{\text{L}}}\right) \tag{3.50a}$$

$$\delta\mathcal{L}_{6H} = -2\left(\lambda_3 + \lambda_5\right)\frac{|\mu_{\Delta}|^2}{M_{\Delta}^4}\left(H^{\dagger}H\right)^3 \tag{3.50b}$$

$$\delta\mathcal{L}_{HD} = \frac{|\mu_{\Delta}|^2}{M_{\Delta}^4}\left(H^{\dagger}\overrightarrow{\tau}\widetilde{H}\right)\left(\overleftarrow{D_\mu}\overrightarrow{D^\mu}\right)\left(\widetilde{H}^{\dagger}\overrightarrow{\tau}H\right) . \tag{3.50c}$$

These terms describe a four fermion coupling, a six scalar coupling and corrections to the kinetic Higgs terms. In contrast to the single fermion type, also the SM gauge fields are involved since

$$D_\mu = \partial_\mu - \text{i}g\frac{\tau_a}{2}W_{a\mu} - \text{i}g' B_\mu Y . \tag{3.51}$$

3.3.3 Type III seesaw

The type III seesaw mechanism is obtained by replacing the fermion singlet of type I with a triplet, so that

$$\mathcal{L}_\Sigma = i\,\overline{\vec{\Sigma}_R} \not{D}\, \vec{\Sigma}_R - \left[\frac{1}{2}\overline{\vec{\Sigma}_R} M_\Sigma \vec{\Sigma}_R^c + \overline{\vec{\Sigma}_R} Y_\Sigma (\widetilde{H}^\dagger \vec{\tau} L_L) + \text{h.c.}\right]. \tag{3.52}$$

After integrating out the heavy fields in analogy to the type I seesaw one obtains

$$\delta\mathcal{L}^{d=5} = \frac{1}{2} c^{d=5}_{\alpha\beta} (\overline{\widetilde{L_{L\alpha}}} \vec{\tau} H)(\widetilde{H}^\dagger \vec{\tau} L_{L\beta}) + \text{h.c.} \tag{3.53}$$

where

$$c^{d=5} = Y_\Sigma^{\mathsf{T}} \frac{1}{M_\Sigma} Y_\Sigma \,. \tag{3.54}$$

This is again equivalent to Eq. (3.28) , so that the neutrino mass is

$$m_\nu = -\frac{v^2}{2} Y_\Sigma^{\mathsf{T}} \frac{1}{M_\Sigma} Y_\Sigma \,. \tag{3.55}$$

For dimension six one obtains

$$\delta\mathcal{L}^{d=6} = c^{d=6}_{\alpha\beta} \left(\overline{L_{L\alpha}} \vec{\tau} \widetilde{H}\right) i \not{D} \left(\widetilde{H}^\dagger \vec{\tau} L_{L\beta}\right) , \tag{3.56}$$

where

$$c^{d=6} = Y_\Sigma^\dagger \frac{1}{M_\Sigma^\dagger} \frac{1}{M_\Sigma} Y_\Sigma \,. \tag{3.57}$$

Compared to type I both cases are very similar but the derivative $\not{\partial}$ is replaced by $\not{D}$. This is because the fields are now in a triplet state and couple to the gauge fields of the SM.

This means that the dimension six operator not only causes corrections of the kinetic terms but also of the couplings to the W and Z bosons.

3.4 Inverse seesaw scenario

In the inverse seesaw scenario [31] we have a parameter μ which is responsible for lepton number violation. If μ vanishes, lepton number conservation, which is an accidental symmetry of the Standard Model, is restored. Hence a small value of μ is T'HOOFT natural [32].

The inverse seesaw generates a $d = 5$ mass term of $\mathcal{O}(v^2\mu/M^2)$. Because of this suppression, M can be much lower, namely at the TeV scale. A realization of this mechanism can be illustrated by a system of a light neutrino and a fermionic singlet that both conserve lepton number and an additional heavy lepton number violating field, which are all Majorana particles (ν_L,N_1,N_2). It is called the inverse seesaw scenario. The mass matrix reads

$$\mathcal{L}_m = (\bar{\nu}^c, \bar{N}_1^c, \bar{N}_2^c) \begin{pmatrix} 0 & m_{D_1} & 0 \\ m_{D_1} & 0 & M_{N_1} \\ 0 & M_{N_1} & \mu \end{pmatrix} \begin{pmatrix} \nu \\ N_1 \\ N_2 \end{pmatrix} . \tag{3.58}$$

Besides the Dirac masses m_{D_1} and M_{N_1} the only Majorana mass is the small entry μ which is associated to the N_2 field. The lightest eigenvalue at order μ/M_{N_1} is

$$m_\nu \propto \mu \frac{Y_1^2 v^2}{M_1^2} , \tag{3.59}$$

assuming that $m_{D_1} \ll M_{N_1}$ and $m_{D_1} \propto Y_1 v/\sqrt{2}$, since the Dirac mass is created by a Yukawa coupling.

There are several ways to realize a flavor structure that is in accordance with neutrino physics. The straight forward approach is to add three generations of the heavy fields. Another possibility that can only be realized in a supersymmetric framework is to generate one neutrino mass by the inverse seesaw with one generation of mediators and a second neutrino mass at one-loop level [33]. A third variant is the minimal inverse seesaw scenario (MISS) [34]. It consists of only two generations of the heavy fields, which narrows down the number of free parameters but requires one light neutrino to have zero mass.

There are various interesting phenomenological effects of this scenario. It is expected that non-unitarity and CP violation can be tested at possible long-baseline neutrino oscillation experiments. Furthermore, one could likely see lepton-flavor-violating (LFV) processes like $\mu \to e\gamma$ at the LHC. Lepton-number-violation in contrast is expected to be as good as invisible since the heavy Majorana neutrinos form pseudo-Dirac particles with suppressed Majorana character. In the case of the MISS it has been found that for an inverse hierarchy non-unitarity effects for μ and τ are of phenomenological interest whereas CP violation is constrained.

3.5 Higher-dimensional effective operators

If one forbids the dimension five operator, for example by introducing a discrete symmetry, under which the fields are charged, neutrino mass can be generated by higher-dimensional effective operators like $d = 7$ or $d = 9$. Since these are suppressed by higher powers of the heavy mass scale, that scale can be lowered down to the TeV range, making such scenarios accessible by current expiremts such as the LHC. But therefore an enhanced Higgs sector is necessary. This is, for example, the case in a Two Higgs Doublet Model (THDM) or the Minimal Supersymmetric Standard Model (MSSM), where we have an additional Higgs doublet with opposite hypercharge. If we would only have H_u, an operator like

$LLH_uH_uH_u^\dagger H_u$ would always induce a $d = 5$ operator since $H_u^*H_u$ has always zero charge.

A systematic study for the THDM scenario can be found in [7]. In Tab. 3.1 an overview of the possible higher dimensional operators is given.

	Op.#	Effective interaction
dim.5	1	LLH_uH_u
	2	$LLH_d^*H_u$
	3	$LLH_d^*H_d^*$
dim.7	4	$LLH_uH_uH_dH_u$
	5	$LLH_uH_uH_d^*H_d$
	6	$LLH_uH_uH_u^*H_u$
	7	$LLH_d^*H_uH_d^*H_d$
	8	$LLH_d^*H_uH_u^*H_u$
	9	$LLH_d^*H_d^*H_d^*H_d$
	10	$LLH_d^*H_d^*H_u^*H_u$
	11	$LLH_d^*H_d^*H_u^*H_d^*$
dim.9	12	$LLH_uH_uH_dH_uH_dH_u$
	13	$LLH_uH_uH_dH_uH_d^*H_d$
	14	$LLH_uH_uH_dH_uH_u^*H_u$
	15	$LLH_uH_uH_d^*H_dH_d^*H_d$
	16	$LLH_uH_uH_d^*H_dH_u^*H_u$
	17	$LLH_uH_uH_u^*H_uH_u^*H_u$
	18	$LLH_d^*H_uH_d^*H_dH_d^*H_d$
	19	$LLH_d^*H_uH_d^*H_dH_u^*H_u$
	20	$LLH_d^*H_uH_u^*H_uH_u^*H_u$
	21	$LLH_d^*H_d^*H_d^*H_dH_d^*H_d$
	22	$LLH_d^*H_d^*H_d^*H_dH_u^*H_u$
	23	$LLH_d^*H_d^*H_u^*H_uH_u^*H_u$
	24	$LLH_d^*H_d^*H_d^*H_u^*H_d^*H_d$
	25	$LLH_d^*H_d^*H_d^*H_u^*H_u^*H_u$
	26	$LLH_d^*H_d^*H_u^*H_d^*H_u^*H_d^*$
dim.11		...

Table 3.1: Effective Operators in the THDM. (See also Tab. 1 of [7])

There are several possible theories that generate the same effective operator, after their heavy fields have been integrated out. A list of the possible decompositions of the $d = 7$ effective operators is given in Tab. 2 of [7].

4 Supersymmetry

In this chapter an introduction to the formal and theoretical aspects of supersymmetric theories is given. It will also define the notation used in later chapters. A more complete description of SUSY can be found in Refs. [35] and [36].

4.1 Motivation

4.1.1 The hierarchy problem

In the Standard Model the scale of all masses depends on the vacuum expectation value (VEV) of the neutral Higgs component[1]

$$v \equiv \left\langle H^0 \right\rangle \approx 174\,\text{GeV}\,, \tag{4.1}$$

which is the expectation value of the field at the minimum of the Higgs potential

$$V = -\mu^2 H^{0\dagger} H^0 + \frac{1}{4}\lambda (H^{0\dagger} H^0)^2\,, \qquad \mu^2 > 0\,, \qquad \lambda > 0\,. \tag{4.2}$$

Its value is experimentally fixed by the mass of the W boson

$$m_W = \frac{g}{\sqrt{2}} v \approx 80\,\text{GeV}\,. \tag{4.3}$$

It is plausible to assume that the Standard Model is only valid up to a certain energy scale at which new physics appears. If one wants to calculate higher order corrections to the masses of particles, this means the loop integrals have a finite energy cutoff Λ. Hence for the next to leading order Higgs mass contribution for example (see Fig. 4.1 a)) one obtains

$$\delta m_H \propto \lambda \int^{\Lambda} \mathrm{d}^4 k \frac{1}{k^2 - m_H^2} \propto \lambda \Lambda^2\,. \tag{4.4}$$

Since this integral is quadratically divergent, one obtains a mass correction which is of the order of the new physics scale. To avoid this problem in all orders of perturbation, the counter terms must almost exactly cancel the tree level mass. That this fine-tuning is realized in Nature is considered very unlikely. Therefore the hierarchy problem is considered as a deficiency of the Standard Model.

[1] In the literature also the definition $v \equiv \sqrt{2} \left\langle H^0 \right\rangle$ is commonly used.

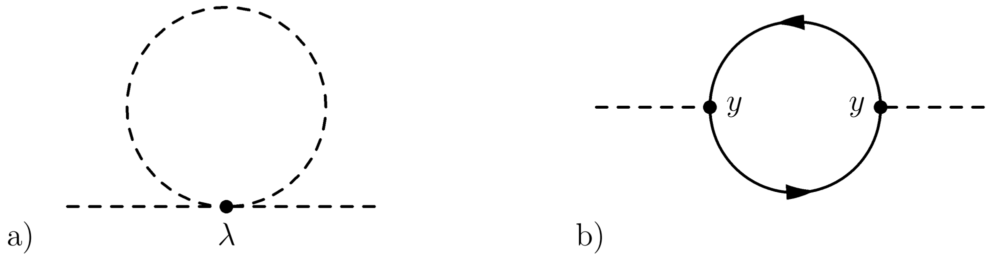


Figure 4.1: Hierarchy problem: Feynman graphs for the Higgs self energy in the SM (a) and in SUSY with fermionic partners of the Higgs fields (b). If the particles in the loop in (b) have the same mass as those in (a) and the condition $\lambda = y^2$ for the couplings is fullfilled, as it is the case for SUSY, both contributions cancel and the hierarchy problem is avoided.

If SUSY is realized on the other hand, an additional contribution to the Higgs self-energy exists, which is shown in Fig. 4.1 b). This fermionic loop naturally cancels the divergences of the SM graph, so that δm_H now is proportional to $\ln \Lambda$ and no more fine-tuning is required. Therefore SUSY is an elegant possibility to avoid the hierarchy problem.

4.1.2 Further indications

One of the topics that deserve the most attention in modern particle physics is Grand Unification. As it was possible in the past to bring electric and magnetic forces and later electromagnetism and the weak interaction together, theoretical physicists nowadays try to unify the gauge groups of the SM.

Due to renormalization, we have a running of gauge couplings. In order to obtain an unification, all three SM gauge couplings should meet at a certain scale. This is not the case in the standard model. In SUSY, however, we have additional particles changing the running of the couplings. Calculations show, that in supersymmetric theories the couplings really meet at the same point at about $2 \cdot 10^{16}$ GeV.

Other hints at SUSY (see Sec. 1.2 of Ref. [35]) are constraints on the Higgs mass and an explanation for the shape of the Higgs potential. The Higgs mass term $\mu^2 H^\dagger H$ naturally becomes negative in SUSY due to renormalization, making spontaneous symmetry breaking possible.

Finally, supersymmetry provides us with a possible dark matter candidate.

4.2 Basic concepts

SUSY is a symmetry between fermions and bosons. Supersymmetric transformations are possible with an operator Q, so that

$$Q|\text{fermion}\rangle = |\text{boson}\rangle \quad \text{and} \quad Q|\text{boson}\rangle = |\text{fermion}\rangle \,. \tag{4.5}$$

The generators satisfy the algebra

$$\{Q_a; Q_b\} = \{Q_a^\dagger; Q_b^\dagger\} = 0\,, \tag{4.6a}$$

$$\{Q_a; Q_b^\dagger\} = (\sigma_\mu)_{ab} P^\mu\,, \tag{4.6b}$$

where P^μ is the 4-momentum operator and σ^μ represents the Pauli matrices.

If supersymmetry is conserved (or *spontaneously* broken), the generators commute with the Hamiltonian. All particles are parts of supermultiplets and have partners which differ by spin 1/2.

4.3 Weyl spinors

It is common to use Weyl spinors in supersymmetric theories. Since they have only two components compared to the four component Dirac spinors and they are chiral eigenstates, they are better suited for the use with SUSY, because right-handed particles are SU(2) singlets whereas left-handed particles are SU(2) doublets, which is also the case for their supersymmetric partners.

In the Weyl representation[2] a four-component Dirac spinor Ψ can be represented by two two-component spinors ψ and χ as

$$\Psi = \begin{pmatrix} \psi \\ \chi \end{pmatrix}\,, \tag{4.7}$$

so that the chiral projections are

$$\Psi_R \equiv P_R \Psi \equiv \frac{1}{2}(1+\gamma_5)\Psi = \begin{pmatrix} \psi \\ 0 \end{pmatrix}\,, \tag{4.8a}$$

$$\Psi_L \equiv P_L \Psi \equiv \frac{1}{2}(1-\gamma_5)\Psi = \begin{pmatrix} 0 \\ \chi \end{pmatrix}\,. \tag{4.8b}$$

From the Dirac equation

$$(\gamma_\mu p^\mu - m)\Psi = 0 \tag{4.9}$$

one can derive the equivalent equations for the Weyl spinors

$$\sigma^\mu p_\mu \psi = m\chi \tag{4.10a}$$

$$\bar{\sigma}^\mu p_\mu \chi = m\psi\,. \tag{4.10b}$$

They transform as

$$\psi \rightarrow \psi' = V\psi\,, \tag{4.11a}$$

$$\chi \rightarrow \chi' = {V^{-1}}^\dagger \chi \tag{4.11b}$$

[2] In the Weyl representation the gamma matrices are $\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ -\sigma^\mu & 0 \end{pmatrix}$ and $\gamma_5 = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix}$, where σ^1, σ^2 and σ^3 are the Pauli matrices and $\sigma^0 = \mathbb{1}$. Furthermore we use $\sigma^\mu = (\sigma^0, \sigma^1, \sigma^2, \sigma^3)$ and $\bar{\sigma}^\mu = (\sigma^0, -\sigma^1, -\sigma^2, -\sigma^3)$.

under Lorentz transformations

$$V = \mathbb{1} + \mathrm{i}\vec{\epsilon}\frac{\vec{\sigma}}{2} - \vec{\eta}\frac{\vec{\sigma}}{2}\,, \tag{4.12}$$

where ϵ refers to rotations and η to boosts. One can easily check that Eq. (4.10) is invariant under these transformations.

With this knowledge, one can now construct Lorentz invariants $\chi^\dagger\psi$ and $\psi^\dagger\chi$ as well as 4-vectors $\psi^\dagger\sigma^\mu\psi$ and $\chi^\dagger\bar{\sigma}^\mu\chi$.

A simple Dirac Lagrangian can be rewritten with Weyl-Spinors as

$$\overline{\Psi}(\mathrm{i}\gamma^\mu\partial_\mu - m)\Psi \quad \rightarrow \quad \psi^\dagger\mathrm{i}\sigma^\mu\partial_\mu\psi + \chi^\dagger\mathrm{i}\bar{\sigma}^\mu\partial_\mu\chi - m(\psi^\dagger\chi + \chi^\dagger\psi)\,. \tag{4.13}$$

Another question is, how to construct a Lorentz invariant from ψ or χ only. Therefore one can define

$$\psi_\chi \equiv \mathrm{i}\sigma_2\chi^* \tag{4.14a}$$

which transforms like ψ, and

$$\chi_\psi \equiv -\mathrm{i}\sigma_2\psi^*\,, \tag{4.14b}$$

which transforms like χ. This means that $\psi_\chi^\dagger\chi$ and $\chi_\psi^\dagger\psi$ are invariants.

There exists a more formal notation using dotted and undotted indices (see e.g. [35] or [36]). We will not use that notation in this work in favor of a more intuitive convention where $\chi\chi \equiv \psi_\chi^\dagger\chi$. This means all products of Weyl spinors are supposed to be Lorentz invariants in order to avoid a confusing bulk of indices.

It is common to use only left handed Weyl Spinors, i.e. χ-like objects, in SUSY. Right handed particles like e_R can be represented by their charge conjugates $\bar{e}_L \equiv (e_R)^c$.

If the Fermions are SU(2) doublets, we will use another notational convention: $A \cdot B \equiv A\mathrm{i}\tau_2 B (\equiv \psi_{\chi_A}^\dagger \mathrm{i}\tau_2\chi_B)$. To combine the doublet in this way is necessary to guarantee the conservation of weak isospin.

4.4 Supersymmetric Lagrangians

The simplest Lagrangian which is invariant under SUSY transformations is

$$\mathcal{L} = \partial_\mu\phi^\dagger\partial^\mu\phi + \chi^\dagger\mathrm{i}\bar{\sigma}_\mu\partial^\mu\chi + F^\dagger F \tag{4.15}$$

for a massles scalar field ϕ and a massles left-handed fermion χ. The auxiliary field F is introduced, since without it the SUSY algebra does not close off-shell. This means that the SUSY transformations get a correction due to the auxiliary field, so that they can be used consistently also off-shell. This can be attributed to the different degrees of freedom of a complex scalar field (2 d.o.f.) compared to a two component spinor (4 d.o.f.) in that case.

The according infinitesimal transformations read

$$\delta_\xi \phi = \xi\chi\,, \tag{4.16a}$$
$$\delta_\xi \chi = -\mathrm{i}\sigma^\mu \mathrm{i}\sigma_2 \xi^* \partial_\mu \phi^* + \xi F\,, \tag{4.16b}$$
$$\delta_\xi F = -\mathrm{i}\xi^\dagger \bar{\sigma}^\mu \partial_\mu \chi\,, \tag{4.16c}$$

where ξ is an infinitesimal spinor. By inserting Eq. (4.16) in Eq. (4.15) one can see, that the Lagrangian is invariant under these transformations.

Besides chiral supermultiplets it is also possible to include vector supermultiplets, i.e., multiplets of a vector boson and a fermion. In a non-abelian gauge theory with gauge fields W^a one can, as usual, define a field strength tensor

$$F^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu - g\epsilon^{abc} W^b_\mu W^c_\nu \tag{4.17}$$

and a covariant derivative

$$D_\mu = \partial_\mu + igT^a W^a_\mu\,, \tag{4.18}$$

where the T^a are the generators of the according symmetry.

The supersymmetric partners of the gauge fields, the gauginos, are denoted λ^a. So the gauge Lagrangian reads (in Wess-Zumino gauge)

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu} a + \mathrm{i}\lambda^{a\dagger} \bar{\sigma}^\mu (D_\mu \lambda)^a + \frac{1}{2} D^a D^a\,, \tag{4.19}$$

where the auxiliary fields D^a are equivalent to the F fields in fermionic supermultiplets.

The fields of the gauge supermultiplet transform like

$$\delta_\xi W^a = \xi^\dagger \bar{\sigma}_\mu \lambda^a + \text{h.c.}\,, \tag{4.20a}$$
$$\delta_\xi \lambda^a = \frac{1}{2} \sigma^\mu \bar{\sigma}^\nu \xi F^a_{\mu\nu} + \xi D^a\,, \tag{4.20b}$$
$$\delta_\xi D^a = -\mathrm{i}\xi^\dagger \bar{\sigma}^\mu (D_\mu \lambda)^a + \text{h.c.} \tag{4.20c}$$

Combining chiral and vector supermultiplets finally leads to the Lagrangian

$$\mathcal{L} = \partial_\mu \phi^\dagger \partial^\mu \phi + \chi^\dagger \mathrm{i}\bar{\sigma}_\mu \partial^\mu \chi + F^\dagger F - \frac{1}{4} F^a_{\mu\nu} F^{\mu\nu} a + \mathrm{i}\lambda^{a\dagger} \bar{\sigma}^\mu (D_\mu \lambda)^a + \frac{1}{2} D^a D^a$$
$$- \sqrt{2} g (\phi^\dagger T^a \chi \lambda^a + \text{h.c.}) - g\phi^\dagger T^a \phi D^a \tag{4.21}$$

This Lagrangian is invariant under the transformations (4.16) and (4.20) with one more modification for $\delta_\xi F$:

$$\delta_\xi F = -\mathrm{i}\xi^\dagger \bar{\sigma}^\mu \partial_\mu \chi \underline{-\sqrt{2} q \lambda^\dagger \xi^\dagger \phi}\,. \tag{4.22}$$

4.5 Superfields and Superpotentials

A chiral superfield Φ provides a linear representation of the SUSY algebra. It contains the component fields ϕ, χ and F and can be written as

$$\hat{\Phi}(x,\theta) = \phi(x) + \theta\chi(x) + \frac{1}{2}\theta \cdot \theta F(x)\,, \tag{4.23}$$

where θ is a GRASSMAN number[3]. It depends on spatial and fermionic degrees of freedom, x and θ.

An infinitesimal supersymmetric transformation of the superfield can be expressed in terms of the SUSY generators as

$$\delta\hat{\Phi} = (-\mathrm{i}\xi Q + \mathrm{i}\xi^* Q^\dagger)\hat{\Phi}\,. \tag{4.24}$$

Using the differential operator expression of the generators

$$Q_a = \mathrm{i}\frac{\partial}{\partial\theta^a}\,, \tag{4.25}$$

one can show that Eq. (4.24) implies the transformations (4.16) and (4.20) of the component fields (see Sec. 9.3 of [35]).

Since $\delta_\xi F$ is only a total derivative, the F-term always generates a SUSY invariant action. Hence a supersymmetric interaction can be expressed as the F-term of the product of superfields.

4.5.1 Products of superfields

The product of two left-handed superfields is

$$\hat{\Phi}_i\hat{\Phi}_j = \phi_{ij} + \theta\chi_{ij} + \frac{1}{2}\theta\theta F_{ij}\,, \tag{4.26}$$

with

$$\phi_{ij} = \phi_i\phi_j\,, \tag{4.27a}$$
$$\chi_{ij} = \chi_i\phi_j + \phi_i\chi_j\,, \tag{4.27b}$$
$$F_{ij} = \phi_i F_j + \phi_j F_i - \chi_i\chi_j\,. \tag{4.27c}$$

In analogy the product of three superfields is

$$\hat{\Phi}_i\hat{\Phi}_j\hat{\Phi}_k = \phi_{ijk} + \theta\chi_{ijk} + \frac{1}{2}\theta\theta F_{ijk}\,, \tag{4.28}$$

with

$$\phi_{ijk} = \phi_i\phi_j\phi_k\,, \tag{4.29a}$$
$$\chi_{ijk} = \chi_i\phi_j\phi_k + \phi_i\chi_j\phi_k + \phi_i\phi_j\chi_k\,, \tag{4.29b}$$
$$F_{ijk} = \phi_i\phi_j F_k + \phi_j\phi_k F_i + \phi_k\phi_i F_j - \chi_i\chi_j\phi_k - \chi_j\chi_k\phi_i - \chi_k\chi_i\phi_j\,. \tag{4.29c}$$

[3] GRASSMANN numbers are anticommutating numbers, i.e. $\{\theta_i;\theta_j\} = 0$. This implies also, that $\theta_i^2 = 0$. The notation used here identifies $\theta = (\theta_1,\theta_2)$ as an object with two components. Hence, for an expansion in θ the highest non-zero order is $\theta\cdot\theta \equiv \theta\mathrm{i}\sigma_2\theta = -2\theta_1\theta_2$.

4.5.2 The superpotential

As stated above, all SUSY invariant Yukawa gauge interactions can be "encoded" as the F-term component of products of superfields. Hence a SUSY model can be described by an expression in terms of these products, which is called the superpotential.

Example: The Wess-Zumino model
The Wess-Zumino model [37] is a simple example of a supersymmetric theory, containing chiral supermultiplets and their interactions. The free Lagrangian for this model is equivalent to Eq. (4.15). Its totally symmetric superpotential reads

$$W_{\mathrm{WZ}} = \frac{1}{2} M_{ij} \hat{\Phi}_i \hat{\Phi}_j + \frac{1}{6} Y_{ijk} \hat{\Phi}_i \hat{\Phi}_j \hat{\Phi}_k \,. \tag{4.30}$$

Using Eq. (4.27c) and Eq. (4.29c) one obtains the interactions

$$\mathcal{L}^{\mathrm{int}}_{\mathrm{WZ}} = \frac{1}{2} M_{ij} (\phi_i F_j + \phi_j F_i - \chi_i \chi_j) + \frac{1}{6} Y_{ijk} (\phi_i \phi_j F_k + \phi_j \phi_k F_i + \phi_k \phi_i F_j - \chi_i \chi_j \phi_k - \chi_j \chi_k \phi_i - \chi_k \chi_i \phi_j) + \mathrm{h.c.} \tag{4.31}$$

4.5.3 Higher order products of superfields

Products of more then three superfields generate terms with $d > 4$, and hence are not considered in general. Due to our use of effective theories, however, they are of interest for us. The evaluation of these products can be accomplished in the same way as for the product of two or three superfields. One point we want to stress here is that only combinations of two fermions and an arbitrary number of scalars—but not of fermions and F-terms—are SUSY invariant. This is because only the former are of order $\theta \cdot \theta$ whereas the latter are always of higher order in θ, which is zero due to the properties of the Grassman numbers.

4.6 The auxiliary fields

The SUSY interactions that are derived from the superpotential still contain the auxiliary fields F and D. These fields, however, can be expressed in terms of the physical observable fields by using their equations of motion. Starting with the former, we only have the plain F fields, and no derivatives. Therefore Euler-Lagrange equation is rather simple,

$$\frac{\partial \mathcal{L}}{\partial F} = 0 \,. \tag{4.32}$$

Since the relevant terms of the Lagrangian are

$$\mathcal{L}^F = F^\dagger F + \mathcal{L}^{\mathrm{int}}(F) \,, \tag{4.33}$$

it follows from Eq. (4.32), that

$$F_i = \left(\frac{\partial \mathcal{L}^{\text{int}}(F)}{\partial F_i}\right)^\dagger \quad \text{and} \tag{4.34a}$$

$$F_i^\dagger = \frac{\partial \mathcal{L}^{\text{int}}(F)}{\partial F_i}\,. \tag{4.34b}$$

In the example of the Wess-Zumino model this means

$$F_i = \frac{1}{2}M^\dagger_{ij}\phi^\dagger_j + \frac{1}{6}Y^\dagger_{ijk}(\phi_j\phi_k)^\dagger\,. \tag{4.35}$$

One can show that, in analogy, also the D-terms of the vector supermultiplets can be replaced. Solving the equations of motion for D for the Lagrangian Eq. (4.21) one obtains

$$D^a = g\phi^\dagger T^a\phi\,. \tag{4.36}$$

4.7 The MSSM

The minimal supersymmetric extension of the standard model, the MSSM (*Minimal Supersymmetric Standard Model*), has one partner for each standard model particle. It is also necessary to introduce a second Higgs doublet (and its superpartner). This is necessary to construct Yukawa couplings for up-type as well as for down-type quarks that conserve supersymmetry as well as electroweak hypercharge. the second doublet makes it also possible to avoid anomalies that would become a problem otherwise.

SUSY models are described by left handed Weyl spinors. But what about the particles that are right-handed in the SM, like the SU(2)-singlet electron e_R? As already mentioned, they can be equivalently represented by their charge conjugated counterparts, which we will write as $\bar{e}_L \equiv (e_R)^c$.

It is specified by the superpotential

$$W_{\text{MSSM}} = \hat{\bar{u}}Y_u\hat{Q}\hat{H}_u - \hat{\bar{d}}Y_d\hat{Q}\hat{H}_d - \hat{\bar{e}}Y_e\hat{L}\hat{H}_d + \mu\hat{H}_d\hat{H}_u\,. \tag{4.37}$$

An overview of the particle content in the MSSM is shown in Tab. 4.1. The superpartners of the SM particles are marked with a tilde.

4.8 Breaking supersymmetry

The symmetry between the components of a supermultiplet implies that their physical properties, including their masses, are equivalent. Since the supersymmetric partners of the known SM particles, however, have not yet been observed in experiments, we know that their masses must be heavier than those of their counterparts. As a consequence of this, SUSY must be broken.

Chiral super-multiplets

Name	Spin 0	Spin 1/2	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
squarks, quarks	$\widetilde{Q} = (\widetilde{u}_L, \widetilde{d}_L)$	$Q = (u_L, d_L)$	$\mathbf{3}$	$\mathbf{2}$	$\frac{1}{6}$
	$\widetilde{\bar{u}}_L$	$\bar{u}_L \sim (u_R)^c$	$\bar{\mathbf{3}}$	$\mathbf{1}$	$-\frac{2}{3}$
	$\widetilde{\bar{d}}_L$	$\bar{d}_L \sim (d_R)^c$	$\bar{\mathbf{3}}$	$\mathbf{1}$	$\frac{1}{3}$
sleptons, leptons	$\widetilde{L} = (\widetilde{\nu}, \widetilde{e}_L)$	$L = (\nu, e_L)$	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{2}$
	$\widetilde{\bar{e}}_L$	$\bar{e}_L \sim (e_R)^c$	$\mathbf{1}$	$\mathbf{1}$	1
Higgs, Higgsinos	$H_u = (H_u^+, H_u^0)$	$\widetilde{H}_u = (\widetilde{H}_u^+, \widetilde{H}_u^0)$	$\mathbf{1}$	$\mathbf{2}$	$\frac{1}{2}$
	$H_d = (H_d^0, H_d^-)$	$\widetilde{H}_d = (\widetilde{H}_d^0, \widetilde{H}_d^-)$	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{2}$

Vector super-multiplets

Name	Spin 1/2	Spin 1	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
gluino, gluon	$\widetilde{g}$	g	$\mathbf{8}$	$\mathbf{1}$	0
winos, W's	$\widetilde{W}^\pm, \widetilde{W}^0$	$W^\pm, W^0$	$\mathbf{1}$	$\mathbf{3}$	0
bino, B	$\widetilde{B}$	B	$\mathbf{1}$	$\mathbf{1}$	0

Table 4.1: Particle content of the MSSM. There are three generations of squarks and sleptons. After symmetry breaking the physical states of the Higgs bosons are h^0, H^0, A and $H^\pm$. W^0 and B mix to Z and the photon. The neutralinos $\widetilde{\chi}_i^0$ and charginos $\widetilde{\chi}_i^\pm$ are a mixture of the Higgsinos with the winos and binos.

In many discussions of SUSY the exact mechanism behind symmetry breaking is not considered in detail. Instead one only specifies some explicit SUSY breaking terms. These terms can be understood as an effective description of an underlying theory at a higher scale.

A condition that is often required is that the symmetry breaking terms are soft [38]. These have only positive mass dimension, hence renormalizability is not spoiled. This also ensures that SUSY still cancels the divergences that otherwise would lead to the hierarchy problem.

The possible soft breaking terms that respect the SM gauge symmetries are:

- **Gaugino mass terms**

$$-\frac{1}{2}(M_1\widetilde{B}\widetilde{B} + M_2\widetilde{W}^a\widetilde{W}^a + M_3\widetilde{g}^\alpha\widetilde{g}^\alpha + \text{h.c.})\,. \tag{4.38a}$$

- **Sfermion mass terms**

$$-m_{\widetilde{Q}}^2\widetilde{Q}^\dagger \cdot \widetilde{Q} - m_{\widetilde{\bar{u}}}^2\widetilde{\bar{u}}^\dagger \cdot \widetilde{\bar{u}} - m_{\widetilde{\bar{d}}}^2\widetilde{\bar{d}}^\dagger \cdot \widetilde{\bar{d}} - m_{\widetilde{L}}^2\widetilde{L}^\dagger \cdot \widetilde{L} - m_{\widetilde{\bar{e}}}^2\widetilde{\bar{e}}^\dagger \cdot \widetilde{\bar{e}}\,, \tag{4.38b}$$

where the family indices have been suppressed. The dot denotes again the SU(2) invariant product.

- **Higgs mass terms**

$$-m_{H_u}^2 H_u^\dagger \cdot H_u - m_{H_d}^2 \cdot H_d^\dagger H_d - (B_H H_u \cdot H_d + \text{h.c.}) \,. \tag{4.38c}$$

- **Trilinear terms**

$$-A_u \widetilde{\bar{u}} \widetilde{Q} \cdot H_u - A_d \widetilde{\bar{d}} \widetilde{Q} \cdot H_d - A_e \widetilde{\bar{e}} \widetilde{L} \cdot H_d + \text{h.c.} \tag{4.38d}$$

Usually all of these terms appear, independent of the underlying breaking mechanism. A consequence of this is the well known problem of a large number of free parameters in SUSY. An important parameter is $\tan\beta = \frac{v_u}{v_d}$, the ratio of the Higgs vacuum expectation values.

4.9 Neutralino and chargino mixing

Since the bino, the neutral wino and the two neutral Higgs fields have the same quantum numbers, they can mix. This means the mass matrix in the basis $(\widetilde{B}, \widetilde{W}^0, \widetilde{H}_d^0, \widetilde{H}_u^0)$ reads

$$\begin{pmatrix} M_1 & 0 & -\cos\beta\sin\theta_W m_Z & \sin\beta\sin\theta_W m_Z \\ 0 & M_2 & \cos\beta\cos\theta_W m_Z & -\sin\beta\cos\theta_W m_Z \\ -\cos\beta\sin\theta_W m_Z & \cos\beta\cos\theta_W m_Z & 0 & -\mu \\ \sin\beta\sin\theta_W m_Z & -\sin\beta\cos\theta_W m_Z & -\mu & 0 \end{pmatrix} . \tag{4.39}$$

M_1 and M_2 are the soft breaking wino and bino masses (Eq. (4.38a)), μ comes from the MSSM superpotential (Eq. (4.37)) and β is a model dependend parameter (see last section). The masses generated by electroweak symmetry breaking are described in terms of the Weinberg angle and the Z boson mass m_Z. The four mass eigenstates of this matrix are called neutralinos $\widetilde{\chi}_i^0$.

The charged components of the Wino and Higgs doublets mix in a similar way, generating the so called charginos $\widetilde{\chi}_i^\pm$. The according mass matrix can be expressed as

$$-\frac{1}{2}\left[(\widetilde{W}^+ H_u^+) M_{\widetilde{\chi}^\pm} \cdot \begin{pmatrix} \widetilde{W}^- \\ H_d^- \end{pmatrix} + (\widetilde{W}^- H_d^-) M_{\widetilde{\chi}^\pm} \cdot \begin{pmatrix} \widetilde{W}^+ \\ H_u^+ \end{pmatrix}\right] + \text{h.c.}\,, \tag{4.40}$$

where

$$M_{\widetilde{\chi}^\pm} = \begin{pmatrix} M_2 & \sqrt{2}\sin\beta m_W \\ \sqrt{2}\cos\beta m_W & \mu \end{pmatrix} . \tag{4.41}$$

4.10 R-Parity

In the Standard Model lepton number and baryon number are conserved by an accidental symmetry. In SUSY, however, it is possible to introduce terms which violate both, but are not forbidden by any other symmetry. Experiments, on the other hand, show that effects of this violation—like the proton becoming unstable—must be very small.

To avoid this problem, one can introduce R-parity, which is defined as

$$R = (-1)^{3B+L+2s}\,, \tag{4.42}$$

where B is the baryon number, L is the lepton number and s the spin of the particle in question. R is $+1$ for the familiar SM particles and -1 for their superpartners.

If R-parity is conserved, it has some phenomenologically important consequences. First, sparticles can only be produced in pairs at collider experiments like the LHC. Second, sparticles can only decay into an odd number of sparticles (and an arbitrary number of $R = +1$ particles).

Another consequence of this concerns dark matter. As we know today, most of the matter in the universe is dark matter, but its nature is still a mystery. Supersymmetry, however, offers a possible solution: If R-parity is conserved, the lightest supersymmetric particle (LSP) must be stable. Such a particle, like the neutralino for example, is massive and interacts only weakly and therefore fulfills the requirements on a dark matter candidate.

5 Higher than d=5 Effective Operators in a Supersymmetric Framework

The various $d = 7$ operators and their possible decompositions have already been studied for a Two Higgs Doublet Model (THDM) [7]. In the following we demonstrate that this is also possible in a supersymmetric framework.

5.1 Prerequisites

Since SUSY can be implemented in various ways, we first have to make some assumptions.

Model

The simplest case of a supersymmetric Model is the MSSM. Therefore we will use it as a reference for the extensions we will make to incorporate neutrino physics. We will therefore use the definition given in Sec. 4.7.

Another possible candidate is the NMSSM (see e.g. Ref. [39] for a review), which circumvents the so called μ-term problem: By introducing an additional scalar field S (and, of course, its supersymmetric partner), which couples to the Higgs fields and has a non-zero vacuum expectation value, the small scale of the μ-term is naturally generated. In this study our results are mostly independent on these details. We will, however, point out the differences, when necessary (see also Sec. 5.2.2).

Discrete symmetry

In order to avoid the $d = 5$ operator as leading contribution to neutrino mass, we have to require a discrete symmetry that forbids this operator (see Sec. 3.5).

There are theoretical motivations for discrete symmetries, but we will not care about its origins here and take it simply as a precondition. But we will show how to implement it.

One should also note that instead of a discrete symmetry a U(1) symmetry could be used. In this case, however, one would also have to care about additional Goldstone bosons, which can appear, since by breaking the electroweak symmetry also the U(1) symmetry is broken.

R-Parity
If R-parity is broken, the sneutrino can get a VEV. Because of the neutrino-sneutrino-neutralino interacton, an additional $d = 5$ effective operator, which contributes to the neutrino mass, is possible [40]. We, however, want to show a qualitatively different model where R-parity is conserved.

As a consequence of this, we have a strong constraint on the possible decompositions of the effective operators.

Holomorphy of the Superpotential
Because of the holomorphy of the superpotential, conjugate fields can only be introduced by F-terms. Since there are no SUSY invariant interactions with both, fermion and F fields (see Sec. 4.5.3), the only possible effective $d = 7$ operator in SUSY is $LL(H_u)^3 H_d$. This is an important difference to the THDM and limits the number of possible decompositions further.

5.2 General remarks

5.2.1 Possible effective operators

As mentioned in section 3.5, there are higher-dimensional effective operators in models with an additional Higgs doublet (like the THDM or the MSSM) and/or an additional SU(2) singlet scalar field. It is also required that all these fields have a non-zero vacuum expectation value. In supersymmetric models, however, the possibilities for these effective operators are limited due to the holomorphy condition stated above. This means that the effective operator only contains Lorentz invariant products of fermions and only non conjugated scalar fields. This can be understood by looking at the products of superfields again, since—as stated in subsection 4.5.3—only combinations of fermions and scalar fields are SUSY invariant, but not of fermions and auxiliary fields. This is different to the THDM where operators such as $LLH_d^* H_u$ are possible.

A list of all possible operators up to dimension 9 is given in Tab. 5.1. The discrete charge of the fields has to be chosen in a way such that the effective operator with the leading contribution to neutrino mass has zero charge. In other words, the effective operator must conserve the discrete symmetry, while all other possible operators of lower dimension break the symmetry explicitly and hence are forbidden.

In the NMSSM we have another situation. The superpotential for this model is

$$W_{\text{NMSSM}} = W_{\text{Yuk}} + \lambda \hat{S} \hat{H}_u \hat{H}_d + \kappa \hat{S}^3 \,, \tag{5.1}$$

where W_{Yuk} denotes the superpotential for the Yukawa couplings, i.e. the MSSM superpotential without the μ-term. Therefore the charge q_S is fixed. Hence the fields cannot be charged under the discrete symmetry in a way that operator #5 is allowed and #1 is forbidden at the same time. This implies that #5 can never

	Op.#	Effective interaction	Cond.#	Charge	$q_S = -(q_{H_u} + q_{H_d})$
dim.5	1	LLH_uH_u	1	$2q_L + 2q_{H_u}$	$2q_L + 2q_{H_u}$
dim.6	2	LLH_uH_uS	2	$2q_L + 2q_{H_u} + q_S$	$2q_L + q_{H_u} - q_{H_d}$
dim.7	3	$LLH_uH_uH_dH_u$	3	$2q_L + q_{H_d} + 3q_{H_u}$	$2q_L + q_{H_d} + 3q_{H_u}$
	4	LLH_uH_uSS	4	$2q_L + 2q_{H_u} + 2q_S$	$2q_L - 2q_{H_d}$
dim. 8	5	$LLH_uH_uH_uH_dS$	1	$2q_L + 3q_{H_u} + q_{H_d} + q_S$	$2q_L + 2q_{H_u}$
	6	LLH_uH_uSSS	5	$2q_L + 2q_{H_u} + 3q_S$	$2q_L - q_{H_u} - 3q_{H_d}$
dim.9	7	$LLH_uH_uH_dH_uH_dH_u$	6	$2q_L + 2q_{H_d} + 4q_{H_u}$	$2q_L + 2q_{H_d} + 4q_{H_u}$
	8	$LLH_uH_uH_dH_uSS$	2	$2q_L + 3q_{H_u} + q_{H_d} + 2q_S$	$2q_L + q_{H_u} - q_{H_d}$
	9	LLH_uH_uSSSS	7	$2q_L + 2q_{H_u} + 4q_S$	$2q_L - 2q_{H_u} - 4q_{H_d}$

Table 5.1: Effective operators generating neutrino mass in the (N)MSSM up to dim. 9. If S is the NMSSM scalar, then its charge q_S is fixed by the term $\lambda S H_u H_d$ (last column). Note that operator #5 has the same condition for the charges as operator #1 (if q_S is fixed). Therefore it is not possible to have only one of them and forbid the other one. The same is true for operator #8 and operator #2. Hence operator #5 and #8 can never be the leading contribution to neutrino mass. If one assumes that the trilinear term $\lambda\hat{S}\hat{S}\hat{S}$ of the NMSSM has zero charge, this also rules out operator #6 as leading contribution.

be the leading contribution to neutrino mass. This is the same for operator #8 compared to operator #2. In all the other listed cases it is possible to find a charge assignment that forbids all other operators. As already noted in Sec. 2 of Ref. [7] it is sufficient to use a $\mathbb{Z}_3$ symmetry in the case of SUSY, to get operator #3 as leading contribution. In the THDM at least a $\mathbb{Z}_5$ symmetry is required, since we have no holomorphy condition. A possible charge assignment is

$$q_{H_u} = 0,\ q_{H_d} = 1,\ q_L = 1,\ (q_S = 0)\,. \tag{5.2}$$

5.2.2 The μ-term and the discrete symmetry

Without singlet fields we have effective operators of the type $(LH_u)^2(H_uH_d)^n$ with an overall discrete charge of zero. Furthermore we know that the charge of $(LH_u)^2$ must be different from zero in order to forbid the $d = 5$ effective operator. Therefore it is obvious that H_uH_d cannot have zero charge either. Hence we must be aware of the fact that the μ-term of the MSSM, $\mu\hat{H}_u\hat{H}_d$, explicitly breaks the discrete symmetry. The reasons for this symmetry violation will not be discussed here. One can think of modifications of the MSSM, however, where this problem can be circumvented. The explicit breaking can be avoided if an additional scalar singlet S is introduced, which couples to the Higgs fields:

$$\lambda\hat{S}\hat{H}_u\hat{H}_d\,. \tag{5.3}$$

If the scalar S gets a VEV, then the μ-term is recreated with $\mu = \lambda\langle S\rangle$. This is the case in the NMSSM. A further advantage of this model is that the scale of the coupling between H_u and H_d can be easily set to the electroweak symmetry breaking scale. This is required for the Higgs mechanism to work. The μ of the MSSM in contrast must by adjusted to this scale by hand.

In Sec. 3.1 of Ref. [7] it was discussed that by introducing a discrete symmetry breaking term $m^2 H_d \cdot H_u$ the external H_d and H_u lines of a $d = 7$ operator can be connected, which unavoidably generates a one-loop $d = 5$ operator. This term, however, is suppressed compared to the tree level $d = 7$ contribution, assuming that m is at the electroweak scale. In the case of the MSSM, however, we have a different case. The term $\mu \hat{H}_u \cdot \hat{H}_d$ in the superpotential corresponds to the scalar terms $\mu H_u^\dagger H_u$ and $\mu H_d^\dagger H_d$ in the Lagrangian and not $\mu H_u \cdot H_d$ (see e.g. Sec. 16 of Ref. [35]). Instead we have a SUSY soft breaking term $B_H H_u \cdot H_d$ from Eq. (4.38c). This means that in our model the soft breaking parameter B_H must be sufficiently small, in order to avoid the $d = 5$ one-loop contribution.

5.2.3 Supersymmetric partners of the mediators

Since we have a supersymmetric partner for each particle, the question arises whether it is possible to construct more "economic" decompositions in SUSY. This means, can we use both fields of the supermultiplet simultaneously as mediators? This could minimize the number of necessary (super-)fields in decompositions where fermionic as well as scalar mediators appear.

The answer is no, at least on tree level and with R-Parity conservation. As all external fields, L, H_u and H_d, have $R = +1$, a mediator with $R = -1$ would cause vertices where R-Parity is violated. As a consequence we have also supersymmetric partners for each mediator field. Therefore a model with minimal particle content has different phenomenology, if R-Parity is conserved, compared to models with R-Parity violation, where the partner of a scalar mediator can also be a fermionic mediator and no additional particles are needed.

5.3 Effective operators with d=6

In Ref. [8] the $d = 6$ operator $(LH_u)^2 S$ was studied within the NMSSM framework. A possible decomposition is shown in Fig. 5.1. The corresponding superpotential reads

$$W = W_{\text{NMSSM}} + Y_N N H_u \cdot L + \frac{\lambda_N}{2} S N' N' + M_N N' N \tag{5.4}$$

In this case the neutrino mass becomes

$$m_\nu = Y_N^2 \frac{\lambda_N \langle S \rangle \langle H_u \rangle^2}{M_N} \,. \tag{5.5}$$

Since $\lambda \langle S \rangle$ is confined to the electroweak scale by the NMSSM and assuming $\lambda_N \approx \lambda$ one obtains Yukawa couplings Y_N of the order $10^{-4} \ldots 10^{-5}$ for a mass

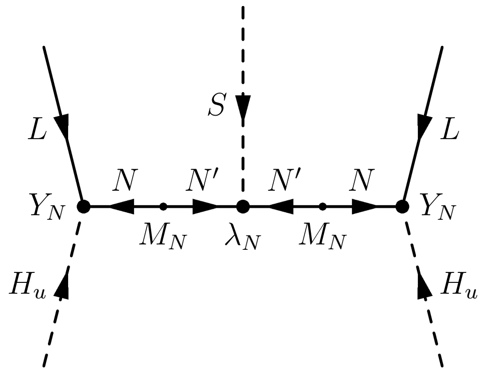


Figure 5.1: Decomposition of a $d = 6$ operator.

scale of the order $1 \dots 10\,\mathrm{TeV}$. We will not discuss $d = 6$ effective operators and their decompositions any further in this work. Instead we will focus on the dimesion seven scenarios, where we have higher suppression and hence can lower the new physics scale further.

5.4 Effective operators with d=7

5.4.1 Topologies

In Fig. 5.2 all possible topologies for the decomposition of the $d = 7$ operators are shown. In SUSY scenarios topologies 3 and 4 can be excluded. This is due to the fact that scalar couplings in SUSY have to be of the type $\phi^\dagger\phi$, $\phi^\dagger\phi\phi$, $\phi^\dagger\phi^\dagger\phi$ or $\phi^\dagger\phi^\dagger\phi\phi$, since they are generated by F-terms and D-terms. This is an implication of the holomorphy condition mentioned before.

Topology 3
The scalar four-vertex has to be of the type HHX^*X^*, where X is a heavy virtual scalar field. The three vertex must be HHX^*. These two vertices can not be connected by a propagator Δ_X. Hence topology 3 can not be realized in SUSY.

Topology 4
The four scalar vertex can only be of the type $H_dH_uH_uX^{(*)}$ or $H_uH_uH_uX^{(*)}$ in order to produce an effective operator of the type $LLH_uH_uH_dH_u$. The only possible scalar four couplings allowed by SUSY, however, are of the type $\phi^\dagger\phi^\dagger\phi\phi$. Hence also topology 4 is not possible.

5.4.2 Extended type II seesaw scenarios

The possible decompositions of the $d = 7$ effective operator can be roughly categorized as extensions of the well known $d = 5$ decompositions, the type I, II and

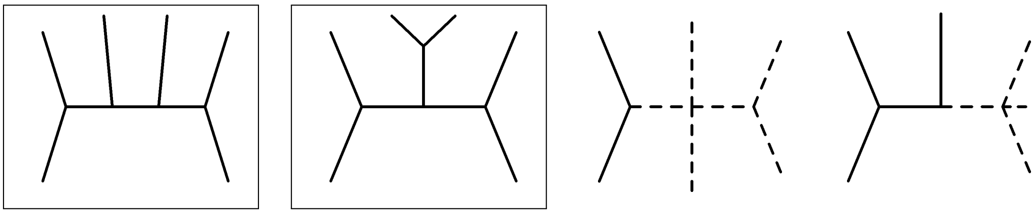


Figure 5.2: Possible topologies for seesaw type II like decompositions of an effective $d = 7$ operator. Topologies 3 and 4 cannot be realized in SUSY. Solid lines are either fermions or scalars, dashed lines are always scalars. (cf. Fig. 2 of [7])

#	Operator	Top.	Mediators
5	$(\overline{L^c}\mathrm{i}\tau^2\vec{\tau}L)(H_d\mathrm{i}\tau^2H_u)(H_u\mathrm{i}\tau^2\vec{\tau}H_u)$	2	$\mathbf{3}^s_{+1}$, $\mathbf{3}^s_{+1}$, $\mathbf{1}^s_0$
6	$(-\mathrm{i}\epsilon_{abc})(\overline{L^c}\mathrm{i}\tau^2\tau_aL)(H_d\mathrm{i}\tau^2\tau_bH_u)(H_u\mathrm{i}\tau^2\tau_cH_u)$	2	$\mathbf{3}^s_{+1}$, $\mathbf{3}^s_{+1}$, $\mathbf{3}^s_0$
21	$(\overline{L^c}\mathrm{i}\tau^2\tau^aL)(H_u\mathrm{i}\tau^2\tau^a)(\tau^bH_d)(H_u\mathrm{i}\tau^2\tau^bH_u)$	1	$\mathbf{3}^s_{+1}$, $\mathbf{2}^s_{+1/2}$, $\mathbf{3}^s_{+1}$
22	$(\overline{L^c}\mathrm{i}\tau^2\tau^aL)(H_d\mathrm{i}\tau^2\tau^a)(\tau^bH_u)(H_u\mathrm{i}\tau^2\tau^bH_u)$	1	$\mathbf{3}^s_{+1}$, $\mathbf{2}^s_{+3/2}$, $\mathbf{3}^s_{+1}$
23	$(\overline{L^c}\mathrm{i}\tau^2\vec{\tau}L)(H_u\mathrm{i}\tau^2\vec{\tau})(H_u)(H_d\mathrm{i}\tau^2H_u)$	1	$\mathbf{3}^s_{+1}$, $\mathbf{2}^s_{+1/2}$, $\mathbf{1}^s_0$
24	$(\overline{L^c}\mathrm{i}\tau^2\tau^aL)(H_u\mathrm{i}\tau^2\tau^a)(\tau^bH_u)(H_d\mathrm{i}\tau^2\tau^bH_u)$	1	$\mathbf{3}^s_{+1}$, $\mathbf{2}^s_{+1/2}$, $\mathbf{1}^s_0$

Table 5.2: Possible type II like decompositions of an effective $d = 7$ operator in SUSY. The parentheses group the external fields that are connected to the same vertex. If $\vec{\tau}$ appears, the fields couple to a triplet mediator; if not, they couple to a singlet. The mediators are specified by their isospin (I) and hypercharge (Y) quantum numbers in the way $(2I + 1)_Y$. The s indicates a scalar mediator. All charged scalar fields must have an additional partner of opposite charge (not listed) to make a mass term possible in the superpotential. (cf. Tab. 2 of [7] for the THDM case, the numbers of the operators are chosen accordingly.)

III seesaw scenarios.

We define a decomposition as extended type II seesaw, if all mediators are scalars (cf. Sec. 2.3 and 3.3 for the normal seesaw). Therefore the only appearing fermions are the external lepton doublets. The only lepton number violating interaction is then

$$(\overline{L^c}\mathrm{i}\tau_2\vec{\tau}L)\vec{\phi}\,, \tag{5.6}$$

where ϕ represents one of the scalar mediators. This vertex violates lepton number by $\Delta L = 2$ and therefore conserves R-Parity.

Tab. 5.2 lists all decompositions possible in SUSY, which are extended type II. Due to the holomorphy condition we need partners with opposite charge for each scalar field in order to obtain SUSY invariant mass terms.

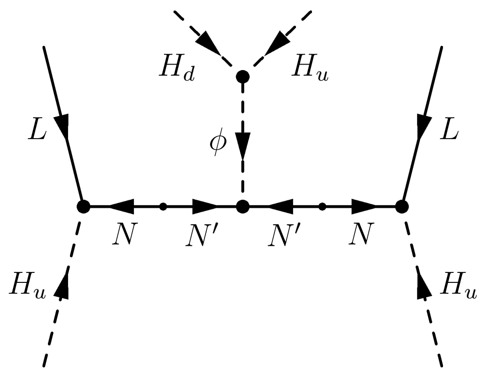


Figure 5.3: Decomposition #1 of the effective $d = 7$ operator.

5.4.3 Extended type I and III seesaw scenarios

All other decompositions that have fermionic mediators can be seen as extensions of the $d = 5$ type I or type III seesaw mechanism. Since we can have several combinations of scalar fields and SU(2) singlet, doublet or triplet fields as mediators, a further distinction will not be made. A list of all type I/III like decompositions of the $d = 7$ effective operator is shown in Tab. 5.3.

Compared to the extended type II, we can have scalars and fermions or solely fermions as mediator. Depending on the topology and the actual realization of these operators, the various decompositions have different characteristics, as the following examples show. They will be compared in Sec. 5.5.4.

5.5 Examples for decompositions of the d=7 effective operator

5.5.1 Decomposition #1

The Feynman graph corresponding to decomposition #1 of the $d = 7$ operator is shown in Fig. 5.3. The corresponding superpotential is

$$W = W_{\text{quarks}} - Y_e \hat{\bar{e}} \hat{L} \cdot \hat{H}_d + Y_N \hat{N} \hat{L} \cdot \hat{H}_u + m_N \hat{N} \hat{N}' + \kappa \hat{N}' \hat{N}' \hat{\phi} + \lambda \hat{H}_u \cdot \hat{H}_d \hat{\phi} \\ + \frac{1}{2} m_\phi \hat{\phi} \hat{\phi} + \mu \hat{H}_u \hat{H}_d \,, \tag{5.7}$$

where $N = N_L = (N_R)^c$ and $N' = {N_L}'$ are fermion singlets and ϕ is a scalar singlet. The charges of the new fields have to be assigned in a way that all interactions are charge neutral.

The terms of the Lagrangian corresponding to Eq. (5.7) that include at least

#	Operator	Top.	Mediators
1	$(H_u\mathrm{i}\tau^2\overline{L^c})(H_u\mathrm{i}\tau^2 L)(H_d\mathrm{i}\tau^2 H_u)$	2	$\mathbf{1}_0^R,\ \mathbf{1}_0^L,\ \mathbf{1}_0^s$
2	$(H_u\mathrm{i}\tau^2\vec{\tau}\overline{L^c})(H_u\mathrm{i}\tau^2 L)(H_d\mathrm{i}\tau^2\vec{\tau}H_u)$	2	$\mathbf{3}_0^R,\ \mathbf{3}_0^L\ \mathbf{1}_0^R,\ \mathbf{1}_0^L,\ \mathbf{3}_0^s$
3	$(H_u\mathrm{i}\tau^2\vec{\tau}\overline{L^c})(H_u\mathrm{i}\tau^2\vec{\tau}L)(H_d\mathrm{i}\tau^2 H_u)$	2	$\mathbf{3}_0^R,\ \mathbf{3}_0^L,\ \mathbf{1}_0^s$
4	$(-\mathrm{i}\epsilon^{abc})(H_u\mathrm{i}\tau^2\tau^a\overline{L^c})(H_u\mathrm{i}\tau^2\tau^b L)(H_d\mathrm{i}\tau^2\tau^c H_u)$	2	$\mathbf{3}_0^R,\ \mathbf{3}_0^L,\ \mathbf{3}_0^s$
7	$(H_u\mathrm{i}\tau^2\overline{L^c})(L\mathrm{i}\tau^2\vec{\tau}H_d)(H_u\mathrm{i}\tau^2\vec{\tau}H_u)$	2	$\mathbf{1}_0^R,\ \mathbf{1}_0^L,\ \mathbf{3}_{-1}^R,\ \mathbf{3}_{-1}^L,\ \mathbf{3}_{+1}^s$
8	$(-\mathrm{i}\epsilon^{abc})(H_u\mathrm{i}\tau^2\tau^a\overline{L^c})(L\mathrm{i}\tau^2\tau^b H_d)(H_u\mathrm{i}\tau^2\tau^c H_u)$	2	$\mathbf{3}_0^R,\ \mathbf{3}_0^L,\ \mathbf{3}_{-1}^R,\ \mathbf{3}_{-1}^L,\ \mathbf{3}_{+1}^s$
9	$(H_u\mathrm{i}\tau^2\overline{L^c})(\mathrm{i}\tau^2 H_u)(L)(H_d\mathrm{i}\tau^2 H_u)$	1	$\mathbf{1}_0^R,\ \mathbf{1}_0^L,\ \mathbf{2}_{-1/2}^R,\ \mathbf{2}_{-1/2}^L,\ \mathbf{1}_0^s$
10	$(H_u\mathrm{i}\tau^2\vec{\tau}\overline{L^c})(\mathrm{i}\tau^2\vec{\tau}H_u)(L)(H_d\mathrm{i}\tau^2 H_u)$	1	$\mathbf{3}_0^R,\ \mathbf{3}_0^L,\ \mathbf{2}_{-1/2}^R,\ \mathbf{2}_{-1/2}^L,\ \mathbf{1}_0^s$
11	$(H_u\mathrm{i}\tau^2\overline{L^c})(\mathrm{i}\tau^2 H_u)(\vec{\tau}L)(H_d\mathrm{i}\tau^2\vec{\tau}H_u)$	1	$\mathbf{1}_0^R,\ \mathbf{1}_0^L,\ \mathbf{2}_{-1/2}^R,\ \mathbf{2}_{-1/2}^L,\ \mathbf{3}_0^s$
12	$(H_u\mathrm{i}\tau^2\tau^a\overline{L^c})(\mathrm{i}\tau^2\tau^a H_u)(\tau^b L)(H_d\mathrm{i}\tau^2\tau^b H_u)$	1	$\mathbf{3}_0^R,\ \mathbf{3}_0^L,\ \mathbf{2}_{-1/2}^R,\ \mathbf{2}_{-1/2}^L,\ \mathbf{3}_0^s$
13	$(H_u\mathrm{i}\tau^2\overline{L^c})(L)(\mathrm{i}\tau^2 H_u)(H_d\mathrm{i}\tau^2 H_u)$	1	$\mathbf{1}_0^R,\ \mathbf{1}_0^L,\ \mathbf{2}_{+1/2}^s,\ \mathbf{1}_0^s$
14	$(H_u\mathrm{i}\tau^2\vec{\tau}\overline{L^c})(\vec{\tau}L)(\mathrm{i}\tau^2 H_u)(H_d\mathrm{i}\tau^2 H_u)$	1	$\mathbf{3}_0^R,\ \mathbf{3}_0^L,\ \mathbf{2}_{+1/2}^s,\ \mathbf{1}_0^s$
15	$(H_u\mathrm{i}\tau^2\overline{L^c})(L)(\mathrm{i}\tau^2\vec{\tau}H_u)(H_d\mathrm{i}\tau^2\vec{\tau}H_u)$	1	$\mathbf{1}_0^R,\ \mathbf{1}_0^L,\ \mathbf{2}_{+1/2}^s,\ \mathbf{3}_0^s$
16	$(H_u\mathrm{i}\tau^2\tau^a\overline{L^c})(\tau^a L)(\mathrm{i}\tau^2\tau^b H_u)(H_d\mathrm{i}\tau^2\tau^b H_u)$	1	$\mathbf{3}_0^R,\ \mathbf{3}_0^L,\ \mathbf{2}_{+1/2}^s,\ \mathbf{3}_0^s$
17	$(H_u\mathrm{i}\tau^2\overline{L^c})(H_d)(\mathrm{i}\tau^2 H_u)(H_u\mathrm{i}\tau^2 L)$	1	$\mathbf{1}_0^R,\ \mathbf{1}_0^L,\ \mathbf{2}_{-1/2}^R,\ \mathbf{2}_{-1/2}^L$
18	$(H_u\mathrm{i}\tau^2\vec{\tau}\overline{L^c})(\vec{\tau}H_d)(\mathrm{i}\tau^2 H_u)(H_u\mathrm{i}\tau^2 L)$	1	$\mathbf{3}_0^R,\ \mathbf{3}_0^L,\ \mathbf{2}_{-1/2}^R,\ \mathbf{2}_{-1/2}^L,\ \mathbf{1}_0^R,\ \mathbf{1}_0^L$
19	$(H_u\mathrm{i}\tau^2\overline{L^c})(H_d)(\mathrm{i}\tau^2\vec{\tau}H_u)(H_u\mathrm{i}\tau^2\vec{\tau}L)$	1	$\mathbf{1}_0^R,\ \mathbf{1}_0^L,\ \mathbf{2}_{-1/2}^R,\ \mathbf{2}_{-1/2}^L,\ \mathbf{3}_0^R,\ \mathbf{3}_0^L$
20	$(H_u\mathrm{i}\tau^2\tau^a\overline{L^c})(\tau^a H_d)(\mathrm{i}\tau^2\tau^b H_u)(H_u\mathrm{i}\tau^2\tau^b L)$	1	$\mathbf{3}_0^R,\ \mathbf{3}_0^L,\ \mathbf{2}_{-1/2}^R,\ \mathbf{2}_{-1/2}^L,$
25	$(H_d\mathrm{i}\tau^2 H_u)(\overline{L^c}\mathrm{i}\tau^2)(\vec{\tau}L)(H_u\mathrm{i}\tau^2\vec{\tau}H_u)$	1	$\mathbf{1}_0^s,\ \mathbf{2}_{+1/2}^L,\ \mathbf{2}_{+1/2}^R,\ \mathbf{3}_{+1}^s$
26	$(H_d\mathrm{i}\tau^2\tau^a H_u)(\overline{L^c}\mathrm{i}\tau^2\tau^a)(\tau^b L)(H_u\mathrm{i}\tau^2\tau^b H_u)$	1	$\mathbf{3}_0^s,\ \mathbf{2}_{+1/2}^L,\ \mathbf{2}_{+1/2}^R,\ \mathbf{3}_{+1}^s$
27	$(H_u\mathrm{i}\tau^2\overline{L^c})(\mathrm{i}\tau^2 H_d)(\vec{\tau}L)(H_u\mathrm{i}\tau^2\vec{\tau}H_u)$	1	$\mathbf{1}_0^R,\ \mathbf{1}_0^L,\ \mathbf{2}_{+1/2}^R,\ \mathbf{2}_{+1/2}^L,\ \mathbf{3}_{+1}^s$
28	$(H_u\mathrm{i}\tau^2\tau^a\overline{L^c})(\mathrm{i}\tau^2\tau^a H_d)(\tau^b L)(H_u\mathrm{i}\tau^2\tau^b H_u)$	1	$\mathbf{3}_0^R,\ \mathbf{3}_0^L,\ \mathbf{2}_{+1/2}^R,\ \mathbf{2}_{+1/2}^L,\ \mathbf{3}_{+1}^s$
29	$(H_u\mathrm{i}\tau^2\overline{L^c})(L)(\mathrm{i}\tau^2\vec{\tau}H_d)(H_u\mathrm{i}\tau^2\vec{\tau}H_u)$	1	$\mathbf{1}_0^R,\ \mathbf{1}_0^L,\ \mathbf{2}_{+1/2}^s,\ \mathbf{3}_{+1}^s$
30	$(H_u\mathrm{i}\tau^2\tau^a\overline{L^c})(\tau^a L)(\mathrm{i}\tau^2\tau^b H_d)(H_u\mathrm{i}\tau^2\tau^b H_u)$	1	$\mathbf{3}_0^R,\ \mathbf{3}_0^L,\ \mathbf{2}_{+1/2}^s,\ \mathbf{3}_{+1}^s$
31	$(\overline{L^c}\mathrm{i}\tau^2\tau^a H_d)(\mathrm{i}\tau^2\tau^a H_u)(\tau^b L)(H_u\mathrm{i}\tau^2\tau^b H_u)$	1	$\mathbf{3}_{+1}^L,\ \mathbf{3}_{+1}^R,\ \mathbf{2}_{+1/2}^L,\ \mathbf{2}_{+1/2}^R,\ \mathbf{3}_{+1}^s$
32	$(\overline{L^c}\mathrm{i}\tau^2\tau^a H_d)(\tau^a L)(\mathrm{i}\tau^2\tau^b H_u)(H_u\mathrm{i}\tau^2\tau^b H_u)$	1	$\mathbf{3}_{+1}^L,\ \mathbf{3}_{+1}^R,\ \mathbf{2}_{+3/2}^s,\ \mathbf{3}_{+1}^s$
33	$(\overline{L^c}\mathrm{i}\tau^2\vec{\tau}H_d)(\mathrm{i}\tau^2\vec{\tau}H_u)(H_u)(H_u\mathrm{i}\tau^2 L)$	1	$\mathbf{3}_{+1}^L,\ \mathbf{3}_{+1}^R,\ \mathbf{2}_{+1/2}^L,\ \mathbf{2}_{+1/2}^R,\ \mathbf{1}_0^L,\ \mathbf{1}_0^R$
34	$(\overline{L^c}\mathrm{i}\tau^2\tau^a H_d)(\mathrm{i}\tau^2\tau^a H_u)(\tau^b H_u)(H_u\mathrm{i}\tau^2\tau^b L)$	1	$\mathbf{3}_{+1}^L,\ \mathbf{3}_{+1}^R,\ \mathbf{2}_{+1/2}^L,\ \mathbf{2}_{+1/2}^R,\ \mathbf{3}_0^L,\ \mathbf{3}_0^R$

Table 5.3: Type I/III like decompositions of the $d = 7$ operator. Besides the fixed sign of the scalars' hypercharges they should be equal to the THDM case. We use the same notation as in Tab. 5.2. R and L indicate right and left handed fermions, where the right handed ones can also be represented by left handed Weyl spinors after charge conjugation. (The numbering of the operators follows again Ref. [7] for better comparison.)

one fermionic field (except the quark-couplings and kinetic terms) are

$$\begin{aligned}\mathcal{L}^{\text{fermionic}} &= Y_e(\bar{e}L\cdot H_d + \widetilde{\bar{e}}L\cdot \widetilde{H}_d + \bar{e}\widetilde{L}\cdot \widetilde{H}_d) - Y_N(NL\cdot H_u + \widetilde{N}L\cdot \widetilde{H}_u + N\widetilde{L}\widetilde{H}_u) \\ &\quad - \kappa(N'N'\phi + N'\widetilde{N}'\widetilde{\phi} + \widetilde{N}'N'\widetilde{\phi}) - \lambda(H_u\widetilde{H}_d\widetilde{\phi} + \widetilde{H}_u H_d\widetilde{\phi} + \widetilde{H}_u\widetilde{H}_d\phi) \\ &\quad - M_N N'N - \frac{1}{2}m_\phi\widetilde{\phi}\widetilde{\phi} + \mu(\widetilde{H}_u^0\widetilde{H}_d^0 - \widetilde{H}_u^+\widetilde{H}_d^-) + \text{h.c.} \qquad (5.8)\end{aligned}$$

After the Higgs fields get a VEV, the relevant fermionic mass terms are

$$\begin{aligned}\mathcal{L}^{\text{fermionic}}_{\text{mass}} &= -Y_e\bar{e}ev_d - Y_N N\nu v_u - \lambda v_u\widetilde{H}_d^0\widetilde{\phi} + \lambda v_d\widetilde{H}_u^0\widetilde{\phi} - M_N N'N - \frac{1}{2}m_\phi\widetilde{\phi}\widetilde{\phi} \\ &\quad - \mu(\widetilde{H}_u^0\widetilde{H}_d^0 - \widetilde{H}_u^+\widetilde{H}_d^-) + \text{h.c.} \qquad (5.9) \\ &= -\frac{1}{2}f^{0\mathsf{T}}M_f^0 f^0 + \text{h.c.} \qquad (5.10)\end{aligned}$$

Hence the mass matrix for the neutral fermions, including gauginos and Higgsinos, reads in the basis $f^0 = (\nu, N, N', \widetilde{B}, \widetilde{W}^0, \widetilde{H}_u^0, \widetilde{H}_d^0, \widetilde{\phi})$

$$M_f^0 = \left(\begin{array}{ccc|ccccc} 0 & Y_N v_u & 0 & 0 & 0 & 0 & 0 & 0 \\ Y_N v_u & 0 & m_N & 0 & 0 & 0 & 0 & 0 \\ 0 & m_N & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & M_1 & 0 & -c_\beta s_W m_Z & s_\beta s_W m_Z & 0 \\ 0 & 0 & 0 & 0 & M_2 & c_\beta c_W m_Z & -s_\beta c_W m_Z & 0 \\ 0 & 0 & 0 & -c_\beta s_W m_Z & c_\beta c_W m_Z & 0 & -\mu & -\lambda v_d \\ 0 & 0 & 0 & s_\beta s_W m_Z & -s_\beta m_Z & -\mu & 0 & \lambda v_u \\ 0 & 0 & 0 & 0 & 0 & -\lambda v_d & \lambda v_u & m_\phi \end{array}\right), \qquad (5.11)$$

where M_1 and M_2 are the soft breaking masses of the gauginos and $\tan\beta = v_u/v_d$. The neutrino and the neutralino sector are independent, since otherwise particles with and without lepton number would mix, which would violate R-parity. To generate neutrino mass in this decomposition it is not sufficient to consider solely the fermion mass matrix. Only after the scalar mediator ϕ is integrated out an effective Majorana mass for N' appears.

The charged fermions $(\bar{e}, e)$ have their usual mass matrix

$$M_e = \begin{pmatrix} 0 & Y_e v_d \\ Y_e v_d & 0 \end{pmatrix}. \qquad (5.12)$$

The chargino mixing matrix remains unchanged.

Taking a closer look at the field ϕ, one might notice that it has the same quantum numbers as the scalar of the NMSSM. It also has the same coupling to the Higgs fields. So is it the same field?

In this case we have to modify our superpotential so that we have an additional term $\kappa'\hat{\phi}^3$ and no quadratic mass term of ϕ as well as no μ-term. But now ϕ can get a VEV v_ϕ. Due to this fact, we now get also a contribution from a $d = 6$ operator of the type LLH_uH_uS where $\phi \equiv S$.

To estimate the contributions of both operators, one can take a look at the Majorana mass of N'. In the dimension six operator the Majorana mass is generated if ϕ has a VEV:

$$m_{N'}^{\text{Maj}} = \kappa' v_\phi \,. \tag{5.13}$$

By integrating out ϕ in the dimension seven operator one obtains

$$m_{N'}^{\text{Maj}} = \lambda\kappa' \frac{v_u v_d}{m_\phi} = \lambda\kappa' \frac{v_u v_d}{\kappa v_s} \,. \tag{5.14}$$

In the case of couplings close to 1 and m_ϕ not too far above the EW scale both operators have similar contributions. It should be mentioned that in the NMSSM, where $\hat{\phi} \equiv \hat{S}$, this $d = 7$ operator is always allowed—if the $d = 6$ is—by the discrete symmetry, since the charge of $\hat{S}$ is set by the term $\lambda\hat{S}\hat{H}_u\hat{H}_d$. The $d = 6$ operator in the NMSSM has been studied in Ref. [8]. As mentionend there, in the case of neutral SM singlets as mediators the production rates of the new particles are rather low, since these do not couple to the gauge bosons. Replacing the fermion singlets with triplets, however, should be straight forward.

5.5.2 Decomposition #5

Decomposition #5 of the effective $d = 7$ operator is shown in Fig. 5.4. It is an extended type II seesaw. The superpotential reads

$$\begin{aligned} W =& W_{\text{quarks}} - Y_e \hat{\bar{e}} \hat{L} \cdot \hat{H}_d + Y_\phi \hat{L} \cdot \vec{\tau} \hat{L} \hat{\vec{\phi}}_a - \kappa \hat{\vec{\phi}}_b \hat{\vec{\phi}}_c \hat{\varphi} - \mu_d \hat{\vec{\phi}}_d \hat{H}_u \cdot \vec{\tau} \hat{H}_u - \mu_\varphi \hat{\varphi} \hat{H}_u \cdot \hat{H}_d \\ &+ m_{ab} \hat{\vec{\phi}}_a \hat{\vec{\phi}}_b + m_{cd} \hat{\vec{\phi}}_c \hat{\vec{\phi}}_d + \frac{1}{2} \hat{\varphi}\hat{\varphi} + \mu \hat{H}_u \hat{H}_d \,, \end{aligned} \tag{5.15}$$

where ϕ_a, ϕ_c with $Y = +1$, ϕ_b, ϕ_d with $Y = -1$ are scalar triplets and φ with zero hypercharge is a scalar singlet.

The relevant terms of the Lagrangian corresponding to Eq. (5.15) are

$$\begin{aligned} \mathcal{L}^{\text{ferm.}} =& Y_e(\bar{e} L \cdot H_d + \tilde{\bar{e}} L \cdot \widetilde{H}_d + \bar{e} \widetilde{L} \cdot \widetilde{H}_d) - Y_\phi (L \cdot \vec{\tau} L \widetilde{\vec{\phi}}_a + \widetilde{L} \cdot \vec{\tau} L \widetilde{\vec{\phi}}_a + L \cdot \vec{\tau} \widetilde{L} \widetilde{\vec{\phi}}_a) \\ &+ \hat{\kappa}(\widetilde{\vec{\phi}}_b \widetilde{\vec{\phi}}_c \varphi + \widetilde{\vec{\phi}}_b \vec{\phi}_c \widetilde{\varphi} + \vec{\phi}_b \widetilde{\vec{\phi}}_c \widetilde{\varphi}) + \mu_d (\vec{\phi}_d \widetilde{H}_u \cdot \vec{\tau} \widetilde{H}_u + \widetilde{\vec{\phi}}_d \widetilde{H}_u \cdot \vec{\tau} H_u + \widetilde{\vec{\phi}}_d H_u \cdot \vec{\tau} \widetilde{H}_u) \\ &+ \mu_\varphi (\varphi \widetilde{H}_u \cdot \widetilde{H}_d + \widetilde{\varphi} \widetilde{H}_u \cdot H_d + \widetilde{\varphi} H_u \cdot \widetilde{H}_d) - m_{ab} \widetilde{\vec{\phi}}_a \widetilde{\vec{\phi}}_b - m_{cd} \widetilde{\vec{\phi}}_c \widetilde{\vec{\phi}}_d \\ &- \frac{1}{2} m_\varphi \widetilde{\varphi}\widetilde{\varphi} + \text{h.c.} \end{aligned} \tag{5.16}$$

After the Higgs fields get a VEV the fermionic mass terms are

$$\begin{aligned} \mathcal{L}^{\text{fermionic}}_{\text{mass}} =& - Y_e \bar{e} e v_d - \mu_d v_u (\widetilde{\phi}_d^0 \widetilde{H}_u^0 + \widetilde{\phi}_d^- \widetilde{H}_u^+) - \mu_\varphi v_d \widetilde{\varphi} \widetilde{H}_u^0 - \mu_\varphi v_u \widetilde{\varphi} \widetilde{H}_d^0 \\ & - m_{ab} \widetilde{\vec{\phi}}_a \widetilde{\vec{\phi}}_b - m_{cd} \widetilde{\vec{\phi}}_c \widetilde{\vec{\phi}}_d - \frac{1}{2} m_\varphi \widetilde{\varphi}\widetilde{\varphi} + \mu (\widetilde{H}_u^0 \widetilde{H}_d^0 - \widetilde{H}_u^+ \widetilde{H}_d^-) + \text{h.c.} \,. \end{aligned} \tag{5.17}$$

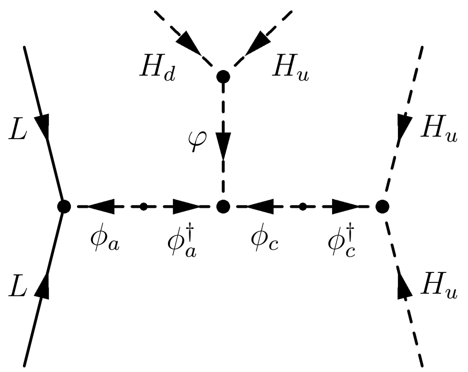


Figure 5.4: Decomposition #5 of the effective $d = 7$ operator.

The mass matrix for the neutral fermions, again with Higgsinos and gauginos, reads in the basis $f^0 = (\nu, \widetilde{\phi}_a^0, \widetilde{\phi}_b^0, \widetilde{\phi}_c^0, \widetilde{\phi}_d^0, \widetilde{W}, \widetilde{B}, \widetilde{H}_u^0, \widetilde{H}_d^0, \widetilde{\varphi})$

$$M_f^0 = \left(\begin{array}{c|cc|cccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & m_{ab} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & m_{ab} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & m_{cd} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_{cd} & 0 & 0 & 0 & \mu_d v_u & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & M_1 & 0 & -c_\beta s_W m_Z & s_\beta s_W m_Z & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & M_2 & c_\beta c_w m_Z & -s_\beta c_W m_Z & 0 \\ 0 & 0 & 0 & 0 & \mu_d v_u & -c_\beta s_W m_Z & c_\beta c_W m_Z & 0 & -\mu & \mu_\varphi v_d \\ 0 & 0 & 0 & 0 & 0 & s_\beta s_W m_Z & -s_\beta m_Z & -\mu & 0 & \mu_\varphi v_u \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_\varphi v_d & \mu_\varphi v_u & m_\varphi \end{array}\right). \tag{5.18}$$

The mass matrix for the single charged leptons is

$$f^{+\mathsf{T}} M_f^{\pm} f^- \tag{5.19}$$

where

$$f^+ = (\bar{e}^+, \widetilde{\phi}_a^+, \widetilde{\phi}_c^+, \widetilde{W}^+, \widetilde{H}_u^+)^\mathsf{T}\,, \tag{5.20a}$$
$$f^- = (e^-, \widetilde{\phi}_b^-, \widetilde{\phi}_c^-, \widetilde{W}^-, \widetilde{H}_d^-)^\mathsf{T} \tag{5.20b}$$

and

$$M_f^{\pm} = \left(\begin{array}{cc|ccc} Y_e v_d & 0 & 0 & 0 & 0 \\ 0 & m_{ab} & 0 & 0 & 0 \\ \hline 0 & 0 & m_{cd} & 0 & 0 \\ 0 & 0 & 0 & M_2 & \sqrt{2} s_\beta m_W \\ 0 & 0 & \mu_d v_u & \sqrt{2} c_\beta m_W & \mu \end{array}\right). \tag{5.21}$$

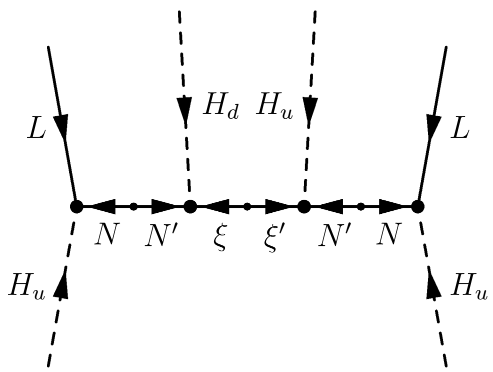


Figure 5.5: Decomposition #17 of the effective $d = 7$ operator.

For the double charged fields we have

$$f^{++\mathsf{T}} M_f^{(2\pm)} f^{--} \tag{5.22}$$

where

$$f^{++} = (\widetilde{\phi}_a^{++}, \widetilde{\phi}_c^{++})^\mathsf{T}\,, \tag{5.23a}$$
$$f^{--} = (\widetilde{\phi}_b^{--}, \widetilde{\phi}_d^{--})^\mathsf{T} \tag{5.23b}$$

and

$$M_f^{(2\pm)} = \begin{pmatrix} m_{ab} & 0 \\ 0 & m_{cd} \end{pmatrix}\,. \tag{5.24}$$

Here we have the same problem as with decomp. #1 if φ gets a VEV or is the NMSSM scalar respectively.

5.5.3 Decomposition #17

Finally we will consider decomposition #17 shown in Fig. 5.5. The corresponding superpotential is

$$W = W_{\text{quarks}} - Y_e \hat{\bar{e}} \hat{L} \cdot \hat{H}_d + Y_N \hat{N} \hat{L} \cdot \hat{H}_u - \kappa_1 \hat{N}' \hat{\xi} \cdot \hat{H}_d + \kappa_2 \hat{N}' \hat{\xi}' \cdot \hat{H}_u + m_N \hat{N} \hat{N}' \\ + m_\xi \hat{\xi} \hat{\xi}' + \mu H_u H_d\,, \tag{5.25}$$

where N and N' are singlets with zero hypercharge and ξ and ξ' are doublets with $Y = \pm\frac{1}{2}$.

The relevant terms of the Lagrangian corresponding to Eq. (5.25) are

$$\mathcal{L}^{\text{ferm.}} = Y_e(\bar{e}L \cdot H_d + \widetilde{\bar{e}}L \cdot \widetilde{H}_d + \bar{e}\widetilde{L} \cdot \widetilde{H}_d) - Y_N(NL \cdot H_u + \widetilde{N}L \cdot \widetilde{H}_u + N\widetilde{L}\widetilde{H}_u) \\ - \kappa_1(N'\xi \cdot H_d + \widetilde{N}'\xi \cdot \widetilde{H}_d + N'\widetilde{\xi} \cdot \widetilde{H}_d) + \kappa_2(N'\xi' \cdot H_u + \widetilde{N}'\xi' \cdot \widetilde{H}_u + N'\widetilde{\xi}' \cdot \widetilde{H}_u) \\ - M_N N'N - m_\xi \xi\xi' - \mu \widetilde{H}_u^0 H_d^0 + \text{h.c.} \tag{5.26}$$

After the Higgs fields get a VEV the fermionic mass terms are

$$\mathcal{L}_{\text{mass}}^{\text{fermionic}} = -Y_e v_d \bar{e}e - Y_N v_u N\nu - \kappa_1 v_d \xi^0 N' - \kappa_2 v_u \xi'^0 N' - m_N NN' - m_\xi \xi\xi' + \text{h.c.} \tag{5.27}$$

The mass matrix for the neutral fermions reads in the basis

$$f^0 = (\nu, N, N', \xi^0, \xi'^0, \widetilde{H}_u^0, \widetilde{H}_d^0, \widetilde{B}, \widetilde{W}^0) \tag{5.28}$$

$$M_f^0 = \left(\begin{array}{ccccc|c} 0 & Y_N v_u & 0 & 0 & 0 & 0 \\ Y_N v_u & 0 & m_N & 0 & 0 & 0 \\ 0 & m_N & 0 & \kappa_1 v_d & \kappa_2 v_u & 0 \\ 0 & 0 & \kappa_1 v_d & 0 & m_\xi & 0 \\ 0 & 0 & \kappa_2 v_u & m_\xi & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & M_{\widetilde{\chi}^0} \end{array}\right). \tag{5.29}$$

In this case $M_{\widetilde{\chi}^0}$ is the unmodified neutralino mixing matrix of the MSSM as in Eq. (4.39).

The mass matrix for the charged fermions is

$$= -\frac{1}{2} f^{0\mathsf{T}} M^{\text{fermionic}} f^0 + \text{h.c.} - \frac{1}{2}(\widetilde{f}^{+\mathsf{T}} M_f^{\pm\mathsf{T}} \widetilde{f}^- + \widetilde{f}^{-\mathsf{T}} M_f^{\pm} \widetilde{f}^+) + \text{h.c.}, \tag{5.30}$$

where $\widetilde{f}^+ = (\bar{e}^+, \xi^+, H_u^+, \widetilde{W}^+)$ and $\widetilde{f}^- = (e^-, \xi'^-, H_d^-, \widetilde{W}^-)$ and

$$M_f^{\pm} = \left(\begin{array}{cc|cc} Y_e v_d & 0 & 0 & 0 \\ 0 & m_\xi & 0 & 0 \\ \hline 0 & 0 & \mu & \sqrt{2} s_\beta m_W \\ 0 & 0 & \sqrt{2} s_\beta m_W & M_2 \end{array}\right) \tag{5.31}$$

The mediators do not mix with the neutralino and chargino fields, since we have only fermionic mediators which have lepton number and otherwise R-Parity would be violated. This decomposition is discussed further in Chapter 6, where also the generation of the neutrino mass is described.

5.5.4 Comparison of the decompositions

If one compares the mass matrices for operator #1 (top. 2, type I), #5 (top. 2, type II) and #17 (top. 1, type I), one can see some substantial differences.

For operator #1 the partner of the neutral scalar mixes with the neutral Higgsinos, whereas the heavy fermions do not, due to R-Parity conservation. Therefore the mixing matrix for the neutralinos has to be modified. In the case of operator

#5 some of the partners of the scalars mix with the charged and neutral Higgsinos, so the mixing of neutralinos and charginos is affected. This is possible, because here we have scalar triplets with electromagnetic charged components as mediators. As a consequence of this mixing with charginos and neutralinos one obtains additional neutralino and chargino states compared to the usual SUSY models, like the MSSM. Also their mass spectrum will be modified, since we obtain new mass eigenstates. This will have phenomenological consequences for processes involving neutralinos and charginos.

Finally for the #17 operator, all the new particles mix with each other and the neutrino fields, but not with the Higgsinos. This is because we have only fermionic mediators.

It also seems that topology 1—and more specifically those decompositions that have only fermionic mediators—can be used to easily avoid a scalar with a VEV that implies a $d = 6$ contribution. Hence we will focus on decomposition #17, with two fermion singlet and doublet mediators, for a more detailed study in the next chapter.

6 Properties of the Decomposition with Two Fermion Singlets and Doublets

6.1 The neutrino mass

The Lagrangian for decomposition #17 is specified in App. A. We can determine the neutrino mass, by integrating out the mediator fields. This would be more complicated, if we were to consider the flavor structure of the decomposition. Here, however, we will only discuss the simpler case of one flavor for each particle (which, of course, is insufficient to reproduce neutrino physics).

First the heavy doublets are integrated out. The relevant terms in the Lagrangian are:

$$\mathcal{L}_\xi = \xi^\dagger \mathrm{i}\bar{\sigma}^\mu D_\mu \xi + \xi'^\dagger \mathrm{i}\bar{\sigma}^\mu D^\mu \xi' - m_\xi \xi \cdot \xi' - \kappa_1 N' \xi \cdot H_d + \kappa_2 N' \xi' \cdot H_u + \text{h.c.} \quad (6.1)$$

The stationary fields are:

$$\xi = \frac{\kappa_2}{m_\xi} N' H_u \quad (6.2a)$$

$$\xi' = -\frac{\kappa_1}{m_\xi} N' H_d \quad (6.2b)$$

So one obtains:

$$\mathcal{L}_N = N^\dagger \mathrm{i}\bar{\sigma}^\mu \partial_\mu N + N'^\dagger \mathrm{i}\bar{\sigma}^\mu \partial_\mu N' - m_N N N' - Y_N N L \cdot H_u - \frac{\kappa_1 \kappa_2}{m_\xi} N' N' H_u \cdot H_d + \text{h.c.} \quad (6.3)$$

This way we have reproduced the inverse seesaw mass matrix which reads in the basis (ν, N, N'):

$$M_\nu = \begin{pmatrix} 0 & Y_N & 0 \\ Y_N & 0 & m_N \\ 0 & m_N & \epsilon \end{pmatrix} \quad (6.4)$$

with $\epsilon = \frac{2\kappa_1\kappa_2}{m_\xi} v_u v_d$ and $v_{u/d} = \langle H_{u/d} \rangle$.

Integrating out the heavy singlets can now be accomplished in analogy to previous calculations. After replacing the parameters accordingly one obtains the neutrino mass:

$$m_\nu = v_u^3 v_d Y_N^2 \frac{\kappa_1 \kappa_2}{m_\xi m_N^2} \tag{6.5}$$

For a neutrino mass $m_\nu \approx 1\,\text{eV}$ and $v \approx 250\,\text{GeV}$ and the heavy mass scale at 1 TeV this means couplings of $\mathcal{O}(10^{-3})$ are required.

Besides the neutrino mass, there are also further possible low energy effects depending on the other parts of the effective low energy Lagrangian. Further effects can occur after integrating out the superpartners of the heavy particles. These can, however, be assumed at a larger mass scale due to SUSY breaking terms.

6.2 Realization of the flavor structure

Due to the neutrino phenomenology described in chapter 2, we know that we have three neutrino flavor states that mix with each other. To reproduce this behavior, we need to add a flavor structure to our model.

Since there are three distinct mass eigenstates, at least two of them must have a finite mass. To simplify the discussion, we will assume that one neutrino has no mass (or a negligible small one). There are two fundamentally different approaches, which will be discussed in the following two subsections.

6.2.1 Flavor Structure created at one-loop

The first possibility is that the two masses are created by different mechanisms, one mass by the seesaw like decomposition of an $d = 7$ operator—as described here for decomposition #17—and the other one by loop corrections. As pointed out by *Hirsch et al.*[33] it is an advantage of SUSY that this is possible at the one-loop level. What they discussed in the case of the inverse seesaw mechanism can be also adapted for our model. The sneutrino couplings and mass matrix can be found in App. B. Since we can derive the mass eigenstates and mixing matrix from that, we can follow closely the procedure of *Hirsch et al.* to determine the neutrino self-energy functions. They can be written as

$$-\mathrm{i}\Sigma_{\nu\nu}^{mn}(p) = -\mathrm{i}\left[(\not{p}\Sigma_V^{mn}(p^2) + \Sigma_S^{mn}(p^2))P_L + (\not{p}\Sigma_V^{mn*}(p^2) + \Sigma_S^{nm*}(p^2))P_R\right] \tag{6.6}$$

and contribute to the neutrino mass matrix

$$M_{mn}^{1-\text{loop}} = m_{\nu_m}(Q)\delta_{mn} + \mathrm{Re}\left[\Sigma_S^{mn}(p^2) + m_{\nu_m}\Sigma_V^{mn}(p^2)\right]_{\Delta=0} \tag{6.7}$$

where Q is the renormalization scale.

The transition from mass states to flavor states now contains tree-level as well as one-loop contributions.

$$\nu_\alpha = (U^{\rm tr} U^{1-\rm loop})_{\alpha i}\nu_i \equiv U^\nu_{\alpha i}\nu_i \tag{6.8}$$

The neutrino-sneutrino-neutralino coupling can be expressed as

$$\mathcal{L}_{\nu\chi^0\tilde\nu} = \bar{\tilde\chi}^0_j (A^R_{mjb}P_R + A^L_{mjb}P_L)\nu_m \widetilde{N}_b + \text{h.c.} \tag{6.9}$$

with the coefficients

$$A^R_{mjb} = -\frac{1}{\sqrt{2}} h^i_\nu U^{\rm tr}_{im} N_{j4}(G_{b4} - iG_{b9})\,, \tag{6.10a}$$

$$A^L_{mjb} = -\frac{g}{2}(N^*_{j2} - \tan\theta_W N^*_{j1})(G_{bi} - iG_{b(i+5)})U^{\rm tr}_{im}\;, \tag{6.10b}$$

where G is the sneutrino and N the neutralino mixing matrix. Due to the combinations of A^R and A^L at each vertex of the loop, the self-energy functions then read

$$\Sigma^{mn}_{S2} = \frac{-m_{\chi^0_j}}{(4\pi)^2}\left[A^L_{mjb}A^L_{njb} + A^{R*}_{mjb}A^{R*}_{njb} + A^{R*}_{mjb}A^L_{njb} + A^L_{mjb}A^{R*}_{njb}\right] B_0(m^2_{\chi^0_j}, m^2_{\tilde N_b})\,, \tag{6.11a}$$

$$\Sigma^{mn}_{V2} = \frac{-1}{(4\pi)^2}\left[A^{L*}_{mjb}A^L_{njb} + A^R_{mjb}A^{R*}_{njb} + A^{L*}_{mjb}A^{R*}_{njb} + A^R_{mjb}A^L_{njb}\right] B_1(m^2_{\chi^0_j}, m^2_{\tilde N_b}) \tag{6.11b}$$

Since the sneutrino mass matrix is influenced by the choice of the soft-breaking parameters, the loop corrections couple in a different way to the neutrinos as the effective operator does. Hence we can obtain a complex flavor structure, which generates the required neutrino phenomenology.

6.2.2 Flavor Structure from two families at tree level

We can also recreate neutrino physics, if we add a flavor structure to our decomposition itself. The easiest approach is to make the N and N' fields to flavor pairs and leave the other mediators as flavor singlets. Thus we obtain a mass matrix

$$M_{\alpha\beta} = v_u^3 v_d (Y_N)_{\alpha i}(M_N^{-1})_{ij}\epsilon_{jk}(M_N^{-1,\intercal})_{kl}(Y_N^\intercal)_{l\beta}\,, \tag{6.12}$$

where

$$\epsilon_{jk} = \frac{1}{m_\xi}\left((\kappa_1)_{jm}(\kappa_2^\intercal)_{mk} + (\kappa_2)_{jm}(\kappa_1^\intercal)_{mk}\right)\;. \tag{6.13}$$

The flavor basis can be chosen in a way that M_N or respectively M_N^{-1} is diagonal, without loss of generality.

The appearing matrices must reproduce the neutrino phenomenology and are hence different for normal and inverted hierarchy.

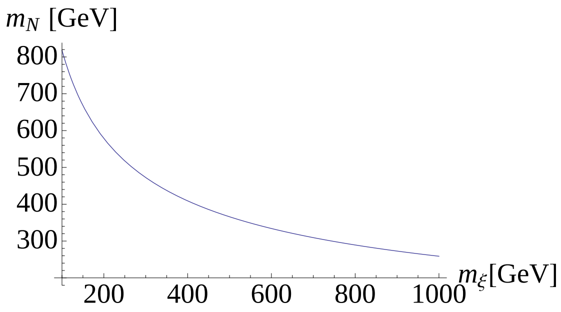


Figure 6.1: Dependence of m_N on m_ξ for $y_N = k_1 = k_2 = 10^{-3}$, $\tan\beta = 1.5$ and $m_2 = 0.00875\,\text{eV}$.

Normal hierarchy

A rather straight forward parameterization is the following:

$$Y_N = y_N \begin{pmatrix} \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} , \quad \kappa_1 = k_1 \begin{pmatrix} -1 \\ 1 \end{pmatrix} , \quad \kappa_2 = k_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} , \quad M_N = m_N \begin{pmatrix} 1 & 0 \\ 0 & \rho \end{pmatrix} , \tag{6.14}$$

where $\rho = \sqrt{m_2/m_3}$ and

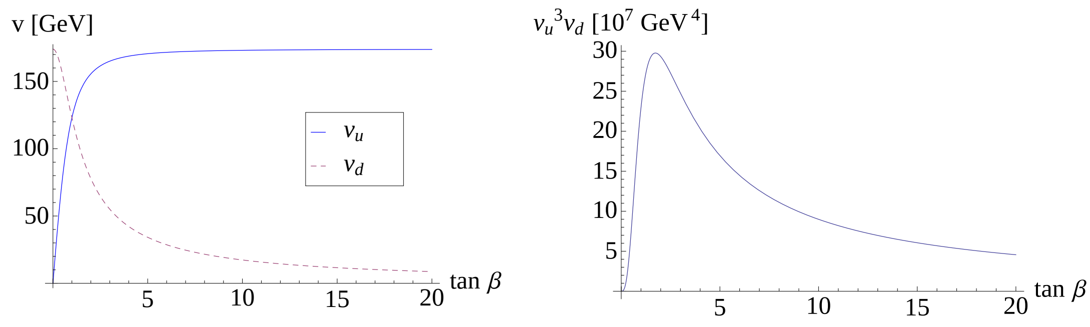
$$2v_u^3 v_d y_N^2 k_1 k_2 / (m_N^2 m_\xi) \stackrel{!}{=} m_2 \, . \tag{6.15}$$

This reproduces the tri-bimaximal mixing pattern and two non-zero mass eigenvalues. The only difference is that we have $-m_2$ instead of m_2. This minus sign can be absorbed in a phase redefinition of the SM neutrino fields, since the effective mass term is of Majorana type. An overview of the possible coefficients which generate the neutrino mass scale can be found in Fig. 6.1.

Since we have more parameters than restrictions due to neutrino physics, there is a certain freedom in the parameters of the couplings. For example one can vary Y_N as long as this is compensated by an according change of M_N and/or κ, as long as M_N remains diagonal, since this is the basis which we choose to work with.

One can conclude, that the tri-bimaximal character—or the flavor mixing of the neutrinos in general—is mostly generated by the Yukawa like couplings Y_N. The mass ratio $\rho = \sqrt{m_2/m_3}$ can be generated by Y_N, M_N or $\kappa_{1/2}$.

The dependence on $\tan\beta$ of the Higgs VEV contribution to the neutrino mass $v_u^3 v_d$ can be seen in Fig. 6.2. A $\tan\beta$ of 20 will introduce an additional suppression factor of about 6 to 7. Going to even larger values like 50 one obtains a suppression factor of about 16.



a) b)

Figure 6.2: Dependence of the Higgs VEVs on $\tan\beta$. Shown separately (a) and in the combination $v_u^3 v_d$ (b), which is the same as in the effective operator. Small values for $\tan\beta$ can be excluded, due to experimental and theoretical constraints such as the top-quark Yukawa coupling, so that small values of $\tan\beta$ can not be used for suppression.

Inverted hierarchy

The inverted hierarchy can be obtained by the parameterization

$$Y_N = y_N \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \end{pmatrix} , \quad \kappa_1 = k_1 \begin{pmatrix} -1 \\ 1 \end{pmatrix} , \quad \kappa_2 = k_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} , \quad M_N = m_N \begin{pmatrix} 1 & 0 \\ 0 & \rho \end{pmatrix} , \tag{6.16}$$

where $\rho = \sqrt{m_1/m_2}$ and

$$2 v_u^3 v_d y_N^2 k_1 k_2 / (m_N^2 m_\xi) \stackrel{!}{=} m_1 \, . \tag{6.17}$$

Here again one eigenvalue is $-m_2$ instead of m_2.

In general, it should be easily possible to generate any mixing pattern for both hierarchies by just changing the Yukawa couplings Y_N accordingly.

6.3 Phenomenology

In this section we want to discuss the phenomenological characteristics of our model with regard to its possible verifiability at the *Large Hadron Collider* (LHC). Even with a minimal flavor structure and the constraints from neutrino physics, we have still many free parameters. A detailed study of the parameter space is beyond the scope of this thesis. Instead we want to present an exemplary discussion of one possible realization of our model.

n_i	n_1	n_2	n_3	n_4	n_5	n_6	n_7	n_8	n_9
m_i [GeV]	0	$-8.50 \cdot 10^{-12}$	$4.99 \cdot 10^{-11}$	-200	200	244	-244	-578	578

Table 6.1: Mass eigenvalues m_i of the neutral fermions and the according mass eigenstates n_i. The three lightest states n_1, n_2 and n_3 are equivalent to the three neutrino mass eigenstates ν_1, ν_2 and ν_3. For the heavy states we have always two pairs with the same mass but opposite sign. Therefore we have pseudo-Dirac pairs, which means the Majorana nature of the mediators is suppressed. n_8 and n_9 are mostly composed of the first generation of N and N' whereas n_6 and n_7 correspond to their second generation. n_3 and n_4 are mostly equivalent to ξ^0 and ξ'^0. All mass states, however, contain also smaller contributions of the other fields because of mixing.

6.3.1 Parameters

We choose the flavor structure of Eq. (6.14) with the parameters

$$y_N = k_1 = k_2 = 10^{-3}\,, \quad m_N = 578\,\text{GeV}\,, \quad \text{and} \quad m_\xi = 200\,\text{GeV}\,. \tag{6.18}$$

We will also use $\tan\beta = 1.5$. This means the heavy fields have a hierarchy $m_N > m_\xi$. The according mass eigenvalues of the neutral fermions are listed in Tab. 6.1. The mixing between ν_1 and the heavy fields is very suppressed. This is due to the fact that ν_1 is considered massless in our model and hence does not couple to the new mass generating particles.

This choice of parameters leads to a rather small mass for m_ξ, so that it is more likely to be produced. The mass is, however, not so small that it would be in a region already excluded by experiments. Couplings closer to one might be preferable but would require higher masses. Couplings of $\mathcal{O}(10^{-3})$ are still in the range of the lepton Yukawa couplings of the SM.

6.3.2 Numerical computation

The processes we want to study in the following sections have up to four final states, which makes it difficult to calculate the according phase space integrals. Therefore it is favorable to use numerical instead of analytical methods to obtain the results we are interested in. We therefore use the software package Whizard [41] (version 1.95), which uses Monte-Carlo methods to integrate over the phase space. The matrix elements Whizard uses are generated by O'Mega [42]. All results are for tree level.

In order to use these programs for our model, we have had to make some modifications. The new particles and their properties had to be specified. Furthermore, their interactions had to be implemented, including couplings and mixing parameters. The neutral fermions have been implemented as Majorana fermions. The charged mediators ξ^+ and $\xi'^- = (\bar{\xi}^+)^c$ can be implemented as the RH and LH components of one Dirac spinor.

6.3.3 Production of the charged heavy fields

An advantage of our model compared to other extensions of the seesaw mechanism—including the inverse seesaw—is that we use mediators that have isospin and hypercharge. As a consequence they couple to the gauge bosons $W^\pm$ and Z. W^+ can be produced by quark-antiquark scattering. We will use the process $pp \to W^+$ here. Since the LHC is a proton-proton collider, the 4-momenta of the initial quarks are unknown in the experiment. Hence we have to take their parton distribution functions (PDFs) into consideration. WHIZARD has already a built-in implementation of PDFs that are based on Ref. [43].

If the produced W^+ is off-shell, it can decay into the two components of the charged mediator doublet ξ^+ and ξ^0, where ξ^0 is composed of the mass eigenstates n_i. Hence the on-shell production cross section is

$$\sigma(pp \to \xi^+ + n_i) = (3.7696 \pm 0.0025) \cdot 10^2\,\text{fb}\,. \tag{6.19a}$$

In analogy one obtains

$$\sigma(pp \to \bar{\xi}^- + n_i) = (1.9647 \pm 0.0037) \cdot 10^2\,\text{fb} \tag{6.19b}$$

for a center of mass energy $\sqrt{s} = 7\,\text{TeV}$. The difference between both values is due to the different PDFs in both cases. For $\sqrt{s} = 14\,\text{TeV}$ one obtains

$$\sigma(pp \to \xi^+ + n_i) = (1.0138 \pm 0.0020) \cdot 10^3\,\text{fb} \tag{6.20a}$$

and

$$\sigma(pp \to \bar{\xi}^- + n_i) = (6.153 \pm 0.012) \cdot 10^2\,\text{fb}\,. \tag{6.20b}$$

The production cross-section for $pp \to \bar{e}^+ + \nu$ via W^+ is $2.21 \cdot 10^6$ fb at 7 TeV, for comparison. Since the gauge couplings are the same in both cases the difference can be explained by the smaller phase space of the heavy particles. They are, however, not stable and will decay further. Therefore, as a next step, we will compute their decay widths.

6.3.4 Decay via $W^\pm$

The particles ξ^+ and $\bar{\xi}^-$ decay into $W^\pm$ and $\nu_{2/3}$, due to the gauge coupling of ξ (see App. A.3) and the mixing of ξ^0 with $\nu_{2/3}$. Since the charged components themselves do not mix, this is the only decay channel for these. The decay widths are:

$$\Gamma^{\text{tot}}_{\xi^+} = \Gamma(\xi^+ \to W^+\nu_2/W^+\nu_3) = 3.0541298 \cdot 10^{-12} \tag{6.21a}$$
$$\Gamma^{\text{tot}}_{\bar{\xi}^-} = \Gamma(\bar{\xi}^- \to W^-\nu_2/W^-\nu_3) = 3.0541298 \cdot 10^{-12} \tag{6.21b}$$

The neutral fields mix to $\nu_e/\nu_\mu/\nu_\tau$ and hence can decay into $e/\mu/\tau$ via $W^\pm$. The partial widths for these decays are listed in Tab. 6.2. One might expect the

Process	Γ[GeV]
$n_4 \rightarrow W^{\pm} + e/\mu/\tau$	$1.1469172 \cdot 10^{-11}$
$n_5 \rightarrow W^{\pm} + e/\mu/\tau$	$7.4481607 \cdot 10^{-13}$
$n_6 \rightarrow W^{\pm} + e/\mu/\tau$	$1.6236987 \cdot 10^{-6}$
$n_7 \rightarrow W^{\pm} + e/\mu/\tau$	$1.6236878 \cdot 10^{-6}$
$n_8 \rightarrow W^{\pm} + e/\mu/\tau$	$3.9673876 \cdot 10^{-6}$
$n_9 \rightarrow W^{\pm} + e/\mu/\tau$	$3.9673876 \cdot 10^{-6}$

Table 6.2: Decay widths for $n_i \rightarrow W^{\pm} + e/\mu/\tau$.

decay to be larger for the states n_4 and n_5, since they correspond to the doublet mediators, which couple to gauge bosons. Here, however, we have in principle a vertex $\nu_e W e$ (accordingly for μ and τ), where ν is composed of the mass states n_i. Since the mass states n_6–n_9 correspond to the two generations of the fields N and N', which have a direct coupling to the neutrinos via Y_N, their decay widths are larger.

6.3.5 Decay of the neutral heavy mass eigenstates via Z

The neutrinos $\nu_e/\nu_\mu/\nu_\tau$ as well as the neutral components of the heavy doublets ξ^0 and ξ'^0 couple to the Z boson (see also App. A.3). The coupling constant is the same in all three cases, but with a different sign for ξ^0, since it has opposite isospin and hypercharge. Since the mass eigenstates mix to these five flavor states, they can decay via a Z boson into $\nu_{2,3}$. Because of the mixing, the coupling for $n_k \rightarrow Z\nu_l$ reads

$$(g'_Z)_{lk} = g_Z \left(U_{\nu_l \nu_e} U^\dagger_{\nu_e n_k} + U_{\nu_l \nu_\mu} U^\dagger_{\nu_\mu n_k} + U_{\nu_l \nu_\tau} U^\dagger_{\nu_\tau n_k} - U_{\nu_l \xi^0} U^\dagger_{\xi^0 n_k} + U_{\nu_l \xi'^0} U^\dagger_{\xi'^0 n_k} \right) , \tag{6.22}$$

where g_Z is the coupling constant as in the Standard model. In Tab. 6.3 g'_Z/g_Z is listed for all cases. The resulting decay widths are listed in Tab. 6.4.

Again one could expect the decay widths of n_4 and n_5 to be larger, since they are related to the doublet fields, which couple to the Z boson. The last two terms in Eq. (6.22) correspond in principle to the vertices $\xi^0 Z \xi^0$ and $\xi'^0 Z \xi'^0$. They are indeed larger for n_4 and n_5, since the mixing of ξ^0 and ξ'^0 to these mass eigenstates is larger than for the others. They are, however, supressed, since ξ^0 and ξ'^0 mix only weakly to the light neutrinos we have in the final state of the decay process. The larger contribution comes from the first three terms, which correspond to the vertices $\nu_{e,\mu,\tau} Z \nu_{e,\mu,\tau}$. Here again n_6–n_9 have a larger mixing to the light neutrinos than n_4 and n_5.

	ν_3	ν_2
n_9	$9.67681{\cdot}10^{-7}$	0.000177005
n_8	$9.67881{\cdot}10^{-7}$	-0.000177005
n_7	-0.000418824	$-2.28863{\cdot}10^{-6}$
n_6	-0.000418826	$2.28863{\cdot}10^{-6}$
n_5	$-8.09644{\cdot}10^{-7}$	$-2.89589{\cdot}10^{-7}$
n_4	$-1.63126{\cdot}10^{-6}$	$-2.7085{\cdot}10^{-7}$

Table 6.3: The correction g'_Z/g_Z for the coupling of the neutral mass eigenstates to Z due to mixing.

Process	Γ[GeV]
$n_4 \to Z + \nu_2/\nu_3$	$6.3768553 \cdot 10^{-12}$
$n_5 \to Z + \nu_2/\nu_3$	$1.7243293 \cdot 10^{-12}$
$n_6 \to Z + \nu_2/\nu_3$	$7.9494161 \cdot 10^{-7}$
$n_7 \to Z + \nu_2/\nu_3$	$7.9493402 \cdot 10^{-7}$
$n_8 \to Z + \nu_2/\nu_3$	$1.9822844 \cdot 10^{-7}$
$n_9 \to Z + \nu_2/\nu_3$	$1.9822844 \cdot 10^{-7}$

Table 6.4: Decay widths for $n_i \to Z + \nu_2/\nu_3$.

6.3.6 Decay of the neutral heavy mass eigenstates via h^0

We make the assumption that the lightest Higgs field h^0, the real scalar field, has a mass of 120 GeV and all other fields are much heavier. The neutral components of the Higgs doublets can be expressed as [36]

$$\mathrm{Re}\, H_u^0 = \left[v_u + \frac{1}{\sqrt{2}}(\cos\alpha\, h^0 + \sin\alpha\, H^0) \right] \tag{6.23a}$$

$$\mathrm{Re}\, H_d^0 = \left[v_u + \frac{1}{\sqrt{2}}(-\sin\alpha\, h^0 + \cos\alpha\, H^0) \right] , \tag{6.23b}$$

where α is the mixing angle between the two neutral scalar Higgs fields h^0 and H^0. Its value can be estimated with the tree-level relation

$$\frac{\tan 2\alpha}{\tan 2\beta} = \frac{m_{A0}^2 + m_Z^2}{m_{A0}^2 - m_Z^2} , \tag{6.24}$$

with $\tan\beta = 1.5$ and $m_{A_0} = 1\,\mathrm{TeV}$. The resulting value is $\alpha = -0.591$. The relevant couplings between the Higgs fields and the neutral neutrino mass eigenstates are

$$-\kappa_1 H_d^0 \xi^0 N' - \kappa_2 H_u^0 \xi'^0 - Y_N H_u^0 N \nu \,. \tag{6.25}$$

	ν_3	ν_2
n_9	$1.55495{\cdot}10^{-6}$	0.0002937
n_8	$-1.65472{\cdot}10^{-6}$	0.000293486
n_7	0.000293433	$1.62735{\cdot}10^{-6}$
n_6	-0.00029334	$1.58194{\cdot}10^{-6}$
n_5	$2.31347{\cdot}10^{-7}$	$-2.63240{\cdot}10^{-7}$
n_4	$1.16996{\cdot}10^{-6}$	$5.84413{\cdot}10^{-8}$

Table 6.5: The coupling $g_{h^0}/\sqrt{2}$ of the neutral mass eigenstates with h^0.

Process	Γ[GeV]
$n_4 \to h^0 + \nu_2/\nu_3$	$8.9454953 \cdot 10^{-12}$
$n_5 \to h^0 + \nu_2/\nu_3$	$8.0064062 \cdot 10^{-13}$
$n_6 \to h^0 + \nu_2/\nu_3$	$9.6296678 \cdot 10^{-7}$
$n_7 \to h^0 + \nu_2/\nu_3$	$9.6357909 \cdot 10^{-7}$
$n_8 \to h^0 + \nu_2/\nu_3$	$3.6280879 \cdot 10^{-6}$
$n_9 \to h^0 + \nu_2/\nu_3$	$3.6332663 \cdot 10^{-6}$

Table 6.6: Decay widths for $n_i \to h^0 + \nu_2/\nu_3$.

By replacing H_u with $\cos\alpha/\sqrt{2}\,h^0$ and H_d with $-\sin\alpha/\sqrt{2}\,h^0$ one obtains the couplings to h^0 for the flavor states. To obtain the couplings of the mass eigenstates, the SM Higgs coupling must be modified:

$$
\begin{aligned}
(g_{h^0})_{n_k\nu_l} = \frac{1}{\sqrt{2}}\ \Big\{ &- \Big[(-U_{n_kN'_a} + U_{n_kN'_b})U^\dagger_{\xi^0\nu_l} + U_{n_k\xi^0}(-U^\dagger_{N'_a\nu_l} + U^\dagger_{N'_b\nu_l})\Big]\, k_1 \sin\alpha \\
&+ \Big[(U_{n_kN'_a} + U_{n_kN'_b})U^\dagger_{\xi'^0\nu_l} + U_{n_k\xi'^0}(U^\dagger_{N'_a\nu_l} + U^\dagger_{N'_b\nu_l})\Big]\, k_2 \cos\alpha \\
&+ \Big[\frac{1}{\sqrt{3}}U_{n_kN_a}(U^\dagger_{\nu_e\nu_l} + U^\dagger_{\nu_\mu\nu_l} + U^\dagger_{\nu_\tau\nu_l}) + \frac{1}{\sqrt{2}}U_{n_kN_b}(-U^\dagger_{\nu_\mu\nu_l} + U^\dagger_{\nu_\tau\nu_l}) \\
&\quad + \frac{1}{\sqrt{3}}(U_{n_k\nu_e} + U_{n_k\nu_{mu}} + U_{n_k\nu_\tau})U^\dagger_{N_a\nu_l} \\
&+ \frac{1}{\sqrt{2}}(-U_{n_k\nu_\mu} + U_{n_k\nu_\tau})U^\dagger_{N_n\nu_l}\Big]\, y_N \cos\alpha \Big\}\ .
\end{aligned}
\tag{6.26}
$$

$g_{h^0}/\sqrt{2}$ is listed in Tab. 6.5. The resulting decay widths are listed in Tab. 6.6.

Particle	Γ[GeV]
n_4	$2.67915226 \cdot 10^{-11}$
n_5	$3.26978599 \cdot 10^{-12}$
n_6	$3.38159619 \cdot 10^{-6}$
n_7	$3.382220091 \cdot 10^{-6}$
n_8	$9.5777599 \cdot 10^{-6}$
n_9	$9.5829383 \cdot 10^{-6}$

Table 6.7: Total decay widths of the neutral mass eigenstates.

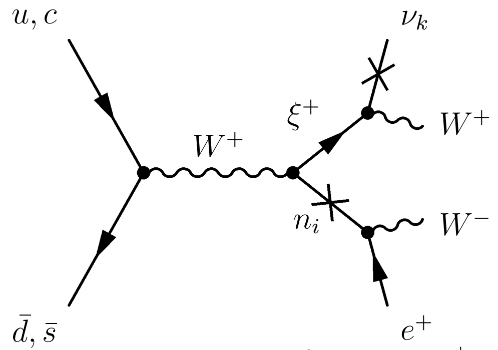


Figure 6.3: Feynman diagram for $pp \to W^+ \nu W^- e^+$.

6.3.7 Total decay widths for the neutral heavy mass eigenstates

By summing over all decay channels one obtains the total decay widths of the neutral heavy mass eigenstates, which are listed in Tab. 6.7. The smallness of these values is due to the weak mixing between the heavy fields and the neutrinos. This is of course a consequence of the same suppression mechanism that keeps the neutrino masses small.

6.3.8 The mediator fields as intermediate states

With the decay widths we can now calculate the cross-section of the processes $pp \to W^+ \nu W^- e^+$ (Fig. 6.3) and $pp \to W^+ \nu W^+ e^-$ (Fig. 6.4) with WHIZARD. Also the corresponding processes with μ and τ instead of e have been studied. The results are listed in Tab. 6.8 for a center of mass energy $\sqrt{s} = 7\,\text{TeV}$ and 14 TeV. These processes are only possible because of the mixing between the heavy fields and the light neutrinos. Hence they are suppressed by the same effects that generate the small neutrino masses.

One can see that the cross-sections for electrons in the final state are between one and two orders of magnitude smaller than for muons and taus. This can

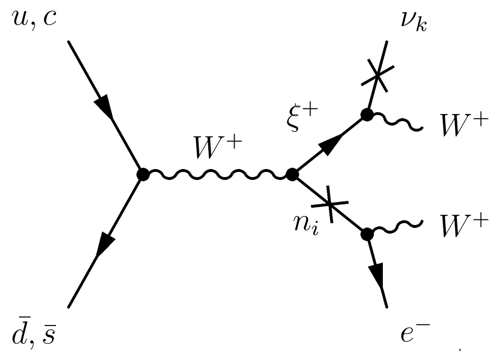


Figure 6.4: Feynman diagram for $pp \to W^+\nu W^+e^-$.

be attributed to the way in which we have implemented tri-bimaximal mixing (Eq. (6.14)). The electron neutrino couples only to one generation of the mediators whereas the others couple to both. Hence the decay of the mass eigenstates $n_{6/7}$ into $W^\pm$+$e^\mp/\mu^\mp/\tau^\mp$ is suppressed for electrons. This offers an easy way to connect this process with neutrino physics.

Another observation one can make is that the processes with W^+W^+ and W^+W^- as final states have both similar cross-sections. This is remarkable, since it requires lepton number violation and hence is another hint at the connection to neutrino physics. This can be better understood if one looks at the same processes where ξ^+ is replaced with e^+ and we have only Dirac neutrinos. In that case only the process with W^+W^+ is allowed whereas the one with W^+W^- would break lepton number and hence is forbidden. In this regard it might be suprising that the pseudo-Dirac nature of the neutral mediators does not have a similar effect in our model. In the inverse seesaw models for example lepton number violating processes are suppressed because of this.

To understand this better, we will shortly describe this mechanism. Consider a Feynman graph where two fermion lines are connected to a propagator via left-handed interactions. An example would be $W^+ \to e^+\nu \to e^+W^+e^-$. We then have a structure $(1+\gamma^5)(\not{p}+m)(1-\gamma^5)$. Due to the algebra of the gamma matrices only $\not{p}$ contributes, whereas all terms with m vanish. If we have a Majorana propagator, we also have the structure $(1+\gamma^5)(\not{p}+m)\mathcal{C}(1-\gamma^5) = (1+\gamma^5)(\not{p}+m)(1+\gamma^5)\mathcal{C}$ for lepton number violating processes. In our example this would correspond to the process $W^+ \to e^+\nu^{\text{maj}} \to e^+W^-e^+$ so that we now get only a contribution from m. For pseudo-Dirac particles, however, we have two Majorana mass states (ν_1^{maj} and ν_2^{maj}) with $m_1 = m$ and $m_2 = -m$. Hence we have one amplitude proportional to $+m$ and one proportional to $-m$ that cancel each other. Therefore the lepton number violating process is suppressed. But how is our model different? Here we have a left handed field ξ^+ and a right handed field ξ'^- that have been combined to one Dirac spinor. This is due to the fact that we have a mass term $m_\xi \xi^+\xi'^-$ and no mixing with the charginos. This is equivalent to the way in which the Weyl spinors of the electron with the mass term $m_e\overline{e_R}^+e_L^-$ ($m_e = Y_e v_d$) are combined to one Dirac spinor $\Psi_e = (e_L, e_R)$. As a consequence the Dirac spinor ξ

Process	σ [fb]
$\sqrt{s} = \mathbf{7\ TeV}$	
$pp \to W^+\nu W^- e^+$	$(5.064 \pm 0.034) \cdot 10^{-2}$
$pp \to W^+\nu W^+ e^-$	$(4.987 \pm 0.044) \cdot 10^{-2}$
$pp \to W^+\nu W^- \mu^+$	1.0622 ± 0.0070
$pp \to W^+\nu W^+ \mu^-$	1.083 ± 0.012
$pp \to W^+\nu W^- \tau^+$	1.507 ± 0.010
$pp \to W^+\nu W^+ \tau^-$	1.529 ± 0.014
$\sqrt{s} = \mathbf{14\ TeV}$	
$pp \to W^+\nu W^- e^+$	0.1767 ± 0.0012
$pp \to W^+\nu W^+ e^-$	0.1747 ± 0.0016
$pp \to W^+\nu W^- \mu^+$	3.4507 ± 0.023
$pp \to W^+\nu W^+ \mu^-$	3.435 ± 0.017
$pp \to W^+\nu W^- \tau^+$	4.903 ± 0.032
$pp \to W^+\nu W^+ \tau^-$	4.943 ± 0.049

Table 6.8: Cross-sections for the studied processes.

has not only left handed but also right handed couplings to the W bosons, since both ξ components have isospin and hypercharge. Hence both processes—the one with W^+W^+ and the one with W^-W^-—have a contribution from both $\not{p}$ and m, where only the m part is affected by above described cancellation. If we look at a process like $W^+ \to e^+ n_i \to e^+ W^\pm e^\mp$ instead, we will see that here the lepton number violating process is strongly suppressed also in our model, since only the left handed component e_L of the Dirac spinor couples to the W boson.

We also calculated the SM background—all diagrams with the same initial and final states, where we can not distinguish neutrinos from anti-neutrinos—for all discussed processes. The results are listed in Tab. 6.9. Here of course we have equal contributions for each flavor, since in the SM the couplings for all generations are of the same size. The different mass of the particles can be neglected since we have a much higher center of mass energy. For muons and taus the background is of the same order as the processes, even without any additional cuts. An experimental verification at the LHC is therefore very likely. This depends of course on this special choice of parameters. In the case of heavier mediators this will become more difficult. A rough estimation of the signal S to background B ratio (for 7 TeV) with $S/\sqrt{B}$ shows, that for 5σ an integrated luminosity of about $30\,\text{fb}^{-1}$ is required to detect the process $pp \to W^+\nu W^-\mu^+$ and about $70\,\text{fb}^{-1}$ for $pp \to W^+\bar{\nu} W^+\mu^-$. The LHC is expected to run at a luminosity of $100\,\text{fb}^{-1}$ per

Process	σ [fb]
$\sqrt{s} = \mathbf{7\ TeV}$	
$pp \to W^+ \nu W^- e^+$	3.082 ± 0.017
$pp \to W^+ \bar{\nu} W^+ e^-$	1.5601 ± 0.0090
$pp \to W^+ \nu W^- \mu^+$	3.075 ± 0.017
$pp \to W^+ \bar{\nu} W^+ \mu^-$	1.5391 ± 0.0093
$pp \to W^+ \nu W^- \tau^+$	3.086 ± 0.017
$pp \to W^+ \bar{\nu} W^+ \tau^-$	1.5623 ± 0.0094
$\sqrt{s} = \mathbf{14\ TeV}$	
$pp \to W^+ \nu W^- e^+$	9.878 ± 0.051
$pp \to W^+ \bar{\nu} W^+ e^-$	5.005 ± 0.029
$pp \to W^+ \nu W^- \mu^+$	9.806 ± 0.053
$pp \to W^+ \bar{\nu} W^+ \mu^-$	4.959 ± 0.029
$pp \to W^+ \nu W^- \tau^+$	9.776 ± 0.052
$pp \to W^+ \bar{\nu} W^+ \tau^-$	4.919 ± 0.030

Table 6.9: SM background for the studied processes.

year later, which makes a verification of this process possible. For τ in the final state this is similar. For e, however, an integrated luminosity of $\mathcal{O}(10^4\,\text{fb}^{-1})$ would be required, which is unrealistic. But the absence of e while μ and τ are present would still be an important result. This picture, however, will have to be modified if one thinks about cuts to suppress the background.

6.4 Open Issues

There are several further points regarding our model that are of interest, but a discussion is beyond the scope of this thesis. They will be the subject of future study.

First of all a systematic study of the parameter space should be accomplished. Especially the case for the inverted neutrino mass hierarchy might be of interest. In the same way the ordering $m_\xi < m_N$ of the heavy masses will be of interest. It should be tested how much the discussed results rely on the choice of parameters we have made. We will have to study further how our model is qualitatively different from the inverse seesaw scenarios.

Another issue are non-unitarity effects. As we discussed in Sec. 3.3 for the standard seesaw scenarios, higher orders in the expansion of the propagator will probably cause non-unitarity effects. We will have to bring these in accordance with experimental constraints. In this regard, we will have to study lepton flavor violating processes such as $\mu \to e\gamma$.

We should also discuss which processes—besides those discussed here—could be used to distinguish our model from others. This should also include the supersymmetric partners of the new fields, if they have sufficiently small masses. Furthermore, we have to find ways to suppress the SM background with appropriate cuts. Since the heavy particles have so small decay widths, it might be worthwhile to consider displaced vertices in that regard.

Finally, it would also be interesting to study models where R-Parity is violated in comparison. Some work in this regard has already been started [44].

7 Summary

In this thesis we studied possible extensions of the seesaw mechanism, using higher than $d = 4$ effective operators in a supersymmetric framework. We have demonstrated that it is indeed possible to realize these operators in SUSY in a similar way as in the THDM. Due to the holomorphy condition of supersymmetry, however, the possibilities are more limited. In models with an additional scalar that has a VEV, such as the NMSSM, we can have additional operators, including some of dimension six or eight. The NMSSM might even be preferred to the MSSM, since it avoids the μ-term, which breaks the discrete symmetry that is necessary to avoid lower-dimensional operators as leading contribution to the neutrino mass. In this study we have focused on models with R-Parity conservation, which has the disadvantage that we cannot reduce the number of additional fields by the use of supermultiplets, but avoids a non-zero VEV of the sneutrino that would lead to additional contributions to the neutrino mass.

We were especially interested in $d = 7$ effective operators. We have given a systematic overview of the possible topologies and decompositions. For several examples we have studied the effective generation of neutrino mass. In this context we have been able to show that—depending on the nature of the mediator fields—we obtain modifications to the neutralino and chargino mixing matrix that can affect SUSY phenomenology. If we use heavy scalars as mediators, we might have the problem of induced $d = 6$ operators, if these fields have a VEV.

Following these general considerations, we have presented a more detailed study of the flavor structure and phenomenology of one example with solely fermionic mediators—two neutral fields and two SU(2) doublets. This decomposition generates an effective neutrino mass of the order $m_\nu = v_u^3 v_d Y_N^2 \kappa_1 \kappa_2 / (m_\xi m_N^2)$. The neutrino mass scale is reproduced for couplings of the order 10^{-3} and the heavy masses at the TeV scale.

Then we discussed two possible realizations of a flavor structure that is in accordance with neutrino physics, under the assumption of tri-bimaximal mixing. We focused on a minimal variant, where one of the neutrinos is massless, in order to avoid more free parameters. The first possibility is to have only one generation of mediators. The flavor structure is then created by neutrino-sneutrino-neutralino interactions at one-loop level, which have different couplings for the two massive neutrinos compared to tree level, due to an additional flavor dependence introduced by soft SUSY breaking terms. The second alternative is to create the whole flavor structure at tree level, which requires a second generation of the singlet mediators. For the latter variant we have shown a possible realization of the matrix structure of the couplings—for the normal as well as for the inverse hierarchy.

Finally we considered exemplary the phenomenology of this decomposition for the case of the normal hierarchy. We found the components of the mediator doublets have a production cross-section of $(3.7696 \pm 0.0025) \cdot 10^2$ fb in the process $u\bar{d} \to W+ \to \xi^+ n_i$ and (1.9647 ± 0.0037) fb in the process $\bar{u}d \to \bar{\xi}^- + n_i$ for a proton-proton collider at $\sqrt{s} = 7$ TeV. For $\sqrt{s} = 14$ TeV we obtained $\sigma(u\bar{d} \to \xi^+ + n_i) = (1.0138 \pm 0.0020) \cdot 10^3$ fb and $\sigma(\bar{u}d \to \bar{\xi}^- + n_i) = (6.153 \pm 0.012) \cdot 10^2$ fb.

Since the heavy mediators decay further, we have finally studied the process $u\bar{d} \to W^+ \nu W^- e^+/\mu^+/\tau^+$ and $u\bar{d} \to W^+ \nu W^+ e^-/\mu^-/\tau^-$, after calculating their decay widths. We have seen, that the processes with electrons are suppressed compared to the others, because of our implementation of the flavor structure. The cross-sections are of the order of 1 fb and therefore likely to be discovered at the LHC. The presence of both types of processes is an indication for lepton flavor violation that is connected with neutrino physics.

To conclude, we have presented how extensions of the seesaw mechanism that are also possible in a supersymmetric framework can be implemented. This does not require couplings smaller than $\mathcal{O}(10^{-3})$. We obtain a structure with some similarities to the inverse seesaw model, but we do not have to introduce a small lepton number violating parameter explicitly. Instead we obtain this term automatically by partially integrating out some of the mediators. Since masses of the mediators are below 1 TeV our models are potentially testable at the LHC. We have also shown that the use of charged doublets as mediators makes it possible to test lepton flavor violating processes at the LHC that are connected to neutrino physics.

Acknowledgements

Finally, there are some people that deserve to be mentioned here. First of all, I like to thank Klaus Klopfer for many helpfull discussions and his help regarding WHIZARD. Lisa Edelhäuser has also given me valuable advice concerning the technical aspects of implementing my model. In this regard, I also thank Christian Speckner for sharing his knowledge of WHIZARD, which was very usefull for solving some problems. I also want to thank Toshihiko Ota for his help with higher-dimensional effective operators and his collaboration. Last but not least, I thank my two supervisors Werner Porod and Walter Winter for making this project possible.

A Couplings in the Lagrangian

From the superpotential Eq. (5.25), one can derive the corresponding coupling terms of the Lagrangian.

A.1 Fermionic couplings

The part of the Lagrangian containing the fermions has been already discussed in the last chapter.

$$\begin{aligned}\mathcal{L}^{\text{fermionic}} = & Y_e(\bar{e}L \cdot H_d + \widetilde{\bar{e}}L \cdot \widetilde{H}_d + \bar{e}\widetilde{L} \cdot \widetilde{H}_d) - Y_N(NL \cdot H_u + \widetilde{N}L \cdot \widetilde{H}_u + N\widetilde{L}\widetilde{H}_u) \\ & - \kappa_1(N'\xi \cdot H_d + \widetilde{N}'\xi \cdot \widetilde{H}_d + N'\widetilde{\xi} \cdot \widetilde{H}_d) + \kappa_2(N'\xi' \cdot H_u + \widetilde{N}'\xi' \cdot \widetilde{H}_u + N'\widetilde{\xi}' \cdot \widetilde{H}_u) \\ & - M_N N'N - m_\xi \xi\xi' - \mu\tilde{H}_u^0 H_d^0 + \text{h.c.} \end{aligned} \tag{A.1}$$

A.2 Scalar couplings

The couplings involving only scalars are also specified by the superpotential.

$$\begin{aligned}\mathcal{L}^{\text{scalar}} = & - Y_e(F_e\widetilde{L} \cdot H_d + \widetilde{\bar{e}}F_L \cdot H_d + \widetilde{\bar{e}}\widetilde{L} \cdot F_d) + Y_N(F_N\widetilde{L} \cdot H_u + \widetilde{N}F_L H_u + Y_N\widetilde{N}\widetilde{L} \cdot F_u) \\ & + \kappa_1(F_{N'}\widetilde{\xi} \cdot H_u + \widetilde{N}'F_\xi \cdot H_u + \widetilde{N}'\widetilde{\xi} \cdot F_u) - \kappa_2(F_{N'}\widetilde{\xi}' \cdot H_d + \widetilde{N}'F_{\xi'} \cdot H_d + \widetilde{N}'\widetilde{\xi}' \cdot F_d) \\ & + m_N\widetilde{N}F_{N'} + m_N F_N\widetilde{N}' + m_\xi\widetilde{\xi}F_{\xi'} + m_\xi F_\xi\widetilde{\xi} + \mu F_u H_d + \mu H_u F_d + \text{h.c.}\end{aligned} \tag{A.2}$$

The auxiliary fields can be determined using their equations of motion.

$$F_e^\dagger = +\widetilde{L} \cdot H_d \tag{A.3a}$$
$$F_L^\dagger = +Y_e\widetilde{\bar{e}}H_d - Y_N\widetilde{N}H_u \tag{A.3b}$$
$$F_u^\dagger = -Y_N\widetilde{N}\widetilde{L} - \kappa_1\widetilde{N}'\widetilde{\xi} - \mu H_d \tag{A.3c}$$
$$F_d^\dagger = +Y_e\widetilde{\bar{e}}\widetilde{L} + \kappa_2\widetilde{N}'\widetilde{\xi}' - \mu H_u \tag{A.3d}$$
$$F_N^\dagger = -Y_N\widetilde{L} \cdot H_u - m_N\widetilde{N}' \tag{A.3e}$$
$$F_{N'}^\dagger = -\kappa_1\widetilde{\xi} \cdot H_u + \kappa_2\widetilde{\xi}' \cdot H_d - m_N\widetilde{N} \tag{A.3f}$$
$$F_\xi^\dagger = -\kappa_1\widetilde{N}' \cdot H_u - m_\xi\widetilde{\xi}' \tag{A.3g}$$
$$F_{\xi'}^\dagger = +\kappa_2\widetilde{N}' \cdot H_d - m_\xi\widetilde{\xi} \tag{A.3h}$$

A.3 The kinetic terms and gauge couplings

The kinetic terms introduce the coupling to the gauge bosons by replacing the derivatives:

$$\begin{aligned}\mathcal{L}^{\text{kin}} &= \mathrm{i}L^\dagger\bar{\sigma}^\mu D_\mu L + (D_\mu\widetilde{L})^\dagger(D^\mu\widetilde{L}) \\ &\quad + \mathrm{i}N^\dagger\bar{\sigma}^\mu D_\mu N + (D_\mu\widetilde{N})^\dagger(D^\mu\widetilde{N}) + \mathrm{i}N'^\dagger\bar{\sigma}^\mu D_\mu N' + (D_\mu\widetilde{N}')^\dagger(D^\mu\widetilde{N}') \\ &\quad + \mathrm{i}\xi^\dagger\bar{\sigma}^\mu D_\mu\xi + (D_\mu\widetilde{\xi})^\dagger(D^\mu\widetilde{\xi}) + \mathrm{i}\xi'^\dagger\bar{\sigma}^\mu D_\mu\xi' + (D_\mu\widetilde{\xi}')^\dagger(D^\mu\widetilde{\xi}')\,, \end{aligned} \tag{A.4}$$

where

$$D_\mu = \partial_\mu + \mathrm{i}gW^a_\mu\frac{\tau^a}{2} + \mathrm{i}g'B_\mu \tag{A.5}$$

is the covariant Derivative of the SM.

So the N fields do not couple directly to the gauge fields. However the ξ fields do. Their couplings to the gauge bosons are similar to those of the SM lepton doublet.

A coupling of the singlets to the gauge bosons can still be realized if the mixing of the mass states is sufficiently high.

A.4 Gaugino couplings

When gauge couplings appear, also their supersymmetric counterparts, the gaugino couplings, must be taken into account. They read

$$\begin{aligned}\mathcal{L}_L^{\text{gaugino}} &= -\sqrt{2}g(\widetilde{L}^\dagger\cdot\frac{\tau^a}{2}L\widetilde{W}^a + \widetilde{W}^{a\dagger}(L^\dagger\cdot\frac{\tau^a}{2}\widetilde{L}) - g\widetilde{L}^\dagger\cdot\frac{\tau^a}{2}\widetilde{L}D^a_W) \\ &\quad -\sqrt{2}g'(\widetilde{L}^\dagger\cdot L\widetilde{B} + \widetilde{B}^\dagger(L^\dagger\cdot\widetilde{L}) - g'\widetilde{L}^\dagger\cdot\widetilde{L}D_B) \end{aligned} \tag{A.6a}$$

$$\begin{aligned}\mathcal{L}_\xi^{\text{gaugino}} &= -\sqrt{2}g(\widetilde{\xi}^\dagger\cdot\frac{\tau^a}{2}\xi\widetilde{W}^a + \widetilde{W}^{a\dagger}(\xi^\dagger\cdot\frac{\tau^a}{2}\widetilde{\xi}) - g\widetilde{\xi}^\dagger\cdot\frac{\tau^a}{2}\widetilde{\xi}D^a_W) \\ &\quad -\sqrt{2}g'(\widetilde{\xi}^\dagger\cdot\xi\widetilde{B} + \widetilde{B}^\dagger(\xi^\dagger\cdot\widetilde{\xi}) - g'\widetilde{\xi}^\dagger\cdot\widetilde{\xi}D_B) \end{aligned} \tag{A.6b}$$

$$\begin{aligned}\mathcal{L}_{\xi'}^{\text{gaugino}} &= -\sqrt{2}g(\widetilde{\xi'}^\dagger\cdot\frac{\tau^a}{2}\xi'\widetilde{W}^a + \widetilde{W}^{a\dagger}(\xi'^\dagger\cdot\frac{\tau^a}{2}\widetilde{\xi}') - g\widetilde{\xi'}^\dagger\cdot\frac{\tau^a}{2}\widetilde{\xi}'D^a_W) \\ &\quad -\sqrt{2}g'(\widetilde{\xi'}^\dagger\cdot\xi'\widetilde{B} + \widetilde{B}^\dagger(\xi'^\dagger\cdot\widetilde{\xi}') - g'\widetilde{\xi'}^\dagger\cdot\widetilde{\xi}'D_B) \end{aligned} \tag{A.6c}$$

The auxiliary fields are again determined by the equations of motion:

$$D^a_W = g(\widetilde{L}^\dagger\cdot\tau^a\widetilde{L} + \widetilde{\xi}^\dagger\cdot\tau^a\widetilde{\xi} + \widetilde{\xi'}^\dagger\cdot\tau^a\widetilde{\xi}') \tag{A.7a}$$

$$D_B = g'(\widetilde{L}^\dagger\cdot\widetilde{L} + \widetilde{\xi}^\dagger\cdot\widetilde{\xi} + \widetilde{\xi'}^\dagger\cdot\widetilde{\xi}') \tag{A.7b}$$

B Contributions to the sneutrino mass

We will also have a closer look at the properties of the sneutrinos in our model. This can be relevant for phenomenological reasons or for higher order effects.

B.1 Scalar potential

The terms in the scalar part of the Lagrangian (Eq. (A.2)) contribute to the sneutrino mass terms after the Higgs fields have obtained a VEV. The relevant terms are:

$$\begin{aligned}\mathcal{L}_{\widetilde{n}}^{\text{scalar}} = & - Y_N^2 v_u^2 \widetilde{\nu}^\dagger \widetilde{\nu} - Y_N m_N v_u \widetilde{N}'^\dagger \widetilde{\nu} - Y_N^2 v_u^2 \widetilde{N}^\dagger \widetilde{N} - Y_N \mu v_d \widetilde{N} \widetilde{\nu} - \kappa_1^2 v_u^2 \widetilde{\xi}'^{0\dagger} \widetilde{\xi}^0 \\ & - \kappa_1 \kappa_2 v_d v_u \widetilde{\xi}^{0\prime\dagger} \widetilde{\xi}^0 - \kappa_1 m_N v_u \widetilde{N}^\dagger \widetilde{\xi}^0 - \kappa_1^2 v_u^2 \widetilde{N}'^\dagger \widetilde{N}' - \kappa_1 m_\xi v_u^2 \widetilde{\xi}'^{0\dagger} \widetilde{\xi}^0 - \kappa_1 \mu v_d \widetilde{\xi}^0 \widetilde{N}' \\ & - \kappa_1 \kappa_2 v_u v_d \widetilde{\xi}^{0\dagger} \widetilde{\xi}^{0\prime} - \kappa_2^2 v_d^2 \widetilde{\xi}^{0\prime\dagger} \widetilde{\xi}^{0\prime} - \kappa_1 m_N v_d \widetilde{N}^\dagger \widetilde{\xi}^{0\prime} - \kappa_2^2 v_d^2 \widetilde{N}'^\dagger \widetilde{N}' - \kappa_2 m_\xi v_d \widetilde{\xi}^{0\dagger} \widetilde{N}' \\ & - \kappa_2 \mu v_u \widetilde{N}' \widetilde{\xi}^{0\prime} - \kappa_1 m_N v_u \widetilde{\nu} \widetilde{N}' - m_N^2 \widetilde{N}'^\dagger \widetilde{N}' - \kappa_2 m_\xi v_d \widetilde{N}'^\dagger \widetilde{\xi}^0 - m_\xi^2 \widetilde{\xi}^{0\dagger} \widetilde{\xi}^0 \\ & - \kappa_1 m_\xi v_u \widetilde{N}'^\dagger \widetilde{\xi}^{0\prime} - m_\xi^2 \widetilde{\xi}^{0\prime\dagger} \widetilde{\xi}^{0\prime} - Y_N \mu v_d \widetilde{N}^\dagger \widetilde{\nu}^\dagger - \kappa_1 \mu v_d \widetilde{N}'^\dagger \widetilde{\xi}^{0\dagger} - \kappa_2 \mu v_u \widetilde{N}'^\dagger \widetilde{\xi}'^\dagger \\ & + \text{h.c.}\end{aligned} \tag{B.1}$$

B.2 Soft-breaking terms

In analogy to Eq. (4.38) we can have bi- and trilinear couplings for the new fields that break supersymmetry. These terms involve only the $R = -1$ components of the supermultiplets and therefore violate SUSY. They must, however, conserve the SM gauge symmetries. Therefore the possible soft-breaking terms are

$$\begin{aligned}\mathcal{L}_{\widetilde{n}}^{\text{soft}} = & - m_{\widetilde{L}}^2 \widetilde{\nu}^\dagger \widetilde{\nu} - m_{\widetilde{N}}^2 \widetilde{N}^\dagger \widetilde{N} - m_{\widetilde{N}'}^2 \widetilde{N}'^\dagger \widetilde{N}' - m_{\widetilde{\xi}}^2 \widetilde{\xi}^{0\dagger} \widetilde{\xi}^0 - m_{\widetilde{\xi}}^2 \widetilde{\xi}'^{0\dagger} \widetilde{\xi}'^0 \\ & + \left(A_{Y_N} v_u \widetilde{\nu} \widetilde{N} + A_{\kappa_1} v_u \widetilde{\xi}^0 \widetilde{N}' + A_{\kappa_2} v_d \widetilde{\xi}'^0 \widetilde{N}' + B_{m_N} \widetilde{N} \widetilde{N}' + B_{m_\xi} \widetilde{\xi}^0 \widetilde{\xi}'^0 + \text{h.c.} \right)\end{aligned} \tag{B.2}$$

B.3 D-terms

The relevant gaugino terms (from Eq. (A.6)) are

$$\begin{aligned}\mathcal{L}_{\tilde{n}}^{D} = \sqrt{2}\Bigg(&\frac{g^2}{2}\widetilde{\nu}^\dagger\widetilde{\nu}D_W^3 - \frac{g'^2}{2}\widetilde{\nu}^\dagger\widetilde{\nu}D_B + \frac{g^2}{2}\widetilde{\xi}^{0\dagger}\widetilde{\xi}^0 D_W^3 - \frac{g'^2}{2}\widetilde{\xi}^{0\dagger}\widetilde{\xi}^0 D_B - \frac{g^2}{2}\widetilde{\xi}'^{0\dagger}\widetilde{\xi}'^0 D_W^3 \\ &+\frac{g'^2}{2}\widetilde{\xi}'^{0\dagger}\widetilde{\xi}'^0 D_B - \frac{g^2}{2}v_u^2 D_W^3 + \frac{g'^2}{2}v_u^2 D_B + \frac{g^2}{2}v_d^2 D_W^3 - \frac{g'^2}{2}v_d^2 D_B\Bigg) \\ &- \frac{1}{2}(D_W^3)^2 - \frac{1}{2}D_B^2 \end{aligned} \tag{B.3}$$

The D fields can be determined using their equations of motion. This leads to the sneutrino mass contributions

$$\frac{1}{2}m_Z\cos(2\beta)(\widetilde{\nu}^\dagger\widetilde{\nu} + \widetilde{\xi}^{0\dagger}\widetilde{\xi}^0 - \widetilde{\xi}'^{0\dagger}\widetilde{\xi}'^0)\,, \tag{B.4}$$

where $m_Z = (g^2+g'^2)(v_u/4)(\sin\beta)^{-2} = (g^2+g'^2)(v_d/4)(\cos\beta)^{-2}$ and $\tan\beta = v_u/v_d$.

B.4 The sneutrino mass matrix

In the basis $\phi = (\widetilde{\nu}_i, \widetilde{N}, \widetilde{N}', \widetilde{\xi}^0, \widetilde{\xi}'^0) \equiv \phi^R + \mathrm{i}\phi^I$ the sneutrino mass basis can be decomposed to

$$\mathcal{L}_{\widetilde{n}}(\phi^R\ \phi^I)\begin{pmatrix} M_+^2 & 0 \\ 0 & M_-^2 \end{pmatrix}\begin{pmatrix}\phi^R \\ \phi^I\end{pmatrix} \tag{B.5}$$

The matrix $M_\pm$ is shown in Eq. (B.6). It will become relevant in the next section when we discuss the possible generation of the neutrino flavor structure via loops involving sneutrinos.

$$\begin{pmatrix} m_{\widetilde{L}}^2 + m_D^2 + \frac{1}{2}m_Z\cos(2\beta) & \mp A_{Y_N}v_u \pm 2\mu m_D\cot\beta & m_D m_N & 0 & 0 \\ & m_{\widetilde{N}}^2 + m_N^2 + m_D^2 & \pm B_{m_N} & 2m_u m_N & 2m_d m_N \\ & & m_{\widetilde{N}'}^2 + m_N^2 + m_u^2 + m_d^2 & \pm A_{\kappa_1}v_u \pm 2\mu m_u\cot\beta + 2m_d m_\xi & \pm A_{\kappa_2}v_d \pm 2\mu m_d\tan\beta + 2m_u m_\xi \\ & & & m_{\widetilde{\xi}}^2 + m_\xi^2 + m_u^2 + \frac{1}{2}m_Z\cos(2\beta) & \pm B_{m_\xi} + m_u m_d \\ & & & & m_{\widetilde{\xi}'}^2 + m_\xi^2 + m_d^2 - \frac{1}{2}m_Z\cos(2\beta) \end{pmatrix} \tag{B.6}$$

where $m_D = Y_N v_u$, $m_u = \kappa_1 v_u$ and $m_d = \kappa_2 v_d$.

The sneutrino mass matrix. The blank entries can be symmetrically completed.

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Erklärung

Gemäß der allgemeinen Studien- und Prüfungsordung für die Bachelor- und Masterstudiengänge an der Julius-Maximilians-Universität Würzburg erkläre ich hiermit, dass ich diese Arbeit selbstständig verfasst, keine anderen als die angegebenen Quellen und Hilfsmittel benutzt habe und die Arbeit bisher keiner anderen Prüfungsbehörde unter Erlangung eines akademischen Grades vorgelegt habe.

Würzburg, den 29. September 2010

Martin Krauß