Large Extra Dimensions: A 5D Standard Model

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Chapter 1

Introduction

Why do we live in four dimensions? This fundamental question still cannot be answered. However, already at the beginning of the 20$^{\text{th}}$ century physicists became aware that maybe the question is ill posed. From the modern physics point of view it seems more appropriate to ask instead: In how many dimensions do we live?

Already in the 1920's Kaluza [1] and Klein [2] proposed theories with an extra space dimension in order to unify the electromagnetic and gravitational force within a single gravitation-like action. Although their theories could not be formulated without inconsistencies their basic ideas survived [3]. Later on, the birth of string theory showed for the first time that a theory can pose constraints on space-time. There are string theories which cannot be consistently defined within the usual four space-time dimensions. The appearance of extra dimensions is, therefore, a necessary ingredient of such string theories. Because we do not recognize those extra dimensions they are thought to be compact and very small, too small to be seen so far.

There is yet another important concept that was born in the context of string theories: It is by no means guaranteed that all particle excitations feel the same space-time. Some of the fields or strings (or whatever is used to describe nature) can be restricted to lower dimensional subspaces of space-time, called branes. This discovery opened physics for a completely new class of ideas. While string theory, for a long time, was thought to be unreachable experimentally due to the enormous Planck scale where gravitation seems to become important, this has changed dramatically during the last years [4, 5]. It was shown that compact extra dimensions cannot be excluded experimentally even in the sub-millimeter range if the strong and electroweak interactions are restricted to branes and, thus, do not "feel" the extra dimension(s). In a theory containing such extremely large extra dimensions the fundamental gravitational scale can be as low as a few tens of TeV. The enormously large Planck scale $M_{\mathrm{Pl}} = G_N^{-1/2} \approx 10^{19}$ GeV ($\hbar = c = 1$ as in the whole thesis) is not a fundamental scale at all, it is just a consequence of the extrapolation of four dimensional physics to distances where extra dimensions have to be taken into account [6]. Therefore, the hierarchy problem, i.e. why are the typical scales of electroweak and gravitational interactions so different, can be solved in an interesting way by lowering the fundamental gravitational scale.

So far we only considered the effects of large extra dimensions on gravitational physics. In this context, naturally, the question arises: Are the standard model fields really restricted to a four dimensional brane leaving us with the well-known standard model physics? Or do they propagate in extra dimensions which have a size not in the millimeter but, let's say, in the TeV range? Such theories have a very appealing feature: As the fundamental gravitational scale, the gauge coupling unification scale is lowered to the TeV range [7], i.e. the interesting physical scales are moved into the reach of future collider experiments.

It is not necessary to implement the idea of large extra dimensions for standard model fields and interactions in a string theoretic framework. Instead, we can try to formulate the standard model field theories in space-time geometries with large extra dimensions. While the ultimate connection to a more fundamental physical theory, like string theory, is lost in this context such a field theoretic approach should, nevertheless, describe physics in the energy range between the scale of the extra dimension(s) and the string scale where more fundamental physics comes into play. The phenomenological consequences of such a theory, in which the standard model physics is simply formulated in a higher dimensional space-time, are interesting and, thus, have been intensively investigated within the past few years [7, 8, 9, 10, 12].

This thesis is devoted to such a field theoretic approach. We will investigate the complete standard model in a special higher dimensional generalization. While the number of large extra dimensions is not known we will concentrate on the simplest possibility, a single extra dimension or five dimensional space-time. In contrast to theories with more extra dimensions it has an appealing feature: It is finite at least at tree level. While any field theory with a finite number of particle species is finite at tree level, this is not guaranteed in higher dimensional models because there are processes described by an infinite number of Feynman graphs. However, in five dimensional space-time, the contributions to observables from the ultraviolet limit of the theory (i.e. from the exchange of extremely heavy gauge bosons) are small and the question at which scale new physics comes into play and at which scale we have to implement an explicit cut-off is not an urgent one. In our tree level analysis we will investigate the complete theory not truncated by any means.

In section 2 we will show how a four dimensional gauge theory can be promoted to five dimensions. Using the simplest gauge theory at hand, namely QED, we will show the basic concepts of theories with extra dimensions and how they manifest themselves in terms of four dimensional field theory. Compactifying the five dimensional gauge theory onto the compact manifold S^1/Z_2 the Kaluza-Klein excitations of the gauge field naturally arise dominating the high energy behavior of the theory. How these heavy gauge bosons couple to fermions crucially depends on the space-time geometry fermions feel. We concentrate on theories in which fermions do not propagate in any extra dimension. We consider them being restricted to a subspace which coincides with the usual four dimensional Minkowski space. This framework is not only especially simple but also extremely predictive (only one free parameter, the compactification scale, has to be chosen).

We will particularly focus on the gauge structure of the investigated theories and implement a generalized R_ξ-gauge which has not been investigated in the literature, yet. While most of the authors work in the so-called unitary gauge (introduced in section 2.4) other gauge fixing prescription can be found for example in [7, 13, 14]. They should be carefully compared to the analysis in section 2.4. Finally we will show in a simple example how large extra dimensions will generically show up in high energy processes.

Chapter 3 is devoted to the higher dimensional analysis of non-Abelian gauge theories like QCD or the electroweak theory. There, we will encounter the typical features of interactions between particles not restricted to a brane.

The specific features of a spontaneously broken gauge theory in the context of large extra dimensions are analyzed in detail in chapter 4, again with a special focus on the gauge fixing prescription. The Higgs field itself can be either considered to be located on a brane or not. Both cases are investigated separately and combined in a generic theory. The five dimensional standard model is completed in chapter 5 where some missing concepts for the standard model fermions in the light of an extra dimension are introduced and we investigate the gauge invariance of certain higher dimensional theories in an exemplary process.

Chapter 6 is devoted to the phenomenological implications of a theory containing a large extra dimension with a certain compactification scale. Observables of precision measurements are investigated in a higher dimensional generalization of the standard model in order to search for the compactification scales allowed by the current data. There, we will also correct some results stated in the literature before we conclude this thesis with chapter 7.

A few appendices finally include some further investigations, basic concepts or can be used for future reference. A vast number of further references on the theories investigated in this thesis and beyond can be found for example in [15, 16].

Chapter 2

QED in 5 Dimensions

We will begin with an analysis of a simple, exemplary model, 5-dimensional QED (5D-QED) [7]. Although 5D-QED is not thought to describe nature by any means it is well suited to introduce the basic concepts and notations inherent to theories with large extra dimensions. Furthermore, in the first sections we will concentrate on the gauge boson part of the 5D-QED Lagrangian, i.e. we consider the physics of a single vector boson living in five space-time dimensions.

First of all, we have to specify, what is really meant by 5D-QED, i.e. we have to specify the Lagrangian of the theory as well as the structure of space-time. The ultimate goal is to introduce a five-dimensional theory embedding 4D-QED as a natural ingredient. On the other hand, simplicity should be a leading principle in the sense that the five dimensional extension should be as minimal as possible. Our starting point is (4+1) dimensional space-time with SO(4,1) symmetry, i.e. there is simply one additional space-like dimension, which has not been compactified yet. In analogy with conventional QED the 5D-QED Lagrangian has to be a scalar with respect to SO(4,1). Moreover, also in analogy with QED, we want the Lagrangian to be invariant under a generalized local U(1) gauge transformation

$$A_M(x,y) \to A_M(x,y) + \partial_M \Theta(x,y)\,, \tag{2.1}$$

where the five dimensional gauge boson field A_M, transforming as a vector under SO(4,1), is introduced. Capital Roman indices denote the Lorentz indices in five dimensions ($M = 0,1,2,3,4$), $x = (x^0, \vec{x})$ the usual coordinates of four dimensional space-time and y the extra coordinate corresponding to the fifth dimension.

The direct generalization of the QED Lagrangian density to five dimensions (automatically fulfilling these requirements) is given by

$$\mathcal{L}(x,y) \;=\; -\frac{1}{4} F_{MN}(x,y) F^{MN}(x,y)\,, \tag{2.2}$$

where F_{MN} denotes the 5-dimensional field strength tensor

$$F_{MN}(x,y) \;=\; \partial_M A_N(x,y) - \partial_N A_M(x,y)\,. \tag{2.3}$$

This is the Lagrangian of 5D-QED as it is investigated below. We will see in the following sections that the right choice for the compactification of the extra dimension will assure that conventional QED is embedded and furnishes the low energy limit of the theory. The issue of compactification is addressed in the next section before we can go on to built up a formulation of 5D-QED in terms of an effective four dimensional field theory in sections 2.3 and 2.4.

2.1 Compactification

So far, 5D-QED lives in four spatial dimensions treated all on the same footing. This is certainly an assumption in contradiction not only to experimental evidence but also to our daily experience. However, we cannot exclude a compact fifth dimension up to a scale of a few TeV even if all the gauge boson fields of the standard model are allowed to propagate in the extra dimension (see chapter 6). A procedure to derive a theory including a compact fifth dimension is known as compactification. In the simple approach introduced here, compactification simply means that the theory described by the Lagrangian eq.(2.2) lives in a space-time where the fifth dimension is a compact manifold in contrast to the other three flat and infinitely extended space dimensions. The symmetry group of this space-time is no longer SO(4,1) but its subgroup SO(3,1), the Lorentz group, together with the internal symmetries of the given manifold which in turn impose some additional constraints on the fields contained within the Lagrangian.

First, we consider the simplest example, compactification on a circle of radius R, i.e. all the fields live on an interval $[0, 2\pi R[$, where the end points 0 and $2\pi R$ are identified. Conventionally, we can extend all the fields to be defined for all values of y. The special topology of the circle is then established by the periodicity constraint

$$A_M(x,y) \;=\; A_M(x, y + 2\pi\, R)\,. \tag{2.4}$$

In order to be well defined on the circle, also gauge transformations have to be parameterized by scalar functions obeying

$$\Theta(x,y) \;=\; \Theta(x, y + 2\pi\, R)\,. \tag{2.5}$$

However, there is a manifold similar to the circle which leads to an even simpler theory in the sense that the degrees of freedom are considerably reduced. Introducing a further constraint on the fields, we assume that they are even or odd with respect to the fifth dimension. Such a constraint allows for the identification of the coordinates y and $-y = 2\pi R - y$ in the sense that it is sufficient to know the field configuration on one half of the circle. Using this additional assumption is referred to as compactification not on a circle but on the orbifold S^1/Z_2. S^1 denotes the one dimensional circle, Z_2 refers to the reflection symmetry of the fields which has the group structure of Z_2 and is called Z_2-parity. Moreover, the orbifold contains two fixed points at $y = 0$ and $y = \pi R$ being natural candidates to host branes which will be introduced later on in section 2.5. This is the space-time geometry most widely investigated in the literature [8, 9, 10, 12].

The first four components of the five dimensional vector field $A_\mu(x, y)$, where $\mu = 0, 1, 2, 3$ denotes the usual Lorentz index in four dimensions, transform as a usual vector field under SO(3,1). They have to be of even Z_2-parity. Otherwise, if we chose odd parity, the theory would not contain a massless gauge boson. The mass of the lightest gauge boson would coincide with the compactification scale as can be inferred from an analysis being equivalent to the one in section 2.3. Therefore, we would not recover four dimensional QED, i.e. known physics, in the low energy limit of the theory. Imposing the corresponding constraint

$$A_\mu(x, y) \;=\; A_\mu(x, -y) \tag{2.6}$$

also restricts the scalar field $\Theta(x, y)$ parameterizing gauge transformations to be even with respect to the extra dimension, i.e.

$$\Theta(x, y) \;=\; \Theta(x, -y)\,. \tag{2.7}$$

Otherwise a gauge transformation would result in a field configuration violating eq.(2.6). This property of $\Theta(x, y)$ in combination with eq.(2.1), in fact, forces the extra component of the gauge boson field to obey

$$A_4(x, y) \;=\; -A_4(x, -y)\,, \tag{2.8}$$

because the derivative of an even function with respect to the extra dimension is odd. A_4 does not mix with the other field components under SO(3,1) transformations and, therefore, furnishes a scalar with respect to the Lorentz group.

To summarize, there is a well defined, gauge invariant theory containing fields of definite parity on the compact orbifold S^1/Z_2 and having the appropriate low energy behavior, provided the fields fulfill the following constraints:

$$\begin{aligned}
A_M(x, y) &= A_M(x, y + 2\pi\, R) & \qquad M &= 0, 1, 2, 3, 4 \\
A_\mu(x, y) &= A_\mu(x, -y) & \qquad \mu &= 0, 1, 2, 3 \\
A_4(x, y) &= -A_4(x, -y) \\
\Theta(x, y) &= \Theta(x, y + 2\pi\, R) \\
\Theta(x, y) &= \Theta(x, -y)
\end{aligned} \tag{2.9}$$

In order to relate the five dimensional fields to four dimensional physics we will expand them with respect to the extra dimension. This is the first step to derive an effective 4D theory. Because the fields have to obey eq.(2.9), it is convenient to expand the gauge fields into a general Fourier series

$$\begin{aligned} A^\mu(x,y) &= \sum_{n=0}^{\infty} \frac{1}{\sqrt{2\pi R}}\, A^\mu_{(n)}(x)\, \cos\left(\frac{ny}{R}\right), \\ A^4(x,y) &= \sum_{n=1}^{\infty} \frac{1}{\sqrt{2\pi R}}\, A^4_{(n)}(x)\, \sin\left(\frac{ny}{R}\right), \end{aligned} \tag{2.10}$$

where the $\sin(ny/R)$ and $\cos(ny/R)$ terms are missing for $A^\mu(x,y)$ and $A^4(x,y)$, respectively, because of the parity constraints. The expansion coefficients $A^\mu_{(n)}(x)$ for $n \geq 1$ are the famous Kaluza-Klein modes of the massless zero mode $A^\mu_{(0)}(x)$ furnishing the photon field of 4D-QED as we will see in section 2.3. Together with the scalar Kaluza-Klein modes $A^4_{(n)}(x)$ $(n \geq 1)$ they reflect the higher dimensional character of the theory in its effective four dimensional formulation and can finally be identified with Kaluza-Klein excitations (see section 2.4). Note, that the 5D-QED gauge boson field has mass dimension 3/2. This can be easily seen from the Lagrangian density eq.(2.2) which has to have mass dimension five in order to yield a dimensionless Lagrangian. The extra factors $1/\sqrt{2\pi R}$ are conventionally inserted because in their presence the Kaluza-Klein modes become vector or scalar fields with mass dimension 1, as usual in four dimensions. Moreover, as we will see below, the kinetic term of the zero mode $A^\mu_{(0)}(x)$ will be canonically normalized.

Although the expansion of the gauge field into Fourier modes may seem to be natural at this point it is not a unique choice. Any complete orthogonal system of functions with definite parities on the interval $[0, 2\pi[$ could be chosen instead provided it can be continued periodically in a continuous way. Nevertheless, the choice of the Fourier expansion will turn out to be very convenient throughout the following analysis. That is the main reason why it is chosen here. We will not encounter the drawbacks of this choice until section 4.1.2. In Appendix A we investigate why a Fourier expansion is adequate here and when it does break down, i.e. why the expansion coefficients correspond to the physical Kaluza-Klein excitations in 5D-QED and why they do not in more general theories.

2.2 Gauge Transformations

5D-QED, as defined in the preceding sections, is gauge invariant by construction even in its compactified version. Nevertheless, it is instructive to briefly digress to the gauge transformation properties of the Kaluza-Klein modes because they are the fundamental fields in the corresponding 4D Lagrangian density to be derived in section 2.3.

The gauge transformation properties of the five dimensional gauge field (see eq.(2.1)) can be rewritten in terms of the corresponding vector and scalar components, i.e.

$$\begin{aligned} A_\mu(x,y) &\to A_\mu(x,y) + \partial_\mu \Theta(x,y)\,, \\ A_4(x,y) &\to A_4(x,y) + \partial_y \Theta(x,y)\,. \end{aligned} \tag{2.11}$$

Like the gauge fields, $\Theta(x,y)$ can be expanded into the Fourier series

$$\Theta(x,y) \;=\; \sum_{n=0}^{\infty} \frac{1}{\sqrt{2\pi R}}\,\Theta_{(n)}(x)\cos\Big(\frac{ny}{R}\Big)\,, \tag{2.12}$$

where the odd Fourier modes are again missing due to the parity constraint in eq.(2.9). Note, that the factor $1/\sqrt{2\pi R}$ relates $\Theta(x,y)$ having mass dimension 1/2 to Fourier modes being conventionally dimensionless. Therefore, eq.(2.11) can be rewritten in terms of Fourier modes as

$$\begin{aligned} A_\mu(x,y) \to A_\mu(x,y) + \partial_\mu\Theta(x,y) &= \sum_{n=1}^{\infty} \frac{1}{\sqrt{2\pi R}}\Big(A_{(n)\mu}(x) + \partial_\mu \Theta_{(n)}(x)\Big)\cos\Big(\frac{ny}{R}\Big)\,, \\ A_4(x,y) \to A_4(x,y) + \partial_y\Theta(x,y) &= \sum_{n=1}^{\infty} \frac{1}{\sqrt{2\pi R}}\Big(A_{(n)4}(x) - \frac{n}{R}\Theta_{(n)}(x)\Big)\sin\Big(\frac{ny}{R}\Big)\,, \end{aligned} \tag{2.13}$$

where we have used eq.(2.10) and eq.(2.12). The gauge transformation properties of the Kaluza-Klein modes can be read off directly from eq.(2.13). They transform like

$$\begin{aligned} A_{(n)\mu}(x) &\to A_{(n)\mu}(x) + \partial_\mu \Theta_{(n)}(x)\,, \\ A_{(n)4}(x) &\to A_{(n)4}(x) - \frac{n}{R}\Theta_{(n)}(x)\,. \end{aligned} \tag{2.14}$$

As we will see in section 2.3 we have to rescale all the Kaluza-Klein modes for $n \geq 1$ according to $A_{(n)\mu} \to \sqrt{2}A_{(n)\mu}$ and $A_{(n)4} \to \sqrt{2}A_{(n)4}$. For future reference, we state the final result for the gauge transformation properties of the rescaled Kaluza-Klein modes ($n \geq 1$):

$$\begin{aligned} A_{(n)\mu}(x) &\to A_{(n)\mu}(x) + \partial_\mu \frac{\Theta_{(n)}(x)}{\sqrt{2}}\,, \\ A_{(n)4}(x) &\to A_{(n)4}(x) - \frac{n}{R}\frac{\Theta_{(n)}(x)}{\sqrt{2}}\,. \end{aligned} \tag{2.15}$$

2.3 The Corresponding 4D Theory

Let us now derive the effective four dimensional theory corresponding to the Lagrangian density eq.(2.2). This is easily accomplished by integrating out the extra dimension, i.e.

integrating over the S^1/Z_2 orbifold. The resulting four dimensional Lagrangian density will finally allow for an interpretation of the theory in terms of particle excitations and interactions as known from standard four dimensional field theory.

In order to explicitly show the scalar gauge field components due to the extra dimension we can recast the Lagrangian density into the form

$$\mathcal{L}(x,y) = -\frac{1}{4}\left[F_{\mu\nu}F^{\mu\nu} + (\partial_y A_\nu - \partial_\nu A_4)(\partial^y A^\nu - \partial^\nu A^4) + (\partial_\mu A_4 - \partial_y A_\mu)(\partial^\mu A^4 - \partial^y A^\mu)\right]. \quad (2.16)$$

All the fields are implicitly understood to depend on both x and y and Greek indices, as usual, run from zero to three ($\mu, \nu = 0, 1, 2, 3$). Collecting terms and replacing the partial derivative (∂^y) by ($-\partial_y$) yields

$$\mathcal{L}(x,y) = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(\partial_y A_\mu)(\partial_y A^\mu) - \frac{1}{2}(\partial_\mu A_4)(\partial^\mu A^4) - (\partial_\mu A_4)(\partial_y A^\mu). \quad (2.17)$$

Using eq.(2.10) this Lagrangian density can be rewritten in terms of the corresponding Kaluza-Klein modes

$$\begin{aligned}\mathcal{L}(x,y) = \frac{1}{2\pi R}\sum_{n,m=0}^{\infty} & -\frac{1}{4}F_{(n)\mu\nu}\,F^{\mu\nu}_{(m)}\cos\left(\frac{ny}{R}\right)\cos\left(\frac{my}{R}\right) \\ & + \left[\frac{1}{2}\frac{nm}{R^2}A_{(n)\mu}A^\mu_{(m)} - \frac{1}{2}(\partial_\mu A_{(n)4})(\partial^\mu A^4_{(m)}) + \frac{m}{R}(\partial_\mu A_{(n)4})A^\mu_{(m)}\right]\sin\left(\frac{ny}{R}\right)\sin\left(\frac{my}{R}\right),\end{aligned} \quad (2.18)$$

where differentiation with respect to the extra dimension has been performed. The field strength tensor $F^{\mu\nu}_{(n)}$ is defined as in QED, where the photon field is replaced by the n^{th} Kaluza-Klein mode. The dependence of the Fourier modes on the four dimensional space-time x only is implicit and suppressed for notational convenience from now on. Finally, we have to perform the integration over the fifth, compact dimension in order to derive the four dimensional Lagrangian density

$$\mathcal{L}(x) = \int_0^{2\pi R} dy\ \mathcal{L}(x,y). \quad (2.19)$$

Starting from eq.(2.18) this is a quite simple task. We could directly make use of the orthogonality properties of the Fourier modes. However, for further generalizations, it is more convenient to use the trigonometric identities

$$\begin{aligned}\cos(my)\cos(ny) &= \frac{1}{2}\Big(\cos((m+n)y) + \cos((m-n)y)\Big), \\ \cos(my)\sin(ny) &= \frac{1}{2}\Big(\sin((m+n)y) - \sin((m-n)y)\Big), \\ \sin(my)\sin(ny) &= \frac{1}{2}\Big(\cos((m-n)y) - \cos((m+n)y)\Big)\end{aligned} \quad (2.20)$$

before integration in order to deal with the products of trigonometric functions in eq.(2.17). The only integrals to be explicitly solved are thus given by

$$
\begin{aligned}
\int_0^{2\pi R} dy \ \cos\Big(\frac{ny}{R}\Big) &= 2\,\pi\,R\,\delta_{n,0} \qquad n = 0, 1, 2, \dots , \\
\int_0^{2\pi R} dy \ \sin\Big(\frac{ny}{R}\Big) &= 0 \qquad\qquad\quad\; n = 0, 1, 2, \dots .
\end{aligned}
\tag{2.21}
$$

Eq.(2.20) and eq.(2.21) render the integration almost trivial and finally yield

$$
\begin{aligned}
\mathcal{L}(x) = -\frac{1}{4} F_{(0)\mu\nu}\, F^{\mu\nu}_{(0)} + \sum_{n=1}^{\infty} \Big[&-\frac{1}{8} F_{(n)\mu\nu}\, F^{\mu\nu}_{(n)} + \frac{1}{4}\,\frac{n^2}{R^2}\, A_{(n)\mu}\, A^{\mu}_{(n)} \\
&+ \frac{1}{4}\,\partial_\mu A_{(n)4}\,(\partial^\mu A_{(n)4}) + \frac{1}{2}\,\frac{n}{R}\,(\partial_\mu A_{(n)4})\, A^{\mu}_{(n)} \Big] .
\end{aligned}
\tag{2.22}
$$

Although the four dimensional Lagrangian density eq.(2.22) formulated in terms of four dimensional field theory is conventionally called "effective", it is still completely equivalent to the five dimensional model in eq.(2.2). The physics does not change whether the extra dimension is integrated out or not because the Lagrangian, not its density, is the fundamental physical quantity anyway.

In the first term of eq.(2.22) we recover, by construction, the free QED-Lagrangian dominating the low energy behavior of the theory in an energy range far below the compactification scale. However, the mixing term in eq.(2.22) between the vector and scalar Kaluza-Klein modes is a sign that not all of the fields represent physical degrees of freedom as we could also expect from the gauge character of the theory. In fact, there is more work to be done in section 2.4 before we can extract the physical particle spectrum from the Lagrangian.

Although gauge invariance of 5D-QED is obvious from our previous considerations we can recast eq.(2.22) into the slightly different form

$$
\mathcal{L}(x) = -\frac{1}{4} F_{(0)\mu\nu}\, F^{\mu\nu}_{(0)} + \sum_{n=1}^{\infty} \left[-\frac{1}{4}\, F_{(n)\mu\nu}\, F^{\mu\nu}_{(n)} + \frac{1}{2} \Big(\frac{n}{R}\, A_{(n)\mu} + \partial_\mu A_{(n)4} \Big)^2 \right] , \tag{2.23}
$$

in which the invariance under the transformations eq.(2.15) is manifest. Moreover, as mentioned before, we had to rescale the Kaluza-Klein fields, in order to canonically normalize the kinetic terms, according to

$$
A_{(n)\mu} \to \sqrt{2} A_{(n)\mu} \qquad \text{and} \qquad A_{(n)4} \to \sqrt{2} A_{(n)4} \tag{2.24}
$$

for $n = 1, 2, \dots$.

2.4 Gauge Fixing and the Physical Particle Spectrum

Mixing terms, as in eq.(2.23), are well-known from models containing spontaneous symmetry breaking. In fact the Lagrangian eq.(2.23) is very similar to the Lagrangian of an Abelian Higgs model [17, 18] (see also section 4.1.1). As in the analysis of an Abelian Higgs model, it is our next task to find a convenient gauge for 5D-QED such that we can separate the physical from the unphysical degrees of freedom and the interpretation of the Lagrangian in terms of particle excitations becomes possible.

By virtue of eq.(2.15) it is possible to simultaneously gauge away all scalar Kaluza-Klein modes $A_{(n)4}$. Simply choose $\Theta_{(n)} = \sqrt{2}(R/n)A_{(n)4}$ for $n \geq 1$ and perform the corresponding gauge transformation. In analogy with theories with spontaneous symmetry breaking, the gauge $A_4 = 0$, where all scalar Kaluza-Klein modes vanish, is called unitary gauge. In this gauge we are left with the Lagrangian

$$\mathcal{L}(x) = -\frac{1}{4}F_{(0)\mu\nu}\,F^{\mu\nu}_{(0)} + \sum_{n=1}^{\infty}\left[-\frac{1}{4}\,F_{(n)\mu\nu}\,F^{\mu\nu}_{(n)} + \frac{1}{2}\left(\frac{n}{R}\right)^2 A^2_{(n)\mu}\right]. \tag{2.25}$$

In contrast to four dimensional QED, where a gauge like $A_0 = 0$ or $A_3 = 0$ is possible but necessarily non-covariant, from the four dimensional point of view, this gauge choice leads to a covariant theory. There is still a freedom to fix the gauge of the massless field $A_{(0)\mu}$ (which can be identified with the photon) via gauge transformations parameterized by $\Theta_{(n)} \neq 0$ only for $n = 0$. Thereby, the first term simply furnishes QED before gauge fixing. Otherwise, the Lagrangian describes an infinite tower of massive Kaluza-Klein modes. The scalar modes are removed from the theory. They are unphysical degrees of freedom and play the role of the would-be Goldstone modes in spontaneously broken theories (see section 4.1.1). In this gauge the gauge structure of the theory is completely hidden.

In the literature [8, 9, 10, 12], unitary gauge is mostly the only gauge considered because it is convenient for a tree level analysis. Nevertheless, it is known from gauge theories with spontaneous symmetry breaking that unitary gauge is not well-suited to investigate theories in higher orders of perturbation theory. Therefore, let us now consider a whole class of more general gauges, also considered for example in [7, 14]. Using a standard technique in path integral quantization (due to deWitt, Faddeev and Popov [19]) we are free to choose a gauge fixing function, i.e. a function of the fields and their derivatives denoted by $F(A)$. The standard choice in four dimensional QED is $F(A) = \partial_\mu A^\mu$, known as Lorentz gauge. The direct generalization of this covariant gauge fixing function to the five dimensional model is

$$F(A) = \partial_M A^M\,. \tag{2.26}$$

In the course of deWitt-Faddeev-Popov quantization [19], this gauge fixing function effectively leads to a gauge fixing term $-(1/2\xi)F^2(A)$ added to the Lagrangian. It is the

Lagrangian including this gauge fixing term which has to be used for the derivation of the Feynman rules (see section 2.6). The five dimensional gauge fixing term in Lorentz gauge is simply given by

$$\mathcal{L}_{\rm GF}(x,y) \;=\; -\frac{1}{2\xi}(\partial^M A_M)^2\,, \tag{2.27}$$

where again the dependence of the fields on five dimensional space-time is implicit and ξ is a non-negative, real constant to be chosen arbitrarily. Therefore, the Lagrangian of 5D-QED in Lorentz gauge reads

$$\mathcal{L}(x,y) \;=\; -\frac{1}{4}F_{MN}F^{MN} - \frac{1}{2\xi}(\partial^M A_M)^2\,. \tag{2.28}$$

The class of gauges obtained by choosing different ξ's is also known as R_ξ-gauge.

To recover the corresponding gauge fixed, four dimensional theory we just have to follow the considerations of section 2.3 for the gauge fixing term. A decomposition in Fourier modes yields

$$\begin{aligned}\partial^M A_M(x,y) \;&=\; \partial^\mu A_\mu(x,y) - \partial_y A_4(x,y)\\ &=\; \partial^\mu A_{(0)\mu} + \sum_{n=1}^{\infty}\frac{1}{\sqrt{2\pi R}}\Big(\partial^\mu A_{(n)\mu} - \frac{n}{R}A_{(n)4}\Big)\,\cos\Big(\frac{ny}{R}\Big)\,.\end{aligned} \tag{2.29}$$

Integrating over the fifth dimension and rescaling the fields according to eq.(2.24) finally results in the gauge fixing terms

$$\begin{aligned}\mathcal{L}_{\rm GF}(x) \;&=\; -\frac{1}{2\xi}\Big[(\partial^\mu A_{(0)\mu})^2 + \sum_{n=1}^{\infty}\Big(\partial^\mu A_{(n)\mu} - \frac{n}{R}A_{(n)4}\Big)^2\Big]\\ &=\; -\frac{1}{2\xi}\Big[(\partial^\mu A_{(0)\mu})^2 + \sum_{n=1}^{\infty}(\partial^\mu A_{(n)\mu})^2 + \Big(\frac{n}{R}\Big)^2 A_{(n)4}^2 - 2\,\frac{n}{R}A_{(n)4}\partial^\mu A_{(n)\mu}\Big]\end{aligned} \tag{2.30}$$

to be added to the Lagrangian eq.(2.23). Therefore, all the scalar Kaluza-Klein modes acquire a gauge dependent mass squared $m_S^2 = \frac{1}{2\xi}\big(\frac{n}{R}\big)$ being a sign that the scalar fields do not correspond to physical degrees of freedom.

Nevertheless, there is yet another gauge fixing function (in close analogy with an Abelian Higgs model) which more obviously separates the physical from the unphysical degrees of freedom. Furthermore, in contrast to the claims in reference [14], unitary gauge can be recovered in a limiting process. Instead of eq.(2.26) we can choose

$$F(A) \;=\; \partial_\mu A^\mu \;-\; \xi\,\partial_4 A_4 \tag{2.31}$$

for a generalized Lorentz gauge where ξ is chosen to be the same constant as in eq.(2.27). Note, that this gauge is still covariant with respect to four dimensional space-time. The corresponding terms to be added to the Lagrangian density are

$$\mathcal{L}_{\rm GF}(x,y) \;=\; -\frac{1}{2\xi}(\partial^\mu A_\mu \;-\; \xi\,\partial_4 A_4)^2\,. \tag{2.32}$$

Following the same steps leading to eq.(2.30) finally yields

$$\mathcal{L}_{\mathrm{GF}}(x) = -\frac{1}{2\xi}(\partial^\mu A_{(0)\mu})^2 - \sum_{n=1}^{\infty}\frac{1}{2\xi}(\partial^\mu A_{(n)\mu})^2 - \frac{\xi}{2}\left(\frac{n}{R}\right)^2 A_{(n)4}^2 + \frac{n}{R}A_{(n)4}\partial^\mu A_{(n)\mu}\,. \quad (2.33)$$

The last term in eq.(2.33) and the mixing term in eq.(2.23) add up to a total derivative for any choice of ξ and can be dropped from the Lagrangian. Therefore, the Lagrangian in this generalized R_ξ-gauge is given by

$$\begin{aligned}
\mathcal{L}(x) \; = \; & -\frac{1}{4}F_{(0)\mu\nu}\,F^{\mu\nu}_{(0)} \; - \; \frac{1}{2\xi}\,(\partial^\mu A_{(0)\mu})^2 \\
& + \sum_{n=1}^{\infty}\left[-\frac{1}{4}\,F_{(n)\mu\nu}\,F^{\mu\nu}_{(n)} + \frac{1}{2}\left(\frac{n}{R}\right)^2 A^\mu_{(n)}\,A_{(n)\mu} \; - \; \frac{1}{2\xi}\,(\partial^\mu A_{(n)\mu})^2\right] \\
& + \sum_{n=1}^{\infty}\left[\frac{1}{2}\,(\partial^\mu A_{(n)4})\,(\partial_\mu A_{(n)4}) \; - \; \frac{1}{2}\,\xi\left(\frac{n}{R}\right)^2 A_{(n)4}{}^2\right].
\end{aligned} \quad (2.34)$$

The first line of eq.(2.34) is the QED Lagrangian of the massless photon. The second line contains the Lagrangian of massive vector-bosons, the Kaluza-Klein excitations, including the corresponding gauge fixing terms, as it is well-known from models with spontaneous symmetry breaking. The third line corresponds to the scalar Kaluza-Klein modes. They again acquire a gauge dependent mass. In this generalized R_ξ-gauge, no mixing terms appear and it is obvious that the scalar modes do not correspond to physical particle excitations. Simply consider the R_ξ-gauge in the limit $\xi \to \infty$ where the scalar modes become infinitely heavy. Because the propagation of a particle is suppressed by its mass (see section 2.6) the scalar modes decouple from the theory. In the limit $\xi \to \infty$, the R_ξ-gauge simply corresponds to the gauge condition $A_4 = 0$, i.e. unitary gauge.

To summarize, the gauge sector of 5D-QED contains a massless vector boson, the photon, and an infinite tower of heavy Kaluza-Klein vector bosons. The scalar Kaluza-Klein modes being present in some gauges are unphysical and only cancel unphysical propagation amplitudes of the physical vector bosons.

2.5 Fermions and Couplings

Having considered the gauge bosons in detail, we will now introduce fermions in 5D-QED. We treat the fermions rather differently from the gauge bosons. While the gauge bosons can propagate in the bulk we assume that the fermions are restricted to a three dimensional brane located at $y = 0$ in the extra dimension. The term "bulk" is understood to denote all 4 space dimensions. The term "brane" is understood to denote a three dimensional subspace consisting of the three flat, uncompactified dimensions. This assumption is mainly made to keep the theory as simple as possible but it is also well motivated by string theoretic considerations [10]. As we will see, fermions on a brane can be described in terms of four dimensional physics without problems. In particular, chiral couplings, as in the standard model, are easily incorporated (see chapter 5). It is no problem to formulate the Dirac theory in five dimensions to obtain bulk fermions by extending the Clifford algebra of Dirac matrices to five dimensions [20]. However, in such a theory there is no analogue to chirality because there is no analogue to Dirac's γ^5-matrix. The concept of chirality is generally not well defined in a space-time with odd dimensions. In the context of large extra dimensions, this problem has been only recently attacked in the literature (see for example [14]). Moreover, fermions on a brane are very promising from the phenomenological point of view because they couple to the whole tower of Kaluza-Klein modes as we will see below.

The kinetic part of the fermionic Lagrangian density is simply the conventional four dimensional theory embedded in the five dimensional space-time at $y = 0$. This intuitively clear concept is formalized by introducing a delta-function $\delta(y)$ restricting the Lagrangian density to be non-zero only at $y = 0$. Therefore, it is given by

$$\mathcal{L}_{\mathrm{F}}(x,y) \,=\, i\,\delta(y)\,\overline{\Psi}(x)\;(\gamma^\mu\,\partial_\mu \,+\, m_f)\;\Psi(x)\,, \tag{2.35}$$

where m_f is the fermion mass, $\Psi(x)$ is the spinor field and γ^μ denote the Dirac matrices. Trivially, this Lagrangian density reduces to the well-known four dimensional theory when the fifth dimension is integrated out. Note, that the choice $y = 0$ for the location of the brane is not as arbitrary as it may seem to be, because the space-time symmetries forbid a single brane at an arbitrary point $y = a$. Only branes at one of the fixed points $y = 0$ and $y = \pi$ do respect the Z_2 symmetry of the extra dimension because they are mapped onto themselves under Z_2. We choose $y = 0$ as it is conventionally done in the literature.

In analogy with conventional QED, we can couple the fermions to the gauge boson field $A_M(x,y)$ by introducing the covariant derivative

$$D_\mu \,=\, \partial_\mu \,+\, i\,e_5\,A_\mu(x,y)\,, \tag{2.36}$$

where e_5 is the gauge coupling of the five dimensional theory. It is a dimensionful quantity of mass dimension -1/2 in contrast to the dimensionless coupling constants in the standard model. Because a derivative with respect to the extra dimension has no meaning on a brane, the extra component $A_4(x,y)$ cannot be coupled to fermions by this prescription. However, because of the antisymmetry of the scalar field any coupling term to a brane field at one of the fixed points would vanish anyway after integration over the extra dimension. Therefore, the natural starting point for investigations of full 5D-QED is the Lagrangian density

$$\mathcal{L}_{\rm QED}(x,y) \;=\; -\frac{1}{4}F_{MN}(x,y)F^{MN}(x,y) \;+\; i\,\delta(y)\,\overline{\Psi}(x)\;(\gamma^\mu\,D_\mu \;+\; m_f)\;\Psi(x)\,. \tag{2.37}$$

This is the generic form of the Lagrangian if a brane field is coupled to the gauge degrees of freedom in the bulk. All the phenomenological discussions in the literature explicitly or implicitly use analogous Lagrangians as a starting point, see for example [8, 9, 10, 11]. We will see below, that the low energy limit of the theory described by the Lagrangian eq.(2.37) is given by conventional QED. Therefore, one of the leading principles in constructing five dimensional theories is fulfilled. The other one, gauge invariance, is a little more intricate because it is not clear what a gauge transformation means for a brane field. Unfortunately, the phenomenological literature takes gauge invariance of fermionic terms for granted and only considers the pure gauge part of the theory when discussing subjects connected to gauge invariance, e.g. gauge fixing. We do not have a deeper understanding of gauge transformations for brane fields in terms of geometry. However, we can define a transformation for the fermion field and explicitly check that it leads to gauge invariance of the corresponding effective theory. As a very natural choice, the fermion field is to transform according to

$$\Psi(x) \to \exp\left(-i\,e_5\,\Theta(x,0)\right)\Psi(x)\,, \tag{2.38}$$

where $\Theta(x,y)$ is introduced in section 2.1. In this setting, invariance under this transformation is obvious on the brane before integrating over the extra dimension by virtue of the transformation properties of the gauge boson field given in eq.(2.11) for $y=0$. Let us now investigate the corresponding effective theory in four dimensions. The kinetic terms of the gauge bosons have already been investigated in detail. The terms, including fermion fields only, simply reduce to the conventional fermionic Lagrangian in a four dimensional theory. Thus, we only have to investigate the interaction term

$$\mathcal{L}_{\rm int}(x,y) \;=\; -\,\delta(y)\,e_5\,\overline{\Psi}(x)\,\gamma^\mu\,\Psi(x)\,A_\mu(x,y)\,. \tag{2.39}$$

Expanding the gauge boson field according to eq.(2.10), integrating out the extra dimension and rescaling the heavy modes according to eq.(2.24) leads to the effective Lagrangian

$$\mathcal{L}_{\rm int}(x) \;=\; \frac{-e_5}{\sqrt{2\pi R}}\,\overline{\Psi}(x)\,\gamma^\mu\,\Psi(x)\,A_{(0)\mu}(x) \;+\; \sum_{n=1}^{\infty}\frac{-e_5}{\sqrt{\pi R}}\,\overline{\Psi}(x)\,\gamma^\mu\,\Psi(x)\,A_{(n)\mu}(x)\,. \tag{2.40}$$

The conventional QED coupling constant can be identified with $e = e_5/\sqrt{2\pi R}$ and the coupling of the heavy Kaluza-Klein modes is enhanced by a factor of $\sqrt{2}$. For notational convenience we will express all the couplings in terms of the conventional QED coupling e from now on, i.e.

$$\mathcal{L}_{\rm int}(x) \;=\; -\,e\,\overline{\Psi}(x)\,\gamma^\mu\,\Psi(x)\,A_{(0)\mu}(x) \;-\; \sqrt{2}\,e\,\sum_{n=1}^{\infty}\overline{\Psi}(x)\,\gamma^\mu\,\Psi(x)\,A_{(n)\mu}(x)\,. \tag{2.41}$$

Let us now return to the question of gauge invariance. It is instructive to rewrite eq.(2.38) into the form

$$\Psi(x) \to \exp\left(-i\,e\sum_{n=0}^{\infty}\Theta_{(n)}(x)\right)\Psi(x)\,, \tag{2.42}$$

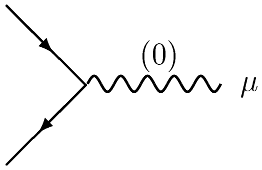
where $\Theta(x,y)$ is expanded according to eq.(2.12). At this point, we can interpret the fermion to be charged with respect to the whole tower of Kaluza-Klein gauge bosons. The interaction with each of the Kaluza-Klein modes has exactly the gauge invariant structure of QED as can be explicitly checked using eq.(2.15). Therefore, the effective theory is gauge invariant and we will refer to the transformation of brane fields in eq.(2.38) as gauge transformations.

To summarize, we introduced the Lagrangian eq.(2.37) being the starting point for all further investigations in 5D-QED. Moreover, it is the generic Lagrangian for the coupling of fermions on a brane to gauge bosons in theories with extra dimensions. While we will encounter theories with more fermion species and different gauge sectors leading to different covariant derivatives later on, the generic form of the interaction Lagrangian will not be altered. In eq.(2.41) we derived how brane fermions interact with a Kaluza-Klein tower of gauge bosons and found the typical enhancement in the coupling to the heavy modes. The scalar fields do not interact with the fermions at all due to their antisymmetry.

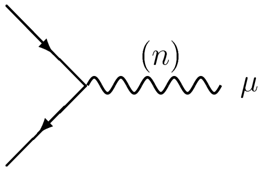
2.6 Feynman Rules

In the preceding sections we have been able to derive an effective four dimensional gauge theory starting from the five dimensional model. At this point we can translate the Lagrangian into Feynman rules, thereby giving the theory predictive power. The Feynman rules for the vertices are quite easy to obtain. The interaction terms of all the Kaluza-Klein modes with fermions are formally equivalent to the well-known QED interaction. Therefore, in analogy with QED [21], we can immediately write down the Feynman rules for the vertices in figure (2.1). The Feynman rules for the external fermion, photon and Kaluza-Klein lines are also completely analogous with QED. We only have to keep in mind that the higher Kaluza-Klein excitations are massive vector bosons and can therefore have three different polarizations.

For the propagators of the Kaluza-Klein modes in an R_ξ-gauge, however, we have to work a little harder. There are no mixing terms between Kaluza-Klein modes of different mass. Therefore, we can evaluate the propagators separately for each mode. Let us start considering the R_ξ-gauge corresponding to the gauge fixing function eq.(2.26). This case is a little more complicated and the propagators in the generalized R_ξ-gauge can be obtained easily

Photon vertex: 

$= -\, i\, e\, \gamma^\mu$

Kaluza-Klein vertices: 

$= -\, i\, \sqrt{2}\, e\, \gamma^\mu$

Figure 2.1: Feynman rules for the vertices in 5D-QED.

after we have worked out this example. As a starting point, we have to consider the kinetic terms in the gauge fixed Lagrangian (the sum of eq.(2.23) and eq.(2.30)) corresponding to a specific Kaluza-Klein mode (n) , i.e.

$$\mathcal{L}_{(n)}(x) \;=\; -\frac{1}{4} F_{(n)\mu\nu}\, F^{\mu\nu}_{(n)} \;+\; \frac{1}{2}\left(\frac{n}{R} A_{(n)\nu} \;+\; \partial_\nu A_{(n)4}\right)^2 \;-\; \frac{1}{2\xi}\left(\partial_\mu A^\mu_{(n)} \;-\; \frac{n}{R} A_{(n)4}\right)^2 . \quad (2.43)$$

Dropping the subscript (n) of the fields, adding total derivatives in order to act with derivatives only on the second field in each term and introducing a matrix notation to be read as usual matrix multiplication eq.(2.43) can be recasted into the form

$$\mathcal{L}_{(n)}(x) = \frac{1}{2}\left(A_\mu, A_4\right) \begin{pmatrix} g^{\mu\nu}\left(\partial^\rho\partial_\rho + \left(\frac{n}{R}\right)^2\right) + \left(\frac{1}{\xi} - 1\right)\partial^\mu\partial^\nu & \left(1 - \frac{1}{\xi}\right)\left(\frac{n}{R}\right)\partial^\mu \\ -\left(1 - \frac{1}{\xi}\right)\left(\frac{n}{R}\right)\partial^\nu & -\partial^\rho\partial_\rho - \frac{1}{\xi}\left(\frac{n}{R}\right)^2 \end{pmatrix} \begin{pmatrix} A_\nu \\ A_4 \end{pmatrix} . \quad (2.44)$$

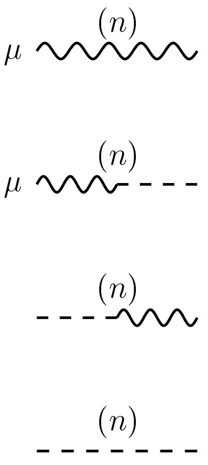
Going to momentum space by Fourier expanding the fields and integrating over space-time the action $S_{(n)} \;=\; \int d^4x\, \mathcal{L}_{(n)}(x)$ is finally given by

$$S_{(n)} \;=\; \frac{1}{2} \int \frac{d^4k}{(2\pi)^4} \quad \left(A_\mu(k), A_4(k)\right) \quad \mathbf{D} \quad \begin{pmatrix} A_\nu(-k) \\ A_4(-k) \end{pmatrix} , \quad (2.45)$$

where $\mathbf{D}$ is the matrix

$$\mathbf{D} \;=\; \begin{pmatrix} g^{\mu\nu}\left(-k^2 + \left(\frac{n}{R}\right)^2\right) + \left(1 - \frac{1}{\xi}\right) k^\mu\, k^\nu & i\left(1 - \frac{1}{\xi}\right)\left(\frac{n}{R}\right) k^\mu \\ -\, i\, \left(1 - \frac{1}{\xi}\right)\left(\frac{n}{R}\right) k^\nu & k^2 - \frac{1}{\xi}\left(\frac{n}{R}\right)^2 \end{pmatrix} \quad (2.46)$$

and $A_{\mu,4}(k)$ denote the Fourier expansion coefficients of the corresponding gauge boson and scalar fields.



vector boson propagators: $= i\left[\frac{-g^{\mu\nu}}{k^2-\left(\frac{n}{R}\right)^2} + \frac{(1-\xi)k^\mu k^\nu}{\left(k^2-\left(\frac{n}{R}\right)^2\right)^2}\right]$

mixing propagators: $= \frac{(1-\xi)\left(\frac{n}{R}\right)k^\mu}{\left(k^2-\left(\frac{n}{R}\right)^2\right)^2}$

mixing propagators: $= \frac{(\xi-1)\left(\frac{n}{R}\right)k^\nu}{\left(k^2-\left(\frac{n}{R}\right)^2\right)^2}$

scalar propagators: $= i\left[\frac{1}{k^2-\left(\frac{n}{R}\right)^2} + \frac{(1-\xi)\left(\frac{n}{R}\right)^2}{\left(k^2-\left(\frac{n}{R}\right)^2\right)^2}\right]$

Figure 2.2: The propagators of the gauge boson Kaluza-Klein modes in 5D-QED in standard R_ξ-gauge up to the $\pm i\epsilon$-terms.

By a general result of path integral quantization [22] the matrix in eq.(2.46) is connected to the Feynman propagator matrix D_F by the simple relation

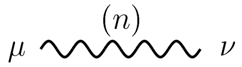
$$D_F \;=\; i\,\mathbf{D}^{-1} \tag{2.47}$$

up to the pole description by $\pm i\epsilon$-terms. Thus, it is the main task to find the inverse of $\mathbf{D}$. In 4D-QED only the upper left block of the matrix $\mathbf{D}$ is present. Here, the analysis is complicated by the appearance of the additional scalar field. Regarding the space-time structure of the matrix $\mathbf{D}$, the most general ansatz for the inverse is

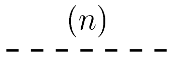
$$\mathbf{D}^{-1} \;=\; \begin{pmatrix} A\,g^{\mu\nu} \;+\; B\,k^\mu\,k^\nu & C\,k^\mu \\ D\,k^\nu & E \end{pmatrix}, \tag{2.48}$$

because $g^{\mu\nu}$ and k^μ are the only tensor and vector with respect to the Lorentz group at hand. The four equations resulting from $D_F\mathbf{D}^{-1} \;=\; \mathbf{1}_{5\times5}$ by block multiplication are sufficient to evaluate the five coefficients. A quite tedious but simple calculation yields

$$\mathbf{D}^{-1} \;=\; \begin{pmatrix} \frac{-g^{\mu\nu}}{k^2-\left(\frac{n}{R}\right)^2} + \frac{(1-\xi)k^\mu k^\nu}{\left(k^2-\left(\frac{n}{R}\right)^2\right)^2} & \frac{i\,(\xi-1)\left(\frac{n}{R}\right)k^\mu}{\left(k^2-\left(\frac{n}{R}\right)^2\right)^2} \\ \frac{i\,(1-\xi)\left(\frac{n}{R}\right)k^\nu}{\left(k^2-\left(\frac{n}{R}\right)^2\right)^2} & \frac{1}{k^2-\left(\frac{n}{R}\right)^2} + \frac{(1-\xi)\left(\frac{n}{R}\right)^2}{\left(k^2-\left(\frac{n}{R}\right)^2\right)^2} \end{pmatrix}. \tag{2.49}$$

Using eq.(2.47) this result allows us to read off the propagators shown in figure 2.2. The mixed propagators only vanish in Feynman gauge ($\xi \;=\; 1$) making actual computation vector boson propagators: 

$= \frac{i}{k^2-\left(\frac{n}{R}\right)^2}\left[-g^{\mu\nu} + \frac{(1-\xi)k^\mu\, k^\nu}{k^2-\xi\left(\frac{n}{R}\right)^2}\right]$

scalar propagators: 

$= \frac{i}{k^2-\xi\left(\frac{n}{R}\right)^2}$

Figure 2.3: The propagators of the gauge boson Kaluza-Klein modes in 5D-QED in generalized R_ξ-gauge up to the $\pm i\epsilon$-terms.

extremely difficult for other choices of ξ. In Feynman gauge the scalar propagator reduces to the usual propagator of a scalar particle of mass $(n/R)^2$. As mentioned before, the zero mode $n = 0$ furnishes 4D-QED where the analysis is less complicated because there is no field $A_{(0)4}$. The photon propagator can formally be obtained by setting $n = 0$ in the vector boson propagator of figure 2.2.

The same analysis can be performed for the generalized R_ξ-gauge corresponding to the gauge fixing function eq.(2.31). The corresponding matrix $\mathbf{D}$ can be derived from eq.(2.34). It is given by

$$\mathbf{D} = \begin{pmatrix} g^{\mu\nu}\left(-k^2 + \left(\frac{n}{R}\right)^2\right) + \left(1 - \frac{1}{\xi}\right) k^\mu\, k^\nu & 0 \\ 0 & k^2 - \xi\left(\frac{n}{R}\right)^2 \end{pmatrix} \quad . \tag{2.50}$$

Its inverse can be computed as before but the calculation is much simpler because of its block diagonal structure. Finally, we obtain the propagators shown in figure 2.3. No mixed propagators appear. This shows the big advantage of using the generalized R_ξ-scheme. Moreover, as already mentioned in section 2.4, it is obvious that the scalar field cannot propagate in the limit $\xi \to \infty$ because the propagator tends to zero. In this limit the gauge boson propagator is the conventional propagator of a massive vector boson in unitary gauge, as known from the standard model. In contrast, the meaning of the analogous limit $\xi \to \infty$ in standard R_ξ-gauge is far from clear.

For $\xi = 1$ the propagators of the two different R_ξ-gauges do not differ because the gauge fixing functions are actually the same. For calculations in gauges different from Feynman gauge it is important not to mix the different gauge fixing prescriptions.

So far, we have investigated the propagators of the Kaluza-Klein modes only at tree level. Therefore, they appear as stable particles. However, the heavy Kaluza-Klein modes will naturally decay, mainly into two fermions (see figure (2.1) for the vertices), as it is allowed kinematically. We will consider this slight complication in section 2.7. Otherwise, knowing the Feynman rules, we have all the ingredients at hand needed for some exemplary

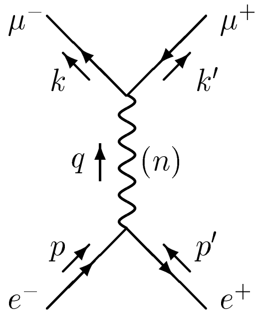


Figure 2.4: Feynman diagram for the process $e^+e^- \to \mu^+\mu^-$ with the n^{th} Kaluza-Klein vector boson in the s-channel.

calculations in 5D-QED.

2.7 An Example in 5D-QED: $e^+e^- \to \mu^+\mu^-$

In this section we will use an exemplary process to introduce some of the computational techniques we will again encounter later on. Moreover, we will see how a large extra dimension will generally show up in experiments at energy scales close to and above the compactification scale $1/R$. There are several common features of cross sections calculated in a higher dimensional framework which are most easily investigated in 5D-QED.

The simplest process in QED, namely $e^+e^- \to \mu^+\mu^-$, is also most suitable to do a model calculation within the Kaluza-Klein version of 5D-QED. In QED there is only one Feynman diagram to lowest order in perturbation theory. For the same process in 5D-QED there is an infinite number of Feynman diagrams (see figure 2.4) distinguished by the Kaluza-Klein mode propagating in the s-channel. As we will see below this is only a slight complication because all the diagrams can be summed analytically.

Using the Feynman rules obtained in section 2.6 it is easy to write down the amplitude $\mathcal{M}_{(n)}$ corresponding to the graph in figure (2.4), where some Kaluza-Klein mode (n) propagates in the s-channel. According to figures 2.1 and 2.3, in generalized R_ξ-gauge the amplitude is simply given by

$$i\,\mathcal{M}_{(n)} = \overline{v}(p')(-i\tilde{e}\gamma^\mu)\,u(p)\,\frac{i}{q^2-\left(\frac{n}{R}\right)^2}\left[-g_{\mu\nu}+\frac{(1-\xi)\,q_\mu\,q_\nu}{q^2-\xi\left(\frac{n}{R}\right)^2}\right]\overline{u}(k)(-i\tilde{e}\gamma^\nu)\,v(k')\,, \quad (2.51)$$

where u, $\overline{u}$, v and $\overline{v}$ are the appropriate spinors of the external particles. The dependence of the spinors on the fermions' spin is suppressed. By means of momentum conservation q is given by $q = p + p'$ and $\tilde{e}$ denotes e or $\sqrt{2}e$ for $n = 0$ or $n \neq 0$, respectively.

In the course of the calculation it turns out that the gauge dependent terms in the amplitude do not contribute if the external fermions are on shell. That is exactly what we expect, the result will be gauge independent. In section 5.3 we will analyze the gauge invariance of fermion scattering in a more general context. Here, for notational convenience, we will drop the gauge dependent terms right from the beginning.

It is easy to formally sum all the amplitudes to obtain $\mathcal{M} = \sum_{n=0}^{\infty} \mathcal{M}_{(n)}$. The square of the amplitudes is then given by

$$
\begin{aligned}
|\mathcal{M}|^2 = \sum_{m,n=0}^{\infty} & \frac{\tilde{e}^4}{\left(q^2 - \left(\frac{m}{R}\right)^2\right)\left(q^2 - \left(\frac{n}{R}\right)^2\right)} \\
& \times \left[\overline{v}(p')\gamma^\mu u(p)\right] \left[\overline{u}(k)\gamma_\mu v(k')\right] \left[\overline{u}(p)\gamma^\nu v(p')\right] \left[\overline{v}(k')\gamma_\nu u(k)\right] ,
\end{aligned}
\tag{2.52}
$$

where $\tilde{e}^4$ has to be understood in a generic way as the product of the corresponding $\tilde{e}^2$'s.

Because we are only interested in the unpolarized cross section, we can sum over the outgoing spins and average over the incoming spins. It is a well-known textbook calculation of introductory quantum field theory [23] to use the completeness relations for spin sums and the rules for handling the resulting traces. There is no difference to the standard calculation in conventional QED. Treating all the fermions as effectively massless because we are not interested in threshold effects anyway eq.(2.52) yields

$$
\frac{1}{4}\sum_{\text{spins}} |\mathcal{M}|^2 = \sum_{m,n=0}^{\infty} \frac{8\tilde{e}^4}{\left(q^2 - \left(\frac{m}{R}\right)^2\right)\left(q^2 - \left(\frac{n}{R}\right)^2\right)} \left[\, (pk)\,(p'k') \, + \, (pk')\,(p'k) \,\right] . \tag{2.53}
$$

In the appropriate center of mass frame, where we have

$$
\begin{aligned}
p &= (E,\, E\hat{z}) , & p' &= (E,\, -E\hat{z}) , & & \\
k &= (E,\, \vec{k}) , & k' &= (E,\, -\vec{k}) & \text{with :} \quad & \vec{k}^2 = E^2 , \\
& & q &= p + p' = 2\,E , & &
\end{aligned}
\tag{2.54}
$$

we finally obtain

$$
\frac{1}{4}\sum_{\text{spins}} |\mathcal{M}|^2 = \sum_{m,n=0}^{\infty} \frac{16\,\tilde{e}^4\,E^4}{\left(4\,E^2 - \left(\frac{m}{R}\right)^2\right)\left(4\,E^2 - \left(\frac{n}{R}\right)^2\right)} \left[1 + \cos^2\theta\right] , \tag{2.55}
$$

where θ is the angle between the incoming and outgoing negatively charged fermion. In order to compare eq.(2.55) to the well-known QED result

$$
\frac{1}{4}\sum_{\text{spins}} |\mathcal{M}_{\text{QED}}|^2 = e^4 \left[1 + \cos^2\theta\right] \tag{2.56}
$$

we can recast it into the form

$$\frac{1}{4}\sum_{\text{spins}}|\mathcal{M}|^2 \;=\; \Sigma(E_{\text{cm}})\,\frac{1}{4}\sum_{\text{spins}}|\mathcal{M}_{\text{QED}}|^2\,. \tag{2.57}$$

The energy dependent factor

$$\Sigma(E_{\text{cm}}) \;=\; 1 \,+\, 4\,E_{\text{cm}}^2\sum_{n=1}^{\infty}\frac{1}{E_{\text{cm}}^2-\left(\frac{n}{R}\right)^2} \,+\, 4\,E_{\text{cm}}^4\sum_{m=1}^{\infty}\frac{1}{E_{\text{cm}}^2-\left(\frac{m}{R}\right)^2}\sum_{n=1}^{\infty}\frac{1}{E_{\text{cm}}^2-\left(\frac{n}{R}\right)^2} \tag{2.58}$$

is due to the additional Kaluza-Klein gauge bosons in the s-channel and encodes all the effects of the large extra dimension in the considered process. A factor of two in the second and a factor of four in the third term arise because the coupling of the heavy Kaluza-Klein modes is enhanced by a factor of $\sqrt{2}$ compared to the photon.

Naturally, phase space does not change in the four dimensional version of 5D-QED and, therefore, we can immediately write down the differential and total cross section for the considered process, i.e.

$$\begin{aligned}\frac{d\sigma}{d\Omega} \;&=\; \Sigma(E_{\text{cm}})\left(\frac{d\sigma}{d\Omega}\right)_{\text{QED}} \;=\; \Sigma(E_{\text{cm}})\,\frac{\alpha^2}{4E_{\text{cm}}^2}\,\left(1+\cos^2\Theta\right)\,,\\ \sigma_{\text{tot}} \;&=\; \Sigma(E_{\text{cm}})\;\;(\sigma_{\text{tot}})_{\text{QED}} \;=\; \Sigma(E_{\text{cm}})\,\frac{4\pi\alpha^2}{3E_{\text{cm}}^2}\,,\end{aligned} \tag{2.59}$$

where $\alpha = e^2/4\pi$ is the fine structure constant.

Eq.(2.59) together with eq.(2.58) present the results already in a transparent way. However, to obtain numerical results we have to perform the infinite sums in eq.(2.58). Fortunately, the theory of complex variables provides a powerful tool for dealing with such infinite sums [27]. It can be shown that the identity

$$\sum_{n=-\infty}^{\infty} f(n) \;=\; -\left\{\text{sum of the residues of } \pi\cot(\pi z)\, f(z) \text{ at the poles of } f(z)\right\} \tag{2.60}$$

is valid basically whenever the sum converges. Using this theorem one can show

$$\sum_{n=1}^{\infty}\frac{1}{E_{\text{cm}}^2-\left(\frac{n}{R}\right)^2} \;=\; \frac{\pi}{2}\frac{R}{E_{\text{cm}}}\cot\left(\pi R E_{\text{cm}}\right) \,-\, \frac{1}{2E_{\text{cm}}^2}, \tag{2.61}$$

where we have used that $f(z)$, in this case, has two simple poles at $z = \pm R E_{\text{cm}}$. A short calculation finally yields

$$\Sigma(E_{\text{cm}}) \;=\; \left[\,\pi R E_{\text{cm}}\cot\left(\pi R E_{\text{cm}}\right)\right]^2. \tag{2.62}$$

Note, that the simplicity of the result crucially depends on the difference in coupling strength between the photon and the heavy Kaluza-Klein modes. In the low energy limit,

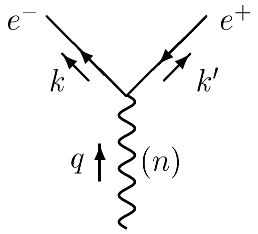


Figure 2.5: Dominant diagram for the decay of heavy Kaluza-Klein vector bosons.

$\Sigma(E_{\text{cm}})$ tends to unity such that we recover conventional QED at energy scales far below the compactification scale as we would have expected.

Because we have used the tree level propagators of the Kaluza-Klein states as derived in section 2.6, the cross section as a function of energy has poles at center of mass energies equal to the masses n/R of the corresponding Kaluza-Klein bosons. As mentioned at the end of section 2.6, the heavy modes can decay. Therefore, they will show up in the cross section as resonances not as poles. If we are interested in the resonances themselves we have to go beyond the tree level analysis for the propagators using some general field theoretic considerations. According to [28], it is a good approximation for narrow resonances to replace the tree level propagators by the appropriate Breit-Wigner propagator, i.e.

$$\frac{-\,i\,g^{\mu\nu}}{k^2-\left(\frac{n}{R}\right)^2} \quad\to\quad \frac{-\,i\,g^{\mu\nu}}{k^2-\left(\frac{n}{R}\right)^2+\,iM\Gamma}\,, \tag{2.63}$$

where M denotes the mass and Γ the decay rate of the decaying particle. Thus, the width of a given resonance depends on the decay rate of the corresponding particle.

Therefore, let us briefly digress and estimate the decay rate of the Kaluza-Klein modes by calculating the decay rate for the dominant decay channel, i.e. the decay into two fermions. This decay is described by the vertex diagram shown in figure 2.5. The corresponding amplitude is given by

$$i\,\mathcal{M}_{(n)} \;=\; \epsilon_\mu(q)\,\overline{u}(k)\left(-i\,\sqrt{2}e\,\gamma^\mu\right)v(k')\,, \tag{2.64}$$

where $\epsilon_\mu(q)$ denotes the polarization vector of the vector boson. It is easy to square this amplitude and sum over final fermion spins. Moreover, we can use the identity

$$\sum_{\lambda=1}^{3}\epsilon_\mu^\lambda(q)\,\epsilon_\nu^{\lambda *}(q) \;=\; -\,g_{\mu\nu}\;+\;\frac{q_\mu q_\nu}{\left(\frac{n}{R}\right)^2} \tag{2.65}$$

for massive vector bosons, where we have to sum over the three physical polarizations vectors fulfilling $q^\mu\epsilon_\mu^\lambda = 0$. Eq.(2.65) allows us to average over the initial polarizations. The second term on the right hand side will not contribute if the fermions the gauge boson is decaying into are on shell (see section 5.3). A standard calculation finally yields

$$\frac{1}{3}\sum_{\text{spins, pol.}} |\mathcal{M}|^2 = \frac{16\, e^2}{3}\,(k\, k')\,, \tag{2.66}$$

where the fermions are again effectively massless compared with gauge boson masses of the order of a few TeV. It is easiest to evaluate this Lorentz invariant probability in the rest frame of the decaying particle where we have

$$\begin{aligned} q &= (M_{(n)},\, 0) = k + k'\,, \\ k &= (M_{(n)}/2,\, \frac{M_{(n)}}{2}\,\vec{e}\,)\,, \qquad k' = (M_{(n)}/2,\, -\frac{M_{(n)}}{2}\,\vec{e}\,)\,. \end{aligned} \tag{2.67}$$

$M_{(n)} = n/R$ is the mass of the n^{th} Kaluza-Klein mode and $\vec{e}$ is a unit vector in some space direction. Therefore, eq.(2.66) simplifies to

$$\frac{1}{3}\sum_{\text{spins, pol.}} |\mathcal{M}|^2 = \frac{8\, e^2}{3}\, M_{(n)}^2 = \frac{8\, e^2}{3}\left(\frac{n}{R}\right)^2. \tag{2.68}$$

Combining this matrix element with the phase space factor $1/16\pi M_{(n)}$ for a two body decay yields

$$\Gamma_{\text{tot}} = \frac{e^2}{6\,\pi}\left(\frac{n}{R}\right) = \frac{2\,\alpha}{3}\left(\frac{n}{R}\right). \tag{2.69}$$

This is the partial decay rate into one fermion species, it has to be multiplied by the number of fermions in the theory to obtain an estimate for the total decay width of the particle. For our needs, it is sufficient to note that the width is of the order of the fine structure constant times the mass of the Kaluza-Klein mode, i.e. roughly one hundredth of the particle's mass.

After calculating the decay rate of the heavy Kaluza-Klein modes we can now return to the question how to take the instability of the vector bosons into account. For the special case of heavy Kaluza-Klein modes in 5D-QED, the modification of the propagators (see eq.(2.63)) is given by

$$\frac{-\,i\,g^{\mu\nu}}{k^2-\left(\frac{n}{R}\right)^2} \quad\to\quad \frac{-\,i\,g^{\mu\nu}}{k^2-\left(\frac{n}{R}\right)^2 + i\tilde{\alpha}\left(\frac{n}{R}\right)^2}\,, \tag{2.70}$$

where $\tilde{\alpha}$ is some constant of order α. In the cross section of the model process only $\Sigma(E_{\text{cm}})$ multiplying the QED result is affected by this substitution. It yields

$$\begin{aligned} \Sigma(E_{\text{cm}}) = 1 &+ \sum_{n=1}^{\infty}\frac{2\,E_{\text{cm}}^2}{\left(E_{\text{cm}}^2-\left(\frac{n}{R}\right)^2\right)+i\tilde{\alpha}\left(\frac{n}{R}\right)^2} + \sum_{n=1}^{\infty}\frac{2\,E_{\text{cm}}^2}{\left(E_{\text{cm}}^2-\left(\frac{n}{R}\right)^2\right)-i\tilde{\alpha}\left(\frac{n}{R}\right)^2} \\ &+ 4\,E_{\text{cm}}^4\sum_{m=1}^{\infty}\frac{1}{E_{\text{cm}}^2-\left(\frac{m}{R}\right)^2+i\tilde{\alpha}\left(\frac{m}{R}\right)^2}\sum_{n=1}^{\infty}\frac{1}{E_{\text{cm}}^2-\left(\frac{n}{R}\right)^2-i\tilde{\alpha}\left(\frac{n}{R}\right)^2}\,. \end{aligned} \tag{2.71}$$

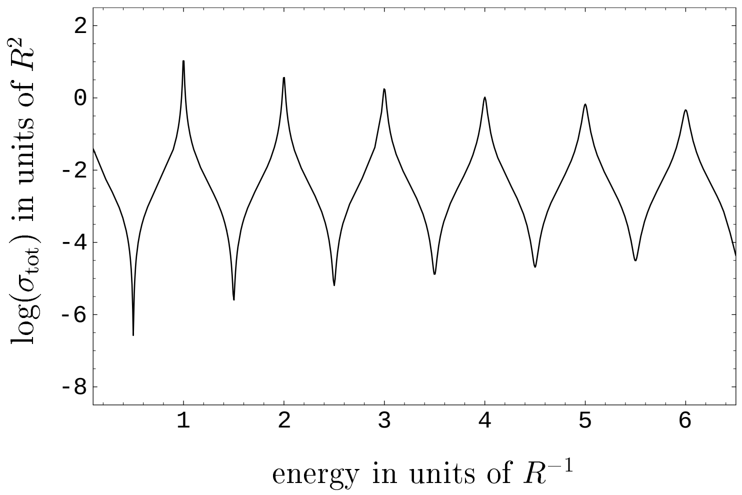


Figure 2.6: Total cross section for $e^+e^- \to \mu^+\mu^-$ as a function of center of mass energy on a logarithmic scale ($\tilde{\alpha} = 1/100$).

To further evaluate this relation we again use the theorem introduced in eq.(2.60). Using the fact that $f(z) = 1/\left(E_{\rm cm}^2 - \left(\frac{z}{R}\right)^2 + i\tilde{\alpha}\left(\frac{z}{R}\right)^2 \right)$ has two simple poles slightly shifted from the real axis into the complex plane due to the finite width of the heavy Kaluza-Klein modes, one can calculate

$$\sum_{n=1}^{\infty} \frac{1}{E_{\rm cm}^2 - \left(\frac{n}{R}\right)^2 + i\tilde{\alpha}\left(\frac{n}{R}\right)^2} = \frac{\pi}{2}\sqrt{\frac{1+i\tilde{\alpha}}{1+\tilde{\alpha}^2}}\frac{R}{E_{\rm cm}} \cot\left(\pi\sqrt{\frac{1+i\tilde{\alpha}}{1+\tilde{\alpha}^2}} R E_{\rm cm}\right) - \frac{1}{2E_{\rm cm}^2}. \quad (2.72)$$

Note, that the argument of the cotangent function is not real any more because we have included the finite width into the propagator. Therefore, $\Sigma(E_{\rm cm})$ does not have any poles. A quite lengthy calculation, where this result is inserted in eq.(2.71), finally leads to

$$\Sigma(E_{\rm cm}) = \frac{R^2\pi^2}{\sqrt{1+\tilde{\alpha}^2}} E_{\rm cm}^2 \frac{\cosh\left(2bcRE_{\rm cm}\right) + \cos\left(2acRE_{\rm cm}\right)}{\cosh\left(2bcRE_{\rm cm}\right) - \cos\left(2acRE_{\rm cm}\right)}, \quad (2.73)$$

where $a = \left(1/2\sqrt{1+\tilde{\alpha}^2} + 1/2\right)^{1/2}$, $b = \left(1/2\sqrt{1+\tilde{\alpha}^2} - 1/2\right)^{1/2}$ and $c = \pi/\sqrt{1+\tilde{\alpha}^2}$.

Figure 2.6 shows the total cross section for $e^+e^- \to \mu^+\mu^-$ on a logarithmic scale. If the compactification radius, for example, is $R = 1\,\text{TeV}^{-1}$, the cross section is shown for center of mass energies from 100 GeV to 6.5 TeV. For $\tilde{\alpha}$ we have chosen the numerical value $\tilde{\alpha} = 1/100$. As the Z boson in the standard model, each of the first heavy Kaluza-Klein modes shows up in a resonance. This is a common feature of all higher dimensional theories. The resonances will be separated approximately by the compactification scale. Comparing different resonances, at first sight it looks like the cross section at the top of the resonances is suppressed by the usual phase space factor $1/E_{\rm cm}^2$. However, this is only approximately

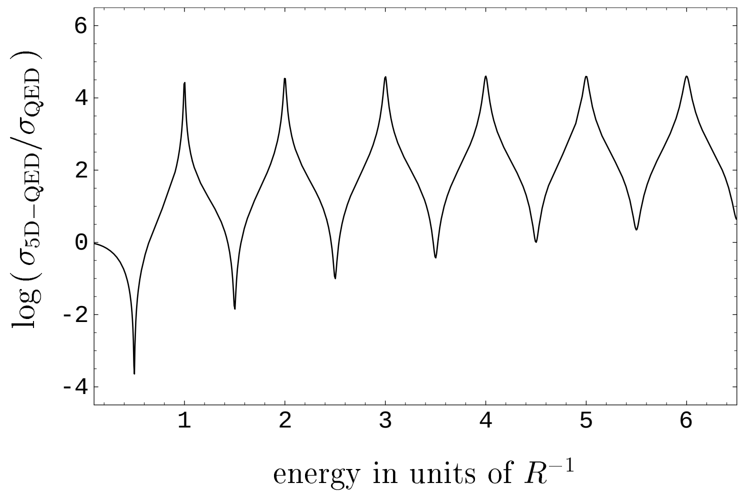


Figure 2.7: Ratio of the cross sections for $e^+e^- \to \mu^+\mu^-$ in 5D-QED and conventional QED on a logarithmic scale ($\tilde{\alpha} = 1/100$).

true for energies of a few R^{-1}. For small $\tilde{\alpha}$ the constant b in eq.(2.73) will also be small. Therefore, we can expand $\cosh(2bcE_{\rm cm})$ up to second order for the first resonances. In this approximation, $\Sigma(E)$ will drop from peak to peak, where $\cos(2acE_{\rm cm}) = 1$, with the inverse square of $E_{\rm cm}$ as we could have expected. In the high energy limit, however, the hyperbolic cosine terms become large. The resonant structure blurs out and the cross section converges to $\sigma_{\rm tot} = 4\pi^3 R^2 \alpha^2/3$.

Figure 2.7 shows the ratio of the 5D-QED theory to conventional QED on a logarithmic scale. For energies being small compared to the compactification scale, the effects of the Kaluza-Klein modes are negligible. This is a common feature of all sensible higher dimensional theories and maybe the reason why we have not observed any large extra dimension in spite of its presence. At intermediate energies well below the first resonance ($E_{\rm cm} \approx 1/2\, R^{-1}$), the cross section is reduced due to destructive interference between the photon and the Kaluza-Klein graphs. This reduction of the cross section as the enhancement in the neighborhood of the resonances are not small effects. In wide energy ranges the cross section in the higher dimensional model deviates several orders of magnitude from the result in conventional QED. This is also a common feature of higher dimensional models: Once collider energies reach the compactification scale, a possible large extra dimension cannot hide any more. In the high energy limit the ratio will diverge because the QED cross section drops to zero while the 5D-QED cross section stays finite.

From the experimental point of view, however, the deviation from the QED-result at low energies (i.e. collider energies) is of greatest interest (see figure 2.8). To investigate this

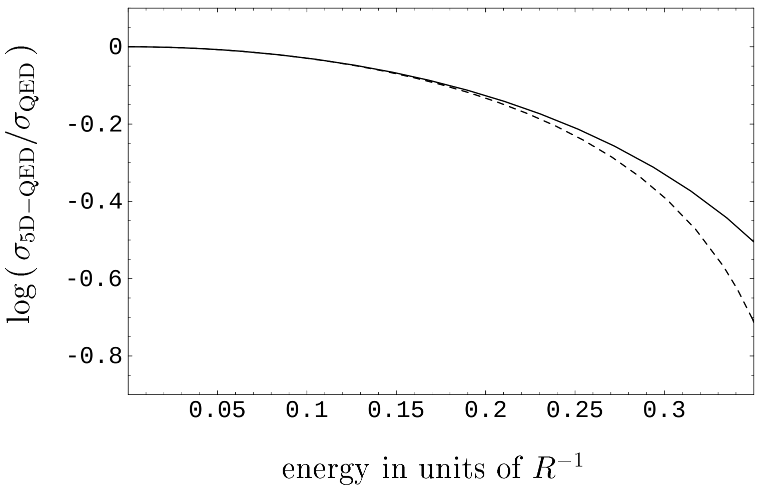


Figure 2.8: Ratio of the cross sections for $e^+e^- \to \mu^+\mu^-$ in 5D-QED and usual QED at small energies on a logarithmic scale ($\tilde{\alpha} = 1/100$). The Solid line shows the ratio of the exact tree level cross sections. For the dashed line the 5D-QED cross section is approximated with the help of eq.(2.74).

deviation it is sufficient to expand the result for the cross section in powers of $E_{\rm cm}R$. When we are interested in the low energy behavior, the width of the Kaluza-Klein modes does not play any role and we can directly expand eq.(2.62). Keeping only terms to second order in $E_{\rm cm}R$ yields

$$\Sigma(E_{\rm cm}) \;=\; 1 \;-\; \frac{2\,\pi^2\,R^2}{3}\,E_{\rm cm}^2\,. \tag{2.74}$$

This should be a good approximation for energies $E_{\rm cm} \ll 1/\pi R$, as shown in figure 2.8.

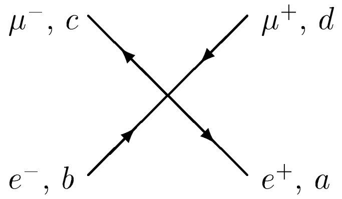
Small deviations from standard predictions in fermion-fermion scattering, like the deviation in eq.(2.74), are conventionally parameterized by an additional four-fermion interaction. Such an effective interaction is described by a Lagrangian

$$\mathcal{L}_{4\rm F} \;=\; -\,\frac{g^2}{M^2}\left(\overline{\Psi}\gamma^\nu\Psi\right)_e\left(\overline{\Psi}\gamma_\nu\Psi\right)_\mu \tag{2.75}$$

with a dimensionless coupling constant g and some mass scale M relevant to the theory. This interaction term leads to the vertex shown in figure 2.9, introducing the additional amplitude

$$\mathcal{M}_{4\rm F} \;=\; -\,\frac{g^2}{M^2}\,\left[\,\overline{v}(p')\,\gamma^\nu\,u(p)\,\right]\left[\,\overline{u}(k)\,\gamma_\nu\,v(k')\,\right] \tag{2.76}$$

to $e^+e^- \to \mu^+\mu^-$ scattering. At low energies this amplitude will be small compared to the QED amplitude and, as a first correction, we only take the interference terms with the



$$= -i\,\tfrac{g^2}{M^2}\,\gamma^\nu\,\gamma_\nu$$

Figure 2.9: Vertex in an effective four-fermion theory.

QED graph into account. Finally, this yields

$$\frac{1}{4}\sum_{\text{spins}}|\mathcal{M}_{\text{QED}+4\text{F}}|^2 \approx \left(1 - 2\,\frac{g^2}{e^2M^2}\,E_{\text{cm}}^2\right)\frac{1}{4}\sum_{\text{spins}}|\mathcal{M}_{\text{QED}}|^2 . \tag{2.77}$$

Comparing this result with eq.(2.74) shows that, at low energies, 5D-QED looks like QED in the presence of an additional effective four-fermion interaction with a coupling constant

$$\frac{g^2}{M^2} = \frac{\pi^2R^2}{3}\,e^2 . \tag{2.78}$$

If we identify $1/R$ with the natural mass scale M of the theory the dimensionless coupling is given by $g = \pi e/\sqrt{3}$. This result can also be derived in a much more general context directly at the Lagrangian level. We just have to apply the considerations in Appendix E to the simple case of 5D-QED. Integrating out the massive Kaluza-Klein modes we can easily derive an effective four-fermion interaction with the correct coupling as a low energy limit of full 5D-QED. Moreover, this result can also be obtained from the full theory at the amplitude level. Just neglect the momenta of the Kaluza-Klein modes in the propagator of eq.(2.51) being extremely small compared with the large compactification scale at small center of mass energies.

Note, that 5D-QED is only a generalization of QED, no electroweak physics is considered in our calculation so far. Therefore, this result cannot be directly compared to experimental data. Nevertheless, we have encountered some of the basic features of a higher dimensional theory in this simple example which can be recovered in any more realistic theory. Having discussed 5D-QED in great detail we can now generalize the results to more complicated theories like non-Abelian gauge theories or theories with spontaneous symmetry breaking.

Chapter 3

Non-Abelian Gauge Theories in 5 Dimensions

There is no principal problem to generalize the basic concepts of a higher dimensional theory, introduced in chapter 2 for QED, to the gauge boson sector of non-Abelian gauge theories. As we will see below, most of the analysis is completely analogous. However, the gauge transformation properties of the Kaluza-Klein modes are more involved in non-Abelian theories and there are gauge boson self-interactions to be analyzed in the five dimensional context which do not appear in 5D-QED.

3.1 The General Structure

Let us briefly summarize some general features of non-Abelian theories before we investigate the higher dimensional extension. In standard four dimensional theories, for a given gauge group with n generators, one has n gauge fields $A^a(x)$ $(a = 1, 2, \ldots, n)$ transforming under the adjoint representation of the gauge group. An infinitesimal gauge transformation parameterized by n scalar fields Θ^a $(a = 1, 2, \ldots, n)$ results in

$$A^a_\mu \quad \to \quad A^a_\mu + \partial_\mu \Theta^a - g f^{abc} \Theta^b A^c_\mu \, , \tag{3.1}$$

where f^{abc} are the structure constants of the gauge group (which are totally antisymmetric for semi-simple, compact Lie-groups exclusively considered here) and g denotes the coupling constant of the theory. Here and in the following, the x-dependence of the fields is again suppressed and a summation over equal gauge group indices is assumed.

Defining the field strength tensors for the non-Abelian gauge theory by

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu \tag{3.2}$$

the gauge invariant Lagrangian for the gauge bosons is given by

$$\mathcal{L}(x) = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} . \tag{3.3}$$

Let us now consider the same theory extended by a single large extra dimension. All the gauge fields live in the bulk and, thus, depend on one extra coordinate y. Starting in a SO(4,1) invariant space-time the gauge boson fields have to transform under the vector representation of SO(4,1) and have to include one extra field component, denoted by $A^a_4(x,y)$, exactly as in the QED case. In five dimensions the gauge transformation in eq.(3.1) reads

$$A^a_M(x,y) \quad \rightarrow \quad A^a_M(x,y) + \partial_M \Theta^a(x,y) - g_5 f^{abc} \Theta^b(x,y) A^c_M(x,y) , \tag{3.4}$$

where M, $N = 0,1,2,3,4$ and g_5 is the coupling constant of the five dimensional theory. The Lagrangian density is given by

$$\mathcal{L}(x,y) = -\frac{1}{4} F^a_{MN} F^{aMN} , \tag{3.5}$$

where the field strength extended to five dimensions reads

$$F^a_{MN} = \partial_M A^a_N - \partial_N A^a_M + g_5 f^{abc} A^b_M A^c_N . \tag{3.6}$$

As in 5D-QED dimensional analysis shows, that the coupling constant g_5 in the five dimensional theory has mass dimension -1/2 while the gauge fields have mass dimension 3/2.

Let us now consider compactification of the non-Abelian gauge theory on the S^1/Z_2 orbifold introduced in chapter 2. The gauge transformation properties in the non-Abelian case will be consistent with the symmetries of the gauge fields if we take the constraints on the fields known from the QED case (eq.(2.9)) to be constraints for each gauge boson field $A^a_M(x,y)$ $(a = 1,2,\ldots,n)$ separately, i.e.:

$$\begin{aligned}
A^a_M(x,y) &= A^a_M(x,y+2\pi R) \qquad && M = 0,1,2,3,4 \\
A^a_\mu(x,y) &= A^a_\mu(x,-y) && \mu = 0,1,2,3 \\
A^a_4(x,y) &= -A^a_4(x,-y) \\
\Theta^a(x,y) &= \Theta^a(x,y+2\pi R) \\
\Theta^a(x,y) &= \Theta^a(x,-y)
\end{aligned} \tag{3.7}$$

This will be shown in section 3.2. Here, again, we choose the vector components $A^a_\mu(x,y)$ to be even in order to recover a four dimensional theory with massless gauge bosons (e.g. QCD) in the low energy limit.

The Lagrangian eq.(3.5) does not only contain the kinetic terms of all the gauge bosons but also the gauge boson self-interactions due to the non-Abelian structure of the theory. Hence, it can be recasted into the form

$$\mathcal{L}(x,y) \;=\; \mathcal{L}_{\rm kin}(x,y) + \mathcal{L}_{\rm s}(x,y)\,, \tag{3.8}$$

where

$$\mathcal{L}_{\rm kin}(x,y) \;=\; -\frac{1}{4}\left(\partial_M A_N^a \,-\, \partial_N A_M^a\right)\left(\partial^M A^{aN} \,-\, \partial^N A^{aM}\right)\,, \tag{3.9}$$

$$\mathcal{L}_{\rm s}(x,y) \;=\; -\frac{1}{2} g_5\, f^{abc} A^{bM} A^{cN}\left(\partial_M A_N^a \,-\, \partial_N A_M^a\right) \,-\, \frac{1}{4} g_5^2\, f^{abc}\, f^{ade} A^{bM} A^{cN} A_M^d A_N^e\,. \tag{3.10}$$

The implications of the kinetic Lagrangian eq.(3.9) and the self-interactions eq.(3.10) are investigated in section 3.3 and section 3.4, respectively.

3.2 Gauge Transformations

For the different field components the infinitesimal gauge transformation eq.(3.4) can be decomposed into

$$A_\mu^a(x,y) \quad\to\quad A_\mu^a(x,y) \,+\, \partial_\mu \Theta^a(x,y) \,-\, g_5 f^{abc}\Theta^b(x,y) A_\mu^c(x,y)\,, \tag{3.11}$$

$$A_4^a(x,y) \quad\to\quad A_4^a(x,y) \,+\, \partial_4 \Theta^a(x,y) \,-\, g_5 f^{abc}\Theta^b(x,y) A_4^c(x,y)\,. \tag{3.12}$$

All the terms on the right hand side of eq.(3.11) are even and all the terms on the right hand side of eq.(3.12) are odd with respect to the extra dimension. Therefore, gauge transformations constrained by eq.(3.7) respect the Z_2-parity of the gauge fields.

The compactification on the S^1/Z_2 manifold again allows us to expand all the fields into corresponding Fourier series

$$\begin{aligned} A^{a\mu}(x,y) \;&=\; \sum_{n=0}^{\infty} \frac{1}{\sqrt{2\pi R}}\, A_{(n)}^{a\mu}(x) \cos\left(\frac{ny}{R}\right)\,, \\ A^{a4}(x,y) \;&=\; \sum_{n=1}^{\infty} \frac{1}{\sqrt{2\pi R}}\, A_{(n)}^{a4}(x) \sin\left(\frac{ny}{R}\right)\,, \end{aligned} \tag{3.13}$$

where we have introduced the Kaluza-Klein modes $A_{(n)}^{aM}(x)$ for the non-Abelian theory. It is our next task to investigate the gauge transformation properties of the Kaluza-Klein modes. Therefore, we also expand the scalar fields $\Theta^a(x,y)$ as in the QED analysis (see eq.(2.12)). Inserting the Fourier series into the gauge transformation equations (3.11) and

(3.12) yields

$$
\begin{aligned}
A^a_\mu(x,y) &= \sum_{n=0}^{\infty} \frac{1}{\sqrt{2\pi R}}\, A^a_{(n)\mu} \cos\Big(\frac{n}{R}y\Big) \\
&\to \sum_{n=0}^{\infty} \frac{1}{\sqrt{2\pi R}} \left[A^a_{(n)\mu} + \partial_\mu \Theta^a_{(n)} - g_5 f^{abc} A^c_{(n)\mu} \sum_{m=0}^{\infty} \frac{\Theta^b_{(m)}}{\sqrt{2\pi R}} \cos\Big(\frac{m}{R}y\Big) \right] \cos\Big(\frac{n}{R}y\Big) , \\
A^a_4(x,y) &= \sum_{n=1}^{\infty} \frac{1}{\sqrt{2\pi R}}\, A^a_{(n)4} \sin\Big(\frac{n}{R}y\Big) \\
&\to \sum_{n=1}^{\infty} \frac{1}{\sqrt{2\pi R}} \left[A^a_{(n)4} - \frac{n}{R}\Theta^a_{(n)} - g_5 f^{abc} A^c_{(n)4} \sum_{m=0}^{\infty} \frac{\Theta^b_{(m)}}{\sqrt{2\pi R}} \cos\Big(\frac{m}{R}y\Big) \right] \sin\Big(\frac{n}{R}y\Big) ,
\end{aligned}
\tag{3.14}
$$

where we have again suppressed the x-dependence of the modes for notational convenience. In contrast to the Abelian theory, it is quite tedious to derive the gauge transformation properties of the Kaluza-Klein modes starting from eq.(3.14). The calculation is presented in detail in Appendix B. As in the Abelian theory, we will see in section 3.3 that we have to rescale the fields according to eq.(2.24). After rescaling, the final result for the gauge transformation properties is given by

$$
\begin{aligned}
A^a_{(0)\mu} &\to A^a_{(0)\mu} + \partial_\mu \Theta^a_{(0)} - \frac{1}{2}\frac{g_5}{\sqrt{2\pi R}} f^{abc} \sum_{m=0}^{\infty} \Theta^b_{(m)} \left((1+\delta_{m,0})\, \sqrt{2}^{\,1-\delta_{m,0}}\, A^c_{(m)\mu} \right) , \\
A^a_{(n)\mu} &\to A^a_{(n)\mu} + \partial_\mu \frac{\Theta^a_{(n)}}{\sqrt{2}} - \frac{1}{2}\frac{g_5}{\sqrt{2\pi R}} f^{abc} \sum_{m=0}^{\infty} \Theta^b_{(m)} \left((1+\delta_{m,n})\sqrt{2}^{\,-\delta_{m,n}} A^c_{(|m-n|)\mu} + A^c_{(m+n)\mu} \right) , \\
A^a_{(n)4} &\to A^a_{(n)4} - \frac{n}{R}\frac{\Theta^a_{(n)}}{\sqrt{2}} - \frac{1}{2}\frac{g_5}{\sqrt{2\pi R}} f^{abc} \sum_{m=0}^{\infty} \Theta^b_{(m)} \left(\mathrm{sign}(n-m)\, A^c_{(|m-n|)4} + A^c_{(m+n)4} \right) ,
\end{aligned}
\tag{3.15}
$$

where $n \geq 1$.

The complicated structure of a gauge transformation at the level of the individual Kaluza-Klein modes is mainly due to the fact that the allowed class of local gauge transformations in a theory with one extra dimension is much larger than in a standard four dimensional theory. To show this it is instructive to digress to a subclass of gauge transformations. For transformations being constant with respect to the extra dimension (i.e. $\Theta^a_{(n)} = 0$ for $n \geq 1$), the transformation properties eq.(3.15) simplify substantially and read

$$
\begin{aligned}
A^a_{(0)\mu} &\to A^a_{(0)\mu} + \partial_\mu \Theta^a_{(0)} - \frac{g_5}{\sqrt{2\pi R}} f^{abc}\, \Theta^b_{(0)}\, A^c_{(0)\mu} , \\
A^a_{(n)\mu} &\to A^a_{(n)\mu} - \frac{g_5}{\sqrt{2\pi R}} f^{abc}\, \Theta^b_{(0)}\, A^c_{(n)\mu} , \\
A^a_{(n)4} &\to A^a_{(n)4} - \frac{g_5}{\sqrt{2\pi R}} f^{abc}\, \Theta^b_{(0)}\, A^c_{(n)4} ,
\end{aligned}
\tag{3.16}
$$

i.e. the zero modes transform exactly as the gauge fields in a four dimensional non-Abelian theory. All further complications arise from local transformations with respect to the extra dimension.

3.3 The Kinetic Part of the Lagrangian

As in the QED example, we investigate the kinetic part of the Lagrangian in order to derive the particle spectrum of a non-Abelian theory in its effective four dimensional formulation. The kinetic Lagrangian eq.(3.9) is equivalent to the Abelian theory (see eq.(2.2) and eq.(2.3)) replicated n times, once for each gauge boson field A^a_M $(a = 1, 2, \ldots, n)$. Hence, the derivation in section 2.3 is also valid for the non-Abelian case and the Lagrangian density in four dimensions after integrating over the extra dimension and rescaling the fields reads

$$\mathcal{L}_{\text{kin}}(x) = -\frac{1}{4}F^a_{(0)\mu\nu}F^{a\mu\nu}_{(0)} + \sum_{n=1}^{\infty}\left[-\frac{1}{4}F^a_{(n)\mu\nu}F^{a\mu\nu}_{(n)} + \frac{1}{2}\left(\frac{n}{R}A^a_{(n)\nu} + \partial_\nu A^a_{(n)4}\right)^2\right]. \quad (3.17)$$

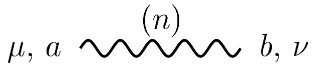
At this point, to find the physical particle spectrum and to calculate the propagators for the gauge bosons, we have to fix the gauge. The analysis concerning the gauge boson fields is analogous with the one in chapter 2. For example, all the remarks about unitary gauge apply.

For more general gauges, however, the formalism of path integral quantization requires the introduction of a new kind of fields called ghosts [24]. The discussion of ghosts is beyond the scope of this thesis but, fortunately, they only play a role at loop-level. We can again choose to work in R_ξ-gauge corresponding to the gauge fixing functions $F(A^a) = \partial^M A^a_M$ or in its generalized form corresponding to the gauge fixing functions $F(A^a) = \partial^\mu A^a_\mu - \xi\partial_4 A^a_4$. Note, that there is a gauge fixing function $F(A^a)$ for each gauge field. In analogy with 5D-QED, the appropriate gauge fixing terms to be added to the Lagrangian are given by

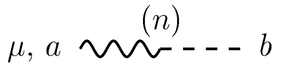
$$\mathcal{L}_{GF}(x,y) = -\frac{1}{2\xi}(\partial^M A^a_M)^2 \quad \text{or} \quad \mathcal{L}_{GF}(x,y) = -\frac{1}{2\xi}(\partial^\mu A^a_\mu - \xi\partial_4 A^a_4)^2 \quad (3.18)$$

for R_ξ-gauge or generalized R_ξ-gauge, respectively. The following analysis for each of the gauge fields does not differ by any means from the Abelian case. Therefore, the kinetic Lagrangian density at a given Kaluza-Klein level n in simple R_ξ-gauge reads (in analogy with eq.(2.43))

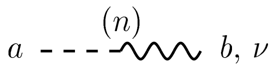
$$\mathcal{L}_{(n)}(x) = -\frac{1}{4}F^a_{(n)\mu\nu}F^{a\mu\nu}_{(n)} + \frac{1}{2}\left(\frac{n}{R}A^a_{(n)\nu} + \partial_\nu A^a_{(n)4}\right)^2 - \frac{1}{2\xi}\left(\partial_\mu A^{a\mu}_{(n)} - \frac{n}{R}A^a_{(n)4}\right)^2. \quad (3.19)$$

vector boson propagators: 

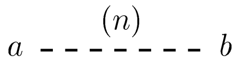
$$= \delta_{a,b}\, i \left[\frac{-g^{\mu\nu}}{k^2-\left(\frac{n}{R}\right)^2} + \frac{(1-\xi)k^\mu k^\nu}{\left(k^2-\left(\frac{n}{R}\right)^2\right)^2} \right]$$

mixing propagators: 

$$= \delta_{a,b} \frac{(1-\xi)\left(\frac{n}{R}\right)k^\mu}{\left(k^2-\left(\frac{n}{R}\right)^2\right)^2}$$

mixing propagators: 

$$= \delta_{a,b} \frac{(\xi-1)\left(\frac{n}{R}\right)k^\nu}{\left(k^2-\left(\frac{n}{R}\right)^2\right)^2}$$

scalar propagators: 

$$= \delta_{a,b}\, i \left[\frac{1}{k^2-\left(\frac{n}{R}\right)^2} + \frac{(1-\xi)\left(\frac{n}{R}\right)^2}{\left(k^2-\left(\frac{n}{R}\right)^2\right)^2} \right]$$

Figure 3.1: The propagators of the gauge boson Kaluza-Klein modes in a five dimensional non-Abelian gauge theory in standard R_ξ-gauge up to the $\pm i\epsilon$-terms. Note, that the propagators are diagonal with respect to the different gauge indices, i.e. there are no mixed propagators between different gauge bosons. The propagators in generalized R_ξ-gauge can be obtained by the same changes leading from figure 2.2 to figure 2.3.

The corresponding kinetic Lagrangian density in generalized R_ξ-gauge is given by

$$\mathcal{L}_{(n)}(x) = -\frac{1}{4} F^a_{(n)\mu\nu} F^{a\mu\nu}_{(n)} + \frac{1}{2}\left(\frac{n}{R}\right)^2 {A^a_{(n)\nu}}^2 - \frac{1}{2\xi}\left(\partial_\mu A^{a\mu}_{(n)}\right)^2 + \left(\partial_\nu A^a_{(n)4}\right)^2 - \frac{\xi}{2}\left(\frac{n}{R}\right)^2 {A^a_{(n)4}}^2. \tag{3.20}$$

At this point, the particle spectrum of the non-Abelian theory is obvious. The considerations following eq.(2.34) are valid for each gauge index separately and, thus, the particle spectrum of 5D-QED is simply replicated n times.

Finally, we can derive the propagators of the Kaluza-Klein modes in the non-Abelian theory. The structure of eq.(3.19) or eq.(3.20) is extremely simple because Kaluza-Klein modes with different gauge indices do not mix at all. Therefore, the calculation of the propagators turns out to be equivalent to the one in section 2.6 and we can immediately state the corresponding Feynman rules, for example in standard R_ξ-gauge, in figure 3.1.

3.4 Gauge Boson Self-Interactions

The gauge boson self-interactions described by the Lagrangian eq.(3.10) are a necessary ingredient of any non-Abelian gauge theory. Because they do not have a counterpart in the Abelian model of chapter 2 we devote this section to a careful investigation of their properties in the context of a large extra dimension. It is convenient to separately consider the interactions involving three and four gauge bosons given in eq.(3.10). Therefore, we rewrite the self-interaction Lagrangian in the form

$$\mathcal{L}_{\mathrm{s}}(x,y) = \mathcal{L}_3(x,y) + \mathcal{L}_4(x,y)\,. \tag{3.21}$$

$\mathcal{L}_3$ and $\mathcal{L}_4$ will be analyzed in detail in the two proceeding sections.

3.4.1 Three Gauge Boson Interactions

We begin with the interactions involving three gauge bosons described by

$$\mathcal{L}_3(x,y) \;=\; -\frac{1}{2}\, g_5\, f^{abc} A^{bM} A^{cN}\, \left(\partial_M A^a_N \;-\; \partial_N A^a_M\right)\,. \tag{3.22}$$

They can be decomposed into

$$\mathcal{L}_3(x,y) \;=\; -\,g_5\, f^{abc} \left(A^{b\mu} A^{c\nu}\, (\,\partial_\mu A^a_\nu\,) \;+\; A^{b4} A^{c\mu}\, \left(\partial_4 A^a_\mu \;-\; \partial_\mu A^a_4\,\right)\right)\,, \tag{3.23}$$

where the antisymmetry of the structure constants $f^{abc} = -f^{acb}$ is used.

In the following, we will derive the corresponding interactions in an effective four dimensional formulation. Fourier expanding all the fields according to eq.(3.13) the first term in eq.(3.23) is given by

$$A^{b\mu} A^{c\nu}\, (\,\partial_\mu A^a_\nu\,) \;=\; \sum_{k,l,m=0}^{\infty} \frac{1}{(2\pi R)^{3/2}}\, A^{b\mu}_{(k)} A^{c\nu}_{(l)}\, \left(\,\partial_\mu A^a_{(m)\nu}\,\right) \cos\!\Big(\frac{k}{R}y\Big)\, \cos\!\Big(\frac{l}{R}y\Big)\, \cos\!\Big(\frac{m}{R}y\Big)\,, \tag{3.24}$$

where the fields on the left hand side depend on x and y while the Kaluza-Klein modes on the right hand side depend on x alone. In the next step we integrate over the extra dimension. Using the identities eq.(2.20) to proof

$$\begin{aligned}\cos(ky)\cos(ly)\cos(my) \;=\; \frac{1}{4}\Big(&\cos((k+l+m)y) + \cos((k+l-m)y)\\ &+ \cos((k-l+m)y) + \cos((k-l-m)y)\Big)\end{aligned} \tag{3.25}$$

it is easy to show

$$\int_0^{2\pi R} dy\, A^{b\mu} A^{c\nu}\, (\partial_\mu A^a_\nu) \;=\; \sum_{k,l,m=0}^{\infty} \frac{1}{\sqrt{2\pi R}}\, A^{b\mu}_{(k)} A^{c\nu}_{(l)}\, \left(\partial_\mu A^a_{(m)\nu}\right) \left(\frac{1}{\sqrt{2}}\right)^{\delta_{k,0}+\delta_{l,0}+\delta_{m,0}+1} \delta_{k,l,m}\,, \tag{3.26}$$

where we have introduced the short hand notation

$$\delta_{k,l,m} = \delta_{k+l+m,0} + \delta_{k+l-m,0} + \delta_{k-l+m,0} + \delta_{k-l-m,0} \tag{3.27}$$

and rescaled the fields according to $A^{aM}_{(n)} \to \sqrt{2} A^{aM}_{(n)}$ for $n \geq 1$ as in the Abelian case.

The analysis for the second term in $\mathcal{L}_3$ is very similar. In terms of Kaluza-Klein modes it is given by

$$
\begin{aligned}
A^{b4} A^{c\mu} \left(\partial_4 A^a_\mu \,-\, \partial_\mu A^a_4 \right) \;=\; - \sum_{\substack{k,m=1\\ l=0}}^{\infty} \frac{1}{(2\pi R)^{3/2}} \, A^b_{(k)4} A^{c\mu}_{(l)} \left(-\frac{m}{R}\, A^a_{(m)\mu} \,-\, \partial_\mu A^a_{(m)4} \right) \\
\sin\!\left(\frac{k}{R} y\right) \cos\!\left(\frac{l}{R} y\right) \sin\!\left(\frac{m}{R} y\right) .
\end{aligned}
\tag{3.28}
$$

Using the identity

$$
\begin{aligned}
\sin(ky)\cos(ly)\sin(my) \;=\; \frac{1}{4}\Big(&\cos((k+l-m)y) - \cos((k+l+m)y) \\
&+ \cos((k-l-m)y) - \cos((k-l+m)y)\Big)
\end{aligned}
\tag{3.29}
$$

(again a consequence of eq.(2.20)) and integrating over the extra dimension finally yield

$$
\begin{aligned}
\int_0^{2\pi R} dy\, A^{b4} A^{c\mu} \left(\partial_4 A^a_\mu \,-\, \partial_\mu A^a_4 \right) \;=\; \sum_{\substack{k,m=1\\ l=0}}^{\infty} \frac{1}{\sqrt{2\pi R}} \, A^b_{(k)4} A^{c\mu}_{(l)} \left(\frac{m}{R}\, A^a_{(m)\mu} \,+\, \partial_\mu A^a_{(m)4} \right) \\
\left(\frac{1}{\sqrt{2}}\right)^{\delta_{l,0}+1} \tilde{\delta}_{k,l,m} \,,
\end{aligned}
\tag{3.30}
$$

where $\tilde{\delta}_{k,l,m}$ stands for

$$
\tilde{\delta}_{k,l,m} = -\delta_{k+l+m,0} + \delta_{k+l-m,0} - \delta_{k-l+m,0} + \delta_{k-l-m,0} \,. \tag{3.31}
$$

Note the difference in signs between this relation and eq.(3.27).

Collecting all the terms of eq.(3.26) and eq.(3.30) the Lagrangian of the self-interactions involving three gauge bosons reads

$$
\begin{aligned}
\mathcal{L}_3(x) \;=\; -\frac{g_5}{\sqrt{2\pi R}}\, f^{abc} \Big(&\sum_{\substack{k,m=1\\ l=0}}^{\infty} A^b_{(k)4} A^{c\mu}_{(l)} \left(\frac{m}{R}\, A^a_{(m)\mu} \,+\, \partial_\mu A^a_{(m)4} \right) \left(\frac{1}{\sqrt{2}}\right)^{\delta_{l,0}+1} \tilde{\delta}_{k,l,m} \\
&+ \sum_{k,l,m=0}^{\infty} A^{b\mu}_{(k)} A^{c\nu}_{(l)} \left(\partial_\mu A^a_{(m)\nu} \right) \left(\frac{1}{\sqrt{2}}\right)^{\delta_{k,0}+\delta_{l,0}+\delta_{m,0}+1} \delta_{k,l,m} \Big) .
\end{aligned}
\tag{3.32}
$$

Hidden in the second term, there is the standard model interaction

$$
\mathcal{L}_{3,SM}(x) \;=\; -\, g\, f^{abc}\, A^{b\mu}_{(0)} A^{c\nu}_{(0)} \left(\partial_\mu A^a_{(0)\nu} \right) \tag{3.33}
$$

if we identify the dimensionless quantity $g_5/\sqrt{2\pi R}$ with the standard model gauge coupling g of the considered gauge group. For example, if we consider QCD, eq.(3.33) represents the gluon self-interaction between three massless gluons. All other interactions described by the second term of eq.(3.32) involve at least two heavy Kaluza-Klein modes.

The first term in eq.(3.32) involves one or two scalar fields $A^a_{(k)4}$. Because there are no scalar Kaluza-Klein modes in the massless sector of the theory, the corresponding interactions do not play any role in the low energy limit of the theory. In unitary gauge this term vanishes identically.

3.4.2 Four Gauge Boson Interactions

The analysis of the interactions involving four gauge bosons is completely analogous with the discussion in the previous section. According to eq.(3.10), the Lagrangian of four gauge boson interactions is given by

$$\mathcal{L}_4(x,y) \;=\; -\frac{1}{4}\, g_5^2\, f^{abc}\, f^{ade} A^{bM} A^d_M A^{cN} A^e_N\,. \tag{3.34}$$

It can be decomposed into

$$\mathcal{L}_4 \;=\; -\frac{1}{4}\, g_5^2\, f^{abc}\, f^{ade} \left(A^{b\mu} A^d_\mu A^{c\nu} A^e_\nu - 2A^{b\mu} A^d_\mu A^c_4 A^e_4 + A^b_4 A^d_4 A^c_4 A^e_4\right)\,, \tag{3.35}$$

where the antisymmetry of the structure constants is used. To derive the corresponding effective interactions we will again consider one term at a time. For the term including only vector fields, the Fourier expansion yields

$$\begin{aligned} A^{b\mu} A^d_\mu A^{c\nu} A^e_\nu \;=\; \sum_{k,l,m,n=0}^{\infty} \frac{1}{(2\pi R)^2}\, A^{b\mu}_{(k)} A^d_{(l)\mu} A^{c\nu}_{(m)} A^e_{(n)\nu} \\ \cos\left(\frac{k}{R}y\right)\cos\left(\frac{l}{R}y\right)\cos\left(\frac{m}{R}y\right)\cos\left(\frac{n}{R}y\right)\,. \end{aligned} \tag{3.36}$$

Again, using eq.(2.20) to proof the quite lengthy but direct generalization of eq.(3.25) to four cosine terms we immediately get

$$\begin{aligned} \int_0^{2\pi R} dy\, A^{b\mu} A^d_\mu A^{b\nu} A^d_\nu \;=\; \sum_{k,l,m,n=0}^{\infty} \frac{1}{2\pi R}\, A^{b\mu}_{(k)} A^d_{(l)\mu} A^{c\nu}_{(m)} A^e_{(n)\nu} \\ \left(\frac{1}{\sqrt{2}}\right)^{\delta_{k,0}+\delta_{l,0}+\delta_{m,0}+\delta_{n,0}+2} \delta_{k,l,m,n}\,, \end{aligned} \tag{3.37}$$

where $\delta_{k,l,m,n}$ is defined analogously with eq.(3.27) and the usual rescaling of the fields is performed.

For the term

$$A^{b\mu}A^d_\mu A^c_4 A^e_4 \;=\; \sum_{\substack{k,l=0\\ m,n=1}}^{\infty} \frac{1}{(2\pi R)^2}\, A^{b\mu}_{(k)}A^d_{(l)\mu}A^c_{(m)4}A^e_{(n)4} \cos\left(\frac{k}{R}y\right)\cos\left(\frac{l}{R}y\right)\sin\left(\frac{m}{R}y\right)\sin\left(\frac{n}{R}y\right) \tag{3.38}$$

involving two vector bosons and two scalars the same reasoning yields

$$\int_0^{2\pi R} dy\, A^{b\mu}A^d_\mu A^b_4 A^d_4 \;=\; \sum_{\substack{k,l=0\\ m,n=1}}^{\infty} \frac{1}{2\pi R}\, A^{b\mu}_{(k)}A^d_{(l)\mu}A^c_{(m)4}A^e_{(n)4}\left(\frac{1}{\sqrt{2}}\right)^{\delta_{k,0}+\delta_{l,0}+2} \tilde{\delta}_{k,l,m,n}\,, \tag{3.39}$$

where

$$\begin{aligned}\tilde{\delta}_{k,l,m,n} =& -\delta_{k+l+m+n,0}+\delta_{k+l+m-n,0}+\delta_{k+l-m+n,0}-\delta_{k+l-m-n,0}\\ &-\delta_{k-l+m+n,0}+\delta_{k-l+m-n,0}+\delta_{k-l-m+n,0}-\delta_{k-l-m-n,0}\,.\end{aligned} \tag{3.40}$$

The third term involving only scalars is symmetric under the exchange of the gauge indices b and c and, thus, vanishes when the sum over the antisymmetric structure constants is performed.

Using eq.(3.37) and eq.(3.39) $\mathcal{L}_4(x)$ can be recasted into the form

$$\begin{aligned}\mathcal{L}_4(x) \;=\; -\frac{1}{4}\left(\frac{g_5}{\sqrt{2\pi R}}\right)^2 f^{abc}\,f^{ade} &\Bigg(\sum_{k,l,m,n=0}^{\infty} A^{b\mu}_{(k)}A^d_{(l)\mu}A^{c\nu}_{(m)}A^e_{(n)\nu}\left(\frac{1}{\sqrt{2}}\right)^{\delta_{k,0}+\delta_{l,0}+\delta_{m,0}+\delta_{n,0}+2}\delta_{k,l,m,n}\\ &-\sum_{\substack{k,l=0\\ m,n=1}}^{\infty} A^{b\mu}_{(k)}A^d_{(l)\mu}A^c_{(m)4}A^e_{(n)4}\left(\frac{1}{\sqrt{2}}\right)^{\delta_{k,0}+\delta_{l,0}}\tilde{\delta}_{k,l,m,n}\Bigg)\,.\end{aligned} \tag{3.41}$$

Again, the first term contains the standard model interaction involving four gauge bosons

$$\mathcal{L}_4(x) \;=\; -\frac{1}{4}\,g^2\,f^{abc}\,f^{ade}A^{b\mu}_{(0)}A^d_{(0)\mu}A^{c\nu}_{(0)}A^e_{(0)\nu}\,, \tag{3.42}$$

where, as before, the dimensionless quantity $g/\sqrt{2\pi R}$ is identified with the standard model gauge coupling g. All other interactions described by eq.(3.41) involve at least two heavy Kaluza-Klein modes.

3.4.3 Feynman Rules

Having derived an effective four dimensional Lagrangian for the gauge boson self-interactions given by eq.(3.32) and eq.(3.41) we can use four dimensional field theory to derive the corresponding Feynman rules. The analysis is quite analogous with the one of a conventional, four dimensional non-Abelian model [24] apart from some subtleties to be pointed out below. For notational convenience we will express all the couplings in terms of g not g_5.

Let us start with the three gauge boson interactions. In the standard model the corresponding Feynman rule describes the interaction of gauge bosons with specific gauge indices, Lorentz indices and momenta. In the five dimensional theory we, generally, also have to specify which Kaluza-Klein mode we consider.

First, let us step back and consider the standard model term. In eq.(3.33) we implicitly sum over the gauge and Lorentz indices. Therefore, we have to identify all the terms in the sum which contribute to a specific Feynman rule, where the Lorentz and gauge indices as well as the momenta are fixed. For the standard model 3-boson vertex in figure 3.2 the $3! = 6$ terms are given by

$$\begin{aligned}
\mathcal{L}_{3,SM}(x) \supset & -g\, f^{cab}\, g^{\nu\rho}\, A^a_{(0)\mu} A^b_{(0)\nu} \left(\partial^\mu A^c_{(0)\rho}\right) - g\, f^{bac}\, g^{\nu\rho}\, A^a_{(0)\mu} A^c_{(0)\rho} \left(\partial^\mu A^b_{(0)\nu}\right) \\
& -g\, f^{abc}\, g^{\mu\rho}\, A^b_{(0)\nu} A^c_{(0)\rho} \left(\partial^\nu A^a_{(0)\mu}\right) - g\, f^{cba}\, g^{\mu\rho}\, A^b_{(0)\nu} A^a_{(0)\mu} \left(\partial^\nu A^c_{(0)\rho}\right) \\
& -g\, f^{acb}\, g^{\mu\nu}\, A^c_{(0)\rho} A^b_{(0)\nu} \left(\partial^\rho A^a_{(0)\mu}\right) - g\, f^{bca}\, g^{\mu\nu}\, A^c_{(0)\rho} A^a_{(0)\mu} \left(\partial^\rho A^b_{(0)\nu}\right) ,
\end{aligned} \tag{3.43}$$

where the gauge and Lorentz indices are fixed and there is no summation over equal indices. Each of the terms can now be translated into a vertex factor according to the conventional rules. For example, we have

$$-g\, f^{cab}\, g^{\nu\rho}\, A^a_{(0)\mu} A^b_{(0)\nu} \left(\partial^\mu A^c_{(0)\rho}\right) \to -i\, g\, f^{cab}\, g^{\nu\rho}(-ik^\mu) \tag{3.44}$$

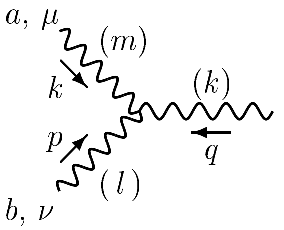
for the first term in eq.(3.43). The derivative contributes a factor $-ik^\mu$ if the particle is considered incoming, i.e the momentum points into the diagram. Adding the vertex factors for the six terms in eq.(3.43) directly results in the 3-boson vertex in figure 3.2 where m, k and l are set to zero for the standard model vertex.

Let us now consider general 3-vertices also including higher Kaluza-Klein vector modes. They correspond to the following terms in eq.(3.32)

$$\mathcal{L}_3(x) \supset \frac{-g_5}{\sqrt{2\pi R}}\, f^{abc} \sum_{k',l',m'=0}^{\infty} A^{b\mu}_{(k')} A^{c\nu}_{(l')} \left(\partial_\mu A^a_{(m')\nu}\right) \left(\frac{1}{\sqrt{2}}\right)^{\delta_{k',0}+\delta_{l',0}+\delta_{m',0}+1} \delta_{k',l',m'} , \tag{3.45}$$

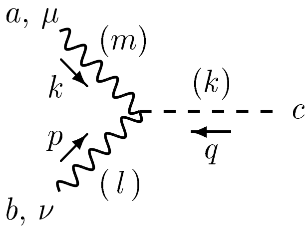
where we sum over the mode indices m', k' and l' for the moment in order not to confuse the summation indices with the fixed Kaluza-Klein modes m, k and l in figure 3.2. If all three Kaluza-Klein modes are different the sum over gauge indices at most contributes a single term to a given vertex if m', k' and l' are fixed in eq.(3.45). However, the explicit sum over the Kaluza-Klein modes again contributes exactly six different terms due to the

3-boson vertex:



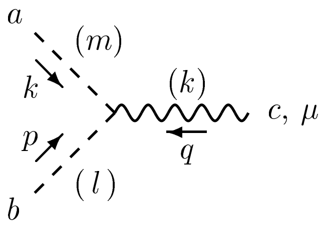
$$= g\left(\frac{1}{\sqrt{2}}\right)^{(\delta_{k,0}+\delta_{l,0}+\delta_{m,0}+1)} \delta_{k,l,m} \; f^{abc}\left[g^{\mu\nu}(k-p)^{\rho} + g^{\nu\rho}(p-q)^{\mu} + g^{\rho\mu}(q-k)^{\nu}\right]$$

vertex with 1 scalar:



$$= -\,i\,g\,f^{abc}\,g^{\mu\nu}\left[\left(\frac{m}{R}\right)\left(\frac{1}{\sqrt{2}}\right)^{(\delta_{l,0}+1)}\tilde{\delta}_{k,l,m} - \left(\frac{l}{R}\right)\left(\frac{1}{\sqrt{2}}\right)^{(\delta_{m,0}+1)}\tilde{\delta}_{k,m,l}\right]$$

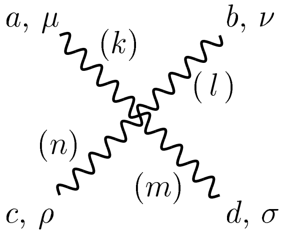
vertex with 2 scalars:



$$= g\left(\frac{1}{\sqrt{2}}\right)^{(\delta_{k,0}+1)} \tilde{\delta}_{l,k,m} \; f^{abc}(p-k)^{\mu}$$

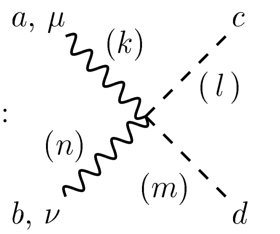
Figure 3.2: Feynman rules for the three gauge boson self-interactions. $\delta_{k,l,m}$ and $\tilde{\delta}_{l,k,m}$ are defined in eq.(3.27) and eq.(3.31), respectively.

4-boson vertex:



$$= -ig^2\delta_{k,l,m,n}\left(\frac{1}{\sqrt{2}}\right)^{(\delta_{k,0}+\delta_{l,0}+\delta_{m,0}+\delta_{n,0}+2)} \left[f^{abe}f^{cde}\left(g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}\right) \; f^{ace}f^{bde}\left(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\sigma}g^{\nu\rho}\right) \; f^{ade}f^{bce}\left(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma}\right)\right]$$

vertex with 2 scalars:



$$= i\,g^2\left(\frac{1}{\sqrt{2}}\right)^{(\delta_{k,0}+\delta_{n,0})}\tilde{\delta}_{k,n,l,m} \; g^{\mu\nu}\,2\left[f^{ace}f^{bde} + f^{ade}f^{bce}\right]$$

Figure 3.3: Feynman rules for the four gauge boson self-interactions. $\delta_{k,l,m,n}$ and $\tilde{\delta}_{k,n,l,m}$ are defined in eq.(3.27) and eq.(3.40), respectively.

possible choices $m' = m$, $k' = k$, $l' = l$ or $m' = m$, $k' = l$, $l' = k$ and the four other permutations. Considering the vertex factors the only difference to the standard model case is given by the factors

$$\left(\frac{1}{\sqrt{2}}\right)^{(\delta_{k',0}+\delta_{l',0}+\delta_{m',0}+1)} \delta_{k',l',m'} \tag{3.46}$$

in each term. However, they are equal as long as m, k and l are only permuted amongst m', k' and l' such that they result in an overall prefactor for the standard model vertex factor.

In any case either the sum over gauge indices or the explicit sum over Kaluza-Klein modes provides for exactly six terms which contribute to a given vertex. Therefore, no further complications arise and the general 3-boson vertex looks exactly as its standard model counterpart apart from the prefactor eq.(3.46) (see figure 3.2). Note, that this prefactor controls which Kaluza-Klein modes interact with each other at all. It ensure selection rules between different Kaluza-Klein modes and is zero for most combinations of m, k and l. These factors are typical for an interaction between bulk fields and could be associated with a special kind of "momentum conservation" with respect to the extra dimension. Note, that the special topology of the S^1/Z_2 manifold does not lead to momentum conservation in the usual sense. In interactions with fermions, as discussed in section 2.6, such terms are absent because the brane where the fermions are localized explicitly alters the space-time structure.

Analogous considerations lead to the other Feynman rules in figure 3.2 and figure 3.3. In the case of the 4-boson vertex there are even $4! = 24$ different terms in the Lagrangian for fixed gauge and Lorentz indices contributing to the vertex factor, however, sets of four are equal to one another. The Feynman rules for vertices including scalars are generally easier to derive because the number of contributing terms in the Lagrangian is largely reduced.

Besides the standard model vertex there are 3-vector boson vertices coupling two identical heavy Kaluza-Klein modes to the massless vector boson. Their coupling equals the standard model gauge coupling. Three heavy Kaluza-Klein modes interact whenever $\delta_{k,l,m} = 1$. The coupling strength of this vertex is suppressed by a factor of $\sqrt{2}$.

Similar consideration apply for the coupling of one scalar to two vector Kaluza-Klein modes. Note, however, that two identical vector modes do not couple to a scalar mode in a 3-vertex at all.

Two identical scalar Kaluza-Klein modes can couple to the massless mode with coupling strength equal to the standard model one. The possible coupling of three heavy modes is again suppressed by a factor of $\sqrt{2}$.

The four gauge boson vertex is also similar to its standard model counterpart. There is no interaction between three massless particles and one heavy mode. Whenever two massless modes are allowed to interact with two heavy modes they interact with gauge coupling strength. If only one massless mode is participating, the coupling is reduced by a factor of $\sqrt{2}$. If all modes are heavy the coupling strength varies between $1/2g$ and $3/2g$. For four different modes the coupling is weakest, for four identical modes the coupling is strongest. Similar relations hold for the interactions between two vector bosons and two scalars. Obviously, only the vector bosons can be massless.

Specifying these results to the important cases, namely QCD, i.e. SU(3), and electroweak theory, i.e. SU(2), only amounts to inserting the appropriate structure constants. In unitary gauge all the scalar components vanish and only two of the vertices remain. In this scheme, QCD has been only recently discussed in the literature. The results derived in this thesis agree with the results obtained in [15].

Chapter 4

Theories with Spontaneously Broken Symmetries in 5 Dimensions

The standard model contains a spontaneously broken gauge theory. The electroweak $SU(2)\times U(1)_Y$ gauge symmetry is broken to $U(1)_{em}$ resulting in massive W and Z bosons. Therefore, if we want to generalize the standard model to extra dimensions, we have to learn how to treat a spontaneously broken theory in this context. In order not to get lost in notational difficulties we will begin with the investigation of an Abelian Higgs model.

4.1 An Abelian Model

To clarify the notation, we will briefly introduce the four dimensional theory first. The easiest model showing spontaneous symmetry breakdown describes the coupling of a U(1) gauge field to a complex scalar in terms of the locally U(1) invariant Lagrangian [17, 18]

$$\mathcal{L}(x) = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + |D_\mu \Phi|^2 - V(\Phi), \tag{4.1}$$

where $F^{\mu\nu}$ is the well-known field strength tensor of a U(1) gauge theory, D_μ is the covariant derivative given by

$$D_\mu = \partial_\mu + i\, e\, A_\mu \tag{4.2}$$

and $V(\Phi)$ denotes the Higgs potential

$$V(\Phi) = \mu^2 |\Phi|^2 + \lambda |\Phi|^4 . \tag{4.3}$$

The U(1) coupling constant is denoted by e and λ is the Higgs self-coupling. If the mass parameter $\mu^2 < 0$ the Higgs potential has the well-known Mexican hat shape and Φ acquires a non-vanishing vacuum expectation value (VEV) in order to minimize the vacuum energy. The minimization condition $\partial_\Phi V(\Phi)|_{\Phi=v} = 0$ yields

$$|v| = \left(\frac{|\mu^2|}{\lambda} \right)^{1/2} \tag{4.4}$$

for the VEV denoted by v, i.e. the VEV of Φ can be only determined up to a phase. As one could have expected, all possible VEVs are related by a U(1) transformation and by virtue of the U(1) symmetry of the theory one is free to choose v real. This leads to the conventional parameterization of the field Φ given by

$$\Phi(x) = \frac{1}{\sqrt{2}} \left(v + h(x) + i\,\chi(x) \right). \tag{4.5}$$

The non-zero VEV implies that the vacuum is not invariant under gauge transformations. Therefore, the usual degeneracies in gauge multiplets are removed such that the particle spectrum does not reflect the underlying symmetry. The symmetry is said to be spontaneously broken or hidden. It is the most prominent effect of symmetry breaking or of the VEV that the gauge bosons become massive. In the Abelian model the gauge boson mass m_A turns out to be $m_A^2 = e^2 v^2$. The field h, known as Higgs field, is also massive and the VEV corresponds to a squared Higgs mass $m_h^2 = 2\lambda v^2$ while the field χ known as would-be Goldstone mode of the theory (corresponding to the broken U(1) generator) turns out to be unphysical. It can be gauged away completely (unitary gauge) or acquires a gauge dependent mass (R_ξ-gauge) and does not correspond to particle excitations which can be measured in experiment. It is said to be absorbed into the gauge boson to make it massive.

4.1.1 Higgs Field in the Bulk

It is straightforward to investigate this Abelian Higgs model in 5 dimensions. Following the analysis in chapter 2 we begin with the five dimensional Lagrangian density

$$\mathcal{L}(x,y) = -\frac{1}{4} F^{MN}(x,y)\, F_{MN}(x,y) + |D_M\, \Phi(x,y)\,|^2 - V(\Phi(x,y))\,, \tag{4.6}$$

where $F^{MN}(x,y)$ is the field strength tensor eq.(2.3), $D_M = \partial_M + i\,e\,A_M$ denotes the covariant derivative in five dimensions, $\Phi(x,y)$ is a complex, scalar field living in five dimensions and

$$V(\Phi) = \mu_5^2\, |\Phi|^2 + \lambda_5\, |\Phi|^4 \tag{4.7}$$

denotes the five dimensional Higgs potential. The physics of the first term in eq.(4.6) has already been discussed in detail in chapter 2, however, the Higgs sector contains new physics. It is possible to analyze the effects of a Mexican hat like potential, i.e. the appearance of a non-zero vacuum expectation value, directly in the five dimensional theory. Nevertheless, the compactified theory has a difficult structure with respect to the compact extra dimension (e.g. a discrete momentum spectrum, internal symmetries). Therefore, we will evaluate the corresponding effective four dimensional theory, investigate spontaneous symmetry breaking due to the 4 dimensional vacuum and try to understand it in the light of the five dimensional theory.

We parameterize the complex scalar field in terms of real fields as

$$\Phi(x,y) \;=\; \frac{1}{\sqrt{2}}\,(\,h(x,y)\,+\,i\,\chi(x,y)\,)\,. \tag{4.8}$$

Note, that the fields h and χ are not necessarily free of non-vanishing vacuum expectation values at this point. After compactification on the orbifold S_1/Z_2 we can expand the fields, as in chapter 2, into the Fourier series

$$\begin{aligned} h(x,y) \;&=\; \sum_{n=0}^{\infty} \frac{1}{\sqrt{2\pi R}}\, h_{(n)}(x)\, \cos\left(\frac{n}{R}y\right), \\ \chi(x,y) \;&=\; \sum_{n=0}^{\infty} \frac{1}{\sqrt{2\pi R}}\, \chi_{(n)}(x)\, \cos\left(\frac{n}{R}y\right). \end{aligned} \tag{4.9}$$

The fields h and χ are taken to be even with respect to the extra dimension. This guarantees that the physics of the 4D Abelian model is reproduced at low energies as we will see below. Higgs fields being odd with respect to the Z_2-symmetry would imply difficulties. For example, as we have seen in section 2.5 odd fields do not interact with fermions on a brane. Thus, there would not be any Yukawa sector being necessary to generate the fermion masses in the standard model.

The complex scalar field transforms under a local U(1) gauge transformation as

$$\Phi(x,y) \;=\; \frac{1}{\sqrt{2}}\,(\,h(x,y)\,+\,i\,\chi(x,y)\,) \quad\to\quad \exp\,(\,-\,i\,e_5\,\Theta(x,y)\,)\,\frac{1}{\sqrt{2}}\,(\,h(x,y)\,+\,i\,\chi(x,y)\,) \tag{4.10}$$

or in terms of an infinitesimal transformation as

$$\begin{aligned} h(x,y) \quad&\to\quad h(x,y)\,+\,e_5\,\Theta(x,y)\,\chi(x,y)\,, \\ \chi(x,y) \quad&\to\quad \chi(x,y)\,-\,e_5\,\Theta(x,y)\,h(x,y)\,. \end{aligned} \tag{4.11}$$

In order to have a well defined parity in the context of gauge transformations, $\Theta(x,y)$ has to be even as we have chosen already in chapter 2. Expanding $\Theta(x,y)$ the gauge transformation properties in terms of the Kaluza-Klein modes are given by

$$\begin{aligned} h_{(n)} &\rightarrow h_{(n)} + \frac{e_5}{\sqrt{2\pi R}} \sum_{m=0}^{\infty} \Theta_{(m)} \left[(1+\delta_{m,n})\chi_{(|m-n|)} + (1-\delta_{n,0})\chi_{(m+n)} \right], \\ \chi_{(n)} &\rightarrow \chi_{(n)} - \frac{e_5}{\sqrt{2\pi R}} \sum_{m=0}^{\infty} \Theta_{(m)} \left[(1+\delta_{m,n})h_{(|m-n|)} + (1-\delta_{n,0})h_{(m+n)} \right], \end{aligned} \tag{4.12}$$

where their derivation exactly follows the line of arguments presented in Appendix B.

After this digression to gauge transformations, we will derive the effective four dimensional theory. The kinetic part of the effective Lagrangian density for the fields h and χ can be calculated even easier than for the gauge sector because no extra field components like A_4 arise. Using identities like

$$(\partial_M h)^2 = \left(\sum_{n=0}^{\infty} \frac{1}{\sqrt{2\pi R}} (\partial_\mu h_{(n)}) \cos\left(\frac{n}{R} y\right) \right)^2 - \left(\sum_{n=1}^{\infty} \frac{n}{R} \frac{1}{\sqrt{2\pi R}} h_{(n)} \sin\left(\frac{n}{R} y\right) \right)^2, \tag{4.13}$$

integrating over the extra dimension and rescaling the Kaluza-Klein modes ($h_{(n)} \rightarrow \sqrt{2} h_{(n)}$ for $n \geq 1$) leads to a free, four dimensional Lagrangian density

$$\mathcal{L}_{\text{kin}}(x) = \frac{1}{2} \sum_{n=0}^{\infty} (\partial_\mu h_{(n)})^2 - \frac{1}{2} \sum_{n=0}^{\infty} \left(\frac{n}{R}\right)^2 h_{(n)}^2 - \frac{1}{2} \sum_{n=0}^{\infty} \mu^2 h_{(n)}^2 + (h \leftrightarrow \chi), \tag{4.14}$$

where the terms proportional to $(n/R)^2$ are due to the Kaluza-Klein structure of the theory while the terms proportional to $\mu^2 = \mu_5^2$ result from the Higgs potential eq.(4.7). Investigating spontaneous symmetry breaking it is interesting to have a closer look at the mass terms of the Higgs fields. Again, the interesting case arises if $\mu^2 < 0$. This can be made explicit by writing $-|\mu^2|$. The mass parameters in the quadratic terms of the Kaluza-Klein modes $h_{(n)}$ and $\chi_{(n)}$ in eq.(4.14) are given by

$$m_{(0)}^2 = -|\mu^2| \qquad \text{and} \qquad m_{(n)}^2 = \left(\frac{n}{R}\right)^2 - |\mu^2| \quad . \tag{4.15}$$

Thus, at least the zero mode fields do not allow for a meaningful perturbation theory such that we have to identify non-vanishing VEVs of the fields $h_{(n)}$ and $\chi_{(n)}$. To evaluate those we have to analyze the Higgs self-interactions arising from the Higgs potential eq.(4.7). In terms of the fields h and χ it is given by

$$V(\Phi) = -\frac{|\mu_5^2|}{2} (h^2 + \chi^2) + \frac{\lambda_5}{4} (h^2 + \chi^2)^2. \tag{4.16}$$

Repeating the same analysis as for the evaluation of the gauge boson self-interactions in section 3.4, a short calculation yields

$$\mathcal{L}_{\Phi^4} = -\frac{\lambda}{4} \sum_{k,l,m,n=0}^{\infty} \left(h_{(k)} h_{(l)} h_{(m)} h_{(n)} + (h \leftrightarrow \chi) + 2 h_{(k)} h_{(l)} \chi_{(m)} \chi_{(n)} \right) c(k,l,m,n), \tag{4.17}$$

where $\lambda = \lambda_5/2\pi R$, $c(k,l,m,n) = \left(1/\sqrt{2}\right)^{\delta_{k,0}+\delta_{l,0}+\delta_{m,0}+\delta_{n,0}+2}\delta_{k,l,m,n}$ and $\delta_{k,l,m,n}$ is given in complete analogy with eq.(3.27). By virtue of the U(1) symmetry we can choose real VEVs and set all the fields $\chi_{(n)}$ to zero. In four dimensions, we thus have to minimize the effective potential

$$V_{\text{eff}} = \frac{1}{2}\sum_{n=0}^{\infty}\left(\left(\frac{n}{R}\right)^2 - |\mu^2|\right)h_{(n)}^2 + \frac{\lambda}{4}\sum_{k,l,m,n=0}^{\infty} h_{(k)}h_{(l)}h_{(m)}h_{(n)}\,c(k,l,m,n)\,, \tag{4.18}$$

where the mass terms due to the Kaluza-Klein structure of the theory are included.

The simplest case arises if $(n/R)^2 > \mu^2$. This assumption seems viable also from the experimental point of view, since the standard model VEV roughly equals 250 GeV while the compactification scale is expected to be in the multi-TeV range. In this case all contributions of higher Kaluza-Klein modes can only increase the overall energy and they will not acquire a non-zero VEV. Therefore, we can set $h_{(n)} = 0$ for $n \geq 1$ in the effective potential eq.(4.18) and the evaluation of the VEV of the field $h_{(0)}$ minimizing the vacuum energy reduces to the problem in the four dimensional Abelian Higgs model. The minimization condition is given by

$$\partial_{h_{(0)}} V_{\text{eff}}\Big|_{h_{(0)}=v} = -|\mu^2|\,h_{(0)} + \lambda\,h_{(0)}^3\Big|_{h_{(0)}=v} = 0\,, \tag{4.19}$$

leading again to the vacuum expectation value $v = \sqrt{|\mu|^2/\lambda}$. From the five dimensional point of view this result can be interpreted as a vacuum expectation value of the field $h(x,y)$ given by $v_5 = \sqrt{|\mu|^2/2\pi R\lambda} = \sqrt{|\mu_5|^2/\lambda_5}$. Therefore, we derived what we could have expected by directly extending the well-known four dimensional result to five dimensions.

The situation becomes more complicated if $|\mu^2| > (1/R)^2$ because higher Kaluza-Klein modes do not seem to be suitable for a sensible perturbation theory, either. For simplicity, let us investigate the case $(1/R)^2 < |\mu^2| < (2/R)^2$. By virtue of the local U(1) symmetry we again consider real VEVs minimizing the potential. However, in this setting also the field $h_{(1)}$ may acquire a non-zero VEV. Therefore, the potential to be minimized (setting $h_{(n)} = 0$ for $n \geq 2$) is given by

$$V_{\text{eff}} = -\frac{|\mu^2|}{2}h_{(0)}^2 + \frac{\lambda}{4}h_{(0)}^4 + \left((1/R)^2 - |\mu^2|\right)h_{(1)}^2 + \frac{3}{2}\frac{\lambda}{4}h_{(1)}^4 + \frac{\lambda}{2}h_{(0)}^2h_{(1)}^2\,, \tag{4.20}$$

where the extra factor of 3/2 is due to the increased coupling of the heavy Kaluza-Klein modes calculated from eq.(4.17). In spite of the different starting point, it turns out that the field configuration of the only minimum is again given by $h_{(0)} = \sqrt{|\mu|^2/\lambda}$ and $h_{(1)} = 0$. No other minimum can be determined, neither by setting $h_{(0)} = 0$ nor by solving the resulting system of linear equations in $h_{(0)}^2$ and $h_{(1)}^2$ for $h_{(0)} \neq 0$ and $h_{(1)} \neq 0$. Therefore, the former result for the VEV of $h_{(0)}$ is recovered. From the five dimensional point of view it means that local variations of the fields cannot lower the vacuum energy. The five dimensional field just acquires a vacuum expectation value as known from four dimensions.

To summarize, as a general result, we only have to redefine the zero mode of the field h because of the spontaneous breakdown of the U(1) symmetry, i.e.

$$h_{(0)} \to v + h_{(0)} \quad \text{or} \quad \Phi_{(0)} = \frac{1}{\sqrt{2}}\big(h_{(0)} + i\chi_{(0)} \big) \to \Phi_{(0)} = \frac{1}{\sqrt{2}}\big(v + h_{(0)} + i\chi_{(0)} \big)\,, \tag{4.21}$$

where again $v = \sqrt{|\mu^2|/\lambda}$. All higher Kaluza-Klein modes already have vanishing VEVs. After the redefinition this is also true for the zero mode $h_{(0)}$. Therefore, in five dimensions the field Φ can be parameterized by

$$\Phi(x,y) = (1/\sqrt{2})(v_5 + h(x,y) + \chi(x,y))\,, \tag{4.22}$$

where $v_5 = v/2\pi R$.

In terms of the zero mode redefined in eq.(4.21) the kinetic Lagrangian of the effective four dimensional theory (up to an unimportant constant) is given by

$$\begin{aligned} \mathcal{L}_{\text{free}} &= \frac{1}{2}\sum_{n=0}^{\infty}\Big[\big(\partial_\mu h_{(n)}\big)^2 - \big((n/R)^2 + 2\lambda v^2 \big)\, h_{(n)}^2 \Big] \\ &+ \frac{1}{2}\sum_{n=0}^{\infty}\Big[\big(\partial_\mu \chi_{(n)}\big)^2 - (n/R)^2\, \chi_{(n)}^2 \Big]\,, \end{aligned} \tag{4.23}$$

where $|\mu^2|$ is replaced by λv^2. The fields $h_{(n)}$ are again called Higgs fields. All the modes acquires positive mass contributions due to the mass terms arising from the vacuum expectation value in the self-interactions eq.(4.17). Although the field χ may be naively thought to be the would-be Goldstone boson of the theory, corresponding to the spontaneous symmetry breakdown, the explicit mass term due to the Kaluza-Klein structure of the theory may be a sign that we have to be careful in interpreting this Lagrangian.

Besides contributions to the mass terms, the VEV also introduces new self-interaction terms. However, they are not important for the following analysis. Therefore, their discussion is postponed to Appendix C.

So far, we have not investigated the couplings to the U(1) gauge fields, i.e.

$$\mathcal{L}_{\text{GC}} = e_5 A^M (\partial_M \chi)(v_5 + h) - e_5 A^M (\partial_M h)\chi + \frac{e_5^2}{2} A^M A_M (v_5 + h)^2 + \frac{e_5^2}{2} A^M A_M \chi^2\,, \tag{4.24}$$

because they are also unimportant for the evaluation of the VEVs. Integration over the extra dimension results in a large number of coupling terms listed in Appendix C but most important for the interpretation of the theory are the bilinear terms in A, h and χ due to the vacuum expectation value.

First of all, all the Kaluza-Klein modes of the gauge bosons acquire additional mass due to the VEV in eq.(4.24). The corresponding terms in the effective four dimensional Lagrangian density are given by

$$\mathcal{L}^{\text{mass}} = \frac{1}{2} e^2 v^2 \sum_{n=0}^{\infty} A^{\mu}_{(n)} A_{(n)\mu} - \frac{1}{2} e^2 v^2 \sum_{n=0}^{\infty} A_{(n)4} A_{(n)4} \,, \tag{4.25}$$

where again $e = e_5/\sqrt{2\pi R}$. Therefore, the zero mode obviously acquires the mass $m^2_{(0)} = e^2 v^2$ as known from the four dimensional theory and the mass of the gauge boson Kaluza-Klein mode (n) is given by $m^2_{(n)} = (n/R)^2 + e^2 v^2$. Moreover, there are mixing terms between the gauge degrees of freedom and the Kaluza-Klein modes of the field χ given by

$$\mathcal{L}_{\text{mixing}} = e\, v \sum_{n=0}^{\infty} A^{\mu}_{(n)} \left(\partial_\mu \chi_{(n)} \right) + e\, v \sum_{n=1}^{\infty} \left(\frac{n}{R} \right) A_{(n)4}\, \chi_{(n)} \,. \tag{4.26}$$

To identify the physical and unphysical degrees of freedom in the theory we write down all the terms bilinear in the gauge and χ fields for a given Kaluza-Klein mode ($n \geq 1$) in a transparent way

$$\begin{aligned} \mathcal{L}_{(n)\text{kin}} =& -\frac{1}{4} F^{\mu\nu}_{(n)} F_{(n)\mu\nu} + \frac{1}{2} \left((n/R)^2 + (ev)^2 \right) A_{(n)\mu} A^{\mu}_{(n)} \\ &+ \frac{1}{2} \left(\partial_\mu A_{(n)4} \right)^2 + \frac{1}{2} \left(\partial_\mu \chi_{(n)} \right)^2 - \frac{1}{2} \left((n/R)\, \chi_{(n)} - (ev)\, A_{(n)4} \right)^2 \\ &+ A^{\mu}_{(n)} \left((n/R)\, \partial_\mu A_{(n)4} + (ev)\, \partial_\mu \chi_{(n)} \right) . \end{aligned} \tag{4.27}$$

Eq.(4.27) shows that neither $\chi_{(n)}$ nor $A_{(n)4}$ but a linear combination of both plays the role of the unphysical would-be Goldstone boson because the gauge fields do not only acquire mass due to spontaneous symmetry breaking but also because of the Kaluza-Klein structure of the theory. As a change of basis we define

$$\begin{aligned} G_{(n)} &= \left((n/R)^2 + (ev)^2 \right)^{-1/2} \left[\left(\frac{n}{R} \right) A_{(n)4} + (ev)\, \chi_{(n)} \right] , \\ a_{(n)} &= \left((n/R)^2 + (ev)^2 \right)^{-1/2} \left[(ev)\, A_{(n)4} - \left(\frac{n}{R} \right) \chi_{(n)} \right] . \end{aligned} \tag{4.28}$$

Within this basis, only the field $G_{(n)}$ is mixed with the vector boson fields. Moreover, this rotation diagonalizes the mass matrix of the fields $G_{(n)}$ and $a_{(n)}$ and eq.(4.27) simplifies to

$$\begin{aligned} \mathcal{L}_{(n)\text{kin}} =& -\frac{1}{4} F^{\mu\nu}_{(n)} F_{(n)\mu\nu} + \frac{1}{2} \left(\left((n/R)^2 + (ev)^2 \right)^{1/2} A_{(n)\mu} + \partial_\mu G_{(n)} \right)^2 \\ &+ \frac{1}{2} \left(\partial_\mu a_{(n)} \right)^2 - \frac{1}{2} \left((n/R)^2 + (ev)^2 \right) a^2_{(n)} \,. \end{aligned} \tag{4.29}$$

Hence, at each Kaluza-Klein level the theory can formally be identified as an Abelian Higgs model with an additional physical scalar field $a_{(n)}$ with mass $m^2_{a(n)} = (n/R)^2 + (ev)^2$. The field $G_{(n)}$ clearly plays the role of the unphysical Goldstone mode, as expected there is no mass term for $G_{(n)}$. This derivation is valid for $n \geq 1$. For $n = 0$ the theory is identical to the simple four dimensional Abelian Higgs model and no further complications occur. We will refer to all the fields $G_{(n)}$ as would-be Goldstone modes in the following, although for $n \geq 1$ the Kaluza-Klein structure of the theory is even more important for their appearance than spontaneous symmetry breaking as can be seen in the limit $v = 0$.

In spite of the non-invariant vacuum, the Lagrangian of the theory is, nevertheless, gauge invariant. To be specific, according to eq.(4.10), a gauge transformation of the scalar fields is given by

$$\frac{1}{\sqrt{2}}\,(\,v_5 + h(x,y) + i\,\chi(x,y)\,) \quad \rightarrow \quad \exp\,(\,-\,i\,e_5\,\Theta(x,y)\,)\,\frac{1}{\sqrt{2}}\,(\,v_5 + h(x,y) + i\,\chi(x,y)\,)\,. \tag{4.30}$$

Therefore, as in chapter 2, one has the freedom to choose a convenient gauge. A particularly simple gauge condition is the unitary gauge. In the context of spontaneous symmetry breaking this means $G_{(n)} = 0$ for all Kaluza-Klein modes (n). Eq.(4.30) and eq.(2.14) show that there is always a gauge transformation leading to $G_{(n)} = 0$. In this gauge, it is most obvious that the Goldstone modes do not correspond to physical particle excitations. As in the four dimensional Abelian Higgs model each Goldstone mode is absorbed by the corresponding vector boson mode to make it massive. Here, in addition, we have linear combinations $a_{(n)}$ of the gauge degrees of freedom $A_{4(n)}$ and the scalar fields $\chi_{(n)}$ carrying mass while their orthogonal linear combinations $G_{(n)}$ are gauged away. In this respect, our 5D Abelian Higgs model closely resembles a model with two Higgs fields.

Nevertheless, for investigations of gauge invariance and for calculations beyond tree level it is necessary to work in a more general gauge. As in section 2.4 one can choose a gauge fixing function. If we replace the field A_4 in section 2.4 by the Goldstone field G and the mass n/R by $\sqrt{(n/R)^2 + (ev)^2}$ the analysis is indeed the same for each mode (n). Before integrating out the extra dimension, the appropriate gauge fixing function in five dimensions, corresponding to generalized Lorentz gauge, is given by

$$F(A,\chi) \,=\, \partial_\mu A^\mu \,-\, \xi\,(\,\partial_4\,A_4 \,+\, ev\,\chi\,)\,. \tag{4.31}$$

It finally leads to a gauge fixed Lagrangian density for mode (n) given by

$$\begin{aligned}\mathcal{L}_{(n)\mathrm{kin}} = &-\frac{1}{4}\,F^{\mu\nu}_{(n)}\,F_{(n)\mu\nu} \,+\, \frac{1}{2}\,m^2_{A(n)}\,\big(A_{(n)\mu}\big)^2 - \frac{1}{2\xi}\Big(\partial_\mu\,A^\mu_{(n)}\Big)^2 + \frac{1}{2}\big(\,\partial_\mu G_{(n)}\big)^2 - \frac{\xi}{2}\,m^2_{A(n)}\,G^2_{(n)}\\ &+\,\frac{1}{2}\big(\,\partial_\mu a_{(n)}\,\big)^2 \,-\, \frac{1}{2}\,m^2_{A(n)}\,a^2_{(n)} \,+\, \frac{1}{2}\big(\,\partial_\mu h_{(n)}\,\big)^2 \,-\, \frac{1}{2}\,m^2_{h(n)}\,h^2_{(n)}\,,\end{aligned} \tag{4.32}$$

where we have introduced $m^2_{A(n)} = (n/R)^2 + (ev)^2$ and $m^2_{h(n)} = (n/R)^2 + 2\lambda v^2$.

To summarize, the theory contains the tower of vector bosons already known from the analysis in chapter 2, two towers of physical scalar fields and the unphysical Goldstone tower which acquires a gauge dependent mass in R_ξ-gauge. The limit $\xi \to \infty$ again corresponds to unitary gauge, because the Goldstone modes become infinitely heavy and decouple from the theory. The masses of the heavy Kaluza-Klein vector bosons are slightly shifted due to the spontaneous symmetry breakdown. The zero mode acquires mass and there is no massless gauge boson as known form four dimensions.

So far, we have not explicitly stated the interactions of the five dimensional Abelian Higgs model. One has to be careful to formulate the interactions in terms of the physically relevant basis of fields. Otherwise, their calculation from eq.(4.6) (using the techniques of section 3.4) is easy. A complete listing of the effective four dimensional Lagrangian of the 5D-Abelian Higgs model is given in Appendix C.

4.1.2 Higgs Field on a Brane

In section 2.5 we discussed a theory in which some of the degrees of freedom, the fermions, do not propagate in the bulk but live on a brane located at $y = 0$. Putting the gauge fields on a brane results in major difficulties concerning the gauge structure of the theory but nothing prevents the Higgs and Goldstone modes from being restricted to a four dimensional subspace of the five dimensional theory. The investigation of such a theory is the content of this section.

The Lagrangian density of this theory is given by

$$\mathcal{L}(x,y) = -\frac{1}{4}F^{MN}(x,y)\,F_{MN}(x,y) + \delta(y)\,|D_\mu\,\Phi(x)\,|^2 - \delta(y)\,V(\Phi(x))\,. \tag{4.33}$$

The covariant derivative and the Higgs potential are given by eq.(4.2) and eq.(4.3) where the gauge field propagates in the bulk. The δ-function again formalizes the idea of restricting the Higgs field to a brane and all the comments in section 2.5 also apply here. Under gauge transformations the brane field Φ transforms like a fermion, i.e.

$$\Phi(x) \quad \to \quad \exp\left(-i\,e_5\,\Theta(x,0)\right)\,\Phi(x)\,, \tag{4.34}$$

making the theory be invariant under local U(1) transformations. As we have seen in section 2.5, the scalar field A^4 does not interact with fields living on a brane at $y = 0$ because of its antisymmetry.

The effective four dimensional Higgs potential is simply equivalent to the four dimensional Abelian Higgs model because the integration over the extra dimension is trivial due to the

δ-function. Therefore, the analysis at the beginning of section 4.1 with respect to the VEV of the field Φ applies without changes and Φ can be parameterized as in eq.(4.5). Note, that neither h nor χ posses Kaluza-Klein excitations because they are brane fields.

After the expansion of the gauge fields and integration over the extra dimension the coupling to the U(1) gauge field results in the following interaction terms

$$\begin{aligned} \mathcal{L}_{\mathrm{GC}} &= \sqrt{2}\, e\, h\, (\partial_\mu \chi) \sum_{n=0}^{\infty} A^\mu_{(n)} \left(\frac{1}{\sqrt{2}}\right)^{\delta_{n,0}} + (h \leftrightarrow \chi) \\ &+ e^2 \left(h^2 + \chi^2 \right) \sum_{m,n=0}^{\infty} A^\mu_{(m)}\, A_{(n)\mu} \left(\frac{1}{\sqrt{2}}\right)^{\delta_{m,0}+\delta_{n,0}}, \end{aligned} \tag{4.35}$$

where the factors of $\sqrt{2}$ are due to the rescaling of the gauge fields as in section 2.5. Because Φ acquires a VEV v (not v_5 because Φ lives on a four dimensional brane) there are additional kinetic terms to be obtained by replacing h by v in eq.(4.35).

First of all, there are mixing terms of the form

$$\mathcal{L}_{\mathrm{mixing}} = e\, v\, (\partial_\mu \chi) A^\mu_{(0)} + \sqrt{2}\, e\, v\, (\partial_\mu \chi) \sum_{n=1}^{\infty} A^\mu_{(n)} \tag{4.36}$$

which have to be removed by a convenient gauge. As in the four dimensional theory, χ is a would-be Goldstone boson of the theory and can be gauged away by virtue of eq.(4.34). Moreover, all the $A_{(n)4}$ can be gauged away simultaneously even if the gauge condition $\chi = 0$ is imposed: Just gauge away all the $A_{(n)4}$ as discussed at the beginning of section 2.4 and use the freedom to choose $\Theta_{(0)}$ to gauge away χ thereafter. This corresponds to unitary gauge. Therefore, neither the mixing terms in eq.(4.36) nor the mixing terms in eq.(2.23) will influence the physical particle spectrum in any gauge. We will consider a more general R_ξ-gauge as in section 4.1.1 later on in this section but, first, let us investigate the physical vector bosons. In the course of this analysis we will encounter some of the techniques also being used in the gauge fixing procedure.

As in the Abelian Higgs model of section 4.1.1, eq.(4.35) implies additional contributions to the masses of all the vector boson modes. Whereas the corresponding mass matrix is diagonal with respect to different Kaluza-Klein modes if Φ is a bulk field, this is no longer true if Φ is localized on a brane. The additional terms in the four dimensional Lagrangian are given by

$$\mathcal{L}^{\mathrm{mass}} = e^2\, v^2 \sum_{m,n=0}^{\infty} A^\mu_{(m)}\, A_{(n)\mu} \left(\frac{1}{\sqrt{2}}\right)^{\delta_{m,0}+\delta_{n,0}}. \tag{4.37}$$

As we will see in the following analysis the appearance of the mixing terms eq.(4.37) is not intrinsic to the theory but a consequence of the Fourier expansion of the five dimensional gauge field, i.e. a consequence of our choice of basis. In Appendix A we show how to choose a convenient basis, i.e. an orthonormal system of functions different from the Fourier modes eq.(2.10), before integrating out the extra dimension in order to avoid the mixing terms right from the beginning.

The terms in eq.(4.37) together with the masses due to compactification result in a mass matrix given by

$$M^2_{\mathrm{VB}} = \begin{pmatrix} m^2 & \sqrt{2}\,m^2 & \sqrt{2}\,m^2 & \sqrt{2}\,m^2 & \cdots \\ \sqrt{2}\,m^2 & 2\,m^2 + (1/R)^2 & 2\,m^2 & 2\,m^2 & \cdots \\ \sqrt{2}\,m^2 & 2\,m^2 & 2\,m^2 + (2/R)^2 & 2\,m^2 & \cdots \\ \sqrt{2}\,m^2 & 2\,m^2 & 2\,m^2 & 2\,m^2 + (3/R)^2 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} , \tag{4.38}$$

where $m^2 = e^2 v^2$ denotes the mass generated by the Higgs mechanism. Therefore, the Kaluza-Klein modes defined in the Fourier expansion eq.(2.10) are no longer mass eigenstates. Also the zero mode mixes with all its Kaluza-Klein modes. For simplicity, it is meant to be included if we refer to Kaluza-Klein modes in the following. Because the gauge interactions are simple in terms of the Fourier modes, we will refer to them as gauge eigenstates. If we want to formulate the theory in terms of the physical mass eigenstates we have to diagonalize the mass matrix. This is complicated by the fact that it is infinite dimensional because there is an infinite tower of Kaluza-Klein modes all mixed by the same off diagonal elements. Nevertheless, the simple structure of the matrix allows for its diagonalization. Actually, the eigenvalues can be derived as follows. The characteristic polynomial of the mass matrix is given by

$$\begin{aligned} \det\left(M^2_{\mathrm{VB}} - \lambda\,\mathrm{I}\right) &= \begin{vmatrix} m^2 - \lambda & \sqrt{2}\,m^2 & \sqrt{2}\,m^2 & \sqrt{2}\,m^2 & \cdots \\ \sqrt{2}\,\lambda & (1/R)^2 - \lambda & 0 & 0 & \cdots \\ \sqrt{2}\,\lambda & 0 & (2/R)^2 - \lambda & 0 & \cdots \\ \sqrt{2}\,\lambda & 0 & 0 & (3/R)^2 - \lambda & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{vmatrix} \\ &= \left(\prod_{n=1}^{\infty}\left(n^2/R^2 - \lambda\right)\right)\left(m^2 - \lambda - 2\,\lambda\,m^2 \sum_{n=1}^{\infty} \frac{1}{(n/R)^2 - \lambda}\right) \end{aligned} \tag{4.39}$$

and can be understood as the limit of a characteristic polynomial of a corresponding $n \times n$-matrix where $n \to \infty$. In eq.(4.39) we used the standard rules for dealing with determinants. In the first step we added the first row multiplied by $\sqrt{2}$ to all other rows before we explicitly evaluated the determinant. It is easy to show that $\lambda = (n/R)^2$ is not a solution for $\det(M^2_{\mathrm{VB}} - \lambda\,\mathrm{I}) = 0$: If one inserts $\lambda = (n/R)^2$ on the right hand side of the first equation in eq.(4.39) the determinant equals a single product of non-zero numbers. Therefore, we have to solve

$$m^2 - \lambda - 2\,\lambda\, m^2 \sum_{n=1}^{\infty} \frac{1}{(n/R)^2 - \lambda} = 0\,. \tag{4.40}$$

Again we can use the theorem eq.(2.60) to perform the infinite sum and arrive at a transcendental equation for the infinite tower of eigenvalues given by

$$\sqrt{\lambda} = \pi\, m^2\, R \cot\left(\pi\sqrt{\lambda}\, R\right). \tag{4.41}$$

If we identify λ with the physical mass squared $\hat{m}^2_{(n)}$ of the corresponding Kaluza-Klein mass eigenstates (where the index $n = 0$ refers to the lightest mode and so on) the equation for these physical masses is given by

$$\hat{m}_{(n)} = \pi\, m^2\, R \cot\left(\pi\, \hat{m}_{(n)}\, R\right). \tag{4.42}$$

Having, at least in principle, determined the eigenvalues we can go on and find the corresponding eigenstates expressed in terms of eigenvectors $\mathbf{E}^{\lambda}$ in the gauge eigenstate basis. Using the ansatz $\mathbf{E}^{\lambda} = (1, \tilde{\mathbf{E}}^{\lambda})$ in the eigenvalue equation $M^2_{\mathrm{VB}}\,\mathbf{E}^{\lambda} = \lambda \mathbf{E}^{\lambda}$ leads to a system of linear equations for the components of $\tilde{\mathbf{E}}^{\lambda}$:

$$\begin{pmatrix} 2m^2 + (1/R)^2 - \lambda & 2\,m^2 & 2\,m^2 & \cdots \\ 2m^2 & 2\,m^2 + (2/R)^2 - \lambda & 2\,m^2 & \cdots \\ 2m^2 & 2\,m^2 & 2\,m^2 + (3/R)^2 - \lambda & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \tilde{\mathbf{E}}^{\lambda} = - \begin{pmatrix} \sqrt{2}m^2 \\ \sqrt{2}m^2 \\ \sqrt{2}m^2 \\ \vdots \end{pmatrix} \tag{4.43}$$

The vector $\tilde{\mathbf{E}}^{\lambda}$ solving these equations can be calculated by Cramer's rule, i.e.

$$\tilde{\mathrm{E}}^{\lambda}_j = \frac{D_j}{D}\,, \tag{4.44}$$

where $\tilde{\mathrm{E}}^{\lambda}_j$ denote the components of $\tilde{\mathbf{E}}^{\lambda}$ and

$$D = \begin{vmatrix} 2\,m^2 + (1/R)^2 - \lambda & 2\,m^2 & 2\,m^2 & \cdots \\ 2\,m^2 & 2\,m^2 + (2/R)^2 - \lambda & 2\,m^2 & \cdots \\ 2\,m^2 & 2\,m^2 & 2\,m^2 + (3/R)^2 - \lambda & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{vmatrix}\,. \tag{4.45}$$

D_j is the same determinant but with column j replaced by the right hand side of eq.(4.43).

These determinants can be evaluated quite straightforwardly. We find

$$\begin{aligned} D &= \left(\prod_{n=0}^{\infty} \left(n^2/R^2 - \lambda \right) \right) \left(1 \, + \, 2\, m^2 \sum_{n=1}^{\infty} \frac{1}{(n/R)^2 \, - \, \lambda} \right), \\ D_j &= \left(\prod_{n=0}^{\infty} \left(n^2/R^2 - \lambda \right) \right) \; \frac{-\sqrt{2}\, m^2}{(j/R)^2 \, - \, \lambda}. \end{aligned} \tag{4.46}$$

Using eq.(4.40) we can finally calculate

$$\tilde{\mathrm{E}}_j^{\lambda} \; = \; \frac{\sqrt{2}\,\lambda}{\lambda \, - \, (j/R)^2} \qquad \text{or} \qquad \tilde{\mathrm{E}}_j^{(n)} \; = \; \frac{\sqrt{2}\, \hat{m}_{(n)}^2}{\hat{m}_{(n)}^2 \, - \, (j/R)^2}\,, \tag{4.47}$$

where we have again substituted $\lambda = \hat{m}_{(n)}^2$. Because we are interested in canonically normalized mass eigenstates, we have to normalize the eigenvectors we just found. Therefore, we have to calculate

$$\left(\mathbf{E}^{(n)} \right)^2 \; = \; 1 \, + \, \left(\tilde{\mathbf{E}}^{(n)} \right)^2 \; = \; 1 \, + \, \sum_{j=1}^{\infty} \left(\frac{\sqrt{2}\, \hat{m}_{(n)}^2}{(j/R)^2 \, - \, \hat{m}_{(n)}^2} \right)^2 . \tag{4.48}$$

Using the theorem eq.(2.60), the identity $\sin^{-2}\alpha \, = \, (1 + \cot^2\alpha)$ and eq.(4.41) a short calculation yields

$$\left(\mathbf{E}^{(n)} \right)^2 \; = \; \frac{1}{2} \, \frac{\hat{m}_{(n)}^2}{m^2} \left(1 \, + \, \pi^2\, m^2\, R^2 + \frac{\hat{m}_{(n)}^2}{m^2} \right). \tag{4.49}$$

As a final result, the mass eigenstates $\hat{A}^{\mu}_{(n)}$ of the Kaluza-Klein modes in terms of the gauge eigenstates are given by

$$\hat{A}^{\mu}_{(n)} \; = \; \left(1 \, + \, \pi^2\, m^2\, R^2 + \frac{\hat{m}_{(n)}^2}{m^2} \right)^{-\frac{1}{2}} \sum_{j=0}^{\infty} \frac{-\,2\,\hat{m}_{(n)}\, m}{\hat{m}_{(n)}^2 \, - \, (j/R)^2} \left(\frac{1}{\sqrt{2}} \right)^{\delta_{j,0}} A^{\mu}_{(j)}\,, \tag{4.50}$$

where we have used the normalized eigenvector $-\mathbf{E}^{(\mathbf{n})}$ instead of $\mathbf{E}^{(\mathbf{n})}$. This will result in positive gauge couplings of the mass eigenstates as we will see below.

Let us now derive the phenomenological effects of these formal results. The mass of the lightest Kaluza-Klein mass eigenstate $\hat{A}^{\mu}_{(0)}$ is reduced in comparison to a model where the Higgs field lives in the bulk as can be seen from the transcendental equation (4.41): $\pi m^2 R \cot(\pi m R) < m$ for all values of $m < (1/R)$ such that there has to be a solution $\hat{m}_{(0)} < m$. If the symmetry breaking scale m^2 is small compared to the compactification scale $M = (1/R)$ we can expand the cotangent in eq.(4.41) and explicitly solve the transcendental equation, at least approximately to second order in m^2/M^2, for the mass of the lightest Kaluza-Klein mode

$$\hat{m}_{(0)} \; \approx \; \left(1 + \frac{\pi^2}{3} \frac{m^2}{M^2} \right)^{-\frac{1}{2}} \; m\,. \tag{4.51}$$

All the masses of the higher Kaluza-Klein modes are slightly increased with respect to their masses due to compactification. If we appropriately expand the transcendental equation to second order in m^2/M^2 they are given by

$$\hat{m}_{(n)} \approx \left(1+\frac{m^2}{(nM)^2}\right)\frac{n}{R}. \tag{4.52}$$

However, not only the masses of the particles described by the theory have to be calculated carefully, the change of basis also affects the couplings of the mass eigenstates. If an interaction

$$\mathcal{L}_{\rm int} = (e\,A^{\mu}_{(0)} + \sqrt{2}\,e\sum_{n=1}^{\infty} A^{\mu}_{(n)})\,J_{\mu} \tag{4.53}$$

with some charged vector current J_μ on a brane (e.g. the fermionic current in eq.(2.41)) is expressed in terms of the mass eigenstates we get (after a resummation)

$$\mathcal{L}_{\rm int} = J_{\mu}\sum_{n=0}^{\infty}\hat{e}_{(n)}\,\hat{A}^{\mu}_{(n)}\,. \tag{4.54}$$

$\hat{A}^{\mu}_{(n)}$ again denote mass eigenstates and the physical couplings $\hat{e}_{(n)}$ of the mass eigenstates expressed in terms of the original gauge coupling e are given by

$$\hat{e}_{(n)} = e\,\mathrm{E}^{(n)}_0 + e\,\sqrt{2}\sum_{j=1}^{\infty}\mathrm{E}^{(n)}_j\,, \tag{4.55}$$

where $\mathrm{E}^{(n)}_j$ are the corresponding coefficients of the fields $A^{\mu}_{(j)}$ in eq.(4.50). A short calculation similar to the one leading to eq.(4.49) finally yields

$$\hat{e}_{(n)} = \sqrt{2}\,e\left(1+\frac{m^2}{\hat{m}^2_{(n)}}+\pi^2\frac{m^2}{M^2}\frac{m^2}{\hat{m}^2_{(n)}}\right)^{-\frac{1}{2}}\,. \tag{4.56}$$

Using eq.(4.51) and the considerations thereafter the physical coupling of the lightest Kaluza-Klein gauge boson which corresponds to the gauge boson of the underlying four dimensional Abelian Higgs model is approximately given by

$$\hat{e}_{(0)} \approx e\left(1+\frac{2}{3}\pi^2\frac{m^2}{M^2}\right)^{-\frac{1}{2}} \approx e\left(1-\frac{1}{3}\pi^2\frac{m^2}{M^2}\right) \tag{4.57}$$

to order $(m/M)^2$. In the same approximation the couplings of the higher Kaluza-Klein mass eigenstates are given by

$$\hat{e}_{(n)} \approx \sqrt{2}\,e\left(1+\frac{m^2}{M^2}\frac{1}{n^2}\right)^{-\frac{1}{2}} \approx \sqrt{2}\,\hat{e}_{(0)}\left(1+\left(\frac{\pi^2}{3}-\frac{1}{2n^2}\right)\frac{m^2}{M^2}\right). \tag{4.58}$$

Here, we encountered for the first time how a Higgs field restricted to a brane can shift the masses and couplings of the Kaluza-Klein mass eigenstates. In chapter 6 similar considerations will become extremely important when we derive bounds on the compactification scale in the context of a 5D standard model.

After deriving the physical particle spectrum of the gauge boson sector, let us now consider a more general gauge fixing procedure in order to deal with the mixing terms in eq.(4.36). As gauge fixing function for the generalized R_ξ-gauge we choose

$$F(A,\chi) \,=\, \partial_\mu A^\mu \,-\, \xi \,(\, \partial_4 A_4 \,+\, e_5 v\, \chi\, \delta(y)\,)\,. \tag{4.59}$$

Note the additional δ-function in comparison to eq.(4.31) because the Higgs field is localized on a brane. The corresponding gauge fixing terms in the effective four dimensional Lagrangian density after integration over the extra dimension are given by

$$\begin{aligned}\mathcal{L}_{\mathrm{GF}} \,=\, \mathcal{L}_{\mathrm{GF}}^{\mathrm{QED}} \,+\, ev\,\chi \left(\partial_\mu A^\mu_{(0)}\right) \,+\, \sqrt{2}\,ev\,\chi \sum_{n=1}^{\infty}\left(\partial_\mu A^\mu_{(n)}\right)\\ +\, \xi\,\sqrt{2}\,ev\,\chi \sum_{n=1}^{\infty}\left(\frac{n}{R}\right) A_{(n)4} \,-\, \frac{\xi}{2}\, e_5^2 v^2 \chi^2 \delta(0)\,,\end{aligned} \tag{4.60}$$

where $\mathcal{L}_{\mathrm{GF}}^{\mathrm{QED}}$ is given by eq.(2.33) and $\delta(0)$ results from the integration over the δ-function squared. Thus, R_ξ-gauge fixing removes all mixing terms of the gauge boson modes up to total derivatives such that our investigation of the physical mass spectrum is unchanged as we would have expected by the gauge invariance of the theory. Nevertheless, new mixing terms between χ and A_4 are introduced which have to be dealt with in the following.

Moreover, the role of $\delta(0)$ is far from clear at this point. Therefore, it is instructive to use a special representation of the δ-function given by

$$\delta(y) \,=\, \frac{1}{2\pi R} \,+\, \sum_{n=1}^{\infty} \frac{1}{\pi R} \cos\left(\frac{ny}{R}\right) \tag{4.61}$$

which is valid with respect to the orbifold S^1/Z_2. To check this relation it is sufficient to remember that all test functions on S^1/Z_2 can be Fourier expanded into sine and cosine terms. Using eq.(4.61) we immediately obtain

$$\delta(0) \,=\, \frac{1}{2\pi R} \,+\, \sum_{n=1}^{\infty} \frac{1}{\pi R}\,. \tag{4.62}$$

Instead of using the five dimensional gauge fixing function eq.(4.59), we could fix the gauge mode by mode after integration over the extra dimension. The natural gauge fixing function for each mode would be

$$F_{(n)}(A_{(n)},\chi) \,=\, \partial_\mu A^\mu_{(n)} \,-\, \xi \left(\left(\frac{n}{R}\right) A_{(n)4} \,+\, \sqrt{2}^{\,(1-\delta_{n,0})}\, ev\,\chi \right). \tag{4.63}$$

It is easy to check that all the gauge fixing terms in the Lagrangian due to these gauge fixing functions add up to the terms in eq.(4.60) if we use $e_5 = \sqrt{2\pi R}\, e$ and eq.(4.62). Therefore, the interpretation of $\delta(0)$ according to eq.(4.62) is consistent.

Now we are ready to investigate the mass terms of the fields $A_{(n)4}$ and χ in the Lagrangian density according to R_ξ-gauge

$$\mathcal{L}_\xi^{\text{mass}}(x) = -\frac{\xi}{2} \left(\chi,\, A_{(1)4},\, A_{(2)4},\, \ldots\right) M_\xi^2 \begin{pmatrix} \chi \\ A_{(1)4} \\ A_{(2)4} \\ \vdots \end{pmatrix}, \tag{4.64}$$

where

$$M_\xi^2 = \begin{pmatrix} e^2 v^2 \left(1 + \sum_{n=1}^{\infty} 2\right) & \sqrt{2}\,(1/R)\, ev & \sqrt{2}\,(2/R)\, ev & \cdots \\ \sqrt{2}\,(1/R)\, ev & (1/R)^2 & 0 & \cdots \\ \sqrt{2}\,(2/R)\, ev & 0 & (2/R)^2 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}. \tag{4.65}$$

The infinite sum in the upper left corner of M_ξ^2 in turn corresponds to $\delta(0)$ in eq.(4.60) according to eq.(4.62).

The diagonalization of this mass matrix in order to find the mass eigenstates is very similar to the calculation following eq.(4.38). However, it is important to note, that none of the masses is physical because the whole matrix is scaled with the gauge dependent parameter ξ. The characteristic polynomial of M_ξ^2 is given by

$$\begin{aligned} \det\left(M_\xi^2 - \lambda\,\mathrm{I}\right) &= \begin{vmatrix} m^2 - \lambda & \sqrt{2}\,(1/R)\, m & \sqrt{2}\,(2/R)\, m & \sqrt{2}\,(3/R)\, m & \cdots \\ \sqrt{2}\, R\, m\, \lambda & (1/R)^2 - \lambda & 0 & 0 & \cdots \\ \sqrt{2}\,(R/2)\, m\, \lambda & 0 & (2/R)^2 - \lambda & 0 & \cdots \\ \sqrt{2}\,(R/3)\, m\, \lambda & 0 & 0 & (3/R)^2 - \lambda & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{vmatrix} \\ &= \left(\prod_{n=1}^{\infty} \left(n^2/R^2 - \lambda\right)\right) \left(m^2 - \lambda - 2\,\lambda\, m^2 \sum_{n=1}^{\infty} \frac{1}{(n/R)^2 - \lambda}\right), \end{aligned} \tag{4.66}$$

where $m = ev$ again denotes the symmetry breaking mass. In the first step we subtracted the n^{th} column multiplied with $\sqrt{2}mR/n$ from the first column. Comparing eq.(4.66) to eq.(4.39) we immediately obtain that, although the mass matrices eq.(4.38) and eq.(4.65) are different, their eigenvalues are the same. To each physical vector boson of mass $\hat{m}_{(n)}$ corresponds an unphysical would-be Goldstone like scalar with gauge dependent mass $\sqrt{\xi}\,\hat{m}_{(n)}$, where $\hat{m}_{(n)}$ is given as a solution of eq.(4.42). This could have been expected from the fact that the gauge dependent longitudinal part of the vector boson propagator can only be compensated by a scalar of exactly the same mass.

Using again $\mathbf{E}^\lambda = (1, \tilde{\mathbf{E}}^\lambda)^{\mathrm{T}}$ as an ansatz in the corresponding eigenvalue equation leads to an especially simple system of linear equations

$$\begin{pmatrix} (1/R)^2 - \lambda & 0 & 0 & \cdots \\ 0 & (2/R)^2 - \lambda & 0 & \cdots \\ 0 & 0 & (3/R)^2 - \lambda & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \tilde{\mathbf{E}}^\lambda = - \begin{pmatrix} \sqrt{2}\,(1/R)\,m \\ \sqrt{2}\,(2/R)\,m \\ \sqrt{2}\,(3/R)\,m \\ \vdots \end{pmatrix} . \tag{4.67}$$

Therefore, the eigenvector components

$$\tilde{\mathrm{E}}_j^{(n)} = \frac{\sqrt{2}\,(j/R)\,m}{\hat{m}_{(n)}^2 - (j/R)^2} \tag{4.68}$$

can be read off immediately. Normalizing the eigenvectors according to the discussion following eq.(4.47) finally yields mass eigenstates corresponding to the would-be Goldstone modes of a spontaneously broken theory

$$G_{(n)} = \left(1 + \pi^2 m^2 R^2 + \frac{\hat{m}_{(n)}^2}{m^2} \right)^{-\frac{1}{2}} \left(\sqrt{2}\,\chi + \sum_{j=1}^{\infty} \frac{2\,(j/R)\,m}{\hat{m}_{(n)}^2 - (j/R)^2} A_{(j)4} \right) , \tag{4.69}$$

where we have used

$$(\mathbf{E}^{(n)})^2 = \frac{1}{2} \left(1 + \pi^2 m^2 R^2 + \frac{\hat{m}_{(n)}^2}{m^2} \right) . \tag{4.70}$$

Comparing the final result eq.(4.69) to the previously evaluated mass eigenstates of the vector bosons (see eq.(4.50)) we recognize that the eigenvectors of the different mass matrices are different while the eigenvalues coincide up to the gauge dependent factor ξ, as we derived before. The physical couplings of the Goldstone modes to brane fields are directly proportional to the coefficient of the field χ in eq.(4.69), because the scalar gauge boson modes do not couple at all to fields on the brane.

To summarize the investigation of the kinetic Lagrangian in an Abelian Higgs model in which the Higgs field is restricted to a four dimensional brane, there is a would-be Goldstone mode with a gauge dependent mass $\sqrt{\xi}\hat{m}_{(n)}$ for each vector boson with mass $\hat{m}_{(n)}$ as expected from spontaneously broken theories in four dimensions. The mass of the zero mode is slightly decreased in comparison to the bulk model while all the higher Kaluza-Klein bosons have a slightly increased mass. How the gauge invariance of the theory expresses itself at the Feynman graph level has to be a topic of further investigations. We will come back to this point in section 5.3 after discussing fermions in spontaneously broken theories in section 5.1.

The interactions of the Higgs sector with the gauge bosons are given in eq.(4.35) in terms of the gauge eigenstates. In terms of mass eigenstates the same interactions can be stated in a formally equivalent way by redefining the coupling constants according to eq.(4.56). For example, the interaction between physical Higgs bosons and two vector bosons is given by

$$\mathcal{L}_{hhAA} = \frac{1}{2}\big(h^2 + 2\,v\,h\big) \sum_{m,n=1}^{\infty} \hat{e}_{(n)}\,\hat{e}_{(m)}\,\hat{A}^{\mu}_{(m)}\,\hat{A}_{(n)\mu}\,. \tag{4.71}$$

On the other hand, expressing the field χ in terms of the Goldstone mass eigenstates introduces an additional sum for each χ contained in an interaction. The model, including all the interaction terms, is summarized in Appendix C.

4.1.3 The General Case

Having considered two different models with Higgs fields living in the bulk and on a brane, respectively, we are ready to introduce a more general model combining the ideas of the preceding sections. The model contains two complex scalar fields. The field Φ_1 is supposed to be a bulk field while Φ_2 is localized on a brane at $y = 0$, as usual. The Lagrangian

$$\mathcal{L}(x,y) = -\frac{1}{4}F^{MN}\,F_{MN}(x,y) + |D_M\,\Phi_1(x,y)\,|^2 + \delta(y)\,|D_\mu\,\Phi_2(x)\,|^2 - V(\Phi_1,\Phi_2)(x,y) \tag{4.72}$$

to be discussed here combines the models we have investigated in sections 4.1.1 and 4.1.2. The covariant derivative is the same as in eq.(4.6) or eq.(4.33) but the Higgs potential $V(\Phi_1,\Phi_2)$ has a much richer structure. The most general Higgs potential allowed by gauge invariance (for a brane field gauge invariance can be understood as gauge invariance on the brane, see section 2.5) is given by [29]

$$\begin{aligned} V(\Phi_1,\Phi_2)(x,y) = {}& \mu_1^2\,(\Phi_1^\dagger\Phi_1) + \lambda_1\,(\Phi_1^\dagger\Phi_1)^2 + \delta(y)\,\Big[\mu_2^2\,(\Phi_2^\dagger\Phi_2) + \lambda_2\,(\Phi_2^\dagger\Phi_2)^2 \\ & + m_{12}^2\,(\Phi_1^\dagger\Phi_2) + m_{12}^{*2}\,(\Phi_2^\dagger\Phi_1) + \lambda_3\,(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4\,(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \\ & + \lambda_5\,(\Phi_1^\dagger\Phi_2)^2 + \lambda_5^*\,(\Phi_2^\dagger\Phi_1)^2 + \lambda_6\,(\Phi_1^\dagger\Phi_1)(\Phi_1^\dagger\Phi_2) + \lambda_6^*\,(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_1) \\ & + \lambda_7\,(\Phi_2^\dagger\Phi_2)(\Phi_1^\dagger\Phi_2) + \lambda_7^*\,(\Phi_2^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)\Big]\,, \end{aligned} \tag{4.73}$$

where all the terms involving the brane field Φ_2 are multiplied by a δ-function. In an Abelian model the dagger simply means complex conjugation but this notation is also valid in a non-Abelian generalization. We will assume that both complex scalar fields acquire a vacuum expectation value. The VEV of the brane field is naturally restricted to the brane on which the field lives and the VEV of the bulk field implies that the field configuration of the vacuum is constant with respect to space-time including the extra dimension. This assumption was found to be valid in the special model considered in section 4.1.1 and allows us to easily implement the VEV before integrating over the extra dimension by parameterizing the scalar fields as

$$\Phi_1(x,y) = \frac{1}{\sqrt{2}}\left(v_{51} + h_1(x,y) + i\,\chi_1(x,y) \right), \quad \Phi_2(x) = \frac{1}{\sqrt{2}}\, e^{i\phi}\left(v_{52} + h_2(x) + i\,\chi_2(x) \right). \tag{4.74}$$

The VEV of one field can be chosen to be real by virtue of gauge invariance while the other VEV will in general be complex. Therefore, we include a global phase ϕ in the definition of Φ_2. Before we go on, let us introduce a commonly used notation in such 2-Higgs models. Instead of v_{51} and v_{52} we will parameterize the VEVs via $v_{51} = v_5 \cos\beta$ and $v_{52} = v_5 \sin\beta$, i.e. $\tan\beta = v_{52}/v_{51}$.

While the Higgs sector of this general model is much more complicated than in the previously considered models the physics of the vector bosons can be extracted quite straightforwardly. After integrating over the extra dimension, the mass matrix M^2_{VB} of the gauge eigenstates $A^\mu_{(n)}(x)$ receives contributions from three different sources: The third term in eq.(4.72) generates the symmetry breaking masses already encountered in eq.(4.37). Together with the mass terms due to the kinetic terms of the gauge bosons they are given (as in the brane model) by eq.(4.38) and will be referred to as M^2_{brane}. Instead of $m^2 = e^2v^2$ we only have to define $m^2 = e^2v^2\sin^2\beta$. As in the bulk model of section 4.1.1, the second term in the Lagrangian introduces additional mass terms given in eq.(4.25). In a matrix notation they can be rewritten as $M^2_{\mathrm{bulk}} = e^2v^2\cos^2\beta\ \mathbf{1}$ and the complete mass matrix is given by

$$M^2_{\mathrm{VB}} = \begin{pmatrix} e^2v^2 & \sqrt{2}\,e^2v^2\,s_\beta^2 & \sqrt{2}\,e^2v^2\,s_\beta^2 & \cdots \\ \sqrt{2}\,e^2v^2\,s_\beta^2 & e^2v^2\left(1+s_\beta^2\right) + (1/R)^2 & 2\,e^2v^2\,s_\beta^2 & \cdots \\ \sqrt{2}\,e^2v^2\,s_\beta^2 & 2\,e^2v^2\,s_\beta^2 & e^2v^2\left(1+s_\beta^2\right) + (2/R)^2 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}, \tag{4.75}$$

where s_β^2 denotes $\sin^2\beta$. Because M^2_{bulk} is proportional to the unit matrix we can repeat the analysis of section 4.1.2 for M^2_{brane} and immediately find that the mass eigenstates of the general model are given by eq.(4.50) where $\hat{m}_{(n)}$ is a solution of eq.(4.42) if $e^2v^2\sin^2\beta$ is inserted for m^2. The mass eigenvalues are also found as in section 4.1.2, but they receive the additional contribution $\Delta\hat{m}^2_{(n)} = e^2v^2\cos^2\beta$ from M^2_{bulk}. For example, the approximate mass of the lightest mode is given by (compare to eq.(4.51))

$$\hat{m}^2_{(0)} = (ev)^2\sin^2\beta\left(1 + \frac{\pi^2}{3}\sin^2\beta\,\frac{(ev)^2}{M^2}\right) + (ev)^2\cos^2\beta\,. \tag{4.76}$$

The coupling to a charged current on a brane can also be derived in complete analogy with section 4.1.2. It is given by

$$\hat{e}_{(n)} \;=\; \sqrt{2}\, e \left(1 \,+\, \frac{(ev)^2 \sin^2\beta}{\hat{m}^2_{(n)} - (ev)^2\cos^2\beta} \,+\, \frac{\pi^2}{M^2}\frac{(ev)^4 \sin^2\beta}{\hat{m}^2_{(n)} - (ev)^2\cos^2\beta} \right)^{-\frac{1}{2}}, \tag{4.77}$$

where, in comparison to eq.(4.56), we had again to replace m^2 by $e^2v^2\sin^2\beta$ and the mass eigenvalue of mode (n) by the mass acquired due to the VEV of the brane Higgs only, because the VEV of the bulk Higgs does not lead to any mixing of different vector boson modes. For the approximate coupling of the zero mode we can thus easily derive

$$\hat{e}_{(0)} \;\approx\; e \left(1 \,-\, \frac{1}{3}\,\pi^2\, \sin^2\beta\, \frac{m^2}{M^2} \right) \tag{4.78}$$

in analogy with eq.(4.57).

However, not only the mass terms due to the two Higgs fields but also the mixing terms with the gauge bosons add up. Reading off the appropriate terms from eq.(4.26) and eq.(4.36) yields

$$\begin{aligned} \mathcal{L}_{\text{mixing}} \;=\; & e\,v\,\cos\beta \sum_{n=0}^{\infty} A^{\mu}_{(n)} \left(\partial_\mu \chi_{1(n)} \right) \,+\, e\,v\,\cos\beta \sum_{n=1}^{\infty} \left(\frac{n}{R} \right) A_{(n)4}\, \chi_{1(n)} \\ & +\, e\,v\,\sin\beta\, (\,\partial_\mu \chi_2\,) A^{\mu}_{(0)} \,+\, \sqrt{2}\, e\,v\,\sin\beta\, (\,\partial_\mu \chi_2\,) \sum_{n=1}^{\infty} A^{\mu}_{(n)}\,. \end{aligned} \tag{4.79}$$

While it was obvious in section 4.1.1 and section 4.1.2 how a unitary gauge (where all the mixing terms disappear) can be chosen, this is not clear at all in the general model. We expect that linear combinations of all the $\chi_{1(n)}$, $A_{(n)4}$ and χ_2 will play the role of the would-be Goldstone modes of the theory. This can be verified by investigating an R_ξ-gauge. As before, we choose a gauge fixing function in order to remove all the mixing terms with the vector boson modes

$$F(A_M, \chi_1, \chi_2) \;=\; \partial_\mu A^\mu \;-\; \xi\, (\,\partial_4\, A_4 \,+\, e_5 v_5 \cos\beta\, \chi_1 \,+\, e_5 v_5 \sin\beta\, \chi_2\, \delta(y)\,)\,. \tag{4.80}$$

One can explicitly check that after integrating over the extra dimension all the mixing terms in eq.(4.79) are cancelled or combine to total derivatives. The corresponding gauge fixing terms in the Lagrangian for mode (n) are given by

$$\mathcal{L}_{(n)\text{GF}} \;=\; -\frac{1}{2\xi} \left(\partial_\mu A^\mu_{(n)} \,-\, \xi \left(\sqrt{(\,n/R\,)^2 + (\,ev\cos\beta\,)^2}\; G_{(n)} \,+\, ev\sin\beta\, \chi_2\, \sqrt{2}^{\,1-\delta_{n,0}} \right) \right)^2, \tag{4.81}$$

where the field $G_{(n)}$ is defined in analogy with eq.(4.28) as

$$G_{(n)} \;=\; \left((\,n/R\,)^2 + (\,ev\cos\beta\,)^2 \right)^{-1/2} \left(\left(\frac{n}{R} \right) A_{(n)4} \,+\, (\,ev\cos\beta\,)\, \chi_{(n)} \right) \tag{4.82}$$

and the factor of $\sqrt{2}$ stems from the δ-function (see eq.(4.61)).

According to eq.(4.81) the ξ-dependent mass terms of the scalar modes in the $G_{(n)}\chi_2$-basis are given by

$$\mathcal{L}_\xi^{\text{mass}}(x) = -\frac{\xi}{2}\left(\chi_2,\, G_{(0)},\, G_{(1)},\, \ldots\right) M_\xi^2 \begin{pmatrix} \chi_2 \\ G_{(0)} \\ G_{(1)} \\ \vdots \end{pmatrix}, \tag{4.83}$$

where

$$M_\xi^2 = \begin{pmatrix} e^2v^2\left(1+\sum_{n=1}^{\infty} 2\right)\sin^2\beta & e^2v^2\sin\beta\cos\beta & \sqrt{2}\,ev\,c_1\,\sin\beta & \cdots \\ e^2v^2\sin\beta\cos\beta & e^2v^2\cos^2\beta & 0 & \cdots \\ \sqrt{2}\,ev\,c_1\,\sin\beta & 0 & (1/R)^2 + e^2v^2\cos^2\beta & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \tag{4.84}$$

and $c_n = \sqrt{(n/R)^2 + (ev\cos\beta)^2}$. The infinite sum in the upper left corner of M_ξ^2 again corresponds to $\delta(0)$ according to eq.(4.62). We expect only the would-be Goldstone modes of the theory to acquire gauge dependent masses due to gauge fixing. Therefore, all mass eigenstates of this mass matrix with non-zero eigenvalues will be would-be Goldstone modes. We will evaluate the eigenvalues and the corresponding eigenstates and check later on, that this is indeed the case. In order to determine the eigenvalues the characteristic polynomial can be rewritten as

$$\det(M_\xi^2 - \lambda\mathbf{1}) = \lambda \begin{vmatrix} e^2v^2\cos^2\beta - \lambda & \sqrt{2}e^2v^2\tan\beta & 0 & \cdots \\ e^2v^2\sin\beta\cos\beta & -1 & \sqrt{2}ev\,c\,\sin\beta & \cdots \\ 0 & \sqrt{2}ev\frac{1}{c}\sin\beta & (1/R)^2 + e^2v^2\cos^2\beta - \lambda & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{vmatrix}, \tag{4.85}$$

where we added each column, multiplied by the appropriate factor to cancel the terms in the infinite sum, to the first column of the determinant. Furthermore, we interchanged the first and second row and the first and second column. Comparing eq.(4.85) to the characteristic polynomial of the vector boson mass matrix eq.(4.75)

$$\det(M_{\text{VB}}^2 - \lambda\mathbf{1}) = \begin{vmatrix} e^2v^2 - \lambda & \sqrt{2}e^2v^2\sin^2\beta & \sqrt{2}e^2v^2\sin^2\beta & \cdots \\ -e^2v^2\cos^2\beta + \sqrt{2}\lambda & (1/R)^2 + e^2v^2\cos^2\beta - \lambda & 0 & \cdots \\ -e^2v^2\cos^2\beta + \sqrt{2}\lambda & 0 & (2/R)^2 + e^2v^2\cos^2\beta - \lambda & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{vmatrix}, \tag{4.86}$$

one can show

$$\det(M_\xi^2 - \lambda \mathbf{1}) \;=\; \lambda \, \det(M_{\rm VB}^2 - \lambda \mathbf{1})\,. \tag{4.87}$$

Thus, there is a would-be Goldstone mass eigenstate with mass $\sqrt{\xi}\hat{m}_{(n)}$ for each vector boson mass eigenstate with mass $\hat{m}_{(n)}$ (see eq.(4.83) for the definition of M_ξ^2). This is exactly what is needed in order to obtain gauge invariant S-matrix elements (see section 5.3). Furthermore, the additional degree of freedom without correspondence in the vector boson sector does not acquire a gauge dependent mass, exactly as one would have expected.

To determine the eigenvectors we can repeat the analysis following eq.(4.67) in section 4.1.2. As a final result, the analogous calculation yields

$$\hat{G}_{(n)} \;=\; E_{\chi_2}^{(n)}\,\chi_2 \;+\; \sum_{j=0}^{\infty}\Big[\, E_{\chi_{1(j)}}^{(n)}\,\chi_{1(j)} \;+\; E_{A_{(j)4}}^{(n)}\,A_{(j)4}\Big]\,, \tag{4.88}$$

where

$$\begin{aligned} E_{\chi_2}^{(n)} = \frac{1}{N}\,, \qquad E_{A_{(j)4}}^{(n)} &= -\frac{1}{N}\frac{\sqrt{2}ev\sin\beta(j/R)}{(j/R)^2 + e^2v^2\cos^2\beta - \hat{m}_{(n)}^2}\,,\\ E_{\chi_{1(0)}}^{(n)} = -\frac{1}{N}\frac{e^2v^2\sin\beta\cos\beta}{e^2v^2\cos^2\beta - \hat{m}_{(n)}^2}\,, \qquad E_{\chi_{1(j)}}^{(n)} &= -\frac{1}{N}\frac{\sqrt{2}e^2v^2\sin\beta\cos\beta}{(j/R)^2 + e^2v^2\cos^2\beta - \hat{m}_{(n)}^2} \end{aligned} \tag{4.89}$$

and $\hat{m}_{(n)}$ denotes the mass of the $n^{\rm th}$ mass eigenstate. The normalization constant N is given by

$$N^2 \;=\; \frac{1}{2}\,\frac{\hat{m}_{(n)}^2}{\hat{m}_{(n)}^2 - e^2v^2\cos^2\beta}\left(1 + \pi^2 e^2v^2\sin^2\beta\,R^2 + \frac{\hat{m}_{(n)}^2 - e^2v^2\cos^2\beta}{e^2v^2\sin^2\beta}\right). \tag{4.90}$$

In a consistent theory these would-be Goldstone modes should not acquire any gauge independent mass. This will be proven in the remaining part of this section. While M_ξ^2 only depends on the gauge fixing, the physical mass matrix of the scalar modes strongly depends on the Higgs potential eq.(4.73). For simplicity, we will consider a theory where the CP-even scalar fields h_1 and h_2 do not mix with the CP-odd fields χ_1 and χ_2. At tree level this can be ensured if $\lambda_5 e^{2i\phi}$, $\lambda_6 e^{i\phi}$ and $\lambda_7 e^{i\phi}$ are real quantities [29]. In this case the Higgs mass matrix is block diagonal and we can concentrate on the CP-odd mass matrix $M_{\rm CP-odd}^2$ because the CP-even fields do not mix with A_4 either. In a quite tedious calculation it can be obtained from eq.(4.73) in the form

$$M_{\rm CP-odd}^2 \;=\; \begin{pmatrix} m_{\chi 11}^2 & m_{\chi 12}^2 \\ m_{\chi 12}^2 & m_{\chi 22}^2 \end{pmatrix}\,, \tag{4.91}$$

where

$$\begin{aligned} m_{\chi 11}^2 \;=\; &-\delta(y)\Big(\tan\beta\,{\rm Re}\big(m_{12}^2 e^{i\phi}\big) + 2v^2\sin^2\beta\,\lambda_5 e^{2i\phi}\\ &+\frac{1}{2}v^2\sin\beta\cos\beta\,\lambda_6 e^{i\phi} + \frac{1}{2}v^2\sin^2\beta\,\tan\beta\,\lambda_7 e^{i\phi}\Big) \end{aligned} \tag{4.92}$$

and the other mass matrix components can be related to $m^2_{\chi 11}$ via

$$m^2_{\chi 11} = m^2_{\chi 22} \tan^2\beta \quad \text{and} \quad m^2_{\chi 12} = -\, m^2_{\chi 22} \tan\beta\,. \tag{4.93}$$

Note, that we only consider the case where $\lambda_5 e^{2i\phi}$, $\lambda_6 e^{i\phi}$ and $\lambda_7 e^{i\phi}$ are all real quantities. In order to evaluate these mass matrix entries we have used the minimization condition on the Higgs potential at tree level, i.e.

$$\left\langle \frac{\partial V(\Phi_1,\Phi_2)}{\partial h_i} \right\rangle\Bigg|_{\mathrm{VEV}} = \left\langle \frac{\partial V(\Phi_1,\Phi_2)}{\partial \chi_i} \right\rangle\Bigg|_{\mathrm{VEV}} = 0\,, \tag{4.94}$$

where $i = 1, 2$. It is very important to note that all $m^2_{\chi ij}$ $(i,j = 1,2)$ include an overall δ-function with respect to the extra dimension. One may not have expected this because Φ_1 is a bulk field. However, all mass terms that do not contain a δ-function, e.g. $\mu_1^2\,(\Phi_1^\dagger\Phi_1)$ in eq.(4.73), are related to terms that include a δ-function via one of the conditions in eq.(4.94) and can be replaced.

When the extra dimension is integrated out, the δ-functions lead to mixing terms between all the fields $\chi_{1(n)}$ and the field χ_2. Moreover, the fields $\chi_{1(n)}$ mix with each other. Finally, we already know from eq.(4.79) how $A_{(n)4}$ mixes with $\chi_{1(n)}$. Therefore, we are ready to investigate the gauge independent mass terms of the scalar fields

$$\mathcal{L}^{\mathrm{mass}}(x) = -\frac{1}{2}\left(\chi_2,\, \chi_{1(0)},\, \chi_{1(1)}, A_{(1)4}\,, \chi_{1(2)}\,, A_{(2)4}\,, \ldots\right) M_{\mathrm{S}}^2 \begin{pmatrix} \chi_2 \\ \chi_{1(0)} \\ \chi_{1(1)} \\ A_{(1)4} \\ \chi_{1(2)} \\ A_{(2)4} \\ \vdots \end{pmatrix}, \tag{4.95}$$

where

$$M_{\mathrm{S}}^2 = \begin{pmatrix} m^2 & -t_\beta\, m^2 & -\sqrt{2}\, t_\beta\, m^2 & 0 & -\sqrt{2}\, t_\beta\, m^2 & 0 & \cdots \\ -t_\beta\, m^2 & t_\beta^2 m^2 & \sqrt{2}\, t_\beta^2 m^2 & 0 & \sqrt{2}\, t_\beta^2 m^2 & 0 & \cdots \\ -\sqrt{2}\, t_\beta\, m^2 & \sqrt{2}\, t_\beta^2 m^2 & 2\, t_\beta^2 m^2 + \left(\frac{1}{R}\right)^2 & -ev\left(\frac{1}{R}\right)c_\beta & 2\, t_\beta^2 m^2 & 0 & \cdots \\ 0 & 0 & -ev\left(\frac{1}{R}\right)c_\beta & e^2 v^2 c_\beta^2 & 0 & 0 & \cdots \\ -\sqrt{2}\, t_\beta\, m^2 & \sqrt{2}\, t_\beta^2 m^2 & 2\, t_\beta^2 m^2 & 0 & 2\, t_\beta^2 m^2 + \left(\frac{2}{R}\right)^2 & -ev\left(\frac{2}{R}\right)c_\beta & \cdots \\ 0 & 0 & 0 & 0 & -ev\left(\frac{2}{R}\right)c_\beta & e^2 v^2 c_\beta & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \tag{4.96}$$

and t_β denotes $\tan\beta$, c_β denotes $\cos\beta$ and $m^2 = m^2_{\chi 22}$. At this point of the analysis we could try to diagonalize the physical mass matrix in order to determine the mass eigenstates. However, the explicit knowledge of the mass eigenstates is of no importance for the question of consistency. Thus, we check that the would-be Goldstone modes eq.(4.88) are indeed massless before gauge fixing without an explicit calculation of all mass eigenstates.

Rewriting M^2_ξ in the $(\chi_{1(n)}, \chi_2, A_{(n)4})$ basis used in eq.(4.95) one can directly calculate

$$M^2_\xi \cdot M^2_{\mathrm{S}} = 0 . \tag{4.97}$$

Because both mass matrices are symmetric, eq.(4.97) implies that M^2_{S} and M^2_ξ commute and, therefore, can be diagonalized simultaneously. Therefore, eq.(4.97) can be rewritten in the mass eigenbasis (in which both M^2_ξ and M^2_{S} are diagonal) and is equivalent to

$$m^\xi_{(i)} \cdot m^{\mathrm{S}}_{(i)} = 0 \tag{4.98}$$

for all i, where $m^\xi_{(i)}$ and $m^{\mathrm{S}}_{(i)}$ denote the mass eigenvalues of M_ξ and M_{S}, respectively, which correspond to some simultaneous mass eigenstate. Each non-zero $m^\xi_{(i)}$ is the gauge dependent mass of some would-be Goldstone mode given by eq.(4.88). Eq.(4.98) immediately shows that would-be Goldstone modes do not acquire mass before gauge fixing. On the other hand, it is guaranteed that the physical scalars do not acquire any gauge dependent mass. Therefore, the gauge fixing implied by eq.(4.80) yields well defined would-be Goldstone modes, as it had to be shown.

To summarize, a generalized Higgs sector only slightly modifies the qualitative results for the Kaluza-Klein vector bosons obtained in section 4.1.2. Moreover, we proved the existence of would-be Goldstone modes in one to one correspondence with the vector bosons. Furthermore, there is a complete Kaluza-Klein tower of physical CP-odd scalars. The mass eigenstates and the masses themselves depend on the specific Higgs potential and can be extracted from transcendental equations similar to eq.(4.40) once $m^2_{\chi 11}$ is specified. We did not comment on the CP-even Higgs fields, so far, because their masses also depend on the Higgs potential at hand. Also the self-interactions of all the scalar modes, physical or unphysical, can only be extracted in a given model if the λ_i $(i = 1, \ldots, 7)$ are specified consistently such that v_1 and v_2 are the VEVs of the true physical vacuum.

4.2 The Electroweak Theory $\mathrm{SU}(2) \times \mathrm{U}(1)_{\mathrm{Y}}$

In this section we will consider the electroweak standard model in five dimensions where a Higgs doublet couples to the $\mathrm{SU}(2) \times \mathrm{U}(1)_{\mathrm{Y}}$ gauge fields. In many respects the analysis of the higher dimensional standard model is very similar to the Abelian Higgs model such that we can use almost all the results of section 4.1.

4.2.1 Higgs Doublet in the Bulk

Before integrating over the extra dimension, we will analyze the five dimensional theory along the same lines known from the conventional electroweak model. It will be formulated such that the results of section 4.1.1 are directly helpful when deriving the effective four dimensional theory.

The Lagrangian density of the 5D Glashow-Salam-Weinberg model is given by

$$\mathcal{L} = -\frac{1}{4} B_{MN} B^{MN} - \frac{1}{4} F^a_{MN} F^{aMN} + (D_M \Phi)^\dagger \left(D^M \Phi\right) - V(\Phi), \tag{4.99}$$

where B_{MN} and F^a_{MN} ($a = 1, 2, 3$ for SU(2)) denote the field strength tensors corresponding to the U(1) and SU(2) gauge fields, respectively. They are discussed in great detail in chapters 2 and 3. The Higgs doublet can be parameterized in terms of four real valued scalar fields

$$\Phi(x, y) = \frac{1}{\sqrt{2}} \begin{pmatrix} \chi^1(x, y) + i\chi^2(x, y) \\ h(x, y) + i\chi^3(x, y) \end{pmatrix}, \tag{4.100}$$

where the notation already anticipates the meaning of the different fields. The Higgs doublet is assumed to be even with respect to Z_2-parity for the same reasons as in the Abelian model.

The Higgs fields have a weak isospin $T^3 = \pm 1/2$ (corresponding to SU(2)) and a weak hypercharge $Y = 1/2$ (corresponding to U(1)) and transform under a combined gauge transformation as

$$\Phi(x, y) \quad \to \quad \exp\left(i\left[g_5\, \Theta^a(x, y)\, \tau^a + (g_5'/2)\, \beta(x, y)\right]\right) \Phi(x, y), \tag{4.101}$$

where Θ^a and β are four arbitrary scalar functions, g_5 and g_5' denote the coupling constants and $\tau^a = \sigma^a/2$ are the three SU(2) generators, where σ^a denotes the Pauli matrices. Therefore, the covariant derivative

$$D_M = \partial_M - i\, g_5\, A^a_M\, \tau^a - i\, \frac{g_5'}{2}\, B_M \tag{4.102}$$

is implied, where A^a and B are the SU(2) and U(1) gauge fields, respectively.

The Higgs potential is given by

$$V(\Phi) = -\left|\mu_5^2\right| \left(\Phi^\dagger \Phi\right) + \lambda_5 \left(\Phi^\dagger \Phi\right)^2, \tag{4.103}$$

similarly to the Abelian Higgs model (see eq.(4.7)), and will lead to spontaneous symmetry breaking. By virtue of the $\mathrm{SU}(2) \times \mathrm{U}(1)_{\mathrm{Y}}$ symmetry we can choose, without loss of generality, that only the field h acquires a non-vanishing vacuum expectation value. According to this choice, we can set $\chi^i = 0$ in the derivation of the VEV and the effective Higgs potential to be considered is the same as in the Abelian analysis following eq.(4.16). Hence, we can easily transfer the results obtained in section 4.1.1 and reparameterize the Higgs doublet in the form

$$\Phi(x,y) \;=\; \Phi_0 \;+\; \frac{1}{\sqrt{2}} \begin{pmatrix} \chi^1(x,y) \;+\; i\chi^2(x,y) \\ h(x,y) \;+\; i\chi^3(x,y) \end{pmatrix}. \tag{4.104}$$

All four real scalar fields in eq.(4.104) have vanishing vacuum expectation values and we have introduced

$$\Phi_0 \;=\; \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_5 \end{pmatrix}, \tag{4.105}$$

where $v_5 = \sqrt{|\mu_5|^2/\lambda_5}$ is the same constant as in eq.(4.22).

All the fields in the 5D standard model live in five dimensions and the vector bosons have an extra component. However, this will be of no importance for the diagonalization of the gauge boson mass matrix arising from the Higgs mechanism. The calculation is completely analogous with the diagonalization in the four dimensional theory and, thus, discussed in great detail by any textbook on quantum field theory. We will only present the important steps in order to introduce the notation.

Because of the vacuum expectation value Φ_0 the gauge bosons acquire mass as can be easily seen by using eq.(4.104) in the Lagrangian eq.(4.99). The mass terms in the Lagrangian are given by

$$\mathcal{L}^{\text{mass}} \;=\; \frac{1}{2}\left(A^1_M,\, A^2_M,\, A^3_M,\, B_M\right) \frac{v_5^2}{2} \begin{pmatrix} g_5^2 & 0 & 0 & 0 \\ 0 & g_5^2 & 0 & 0 \\ 0 & 0 & g_5^2 & -g_5 g_5' \\ 0 & 0 & -g_5 g_5' & g_5'^{\,2} \end{pmatrix} \begin{pmatrix} A^{1M} \\ A^{2M} \\ A^{3M} \\ B^M \end{pmatrix}. \tag{4.106}$$

The mass matrix in eq.(4.106) is diagonalized by the transformation

$$\begin{aligned} W^{\pm}_M \;&=\; \frac{1}{\sqrt{2}}\left(A^1_M \mp i\,A^2_M\right), \\ Z_M \;&=\; \frac{1}{\sqrt{g_5^2+g_5'^{\,2}}}\left(g_5\,A^3_M \;-\; g_5' B_M\right), \\ A_M \;&=\; \frac{1}{\sqrt{g_5^2+g_5'^{\,2}}}\left(g_5'\,A^3_M \;+\; g_5 B_M\right), \end{aligned} \tag{4.107}$$

where in five dimensions the complex fields $W^{\pm}_M$ correspond to charged W bosons and the real field Z_M to Z bosons with masses

$$m_W \;=\; g_5 v_5/2 \quad \text{and} \quad m_Z \;=\; (v_5/2)\sqrt{g_5^2+g_5'^{\,2}}\,, \tag{4.108}$$

respectively. The real field A_M corresponds to massless photons.

To adopt the conventional notation used in electroweak theory, we define the electromagnetic coupling constant e_5 in five dimensions and the electroweak mixing angle θ_w by

$$e_5 = \frac{g_5 g_5'}{\sqrt{g_5^2+g_5'^2}}\,, \quad \cos\theta_w = \frac{g_5}{\sqrt{g_5^2+g_5'^2}} \quad \text{and} \quad \sin\theta_w = \frac{g_5'}{\sqrt{g_5^2+g_5'^2}}\,. \tag{4.109}$$

In terms of these quantities the covariant derivative is conventionally recasted into the form

$$D_M = \partial_M - i\frac{g_5}{\sqrt{2}}\left(W_M^+\tau^+ + W_M^-\tau^-\right) - i\frac{g_5}{\cos\theta_w}Z_M\left(\tau^3-\sin^2\theta_w\,Q\right) - i\,e_5\,A_M\,Q\,, \tag{4.110}$$

where the last term represents the electromagnetic coupling of the photon field and $Q=\tau^3+Y$.

This is a convenient point to begin with the investigation of the corresponding effective four dimensional theory. Therefore, we Fourier expand all the fields into their Kaluza-Klein modes, integrate over the extra dimension and rescale the fields as before. As usual, for notational convenience, we will use the constants

$$\lambda = \lambda_5/(2\pi R)\,, \quad g' = g_5'/\sqrt{2\pi R}\,, \quad g = g_5/\sqrt{2\pi R} \quad \text{and} \quad v = \sqrt{2\pi R}\,v_5\,. \tag{4.111}$$

After a standard calculation, the kinetic terms of the Higgs sector fields for mode (n) are given by

$$\begin{aligned}\mathcal{L}^{\mathrm{H}}_{(n)}(x) = {}& \frac{1}{2}\left(\partial_\mu h_{(n)}\right)^2 - \left(\left(\frac{n}{R}\right)^2 + 2\lambda v^2\right)h_{(n)}^2 + \frac{1}{2}\left(\partial_\mu\chi^3_{(n)}\right)^2 - \left(\frac{n}{R}\right)^2\left(\chi^3_{(n)}\right)^2 \\ & + \left(\partial_\mu\chi^+_{(n)}\right)\left(\partial^\mu\chi^-_{(n)}\right) - \left(\frac{n}{R}\right)^2\chi^+_{(n)}\,\chi^-_{(n)}\,,\end{aligned} \tag{4.112}$$

where we have introduced the complex scalar fields $\chi^\pm_{(n)} = 1/\sqrt{2}\big(\chi^1_{(n)} \pm i\chi^2_{(n)}\big)$. All mass terms (n/R) result from the Kaluza-Klein structure of the theory. The neutral Higgs fields $h_{(n)}$ acquire additional mass due to the Higgs mechanism. The appearance of the mass terms for the fields $\chi^\pm_{(n)}$ and $\chi^3_{(n)}$ for $n\geq 1$ again shows that the latter are not the would-be Goldstone modes of the theory.

The kinetic terms, resulting from the field strength tensors in eq.(4.99), for the real vector fields A_M and Z_M are given by eq.(2.23). The corresponding terms for mode (n) of the complex vector field W_M^+

$$\mathcal{L}^{\mathrm{W}}_{(n)}(x) = -\frac{1}{2}\left(\partial_\mu W^+_{(n)\nu} - \partial_\nu W^+_{(n)\mu}\right)\left(\partial^\mu W^{-\nu}_{(n)} - \partial^\nu W^{-\mu}_{(n)}\right) + \left|\frac{n}{R}W^+_{(n)\nu} + \partial_\mu W^+_{(n)4}\right|^2 \tag{4.113}$$

can be calculated analogously.

By definition, the mass terms of the fields A_M, Z_M and $W_M^\pm$ (see eq.(4.107)) due to the Higgs mechanism are diagonal in five dimensions. If we integrate over the extra dimension and rescale the fields as usual these mass terms simply result in mass contributions for each Kaluza-Klein mode, as in the Abelian model. The terms for the real field Z_M are completely analogous with those in eq.(4.25)

$$\mathcal{L}_{(n)}^{\rm mass} = \frac{1}{2}\left(\frac{gv}{2\cos\beta}\right)^2 Z^{\mu}_{(n)} Z_{(n)\mu} - \frac{1}{2}\left(\frac{gv}{2\cos\beta}\right)^2 Z_{(n)4} Z_{(n)4}\,. \tag{4.114}$$

For the complex gauge boson field we obtain

$$\mathcal{L}_{(n)}^{\rm mass}(x) = \left(\frac{gv}{2}\right)^2 W^{+}_{(n)\mu} W^{-\mu}_{(n)} - \left(\frac{gv}{2}\right)^2 W^{+}_{(n)4} W^{-}_{(n)4}\,. \tag{4.115}$$

As in the standard model there are no mass contributions for the photon field and its higher Kaluza-Klein modes due to symmetry breaking.

The mixing of the gauge bosons with the Higgs sector fields is again most important for the interpretation of the field content in terms of particle excitations. In the 5D electroweak theory, the covariant derivative eq.(4.110) induces the mixing terms

$$\begin{aligned}\mathcal{L}_{(n)}^{\rm mixing} = & -i\,\frac{gv}{2}\left[W^{+}_{(n)\mu}\left(\partial^\mu \chi^{-}_{(n)}\right) - W^{-}_{(n)\mu}\left(\partial^\mu \chi^{+}_{(n)}\right)\right] + \frac{g\,v}{2\cos\theta_w}\, Z_{(n)\mu}\left(\partial^\mu \chi^{3}_{(n)}\right) \\ & -i\,\frac{gv}{2}\left(\frac{n}{R}\right)\left[W^{+}_{(n)4}\,\chi^{-}_{(n)} - W^{-}_{(n)4}\,\chi^{+}_{(n)}\right] + \frac{g\,v}{2\cos\theta_w}\left(\frac{n}{R}\right) Z_{(n)4}\,\chi^{3}_{(n)}\,.\end{aligned} \tag{4.116}$$

At this point, we have derived all the kinetic terms in the effective Lagrangian of the 5D standard model. Considering all the terms containing the fields $Z^{\mu}_{(n)}$, $Z_{(n)4}$ and $\chi^{3}_{(n)}$, we recover exactly the structure of the Lagrangian eq.(4.27). Therefore, we can directly use the results obtained in section 4.1.1: Before gauge fixing, there are massless would-be Goldstone modes $G^{3}_{(n)}$ and physical massive modes $a^{3}_{(n)}$ being linear combinations of the fields $Z_{(n)4}$ and $\chi^{3}_{(n)}$ for each Kaluza-Klein mode ($n \geq 1$)

$$\begin{aligned} G^{3}_{(n)} &= \left(\left(\frac{n}{R}\right)^2 + \left(\frac{gv}{2\cos\theta_w}\right)^2\right)^{-1/2}\left[\left(\frac{n}{R}\right) Z_{(n)4} + \left(\frac{gv}{2\cos\theta_w}\right)\chi^{3}_{(n)}\right], \\ a^{3}_{(n)} &= \left(\left(\frac{n}{R}\right)^2 + \left(\frac{gv}{2\cos\theta_w}\right)^2\right)^{-1/2}\left[\left(\frac{gv}{2\cos\theta_w}\right) Z_{(n)4} - \left(\frac{n}{R}\right)\chi^{3}_{(n)}\right]. \end{aligned} \tag{4.117}$$

For the W-boson modes we recover a complex version of eq.(4.27) and the basis of the physically relevant mass eigenstates is given by

$$\begin{aligned} G^{\pm}_{(n)} &= \left((n/R)^2 + \left(\frac{gv}{2}\right)^2\right)^{-1/2}\left[\left(\frac{n}{R}\right) W^{\pm}_{(n)4} \pm \left(\frac{gv}{2}\right)\chi^{\pm}_{(n)}\right], \\ a^{\pm}_{(n)} &= \left((n/R)^2 + \left(\frac{gv}{2}\right)^2\right)^{-1/2}\left[\left(\frac{gv}{2}\right) W^{\pm}_{(n)4} \mp \left(\frac{n}{R}\right)\chi^{\pm}_{(n)}\right]. \end{aligned} \tag{4.118}$$

Considering the zero modes, $\chi^{\pm}_{(0)}$ and $\chi^3_{(0)}$ are would-be Goldstones of the theory as expected from the standard model which is completely recovered as the low energy limit of this model. Thus, counting real degrees of freedom, the kinetic terms simply furnish three copies of the Abelian model discussed in section 4.1.1: a real copy containing the Z boson and χ^3 and a complex one containing the W bosons and $\chi^{\pm}$.

As in the Abelian model, we can choose, by virtue of the SU(2)×U(1) gauge invariance, a gauge in which the would-be Goldstone modes completely disappear from the theory. This is the simplest gauge condition, again called unitary gauge, where the physical particle content is obvious. The unbroken $\mathrm{U}(1)_{\mathrm{em}}$ symmetry is not involved in the gauge fixing of the massive W and Z bosons. Together with the photon and its higher Kaluza-Klein modes it furnishes 5D-QED as discussed in chapter 2.

Again, other gauges simplifying the analysis can be found by choosing an appropriate gauge fixing function. The natural generalization of the gauge fixing function in the Abelian model (see eq.(4.31)) is given by

$$\begin{aligned} F(A^a) &= \partial_\mu A^{a\mu} - \xi\Big[\partial_4 A^a_4 - i\,g_5\left(\Phi^\dagger\tau^a\Phi_0 - \Phi_0^\dagger\tau^a\Phi\right)\Big], \\ F(B) &= \partial_\mu B^\mu - \xi\Big[\partial_4 B_4 - i\,g_5'\left(\Phi^\dagger\Phi_0 - \Phi_0^\dagger\Phi\right)\Big], \end{aligned} \tag{4.119}$$

where we have used the original gauge fields of the two gauge groups SU(2) and $\mathrm{U}(1)_Y$. The gauge fixing terms

$$\mathcal{L}_{\mathrm{GF}} = -\frac{1}{2\xi}F(A^a)^2 - \frac{1}{2\xi}F(B)^2 \tag{4.120}$$

to be added to the Lagrangian can be written in terms of the physical Z and W bosons. For each Kaluza-Klein mode (n) of the Z-boson field they are given by

$$\begin{aligned} \mathcal{L}_{(n)\,\mathrm{GF}} = &-\frac{1}{2\xi}\left(\partial_\mu Z^\mu_{(n)}\right)^2 + \frac{gv}{2\cos\theta_w}\left(\partial_\mu Z^\mu_{(n)}\right)\chi^3_{(n)} \\ &+\frac{1}{2}\left(\frac{n}{R}\right)\left(\partial_\mu Z^\mu_{(n)}\right)Z_{(n)4} - \frac{\xi}{2}\left(\left(\frac{n}{R}\right)Z_{(n)4} + \frac{gv}{2\cos\theta_w}\chi^3_{(n)}\right)^2. \end{aligned} \tag{4.121}$$

The last term in eq.(4.121) is proportional to one of the would-be Goldstone modes of the theory defined in eq.(4.117). Analogous terms appear for the complex W-boson fields. The gauge fixing terms for the photon do not include any mixing terms with the Higgs sector fields and are given by eq.(2.33). The gauge fixing functions are designed to cancel all the mixing terms between the vector bosons and any scalar fields up to total derivatives. In terms of the scalar fields as defined in eq.(4.117) and eq.(4.118) there is no mixing between scalars, either.

The resulting kinetic Lagrangian for each field is exactly of the form already encountered either in eq.(2.34) or in eq.(4.32). It is equivalent to the kinetic Lagrangian stated in

Appendix D if the masses $\hat{m}_{Z(n)}$ and $\hat{m}_{W(n)}$ given in Appendix D are replaced by

$$m_{Z(n)}^2 = (\,n/R\,)^2 + (\,ev/(\,2\cos\theta_w\,)\,)^2 \quad \text{and} \quad m_{W(n)}^2 = (\,n/R\,)^2 + (\,ev/2\,)^2\,. \tag{4.122}$$

Moreover, the additional physical scalars are described by

$$\mathcal{L}_{(n)\mathrm{kin}} = \frac{1}{2}\big(\,\partial_\mu a_{(n)}^3\,\big)^2 - \frac{1}{2}\,m_{Z(n)}^2\,\big(\,a_{(n)}^3\,\big)^2 + \frac{1}{2}\Big(\,\partial_\mu a_{(n)}^\pm\,\Big)^2 - \frac{1}{2}\,m_{W(n)}^2\,\Big(\,a_{(n)}^\pm\,\Big)^2\,. \tag{4.123}$$

Note, that the would-be Goldstone modes, as in the Abelian model, acquire a gauge dependent mass being equal to the mass of the corresponding vector boson in Feynman gauge.

The physical particle spectrum can be directly read off the gauge fixed Lagrangian. To summarize, there are three massive and one massless gauge boson along with the corresponding Kaluza-Klein towers of heavy modes. Moreover, there are the physical scalar modes $h_{(n)}$ $(n \geq 0)$ of the Higgs field. The scalar fields $A_{(n)4}$ $(n \geq 1)$ are unphysical and can be gauged away as in 5D-QED. The fields $\chi_{(n)}^\pm$, $\chi_{(n)}^3$, $W_{(n)4}^\pm$ and $Z_{(n)4}$ $(n \geq 1)$ are no mass eigenstates. Their linear combinations $G_{(n)}^\pm$ and $G_{(n)}^3$ defined in eq.(4.117) and eq.(4.118) along with $\chi_{(0)}^\pm$ and $\chi_{(0)}^3$ are the unphysical would-be Goldstone modes of the model. The orthogonal linear combinations $a_{(n)}^\pm$ and $a_{(n)}^3$ are physical, massive scalar fields with mass $m_{W(n)}^2$ and $m_{Z(n)}^2$, respectively. The corresponding propagators are of the form discussed in chapter 2 and chapter 3 and also stated in Appendix D if the appropriate masses eq.(4.122) are inserted.

So far, we have only considered the kinetic terms in the Lagrangian. However, in addition, there are the self-interactions between the gauge bosons and the interactions in the Higgs sector even before we consider fermions. The gauge boson self-interactions are derived in section 3.4: Simply insert the SU(2) structure constants and change the basis to the Z- and W-boson fields according to eq.(4.107). The interactions in the Higgs sector are completely analogous with the interactions in the Abelian model. It should be easy to generalize the results summarized in Appendix C to the non-Abelian case if they are needed.

4.2.2 Higgs Doublet on a Brane

In section 4.1.2 we investigated an Abelian Higgs model involving a Higgs field localized on a brane. Here, we will generalize this analysis to a higher dimensional standard model. Using the results of section 4.1.2 for a Higgs on a brane and section 4.2.1 for questions concerning electroweak physics will considerably simplify the discussion. If the Higgs field is restricted to a brane the electroweak Lagrangian in five dimensions is given by

$$\mathcal{L} = -\frac{1}{4}\,B_{MN}\,B^{MN} - \frac{1}{4}\,F_{MN}^a F^{aMN} + \delta(y)\,(\,D_\mu\,\Phi\,)^\dagger\,(\,D^\mu\,\Phi\,) - \delta(y)\,V(\Phi)\,, \tag{4.124}$$

where $\Phi(x)$ is the conventional Higgs doublet which does not depend on the extra dimension and all other ingredients correspond to those in eq.(4.99). The gauge transformation properties of the Higgs doublet are given by a direct generalization of eq.(4.34) to electroweak theory

$$\Phi(x) \quad \rightarrow \quad \exp\left(i \left[g_5\, \Theta^a(x,0)\, \tau^a \,+\, (g_5'/2)\, \beta(x,0) \right] \right) \Phi(x)\,. \tag{4.125}$$

As in the Abelian model, there are no interactions of the Higgs fields with the extra components of the gauge fields. The Higgs potential is the same as in the standard model and the vacuum expectation value of the Higgs doublet

$$\Phi_0 \,=\, \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_5 \end{pmatrix} \tag{4.126}$$

can be derived as in the four dimensional theory.

Choosing the basis eq.(4.107) as in section 4.2.1, the kinetic Lagrangian consists of several separate pieces which can be investigated one at a time: First, there are the terms including the fields Z_M and χ^3. They are completely analogous with the Abelian model discussed in section 4.1.2. Second, the terms including the fields $W^\pm_M$ and $\chi^\pm$ combine to a complex version of the same model. Third, the photon is not affected by the symmetry breakdown and 5D-QED is reproduced as in chapter 2. For example, in analogy with eq.(4.37), the mass terms due to the brane VEV are given by

$$\mathcal{L}^{\mathrm{mass}} \,=\, \sum_{m,n=0}^{\infty} \left[\left(\frac{e\, v}{2\cos\theta_w} \right)^2 Z^\mu_{(m)}\, Z_{(n)\mu} \,+\, 2 \left(\frac{e\, v}{2} \right)^2 W^{+\mu}_{(m)}\, W^-_{(n)\mu} \right] \left(\frac{1}{\sqrt{2}} \right)^{\delta_{m,0}+\delta_{n,0}} . \tag{4.127}$$

As in the Abelian model they mix the Kaluza-Klein gauge eigenstates of the vector bosons. Because there are no terms involving both $Z_{(n)}$ and $W^\pm_{(m)}$, the Kaluza-Klein mass eigenstates of the Z and W bosons can be determined exactly as in the Abelian analysis. One only has to use the symmetry breaking mass $m = gv/2$ for the W bosons and $m = gv/(2\cos\theta_w)$ for the Z bosons, respectively, instead of $m = ev$. Finally, the mass eigenvalues of the different mass eigenstates are the solutions of the transcendental equations (see eq.(4.42))

$$\begin{aligned} \hat{m}_{W(n)} \,&=\, \pi \left(\frac{gv}{2} \right)^2 R\, \cot\left(\pi\, \hat{m}_{W(n)}\, R \right), \\ \hat{m}_{Z(n)} \,&=\, \pi \left(\frac{gv}{2\cos\theta_w} \right)^2 R\, \cot\left(\pi\, \hat{m}_{Z(n)}\, R \right). \end{aligned} \tag{4.128}$$

The analysis of the mixing terms is also completely analogous with the Abelian case because one can consider the terms involving the fields $Z_{(n)}$ separately from those involving the fields $W^\pm_{(n)}$. The discussion following eq.(4.36) in the Abelian case shows that $\chi^\pm$, χ^3 and all the scalar gauge boson fields are unphysical degrees of freedom. They can be gauged away simultaneously in unitary gauge. The remaining $\mathrm{U}(1)_{\mathrm{em}}$ symmetry together with the photon and its Kaluza-Klein modes again result in 5D-QED.

For completeness, we state the gauge fixing functions for a more general R_ξ-gauge. They are given by the analogue of eq.(4.59) for the electroweak model

$$\begin{aligned} F(A^a) &= \partial_\mu A^{a\mu} - \xi\Big[\partial_4 A_4^a - i\,g_5\left(\Phi^\dagger\tau^a\Phi_0 - \Phi_0^\dagger\,\tau^a\,\Phi\right)\delta(y)\Big]\,, \\ F(B) &= \partial_\mu B^\mu - \xi\Big[\partial_4 B_4 - i\,g_5'\left(\Phi^\dagger\,\Phi_0 - \Phi_0^\dagger\,\Phi\right)\delta(y)\Big]\,. \end{aligned} \tag{4.129}$$

The mass eigenstates of the would-be Goldstone modes in this R_ξ-gauge can be also derived as in the analysis of section 4.1.2. Again, there are modes of mass $\sqrt{\xi}\,\hat{m}_{W(n)}$ and $\sqrt{\xi}\,\hat{m}_{Z(n)}$ for each vector boson mode of mass $\hat{m}_{W(n)}$ and $\hat{m}_{Z(n)}$, respectively.

To summarize, the particle content of the vector bosons is equivalent to the one of the bulk model. Only the masses and couplings of the Kaluza-Klein mass eigenstates are slightly shifted for the Z and W bosons as explained in the Abelian model. There is a single massive physical Higgs boson without Kaluza-Klein excitations. There are no additional massive physical scalar fields. All other scalar modes combine to would-be Goldstone fields. The complete kinetic Lagrangian in R_ξ-gauge and the corresponding propagators are equivalent to the ones stated in Appendix D if the masses defined by eq.(4.128) are inserted instead of $m^2_{W(n)}$ and $m^2_{Z(n)}$. Concerning the interactions, the comments at the end of section 4.2.1 apply.

4.2.3 The General Case

Finally, we are ready to investigate the most general model: the five dimensional electroweak theory with two Higgs doublets, one living in the bulk and one localized on a brane. The Lagrangian of this theory is given by

$$\begin{aligned} \mathcal{L} = &-\frac{1}{4}\,B_{MN}\,B^{MN} - \frac{1}{4}\,F^a_{MN}F^{aMN} \\ &+ (\,D_M\,\Phi_1\,)^\dagger\left(D^M\,\Phi_1\right) + \delta(y)(\,D_\mu\,\Phi_2\,)^\dagger\,(\,D^\mu\,\Phi_2\,) - V(\Phi_1,\Phi_2)\,, \end{aligned} \tag{4.130}$$

where the field strength tensors are given by eq.(4.99), D_M by eq.(4.102) and the Higgs potential by eq.(4.73). $\Phi_1(x,y)$ is a bulk field, $\Phi_2(x)$ a brane field parameterized by

$$\Phi_1(x,y) = \begin{pmatrix} \chi_1^+ \\ \frac{1}{\sqrt{2}}\,(v_{51} + h_1 + i\chi_1^3) \end{pmatrix}, \quad \Phi_2(x) = e^{i\phi}\begin{pmatrix} \chi_2^+ \\ \frac{1}{\sqrt{2}}\,(v_{52} + h_2 + i\chi_2^3) \end{pmatrix}, \tag{4.131}$$

where v_{51} and v_{52} are the vacuum expectation values of the two Higgs fields. By virtue of gauge invariance, we can choose the parameterization for Φ_1 without loss of generality.

Any non-vanishing VEV in the charged sector is ruled out because it would give mass to the photon. Therefore, as in the Abelian model, we only allow for a phase ϕ in the parameterization of Φ_2.

As in section 4.2.1 or section 4.2.2 the choice of basis eq.(4.107), defining the Z and W bosons, simplifies the analysis. The mixing terms between the Z-boson field and the neutral scalar fields χ_1^3 and χ_2^3 and the mixing terms between the W-boson field and the charged scalar fields $\chi_1^{\pm}$ and $\chi_2^{\pm}$, discussed in sections 4.2.1 and 4.2.2, respectively, simply add. The appropriate gauge fixing functions

$$
\begin{aligned}
F(A^a) &= \partial_\mu A^{a\mu} - \xi\Big[\partial_4 A_4^a - ig_5\Big(\Phi_1^\dagger\tau^a\Phi_0 - \Phi_0^\dagger\tau^a\Phi_1\Big)\cos\beta \\
&\qquad\qquad - ig_5\Big(\Phi_2^\dagger\tau^a\Phi_0 - \Phi_0^\dagger\tau^a\Phi_2\Big)\sin\beta\,\delta(y)\Big]\,, \\
F(B) &= \partial_\mu B^\mu - \xi\Big[\partial_4 B_4 - ig_5'\Big(\Phi_1^\dagger\Phi_0 - \Phi_0^\dagger\Phi_1\Big)\cos\beta \\
&\qquad\qquad - ig_5'\Big(\Phi_2^\dagger\Phi_0 - \Phi_0^\dagger\Phi_2\Big)\sin\beta\,\delta(y)\Big]\,,
\end{aligned}
\tag{4.132}
$$

where Φ_0 is defined in eq.(4.105), take this into account and make the particle content of this model become transparent by cancelling all the mixing terms up to total derivatives. The analysis for the Z-boson modes is again completely analogous with the Abelian case. Because the W-boson field does not mix with the neutral CP-odd scalars but with the charged fields $\chi_1^{\pm}$ and $\chi_2^{\pm}$, the mass matrix M^2_{ch} replaces $M^2_{\mathrm{CP-odd}}$ in the analysis for the charged part of the theory. It is again a straightforward calculation to obtain M^2_{ch} from the Lagrangian. Using the minimization conditions on the fields and working again in a model, where $\lambda_5 e^{2i\phi}$, $\lambda_6 e^{i\phi}$ and $\lambda_7 e^{i\phi}$ in the Higgs potential are all real quantities, a short computation yields

$$
M^2_{\mathrm{charged}} = m^2 \begin{pmatrix} \tan^2\beta & -\tan\beta \\ -\tan\beta & 1 \end{pmatrix}, \tag{4.133}
$$

where

$$
\begin{aligned}
m^2 = -\,\delta(y)\,\frac{1}{\tan^2\beta}\Big(&\tan\beta\,\mathrm{Re}\big(m_{12}^2 e^{i\phi}\big) + \frac{1}{2}v^2\sin^2\beta\lambda_4 + v^2\sin^2\beta\,\lambda_5 e^{2i\phi} \\
&+\frac{1}{2}v^2\sin\beta\cos\beta\,\lambda_6 e^{i\phi} + \frac{1}{2}v^2\sin^2\beta\tan\beta\,\lambda_7 e^{i\phi}\Big)\,,
\end{aligned}
\tag{4.134}
$$

i.e. M^2_{ch} is proportional to $M^2_{\mathrm{CP-odd}}$. The analysis for the Abelian model does not rely on the overall numerical value of the mass matrix. Only the structure of the matrix components is important. Therefore, the analysis for the W bosons can be also done in complete analogy with the Abelian case.

To summarize, the five dimensional electroweak theory in the context of a 2-Higgs doublet model contains the same neutral and charged vector boson mass eigenstates found already in section 4.2.2. Only their mass eigenvalues are shifted due to the VEV of a bulk Higgs field. There is a neutral would-be Goldstone mode for each of the neutral and a charged one for each of the charged vector boson modes. Their masses equal the corresponding gauge boson masses in Feynman gauge. The corresponding gauge fixed kinetic Lagrangian and the propagators are stated in Appendix D. Furthermore, there are two charged, two CP-even and one CP-odd neutral scalars at each Kaluza-Klein level. Their couplings to the gauge degrees of freedom are fixed by the model but their masses and self-interactions again depend on the Higgs potential specified in a given model.

Chapter 5

The Fermion and Yukawa sector of 5D Theories

We already encountered fermions in section 2.5. In this chapter we will not alter the basic concepts introduced in 5D-QED. We still consider fermions restricted to a brane at $y = 0$. However, in contrast to section 2.5, we will introduce chiral couplings and consider more general gauge sectors the fermions couple to. While the generalization to non-Abelian theories is quite straightforward we have to discuss fermions in spontaneously broken theories in more detail. In particular, the question of gauge invariance of fermion interactions will be investigated in a simple model process in section 5.3.

As usual, we will introduce the basic concepts in an Abelian model before we comment on the final 5D electroweak theory. Some of the considerations are very similar to familiar 4D physics because the fermions do not propagate in the bulk. Therefore, the first part of section 5.1 mainly serves to introduce the notation.

5.1 A Spontaneously Broken Abelian Model

In this section we will include a fermion sector in the different five dimensional Abelian Higgs models discussed in detail in chapter 4. We will keep the discussion as general as possible and we will specify the concrete model only if it is necessary. The Abelian case should be considered as a toy model to show the special features of the extra dimension in the easiest possible theory.

Because the standard model electroweak theory contains chiral couplings, in an interesting model theory, the fermions should couple to the gauge bosons also in a chiral way, e.g. only left-handed fermions interact with the gauge fields. The corresponding Lagrangian is given by

$$\mathcal{L}_{\rm F}(x,y) \;=\; \delta(y)\,\overline{\Psi}_{\rm L}(x)\,i\,\gamma^\mu\,D_\mu\,\Psi_{\rm L}(x) \;+\; \delta(y)\,\overline{\Psi}_{\rm R}(x)\,i\,\gamma^\mu\,\partial_\mu\,\Psi_{\rm R}(x)\,, \tag{5.1}$$

where the covariant derivative is $D_\mu = \partial_\mu + ie_5 A_\mu(x,y)$ and $\Psi_{\rm L,R}$ denote left- and right-handed spinors, respectively. If we regard the left- and right-handed fermion fields as separate degrees of freedom this is a direct generalization of the Lagrangian eq.(2.37) to two fermion species with given charges. According to their couplings the left-handed fermions are charged under the Abelian U(1) symmetry and transform under gauge transformations as the fermion fields in section 2.5, the right-handed components are not charged and do not transform. Using the fermion fields $\Psi = \Psi_{\rm L} + \Psi_{\rm R}$ and the projection operators $1/2(1 \pm \gamma^5)$, eq.(5.1) can be recasted into the form

$$\mathcal{L}_{\rm F}(x,y) \;=\; \delta(y)\,\overline{\Psi}(x)\,i\,\gamma^\mu\,\left(\partial_\mu \,+\, i\,\frac{1}{2}\left(1-\gamma^5\right)e_5\,A_\mu\right)\,\Psi(x)\,. \tag{5.2}$$

Because of the different gauge transformation properties of $\Psi_{\rm L}$ and $\Psi_{\rm R}$, there is no gauge invariant mass term, a fact well-known from the standard model. However, there are gauge invariant interactions between fermions and the Higgs fields of the theory known as Yukawa sector. Let us first introduce the Yukawa sector of a theory with only one Higgs field, as discussed in section 4.1.1 and section 4.1.2. Its Lagrangian density is given by

$$\mathcal{L}_{\rm Y}(x,y) \;=\; -\,\lambda_{f5}\,\delta(y)\left(\overline{\Psi}_{\rm L}(x)\,\Phi\,\Psi_{\rm R}(x) \;+\; \overline{\Psi}_{\rm R}(x)\,\Phi^\dagger\,\Psi_{\rm L}(x)\right), \tag{5.3}$$

where λ_{f5} is a coupling constant and Φ denotes the Higgs field living either in the bulk or on a brane according to section 4.1.1 and 4.1.2, respectively. Here, gauge invariance on the brane is obvious because, concerning gauge transformations, this Lagrangian density is equivalent to its counterpart in four dimensional physics once the extra dimension is integrated out. This can be seen as follows: At $y = 0$, gauge transformations for both the fermion and the scalar fields are parameterized by $\Theta(x,0)$, i.e. by some scalar function depending only on four dimensional space-time. Moreover, concerning gauge transformations, we can identify a possible bulk field on the brane, i.e. $\Phi(x,0)$, with the Higgs field in a four dimensional theory. Therefore, all gauge invariant terms known from four dimensional physics can be promoted to gauge invariant terms in five dimensional theories.

Because of the vacuum expectation value v_5 of the Higgs field, the fermion acquires a mass m_f. The corresponding mass term is given by

$$\mathcal{L}_{\rm mass}(x,y) \;=\; -\,\frac{v_5\,\lambda_{f5}}{\sqrt{2}}\,\delta(y)\left(\overline{\Psi}_{\rm L}\,\Psi_{\rm R} \,+\, \overline{\Psi}_{\rm R}\,\Psi_{\rm L}\right) \;=\; -\,\frac{v_5\,\lambda_{f5}}{\sqrt{2}}\,\delta(y)\,\overline{\Psi}\,\Psi = -\,m_f\,\delta(y)\,\overline{\Psi}\,\Psi\,. \tag{5.4}$$

After integration over the extra dimension, this mass term trivially reduces to the one known from four dimensional theory. Therefore, all terms bilinear in the fermions are equivalent to the corresponding terms in a conventional four dimensional model.

Let us now consider the case where $\Phi(x,y)$ is a bulk field. After expanding the scalar field Φ we can integrate over the extra dimension and rescale the Kaluza-Klein modes, as usual. Using the definitions $\lambda_f = \lambda_{f5}/\sqrt{2\pi R}$ and $v = \sqrt{2\pi R}\, v_5$ (see section 4.1.1) the Lagrangian eq.(5.3) results in

$$\mathcal{L}_{\rm Y}(x) \;=\; -\,\overline{\Psi}(x)\, m_f\, \Psi \,-\, \frac{e\, m_f}{m_A}\, \overline{\Psi}(x) \left[h_{(0)} \,+\, i\,\gamma^5\, \chi_{(0)} \,+\, \sqrt{2} \sum_{n=1}^{\infty} \left(h_{(n)} \,+\, i\,\gamma^5\, \chi_{(n)} \right) \right] \Psi(x)\,, \tag{5.5}$$

where $m_A = ev$ denotes the symmetry breaking mass. If the Higgs sector is restricted to a brane, instead, integrating over the extra dimension becomes trivial and the corresponding Lagrangian is given by

$$\mathcal{L}_{\rm Y}(x) \;=\; -\,\overline{\Psi}(x)\, m_f\, \Psi \,-\, \frac{e\, m_f}{m_A}\, \overline{\Psi}(x) \left(h \,+\, i\,\gamma^5\, \chi \right) \Psi\,, \tag{5.6}$$

where we have used, in this case, $\lambda_f \,=\, \lambda_{f5}$ and $v \,=\, v_5$. In both cases the chiral coupling to the Kaluza-Klein modes of the gauge field given in eq.(5.2) is described in its four dimensional effective form by

$$\mathcal{L}_{\rm GC}(x,y) \;=\; -\,e\, \overline{\Psi}(x)\, \gamma^\mu\, \frac{1}{2} \left(1 - \gamma^5 \right) \Psi(x) \left(A_{(0)\mu} \,+\, \sqrt{2} \sum_{n=1}^{\infty} A_{(n)\mu} \right)\,, \tag{5.7}$$

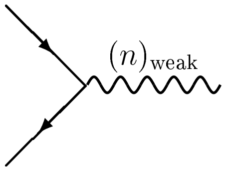
where we have, as usually, used $e \,=\, e_5/\sqrt{2\pi R}$. As expected, we recover the general structure of interactions between fermions and gauge bosons we already found in section 2.5. The interaction of the Kaluza-Klein zero mode dominating the low energy regime is supplemented by a whole Kaluza-Klein tower of interactions with the same chiral coupling structure but a coupling increased by a factor of $\sqrt{2}$.

This theory easily generalizes to the 2-Higgs model of section 4.1.3. The Lagrangian eq.(5.3) simply occurs twice, once for the bulk field Φ_1 and once for the brane field Φ_2. The coupling constants are generally different and denoted by λ_{1f5} and λ_{2f5}, respectively. The fermion mass m_f in such a model is analogously given by

$$m_f \;=\; \frac{v}{\sqrt{2}} \left(\lambda_{1f}\, \cos\beta \,+\, \lambda_{2f}\, \sin\beta \right)\,, \tag{5.8}$$

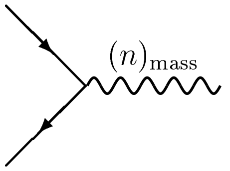
where we have used the notation for the vacuum expectation values introduced in section 4.1.3. Both scalar fields interact with the fermions as in the corresponding 1-Higgs model. However, it is no longer possible to express the coupling constants λ_{1f} and λ_{2f} only in terms of the symmetry breaking mass m_A and the fermion mass m_f unless they are equal.

It is important to note, that all the interactions, so far, are stated in terms of the weak eigenstates of the theory. As we have seen throughout chapter 4 these weak eigenstates are not necessarily mass eigenstates. The CP-odd scalars of the theory always mix with weak eigenbasis:



$$\mu \;=\; -\, i\, e\, \tfrac{1}{2}\,(\,1-\gamma^5\,)\,\gamma^\mu\,\sqrt{2}^{\,1-\delta_{n,0}}$$

mass eigenbasis:



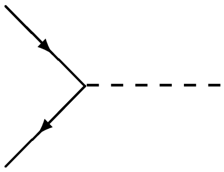
$$\mu \;=\; -\, i\, e\, \tfrac{1}{2}(\,1-\gamma^5\,)\,\gamma^\mu \Big(\, \mathrm{E}_0^{(n)} + \sqrt{2}\,\textstyle\sum_{j=1}^{\infty} \mathrm{E}_j^{(n)} \,\Big)$$

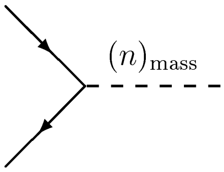
Figure 5.1: Vertices for gauge interactions between fermions and the vector bosons in a theory containing a brane field with non-vanishing vacuum expectation value. The upper vertex is stated in the weak eigenbasis, the lower vertex in the mass eigenbasis. $\mathrm{E}_j^{(n)}$ are the expansion coefficients of the gauge eigenstates in eq.(4.50).

the scalar component of the gauge fields. If there is a non-vanishing VEV of a brane field, moreover, different Kaluza-Klein modes of the vector bosons and scalars mix as we have seen, for example, in section 4.1.2. In actual calculations it is necessary to work in the mass eigenbasis because along with the mass matrix the propagators become diagonal with respect to different Kaluza-Klein modes and, thereby, simplify considerably. Naturally, the choice of this basis will complicate the vertex structure but this is easier to handle than mixed propagators.

Figure 5.1 shows the vertices for the gauge interactions of fermions in both basis systems. Note, that the vertex factor for the interactions of fermions with mass eigenstate can be written in the compact notation $-\, i\, \hat{e}_{(n)}\, \frac{1}{2}\,(\,1-\gamma^5\,)\,\gamma^\mu$ where $\hat{e}_{(n)}$ is defined in eq.(4.55).

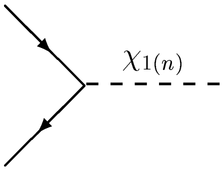
In figure 5.2 the coupling of the CP-odd field χ to fermions in a theory with a single Higgs field on a brane is shown. Note, that the scalar gauge fields $A_{(n)4}$ do not couple to the fermions because of their antisymmetry. Nevertheless, all the would-be Goldstone modes $G_{(n)}$ being linear combinations of these fields do couple to the fermions. Figure 5.3 shows the generalization to the 2-Higgs model discussed in section 4.1.3.

At this point, these Feynman rules are mainly introduced to show how the difference in weak and mass eigenstates affects actual calculations. However, we have also explicitly stated these vertex rules because they will play an important role in section 5.3 where we will show the gauge invariance of such theories in a specific process. The interactions with the physical Higgs fields are assumed to be negligible in most cases because their couplings are suppressed by a factor of m_f/m_A in 1-Higgs models. Note, that also the additional scalars (see for example eq.(4.28)) do not couple with gauge coupling strength, although weak eigenbasis:  $= \frac{e\, m_f}{m_A}\, \gamma^5$

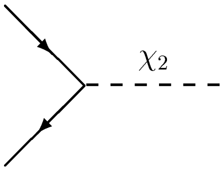
mass eigenbasis: 

$= \frac{e\, m_f}{m_A}\, \gamma^5\, \mathrm{E}_{\chi}^{(n)}$

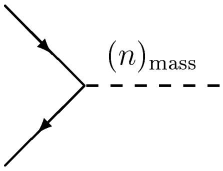
Figure 5.2: The upper vertex corresponds to the interaction of the scalar field χ with fermions in the weak eigenbasis if the considered theory contains a single scalar field with a VEV restricted to a brane. The lower vertex corresponds to the interaction of the would-be Goldstone modes to fermions in the mass eigenbasis. $\mathrm{E}_{\chi}^{(n)}$ denotes the expansion coefficient of χ in eq.(4.69).

weak eigenbasis: 

$= \left(\frac{1}{\sqrt{2}}\right)^{\delta_{n,0}} \lambda_{2f}\, \gamma^5$

weak eigenbasis: 

$= \frac{\lambda_{1f}}{\sqrt{2}}\, \gamma^5$

mass eigenbasis: 

$$= \left(\frac{1}{\sqrt{2}}\right)^{\delta_{n,0}} \gamma^5 \left(\lambda_{2f} \mathrm{E}_{\chi_2}^{(n)} + \lambda_{1f} \left(\mathrm{E}_{\chi_{1(0)}}^{(n)} + \sqrt{2} \sum_{j=1}^{\infty} \mathrm{E}_{\chi_{1(j)}}^{(n)} \right) \right)$$

Figure 5.3: The generalization of figure 4.69 for a 2-Higgs model. The vertices in the weak eigenbasis can be read off directly from eq.(5.5) and eq.(5.6). $\mathrm{E}_{\chi_2}^{(n)}$, $\mathrm{E}_{\chi_{1(0)}}^{(n)}$ and $\mathrm{E}_{\chi_{1(j)}}^{(n)}$ are given in eq.(4.88).

they involve the gauge fields $A_{(n)4}$, because of the antisymmetry of the latter.

5.2 Fermions in 5D Electroweak Theory

In 5D electroweak theory all the basic features of the preceding section are also valid. All the necessary further generalisations are due to the enlarged gauge group $\mathrm{SU}(2) \times \mathrm{U}(1)_{\mathrm{Y}}$ and not specifically connected to the appearance of a large extra dimension. Because we want the standard model to be embedded in the five dimensional theory we simply have to promote the whole fermionic particle content of the standard model onto a brane in five dimensional space. All the quantum numbers of the fermions are unchanged.

The left-handed fermions come in doublets Ψ_L, transforming under the fundamental representation of SU(2). A complete gauge transformation including $\mathrm{U}(1)_{\mathrm{Y}}$ is given by

$$\Psi_L(x) \quad \rightarrow \quad \exp\Big(i\,(g_5\,\Theta^a(x,0)\,\sigma^a/2 \,+\, g_5'\,Y^L\,\beta(x,0))\Big)\,\Psi_L(x)\,, \tag{5.9}$$

where σ^a are again the Pauli matrices and Y^L the hypercharge of the given doublet. The right-handed fermions are singlets with respect to SU(2). They transform like

$$\Psi_R(x) \quad \rightarrow \quad \exp\Big(i\,g_5'\,Y^R\,\beta(x,0)\Big)\,\Psi_R(x)\,. \tag{5.10}$$

We have seen in section 5.1 that the mass generation due to the VEV of the Higgs field is completely equivalent to a four dimensional theory. That does not change in electroweak theory. Simply promote the gauge invariant terms of the 4D Yukawa sector on a brane according to section 5.1. The corresponding terms are explicitly stated in Appendix D. Following the analysis of section 5.1 it is easy to show that all the kinetic terms of the fermions in the effective four dimensional theory are given by their standard model counterparts.

According to their quantum numbers, the fermion interactions with the gauge bosons (in analogy with eq.(5.1)) are described by a generic Lagrangian

$$\mathcal{L}_{\mathrm{F}}(x,y) \,=\, \delta(y)\,\overline{\Psi}_{\mathrm{L}}(x)\,i\,\gamma^\mu\,D^L_\mu\,\Psi_{\mathrm{L}}(x) \,+\, \delta(y)\,\overline{\Psi}_{\mathrm{R}}(x)\,i\,\gamma^\mu\,D^R_\mu\,\Psi_{\mathrm{R}}(x)\,, \tag{5.11}$$

where the covariant derivatives are given by

$$\begin{aligned} D^L_\mu &= \partial_\mu \,-\, i\,g_5\,A^a_\mu(x,y)\,\tau^a \,-\, i\,g_5'\,Y^L\,B_\mu(x,y)\,, \\ D^R_\mu &= \partial_\mu \,-\, i\,g_5'\,Y^R\,B_\mu(x,y) \end{aligned} \tag{5.12}$$

for the left-handed and right-handed fermions respectively. In order to derive all the interactions of the different fermion and gauge boson fields one has to analyze this Lagrangian exactly as in the standard model where $A^a_\mu(x,y)$ and $B_\mu(x,y)$ are given in terms of the usual W- and Z-boson fields by eq.(4.107).

Finally, all interactions between fermions and gauge bosons in the higher dimensional standard model are of the generic form

$$\mathcal{L}_{\mathrm{F}}(x,y) \;=\; \delta(y)\,\overline{\Psi}_k(x)\,\gamma^\mu\left[\,\left(g_V + g_A\gamma^5\right) A_{i\mu}(x,y)\right]\Psi_l(x)\,, \tag{5.13}$$

where $A_{i\mu}(x,y)$ denote the five dimensional gauge boson fields, i.e. photon, Z and W bosons, and $\overline{\Psi}_{k,l}(x)$ one of the fermion fields. Hence, after integration over the extra dimension, each standard model interaction is supplemented with the usual tower of interactions with the higher Kaluza-Klein modes (see eq.(5.7)). All the comments in section 5.1 about weak and mass eigenstates also apply. In Appendix D we state all the interaction terms of the fermion sector and the corresponding Feynman rules in the mass eigenbasis of the gauge bosons. An analogous analysis can be carried out for the interactions in the Yukawa sector leading to direct generalizations of eq.(5.5) and eq.(5.6). We state the would-be Goldstone interactions with the fermions which do not depend on a given Higgs potential in Appendix D.

5.3 Gauge Invariance

All the theories discussed here, are constructed in a manifestly gauge invariant way. At the five dimensional level the invariance is obvious but once we have integrated over the extra dimension it is obscured. Therefore, we will investigate the easiest possible example for a scattering process, $e^+\,e^- \;\rightarrow\; \mu^+\,\mu^-$ in the Abelian models of section 5.1, in order to see how gauge invariance shows up. We will recover the usual interplay between the gauge bosons and the would-be Goldstone modes. However, the interesting point is that in a model with a single scalar field on a brane there is only one weak eigenstate, i.e. χ, which has to cancel the gauge dependent contributions of the complete Kaluza-Klein tower of gauge bosons.

Figure 5.4 shows the two towers of graphs which describe the fermion scattering process at the Born level, where we have chosen the basis such that the mass eigenstates propagate in the s-channel. At this point it is unimportant which specific model we investigate, e.g. a one or a 2-Higgs model, only the expansion of the mass eigenstates in terms of the weak eigenstates changes. We will specify the mass eigenstates later on when we have to.

The amplitude corresponding to the gauge boson mass eigenstate (n) in the s-channel is given by

$$i\,\mathcal{M}_{\hat{A}^\mu_{(n)}} = L^\rho_{e^+e^-\hat{A}^\mu_{(n)}}\left(\frac{-i\,g_{\rho\sigma} + i\,q_\rho\,q_\sigma\frac{(1-\xi)}{q^2-\xi\hat{m}^2_{(n)}}}{q^2-\hat{m}^2_{(n)}}\right)L^\sigma_{\mu^+\mu^-\hat{A}^\mu_{(n)}}\,, \tag{5.14}$$

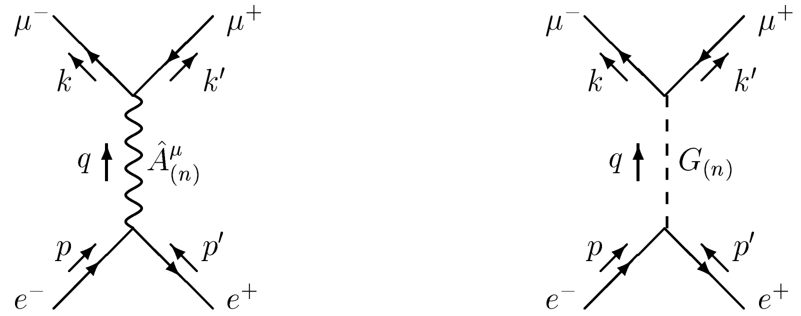


Figure 5.4: The two towers of Feynman graphs describe fermion scattering in the Abelian model at the Born level.

where we have introduced the leptonic currents $L^{\rho}_{f^+f^-\hat{A}^\mu_{(n)}}$ denoting the vertex factor dotted into the appropriate spinors, e.g.

$$\begin{aligned} L^{\rho}_{e^+e^-\hat{A}^\mu_{(n)}} &= -\,i\,e\,\overline{v}(p')\,\frac{1}{2}\big(1-\gamma^5\big)\,\gamma^\rho\left(\mathrm{E}_0^{(n)}+\sqrt{2}\sum_{j=1}^{\infty}\mathrm{E}_j^{(n)}\right)u(p)\,, \\ L^{\rho}_{\mu^+\mu^-\hat{A}^\mu_{(n)}} &= -\,i\,e\,\overline{u}(k)\,\frac{1}{2}\big(1-\gamma^5\big)\,\gamma^\rho\left(\mathrm{E}_0^{(n)}+\sqrt{2}\sum_{j=1}^{\infty}\mathrm{E}_j^{(n)}\right)v(k')\,. \end{aligned} \tag{5.15}$$

It is convenient to split the gauge boson propagator into its transversal part and a ξ-dependent longitudinal contribution, i.e. we use the elementary identity

$$\frac{-i\,g_{\rho\sigma}+i\,q_\rho\,q_\sigma\frac{(1-\xi)}{q^2-\xi\hat{m}^2_{(n)}}}{q^2-\hat{m}^2_{(n)}} = \frac{-i}{q^2-\hat{m}^2_{(n)}}\left(g_{\rho\sigma}-\frac{q_\rho\,q_\sigma}{\hat{m}^2_{(n)}}\right)-\frac{i\,q_\rho\,q_\sigma}{\hat{m}^2_{(n)}}\,\frac{1}{q^2-\xi\hat{m}^2_{(n)}}\,. \tag{5.16}$$

Our ultimate goal is to show that the gauge or ξ-dependent part of the amplitude

$$i\,\mathcal{M}^{\xi}_{\hat{A}^\mu_{(n)}} = L^{\rho}_{e^+e^-\hat{A}^\mu_{(n)}}\left(-\frac{i\,q_\rho\,q_\sigma}{\hat{m}^2_{(n)}}\,\frac{1}{q^2-\xi\hat{m}^2_{(n)}}\right)L^{\sigma}_{\mu^+\mu^-\hat{A}^\mu_{(n)}} \tag{5.17}$$

is cancelled by the amplitude of the corresponding n^{th} would-be Goldstone mode of the theory

$$i\,\mathcal{M}_{G_{(n)}} = L_{e^+e^-G_{(n)}}\left(\frac{i}{q^2-\xi\hat{m}^2_{(n)}}\right)L_{\mu^+\mu^-G_{(n)}}\,, \tag{5.18}$$

i.e.

$$\mathcal{M}_{G_{(n)}} = -\,\mathcal{M}^{\xi}_{\hat{A}_{(n)}}\,. \tag{5.19}$$

It is obvious that eq.(5.19) holds if the currents fulfill

$$\begin{aligned} i\,L_{e^+e^-G_{(n)}} &= \frac{q_\rho}{\hat{m}_{(n)}}\,L^{\rho}_{e^+e^-\hat{A}^\mu_{(n)}}\,, \\ i\,L_{\mu^+\mu^-G_{(n)}} &= -\,\frac{q_\rho}{\hat{m}_{(n)}}\,L^{\rho}_{\mu^+\mu^-\hat{A}^\mu_{(n)}}\,. \end{aligned} \tag{5.20}$$

We will show in the following that these two conditions can be easily casted into only one condition. The leptonic current for the annihilation has the generic structure

$$L^{\rho}_{e^+e^-\hat{A}^{\mu}_{(n)}} = \overline{v}(p')\left(g_V + g_A\gamma^5\right)\gamma^{\rho}u(p)\,, \tag{5.21}$$

where g_V denotes the vector coupling and g_A the axial vector coupling. Using the fact that the spinors obey the Dirac equation in momentum space, i.e.

$$(\gamma^{\rho}p_{\rho} - m_f)u(p) = \overline{v}(p)(\gamma^{\rho}p_{\rho} + m_f) = 0\,, \tag{5.22}$$

and the momentum relation $q_{\rho} = p_{\rho} + p'_{\rho}$ it is easy to proof

$$q_{\rho}\,L^{\rho}_{e^+e^-\hat{A}^{\mu}_{(n)}} = 2\,m_f\,g_A\,\overline{v}(p')\,\gamma^5\,u(p)\,. \tag{5.23}$$

Repeating the above arguments for the production vertex yields the analogous relation

$$q_{\rho}\,L^{\rho}_{\mu^+\mu^-\hat{A}^{\mu}_{(n)}} = -\,2\,m_f\,g_A\,\overline{u}(p')\,\gamma^5\,v(p)\,. \tag{5.24}$$

The vertex factors in figure 5.2 or figure 5.3 show that the generic structure of the leptonic currents which couple to the would-be Goldstone modes is given by

$$L_{e^+e^-G_{(n)}} = g_G\,\overline{v}(p')\,\gamma^5 u(p) \quad \text{and} \quad L_{\mu^+\mu^-G_{(n)}} = g_G\,\overline{u}(p')\,\gamma^5 v(p)\,, \tag{5.25}$$

where g_G denotes the corresponding coupling constant in a given model of section 5.1. Comparing eq.(5.25) with eq.(5.23) and eq.(5.24) it is obvious that eq.(5.20) is equivalent to

$$i\,g_G = 2\,\frac{m_f}{\hat{m}_{(n)}}\,g_A\,. \tag{5.26}$$

At this point we have to specify a model. For a 1-Higgs model where the scalar fields live in the bulk the proof is rather trivial, therefore, we immediately investigate a model with one Higgs on a brane. Reading off g_A from eq.(5.15) or figure 5.1 and g_G from figure 5.2 we are left with

$$i\,e\,\frac{m_f}{m_A}\,\gamma^5\,\mathrm{E}^{(n)}_{\chi} = i\,e\,\frac{m_f}{\hat{m}_{(n)}}\,\gamma^5\left(\mathrm{E}^{(n)}_0 + \sqrt{2}\sum_{j=1}^{\infty}\mathrm{E}^{(n)}_j\right), \tag{5.27}$$

where we have suppressed the identical spinors on both sides. This is still a non-trivial identity where we have to insert the explicit expansion coefficients from eq.(4.50) and eq.(4.69) ($m_A = (ev)^2 = m^2$ in the notation of section 4.1.2), respectively, to proof it. Inserting the expansion coefficients and cancelling identical factors yields

$$\frac{\hat{m}_{(n)}}{m_A} = \frac{m_A}{\hat{m}_{(n)}} + 2\hat{m}_{(n)}\,m_A\sum_{j=1}^{\infty}\frac{1}{\hat{m}^2_{(n)} - (j/R)^2} \tag{5.28}$$

as the identity to proof. The final step to show the gauge invariance of the theory in this simple example is to exploit the transcendental equation (4.40) for the mass eigenvalues. An easy calculation shows that eq.(5.28) holds and, thus, the gauge invariance of the considered fermion scattering is insured.

We could go on and investigate the 2-Higgs model. Following the same line of arguments we can, again, proof gauge invariance. Only the algebra is much more tedious because the Goldstone vertex in the mass eigenbasis becomes more complicated than in the 1-Higgs example (see figure 5.3).

A completely analogous analysis can be done for the Z-boson modes and the corresponding neutral would-be Goldstone modes in electroweak theory. For the W-boson modes and the corresponding charged would-be Goldstone modes the analysis is only slightly different because the vertices of the W boson involve two different fermion species.

Chapter 6

Phenomenology in the 5D Standard Model

So far we have worked out the theoretical foundations of the standard model with one extra dimension. This model contains two additional free parameters, namely the compactification radius R and the angle β parameterizing the ratio of vacuum expectation values of the bulk and brane Higgs fields. This chapter is devoted to the the constraints on these parameters from low energy experiments or precision measurements of the Z-pole observables measured, for example, at LEP.

Generally, for a sufficiently large compactification radius R, one expects the predictions of the present theory to differ from predictions of the conventional 4D standard model. A compactification scale at the order of the electroweak scale (100 GeV) leading to large discrepancies to the very successful standard model is excluded by the existing data. On the other hand, for a sufficiently small compactification radius we expect correspondingly small deviations. Therefore, we will cast the results of the calculations for an observable $\mathcal{O}$ into small correction factors Δ^{HDSM} relating the standard model prediction $\mathcal{O}^{\mathrm{SM}}$ to the prediction $\mathcal{O}^{\mathrm{HDSM}}$ within the 5D standard model, i.e.

$$\mathcal{O}^{\mathrm{HDSM}} = \mathcal{O}^{\mathrm{SM}} \left(1 + \Delta^{\mathrm{HDSM}} \right) . \tag{6.1}$$

In order to extract the bounds on R and $\tan\beta$ from the existing data we will carry out a tree level analysis for all the observables taken into account. Knowing the tree level correction due to a large extra dimension we still have to include radiative corrections in order to be able to compare the predictions to actual measurements. It is beyond the scope of this thesis to implement the radiative corrections in the full higher dimensional theory. Instead, we will completely neglect the extra dimension in the determination of radiative corrections as a second order effect because its tree level contributions will turn out to be already small compared with radiative corrections calculated in the standard model framework. This is the type of analysis widely used in the literature (for a consistent treatment see, for example, [12]).

In section 6.1 we introduce the general framework of our calculations before we go on to extract the bounds from the various precision measurements.

6.1 General Framework

First of all, we have to carefully define the notation. As in the preceding chapters, the coupling constants g, g' and e, the weak mixing angle θ_w relating those and the mass parameters m_Z and m_W, introduced in section 4.2 (see eq.(4.108), eq.(4.109) and eq.(4.111)), denote the basic parameters of the Lagrangian. As in the conventional standard model only three of the parameters are independent such that there are several tree level relations between them. As we have already seen in section 4.1.2 (see for example eq.(4.51) or eq.(4.56)), even at tree level these parameters are in general not identical to any physical couplings and masses in the effective four dimensional theory because of the effects from the extra dimension.

The constants $\hat{g}_{(n)}$, $\hat{e}_{(n)}$, and $\hat{m}_{(n)}$ denote the coupling constants and masses of the corresponding n^{th} Kaluza-Klein mode which include all the tree level corrections due to the extra dimension. They still differ from the physical observables due to radiative corrections but at tree level they are the couplings and masses measured in experiment. For example, $\hat{m}_Z = \hat{m}_{Z(0)}$ and $\hat{m}_W = \hat{m}_{W(0)}$ denote the masses of the ground modes of the Z and W field which have to be identified with the observed Z and W boson masses, respectively.

We will now summarize the relations between the basic parameters of the Lagrangian and the tree level observables because they will be used in the following. Considering the $W-$ and Z-boson masses in a general 2-Higgs model, we have to generalize the result eq.(4.76) of the Abelian case investigated in section 4.1.3. As discussed in the following of eq.(4.127), this can be easily accomplished by replacing ev in eq.(4.76) with m_W or m_Z, respectively. Thus, we find

$$\begin{aligned} \hat{m}_W^2 &= m_W^2 \sin^2\beta \left(1 - \frac{\pi^2}{3} \sin^2\beta \, \frac{m_W^2}{M^2}\right) + m_W^2 \cos^2\beta \\ &= m_W^2 \left(1 - \frac{\pi^2}{3} \sin^4\beta \, \frac{m_W^2}{M^2}\right), \end{aligned} \tag{6.2}$$

where the equality has to be understood to first order in the parameter m_W^2/M^2. We will carry out the whole analysis in this approximation. Analogously, we find

$$\hat{m}_Z^2 = m_Z^2 \left(1 - \frac{\pi^2}{3} \sin^4\beta \, \frac{m_Z^2}{M^2} \right). \tag{6.3}$$

Reversely, one immediately obtains

$$\begin{aligned} m_W^2 &= \hat{m}_W^2 \left(1 + \frac{\pi^2}{3} \sin^4\beta \, \frac{\hat{m}_W^2}{M^2} \right), \\ m_Z^2 &= \hat{m}_Z^2 \left(1 + \frac{\pi^2}{3} \sin^4\beta \, \frac{\hat{m}_Z^2}{M^2} \right), \end{aligned} \tag{6.4}$$

to the order in $m_{W,Z}^2/M^2$ considered.

Similar considerations apply to the couplings. Generally, as in eq.(4.110), we will express the couplings of the massive electroweak vector bosons in terms of g or $\hat{g}_{W,Z(n)}$ and the weak mixing angle θ_w. The effects of the extra dimension on the couplings of the mass eigenstates are taken into account by $\hat{g}_{W,Z(n)}$ only. An effective mixing angle $\hat{\theta}_w$ will be defined below for other reasons. Generalizing the considerations leading to eq.(4.78) for the couplings $\hat{g}_{W,Z} = \hat{g}_{W,Z(0)}$ of the W and Z boson yields

$$\begin{aligned} \hat{g}_Z &= g \left(1 - \frac{\pi^2}{3} \sin^2\beta \, \frac{m_Z^2}{M^2} \right), \\ \hat{g}_W &= g \left(1 - \frac{\pi^2}{3} \sin^2\beta \, \frac{m_W^2}{M^2} \right) = g \left(1 - \frac{\pi^2}{3} \sin^2\beta \, \cos^2\theta_w \, \frac{m_Z^2}{M^2} \right). \end{aligned} \tag{6.5}$$

Note, that the couplings of the two vector bosons are different. In the second line we have used the tree level relation $m_W^2 = m_Z^2 \cos^2\theta_w$ being a direct consequence of eq.(4.108) and eq.(4.109). Again, we can invert the above relations to obtain

$$\begin{aligned} g &= \hat{g}_Z \left(1 + \frac{\pi^2}{3} \sin^2\beta \, \frac{\hat{m}_Z^2}{M^2} \right), \\ g &= \hat{g}_W \left(1 + \frac{\pi^2}{3} \sin^2\beta \, \cos^2\hat{\theta}_w \, \frac{\hat{m}_Z^2}{M^2} \right), \end{aligned} \tag{6.6}$$

where we have replaced $\cos^2\theta_w$ by $\cos^2\hat{\theta}_w$. This is correct to first order in $m_{W,Z}^2/M^2$ as can be seen below from eq.(6.16). The coupling $\hat{e} = \hat{e}_{(0)}$ of the photon is not affected by mixing such that we have

$$\hat{e} = e \,. \tag{6.7}$$

As we mentioned above, in addition to the compactification scale M and $\tan\beta$ the considered 5D electroweak theory contains three independent parameters to be measured in experiment as input for predictions of other observables. As in conventional electroweak theory one could choose g, g' and the vacuum expectation value v as these input parameters. However, one usually uses the best measured observables, namely the Fermi constant

G_F (measured in muon decay), the fine structure constant α and the Z-boson mass M_Z. Therefore, we have to investigate how the parameters of the considered 5D standard model discussed before are related to the input parameters, i.e. what is really measured in experiment. For α and M_Z we can immediately state the answer at tree level. We have

$$\alpha = \frac{\hat{e}^2}{4\pi} \quad \text{and} \quad M_Z = \hat{m}_Z \,. \tag{6.8}$$

For the Fermi constant G_F we have to work a little harder. In addition to the W boson all its Kaluza-Klein excitation can mediate the muon decay. Using the effective Lagrangian in which all the heavy gauge bosons, including the zero modes, are integrated out the Fermi constant G_F is given by

$$G_F = \frac{\hat{g}_W^2}{4\sqrt{2}\hat{m}_W^2} + \sum_{n=1}^{\infty} \frac{\hat{g}_{W(n)}^2}{4\sqrt{2}\hat{m}_{W(n)}^2} = \frac{\hat{g}_W^2}{4\sqrt{2}\hat{m}_W^2}\left(1 + \sum_{n=1}^{\infty} \frac{\hat{g}_{W(n)}^2}{\hat{g}_W^2}\frac{n^2M^2}{\hat{m}_{W(n)}^2}\frac{\hat{m}_W^2}{n^2M^2}\right). \tag{6.9}$$

This relation is derived in Appendix E. Because the second term in parenthesis is already of order $\hat{m}_W^2/M^2$ the corrections of this order to $\hat{g}_{W(n)}$ and $\hat{m}_{W(n)}^2$ (to be derived generalizing eq.(4.52) and eq.(4.58)) are negligible. Inserting $\hat{g}_{W(n)} \approx \sqrt{2}g \approx \sqrt{2}\hat{g}_W$ and $\hat{m}_{W(n)}^2 \approx n^2M^2$ finally yields

$$G_F = \frac{\hat{g}_W^2}{4\sqrt{2}\hat{m}_W^2}\left(1 + \sum_{n=1}^{\infty}\frac{2\hat{m}_W^2}{n^2M^2}\right) = \frac{\hat{g}_W^2}{4\sqrt{2}\hat{m}_W^2}\left(1 + \frac{\pi^2}{3}\frac{\hat{m}_W^2}{M^2}\right). \tag{6.10}$$

Using eq.(6.8) and eq.(6.10) it will be quite simple to write the predictions for precision measurements only in terms of measured input parameters as long as $\sin\theta_w$ is not involved. If $\sin\theta_w$ is involved a quite lengthy calculation has to be performed in order to relate it to the input parameters which will be done at this point for future reference. Using $g = e/\sin\theta_w$ (see eq.(4.109) and eq.(4.111)), eq.(6.5) and eq.(6.7) the Fermi constant can be rewritten as

$$G_F = \frac{\hat{e}^2}{4\sqrt{2}\sin^2\theta_w\,\hat{m}_W^2}\left(1 + \left(1 - 2\sin^2\beta\right)\frac{\pi^2}{3}\frac{\hat{m}_W^2}{M^2}\right). \tag{6.11}$$

While $m_W^2 = \cos^2\theta_w m_Z^2$ holds by virtue of eq.(4.108) and eq.(4.109) this is not true for the tree level masses of the W and Z boson. Instead we have

$$\hat{m}_W^2 = \cos^2\theta_w\hat{m}_Z^2\left(1 + \frac{\pi^2}{3}\sin^4\beta\,\sin^2\theta_w\frac{\hat{m}_Z^2}{M^2}\right), \tag{6.12}$$

where we have used eq.(6.2) and eq.(6.4). Plugging this result into eq.(6.11) we finally arrive at

$$G_F = \frac{\pi\alpha}{\sqrt{2}\sin^2\theta_w\,\cos^2\theta_w\,\hat{m}_Z^2}\left(1 + \cos^2\theta_w\left(1 - 2\sin^2\beta - \frac{\sin^2\theta_w}{\cos^2\theta_w}\sin^4\beta\right)\frac{\pi^2}{3}\frac{\hat{m}_Z^2}{M^2}\right), \tag{6.13}$$

where we have replaced $\hat{m}_W$ by $\hat{m}_Z \cos\theta_w$ in terms being already of order m_W^2/M^2 and used $\alpha = \hat{e}^2/4\pi$. At this point it is very convenient to define an effective angle $\hat{\theta}_w$ by

$$G_F \;=\; \frac{\pi\alpha}{\sqrt{2}\sin^2\hat{\theta}_w\,\cos^2\hat{\theta}_w\,\hat{m}_Z^2} \tag{6.14}$$

being related to θ_w via

$$\sin^2\hat{\theta}_w\,\cos^2\hat{\theta}_w \;=\; \sin^2\theta_w\,\cos^2\theta_w\left(1-\cos^2\theta_w\left(1-2\sin^2\beta-\frac{\sin^2\theta_w}{\cos^2\theta_w}\sin^4\beta\right)\frac{\pi^2}{3}\,\frac{\hat{m}_Z^2}{M^2}\right). \tag{6.15}$$

Solving for $\sin^2\hat{\theta}_w$ finally yields the relation

$$\sin^2\hat{\theta}_w \;=\; \sin^2\theta_w\left(1-\frac{\cos^4\theta_w}{\cos 2\theta_w}\left(1-2\sin^2\beta-\frac{\sin^2\theta_w}{\cos^2\theta_w}\sin^4\beta\right)\frac{\pi^2}{3}\,\frac{\hat{m}_Z^2}{M^2}\right) \tag{6.16}$$

or reversely

$$\sin^2\theta_w \;=\; \sin^2\hat{\theta}_w\left(1+\frac{\cos^4\hat{\theta}_w}{\cos 2\hat{\theta}_w}\left(1-2\sin^2\beta-\frac{\sin^2\hat{\theta}_w}{\cos^2\hat{\theta}_w}\sin^4\beta\right)\frac{\pi^2}{3}\,\frac{\hat{m}_Z^2}{M^2}\right), \tag{6.17}$$

again to first order in m_W^2/M^2. Therefore, we succeeded to express $\sin\theta_w$ in terms of input parameters only: Just use eq.(6.17) to express it in terms of the effective mixing angle being defined as a function of input parameters in eq.(6.14). The effective mixing angle $\hat{\theta}_w$ is related to the input parameters as the weak mixing angle in the standard model where eq.(6.14) is a well known tree level relation for θ_w. That means: If we write our predictions as introduced in eq.(6.1) it is sufficient to recover the standard model prediction $\mathcal{O}^{\rm SM}$ in terms of G_F, α, M_Z and $\sin\hat{\theta}_w$, where $\sin\hat{\theta}_w$ replaces $\sin\theta_w$ in the result of a standard model tree level analysis.

Beyond tree level, radiative corrections will further affect all the predicted observables. Radiative corrections due to standard model loops are taken into account simply by using the radiatively corrected standard model predictions as $\mathcal{O}^{\rm SM}$ in eq.(6.1) which can be found in the literature (see Appendix F). As mentioned before, all radiative corrections due to the extra dimension are neglected as a higher order effect. All the quantities in the correction term $\Delta^{\rm HDSM}$ due to the extra dimension are simply replaced by their measured values without introducing radiative corrections in our leading order analysis.

6.2 Bounds from Precision Measurements

In the following sections we will use the relations presented in section 6.1 to evaluate the deviation from the 4D standard model prediction of several precision observables in the

5D standard model. From that we will be able to restrict the possible parameter space for the compactification scale R and the angle β. The numerical values for the standard model predictions and precision measurements are taken from the Particle Data Group publications [31] and listed in Appendix F. We consider the prediction $\mathcal{O}^{\mathrm{HDSM}}$ to be compatible with the data $\mathcal{O}^{\mathrm{EXP}}$ at a significance level σ if

$$\sigma^2 > \frac{\left(\mathcal{O}^{\mathrm{EXP}} - \mathcal{O}^{\mathrm{HDSM}} \right)^2}{\left(\Delta\mathcal{O} \right)^2}, \tag{6.18}$$

where the error $\Delta\mathcal{O}$ is obtained by quadratically adding the theoretical and experimental errors given in Appendix F. In the literature the theoretical uncertainty is not always taken into account in this way. However, the effect on the bounds is not large if it is neglected because the experimental error is dominating in most cases anyway. Note, that the additional errors due to the approximations concerning the tree level analysis and the radiative corrections are not taken into account at all.

6.2.1 The Mass M_W of the W Boson

At tree level, the W boson mass M_W is given by $\hat{m}_W$. Expressing $\hat{m}_W$ in terms of input parameters we will now derive the corresponding tree level prediction. In eq.(6.12) we already found a relation between $\hat{m}_W$ and the input parameter $\hat{m}_Z$. Using eq.(6.17) one can easily derive

$$\hat{m}_W^2 = m_W^{\mathrm{SM}\,2} \left(1 + \left[\mathrm{s}^4\beta\ \mathrm{s}^2\hat{\theta}_w - \frac{\mathrm{c}^2\hat{\theta}_w\, \mathrm{s}^2\hat{\theta}_w}{\mathrm{c}2\hat{\theta}_w} \left(1 - 2\mathrm{s}^2\beta - \frac{\mathrm{s}^2\hat{\theta}_w}{\mathrm{c}^2\hat{\theta}_w} \mathrm{s}^4\beta \right) \right] \frac{\pi^2}{3} \frac{\hat{m}_Z^2}{M^2} \right), \tag{6.19}$$

where we have introduced the standard model tree level prediction for the W-boson mass $m_W^{\mathrm{SM}\,2} = \hat{m}_Z^2 \cos^2\hat{\theta}_w$. Note, that we have to use $\hat{m}_Z^2$ and $\hat{\theta}_w$ on the right hand side: In a tree level analysis, the Z-boson mass is an input parameter itself and $\hat{\theta}_w$ is given in terms of G_F, α and m_Z^{ph} by eq.(6.14) as in the conventional standard model. Including radiative corrections in the used approximation makes $m_W^{\mathrm{SM}\,2}$ deviate from its numerical value at tree level. The result for the radiatively corrected standard model prediction can be taken from Appendix F.

Having eq.(6.19) at hand, it is easy to scan the parameters $M = 1/R$ and $\sin^2\beta$ for values being compatible with the precision measurements at a given significance level according to eq.(6.18).

Figure 6.1 shows which parameters are allowed by the precision data on the W-boson mass for the significance levels 2σ, 3σ and 5σ. For $\sin^2\beta = 0$, i.e. a single relevant Higgs living in the bulk, the compactification scale has to be larger than 2.45 TeV at the 2σ-level. For

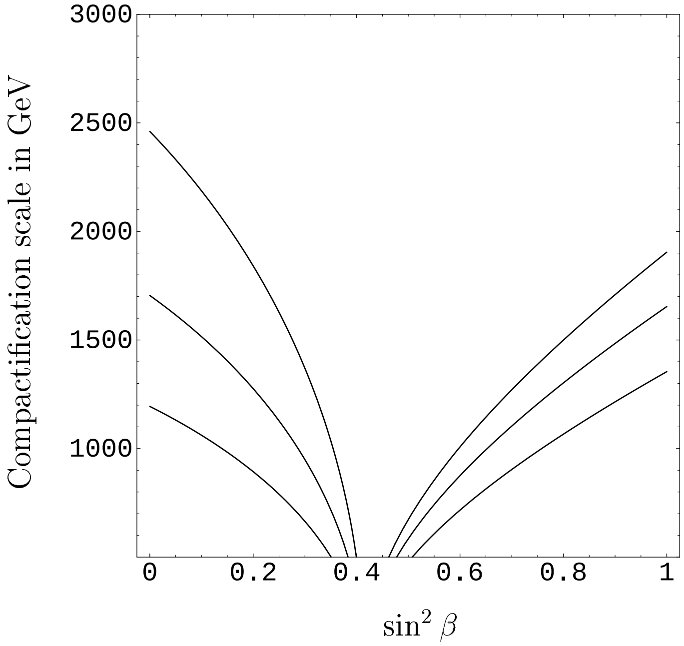


Figure 6.1: The parameter space below the curves is excluded by precision measurements of the W-boson mass at a significance level of 2, 3 and 5σ from the top to the bottom curve. For intermediate values of $\sin^2\beta$ the expansion in m_Z^2/M^2 becomes meaningless because the approximations used in the calculation break down.

a brane Higgs ($\sin^2\beta = 1$) the bound is slightly smaller ($M > 1.9$TeV). For intermediate $\sin^2\beta$ the corrections due to the extra dimension become smaller and the bounds become so close to the weak scale that the approximation to work in first order in m_Z^2/M^2 becomes questionable. In this region one would have to take higher order effects into account. Nevertheless, we will see below that other observables restrict the compactification scale to be large even in the intermediate region such that we do not have to worry about that too much.

6.2.2 The Invisible Z Width $\Gamma_Z(\nu\overline{\nu})$

In this section we will investigate another precisely measured observable, namely the invisible width of the Z boson, or to be more precise, the invisible width of the zero mode of the Z-boson Kaluza-Klein tower to be identified with the standard model vector boson. In this context invisible width means the partial width due to decays into neutrinos which cannot be observed in standard collider experiments. As for $\hat{m}_W$ we will carry out a tree level analysis and express the invisible width in terms of the input parameters of the theory.

In section 2.7 we calculated the decay of a massive vector boson into fermions in a specific model. In electroweak theory the analysis becomes a little more complicated because of the presence of axial vector couplings. We will redo the analysis of section 2.7 in a generic way which can also be used in sections 6.2.3 and 6.2.4. The direct generalization of the decay amplitude eq.(2.64) to a general vector axial vector coupling is given by

$$i\,\mathcal{M} \;=\; \epsilon_\mu(q)\,\overline{u}^s(k)\left(i\,\gamma^\nu\left(\hat{g}_V+\hat{g}_A\gamma^5\right)\right)v^{s'}(k')\,. \tag{6.20}$$

Squaring the amplitude, averaging over the initial polarizations and summing over the fermion spins as in section 2.7 we arrive at

$$\begin{aligned}|\mathcal{M}|^2 &= -\frac{1}{3}\,g_{\mu\nu}\,\mathrm{Tr}\big[\,\gamma^\nu\left(\hat{g}_V+\hat{g}_A\gamma^5\right)\not{p}\;\gamma^\nu\left(\hat{g}_V+\hat{g}_A\gamma^5\right)\not{p}'\,\big]\\ &= -\frac{4}{3}\,g_{\mu\nu}\big[\left(\hat{g}_V^2+\hat{g}_A^2\right)\left(p^\mu p'^\nu-g^{\mu\nu}(pp')+p^\nu p'^\mu\right)+2\,i\,\hat{g}_V\hat{g}_A\,p_\alpha p'_\beta\,\epsilon^{\mu\alpha\nu\beta}\,\big].\end{aligned} \tag{6.21}$$

The term proportional to $\epsilon^{\mu\alpha\nu\beta}$ does not contribute because $g_{\mu\nu}\epsilon^{\mu\alpha\nu\beta}=0$. In the rest frame (see eq.(2.67)) of the decaying particle eq.(6.21) can be restated as

$$|\mathcal{M}|^2 \;=\; \frac{4}{3}\left(\hat{g}_V^2+\hat{g}_A^2\right)\hat{m}_Z^2\,, \tag{6.22}$$

where we have worked in the limit of massless decay products. Including the appropriate phase space factor the partial width for the decay into two specific fermions is given by

$$\Gamma_Z \;=\; \frac{1}{12\pi}\left(\hat{g}_V^2+\hat{g}_A^2\right)\hat{m}_Z\,. \tag{6.23}$$

To calculate the invisible width we have to specify the coupling of the Z boson to neutrinos. Using

$$\hat{g}_V^2 \;=\; \hat{g}_A^2 \;=\; \frac{\hat{g}_Z^2}{16\cos^2\theta_w} \tag{6.24}$$

(see Appendix D) the invisible Z width at tree level is given by

$$\Gamma_Z(\nu\overline{\nu}) \;=\; \frac{1}{32\pi}\,\frac{\hat{g}_Z^2}{\cos^2\theta_w}\,\hat{m}_Z\,, \tag{6.25}$$

where a factor of three is due to the three neutrino species. It is the next step in the analysis to express the invisible width in terms of the input parameters G_F, α and $\hat{m}_Z$. We can use eq.(6.5), $g=e/\sin\theta_w$ and eq.(6.7) to derive

$$\Gamma_Z(\nu\overline{\nu}) \;=\; \frac{1}{32\pi}\,\frac{\hat{e}^2}{\sin^2\theta_w\,\cos^2\theta_w}\,\hat{m}_Z\left(1-2\,\sin^2\beta\,\frac{\pi^2}{3}\frac{\hat{m}_Z^2}{M^2}\right). \tag{6.26}$$

However, only the angle $\hat{\theta}_w$ is directly related to the input parameters such that we have to use eq.(6.15) to replace θ_w in eq.(6.26). This yields

$$\Gamma_Z(\nu\overline{\nu}) \;=\; \frac{1}{8}\,\frac{\alpha}{\sin^2\hat{\theta}_w\,\cos^2\hat{\theta}_w}\,\hat{m}_Z\left(1+\left[\sin^2\hat{\theta}_w\left(\sin^2\beta-1\right)^2-1\right]\frac{\pi^2}{3}\frac{\hat{m}_Z^2}{M^2}\right) \tag{6.27}$$

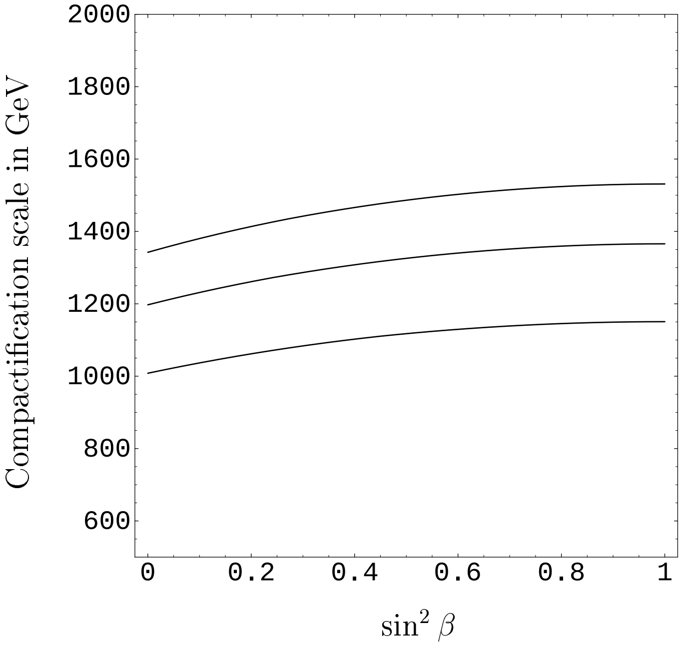


Figure 6.2: The 2, 3 and 5σ bounds on the compactification scale (from top to bottom) from precision measurements of the invisible Z width.

and can be easily rewritten only in terms of input parameters as

$$\Gamma_Z(\nu\overline{\nu}) \;=\; \frac{\sqrt{2}}{8\pi}\, G_F\, \hat{m}_Z^3 \left(1 \,+\, \left[\sin^2\hat{\theta}_w \left(\sin^2\beta - 1 \right)^2 - 1 \right] \frac{\pi^2}{3} \frac{\hat{m}_Z^2}{M^2} \right), \tag{6.28}$$

where we have used eq.(6.14).

While the factor in parenthesis parameterizes the effect of an extra dimension, the prefactor is the tree level standard model prediction expressed in terms of the input parameters. According to the scheme of the calculation, including radiative corrections only effects the standard model prediction and we can cast the prediction of the invisible Z width in a theory with an extra dimension into the form

$$\Gamma_Z(\nu\overline{\nu}) \;=\; \Gamma_Z^{\rm SM}(\nu\overline{\nu}) \left(1 \,+\, \left[\sin^2\hat{\theta}_w \left(\sin^2\beta - 1 \right)^2 - 1 \right] \frac{\pi^2}{3} \frac{\hat{m}_Z^2}{M^2} \right). \tag{6.29}$$

As for the analysis of the W-boson mass in section 6.2.1 we can use the numerical values of the standard model prediction and the experimental width given in Appendix F to find the compactification scales compatible with experiment for a given value of $\sin\beta$. They are shown in figure 6.2. For a bulk Higgs ($\sin\beta = 0$) the compactification scale has to be larger than 1.35 TeV at the 2σ-level, for a brane Higgs the bound is approximately $M > 1.55$ TeV. In contrast to the analysis of the W-boson mass, the invisible Z width restricts the compactification scale for all values of $\sin^2\beta$.

In order to check the consistency of the calculations we can recalculate eq.(6.29) in a completely different way which does not involve the effective mixing angle at all. Starting from eq.(6.25) we use eq.(6.5) to rewrite the invisible width as

$$\Gamma_Z(\nu\overline{\nu}) \;=\; \frac{1}{32\pi}\,\frac{\hat{g}_W^2}{\cos^2\theta_w}\,\hat{m}_Z\left(1 - 2\,\sin^2\beta\,\sin^2\hat{\theta}_w\,\frac{\pi^2}{3}\frac{\hat{m}_Z^2}{M^2}\right), \tag{6.30}$$

because G_F is directly related to $\hat{g}_W$. Using the definition of the Fermi constant at tree level (see eq.(6.10)) yields

$$\Gamma_Z(\nu\overline{\nu}) \;=\; \frac{1}{32\pi}\,\frac{4\sqrt{2}\,\hat{m}_W^2\,G_F}{\cos^2\theta_w}\,\hat{m}_Z\left(1 - \left(\cos^2\hat{\theta}_w \,+\, 2\,\sin^2\beta\,\sin^2\hat{\theta}_w\right)\frac{\pi^2}{3}\frac{\hat{m}_Z^2}{M^2}\right). \tag{6.31}$$

Furthermore, we can express the W-boson mass in terms of the Z-boson mass using eq.(6.12). This finally yields

$$\begin{aligned}\Gamma_Z(\nu\overline{\nu}) &= \frac{\sqrt{2}}{8\pi}\,G_F\,\hat{m}_Z^3\left(1 - \left(\cos^2\hat{\theta}_w \,+\, 2\,\sin^2\beta\,\sin^2\hat{\theta}_w \,-\, \sin^4\beta\,\sin^2\hat{\theta}_w\right)\frac{\pi^2}{3}\frac{\hat{m}_Z^2}{M^2}\right)\\ &= \frac{\sqrt{2}}{8\pi}\,G_F\,\hat{m}_Z^3\left(1 \,+\, \left[\sin^2\hat{\theta}_w\left(\sin^2\beta - 1\right)^2 - 1\right]\frac{\pi^2}{3}\frac{\hat{m}_Z^2}{M^2}\right)\end{aligned} \tag{6.32}$$

and, thus, the prediction is indeed consistent with the result eq.(6.29).

6.2.3 The Leptonic Z Width $\Gamma_Z(l^+l^-)$

The investigation of the leptonic Z width, i.e. the partial width of the Z boson decaying into lepton-anti-lepton pairs, is quite analogous with the analysis of the invisible width in section 6.2.2. There is only one further complication because the interaction of the Z boson with charged leptons is not described by a pure V-A-coupling. Therefore, the weak mixing angle explicitly comes into play, a fact which has not been appropriately dealt with in the literature [8, 9, 10], so far.

Substituting the couplings to charged leptons (see Appendix D)

$$\hat{g}_V^2 \;=\; \frac{\hat{g}_Z^2}{16\cos^2\theta_w}\left(-1 + 4\sin^2\theta_w\right)^2 \qquad \text{and} \qquad \hat{g}_A^2 \;=\; \frac{\hat{g}_Z^2}{16\cos^2\theta_w} \tag{6.33}$$

in eq.(6.23), one obtains in analogy with the calculation in section 6.2.2

$$\Gamma_Z(l^+l^-) = \frac{\sqrt{2}}{16\pi}G_F\,\hat{m}_Z^3\left(\left(-1 + 4\mathrm{s}^2\theta_w\right)^2 + 1\right)\left(1 + \left[\mathrm{s}^2\hat{\theta}_w\left(\mathrm{s}^2\beta - 1\right)^2 - 1\right]\frac{\pi^2}{3}\frac{\hat{m}_Z^2}{M^2}\right). \tag{6.34}$$

According to our previous discussions, the tree level standard model prediction for this observable is given by

$$\Gamma_Z^{\mathrm{SM}}(l^+l^-) = \frac{\sqrt{2}}{16\pi}G_F\,\hat{m}_Z^3\left(\left(-1 + 4\mathrm{s}^2\hat{\theta}_w\right)^2 + 1\right). \tag{6.35}$$

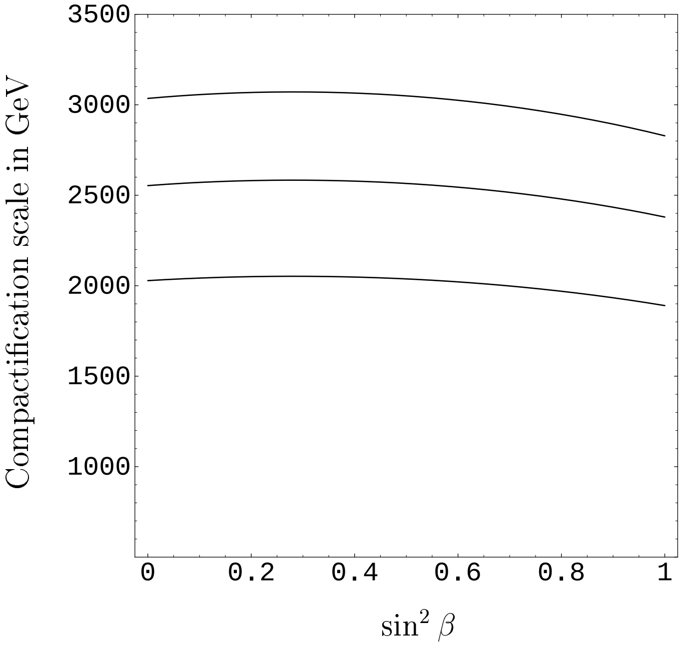


Figure 6.3: The 2, 3 and 5σ bounds on the compactification scale (from top to bottom) from precision measurements of the leptonic Z width.

Hence, eq.(6.34) can be written as

$$\Gamma_Z(l^+l^-) = \Gamma_Z^{\rm SM}(l^+l^-)\,\frac{(-1+4{\rm s}^2\theta_w)^2+1}{(-1+4{\rm s}^2\hat\theta_w)^2+1}\left(1+\left[{\rm s}^2\hat\theta_w\left({\rm s}^2\beta-1\right)^2-1\right]\frac{\pi^2}{3}\frac{\hat m_Z^2}{M^2}\right). \quad (6.36)$$

Again using eq.(6.17), a short calculation finally yields the correct correction of the standard model prediction due to an extra dimension

$$\Gamma_Z(l^+l^-) = \Gamma_Z^{\rm SM}(l^+l^-)\left(1+(\Delta_{\rm u}+\Delta_{\rm l})\frac{\pi^2}{3}\frac{\hat m_Z^2}{M^2}\right), \quad (6.37)$$

where $\Delta_{\rm u}$ parameterizes the deviation from the standard model prediction which is universal for all partial Z widths while $\Delta_{\rm l}$ parameterizes the specific corrections for the decay into charged leptons. $\Delta_{\rm u}$ and $\Delta_{\rm l}$ are given by

$$\begin{aligned}\Delta_{\rm u} &= {\rm s}^2\hat\theta_w\left({\rm s}^2\beta-1\right)^2-1\,,\\ \Delta_{\rm l} &= 8{\rm s}^2\hat\theta_w\,\frac{-1+4{\rm s}^2\hat\theta_w}{(-1+4{\rm s}^2\hat\theta_w)^2+1}\,\frac{{\rm c}^4\hat\theta_w}{{\rm c}^2 2\hat\theta_w}\left(1-2{\rm s}^2\beta-\frac{{\rm s}^2\hat\theta_w}{{\rm c}^2\hat\theta_w}{\rm s}^2\beta\right),\end{aligned} \quad (6.38)$$

where we have again neglected terms of order $\hat m_Z^4/M^4$ throughout the calculation. Eq.(6.37) finally allows us to extract bounds on the compactification scale as a function of $\sin\beta$ from the precision measurement of the leptonic width. Figure 6.3 shows that compactification

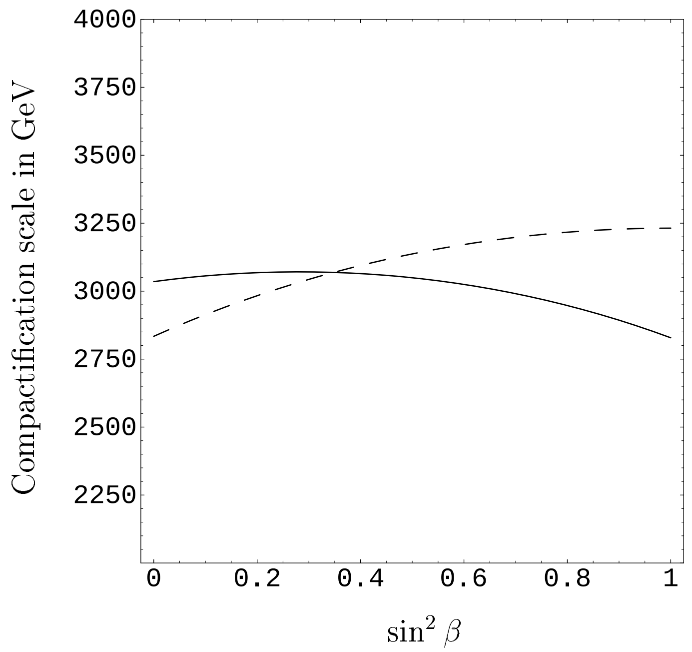


Figure 6.4: 2σ bounds on the compactification scale from the leptonic Z width where either Δ_u and Δ_l (full) or only Δ_u (dashed) are taken into account.

scales below 2-3 TeV are excluded. Moreover, the dependence on $\sin\beta$ is not very strong such that a restrictive bound on the compactification scale is found in a model independent way. The bounds are large enough with respect to the weak scale such that the approximation to expand the results to order $\hat{m}_Z^2/M^2$ seems to be appropriate.

The correction parameterized by Δ_l which has been omitted in the literature, so far, is not only important for consistency, it is also of numerical importance as can be seen in figure 6.4. The bounds differ up to 0.4 TeV if Δ_l is not taken into account. The bound on the compactification scale is slightly increased in the correct approach for a vacuum expectation value dominated by a bulk Higgs. In the region where $\sin\beta$ is large it is decreased.

6.2.4 The Hadronic Z Width $\Gamma_Z(\mathrm{had})$

Investigating the hadronic Z width, being the sum over all the partial widths of the Z boson decaying into the kinematically allowed quark-anti-quark pairs, follows exactly the same line of arguments as the investigation of the leptonic width. There is only one slight modification because the quark-anti-quark pairs couple differently to the Z boson due to their different isospin and charge quantum numbers. While the couplings of the up- and

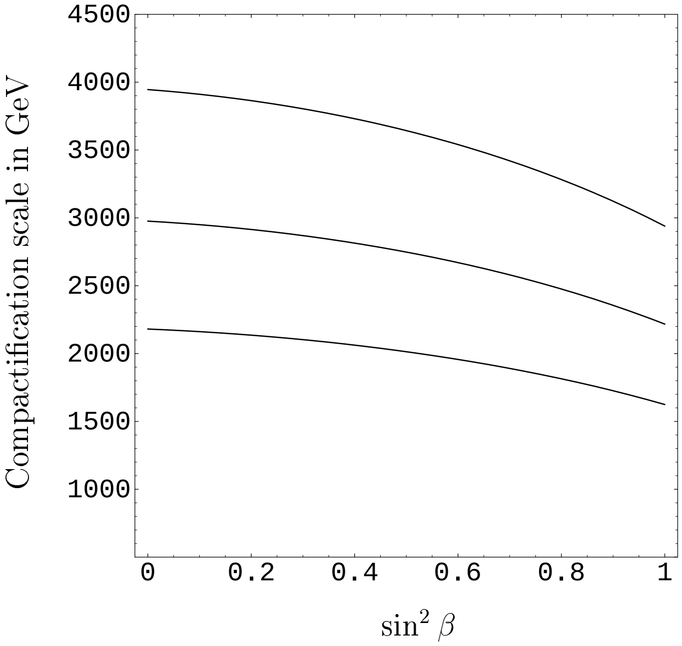


Figure 6.5: The 2, 3 and 5σ bounds on the compactification scale (from top to bottom) from precision measurements of the hadronic Z width.

the charm-quark are given by (see Appendix D)

$$\hat{g}_V^2 = \frac{\hat{g}_Z^2}{16\cos^2\theta_w}\left(1 - \frac{8}{3}\sin^2\theta_w\right)^2 \qquad \text{and} \qquad \hat{g}_A^2 = \frac{\hat{g}_Z^2}{16\cos^2\theta_w}\,, \tag{6.39}$$

the down-, strange- and bottom-quarks couple according to

$$\hat{g}_V^2 = \frac{\hat{g}_Z^2}{16\cos^2\theta_w}\left(-1 + \frac{4}{3}\sin^2\theta_w\right)^2 \qquad \text{and} \qquad \hat{g}_A^2 = \frac{\hat{g}_Z^2}{16\cos^2\theta_w}\,. \tag{6.40}$$

From these couplings the analogue of eq.(6.36) for the hadronic width can be easily calculated. It is given by

$$\Gamma_Z(\text{had}) = \Gamma_Z^{\text{SM}}(\text{had})\,\frac{2\big[\,(1-\frac{8}{3}\,\mathrm{s}^2\theta_w)^2+1\,\big]+3\big[\,(-1+\frac{4}{3}\,\mathrm{s}^2\theta_w)^2+1\,\big]}{2\Big[\,(1-\frac{8}{3}\,\mathrm{s}^2\hat{\theta}_w)^2+1\,\Big]+3\Big[\,(-1+\frac{4}{3}\,\mathrm{s}^2\hat{\theta}_w)^2+1\,\Big]}\left(1+\Delta_{\mathrm{u}}\frac{\pi^2}{3}\frac{\hat{m}_Z^2}{M^2}\right), \tag{6.41}$$

where the factors of 2 and 3 are due to the different number of quark-anti-quark pairs with given couplings the Z boson can decay into. Recasting this result into the form of eq.(6.37)

$$\Gamma_Z(\text{had}) = \Gamma_Z^{\text{SM}}(\text{had})\left(1+(\,\Delta_{\mathrm{u}}+\Delta_{\mathrm{h}}\,)\frac{\pi^2}{3}\frac{\hat{m}_Z^2}{M^2}\right), \tag{6.42}$$

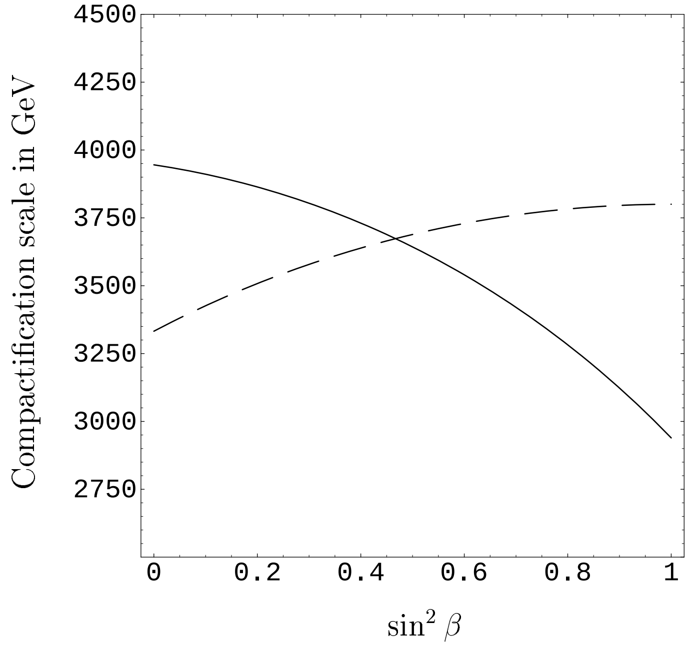


Figure 6.6: 2σ bounds on the compactification scale from the hadronic Z width where either Δ_{u} and Δ_{h} (full) or only Δ_{u} (dashed) are taken into account.

where Δ_{u} is given in eq.(6.38), we can derive the specific corrections Δ_{h} for the hadronic width using again eq.(6.17). After some algebra we finally find

$$\Delta_{\mathrm{h}} = \mathrm{s}^2\hat{\theta}_w \, \frac{-(56/3) + (352/9)\,\mathrm{s}^2\hat{\theta}_w}{10 - (56/3)\,\mathrm{s}^2\hat{\theta}_w + (176/9)\,\mathrm{s}^4\hat{\theta}_w} \, \frac{\mathrm{c}^4\hat{\theta}_w}{\mathrm{c}^2 2\hat{\theta}_w} \left(1 - 2\mathrm{s}^2\beta - \frac{\mathrm{s}^2\hat{\theta}_w}{\mathrm{c}^2\hat{\theta}_w}\mathrm{s}^2\beta \right). \qquad (6.43)$$

Comparing the prediction with the precision measurements of the hadronic Z width we can extract the bounds on the compactification scale shown in figure 6.5. It decreases from 4.0 to 3.0 TeV for increasing $\sin\beta$ and is even stronger than the bound previously found due to the leptonic Z width.

Figure 6.6 again compares our result to the ones previously found in the literature where the specific hadronic correction Δ_{h} is omitted. The deviations are similar to those encountered in section 6.2.3. Their numerical value is even more drastic and almost reaches the 1 TeV level.

6.2.5 The Weak Charge of Cesium

So far, we have considered observables accessible by high energy experiments. In this section we will discuss a low energy observable, the so-called weak charge which is most accurately known for the cesium atom. It is a measure for atomic parity violation due to the exchange of vector bosons with axial couplings between the electrons and the nucleus in a given atom. Within the standard model the weak charge is only due to Z-boson exchange. Below we investigate the predictions in the presence of a compactified extra dimension.

Because the momentum transfer in atomic processes is absolutely negligible compared with the heavy Z mass and the masses of the corresponding Kaluza-Klein modes, all the massive degrees of freedom can be integrated out (see Appendix E). Within the standard model, the part of the effective four-fermion Lagrangian important for atomic parity violation is given by [32]

$$\mathcal{L}_{\mathrm{APV}} = \frac{G_F}{\sqrt{2}} \sum_{q=u,d} \left[C_{1q} \left(\overline{\Psi}_e \gamma_\mu \gamma^5 \Psi_e \right) \left(\overline{\Psi}_q \gamma^\mu \Psi_q \right) + C_{2q} \left(\overline{\Psi}_e \gamma_\mu \Psi_e \right) \left(\overline{\Psi}_q \gamma^\mu \gamma^5 \Psi_q \right) \right], \qquad (6.44)$$

where we sum over the up and down quarks present in the nucleus. The coefficients of the four-fermion interaction terms can be easily derived from the Z-boson interactions (see Appendix D). They are given by

$$C_{1q} \;=\; -T_{3q} + 2\, Q_q \sin^2\theta_w \qquad \text{and} \qquad C_{2q} \;=\; T_{3q}\left(-1 + 4\, \sin^2\theta_w \right), \qquad (6.45)$$

where the isospin quantum number and the charges are given by $T_{3u} = 1/2$, $T_{3d} = -1/2$ and $Q_u = +2/3$, $Q_d = -1/3$, respectively. In terms of these quantities the weak charge Q_w of an atom with P protons and N neutrons is defined at tree level as [31, 32]

$$Q_w(P, N) \;=\; -\,2\left[(\,2P + N\,)C_{1u} \;+\; (\,P + 2N\,)C_{1d} \right]. \qquad (6.46)$$

Observable effects due to atomic parity violation (e.g. a transition rate for an atomic transition forbidden by parity conservation) are calculated from a generic starting point, i.e. the Lagrangian eq.(6.44), in terms of $Q_w(P, N)$.

By rewriting the Lagrangian of the 5D standard model into the form of eq.(6.44) the deviations from the standard model predictions of Q_w can be parameterized in terms of corrections to the coefficients C_{1q}. The natural starting point of the analysis is the Lagrangian where all the massive gauge boson modes are integrated out. The generic form of this Lagrangian is given in eq.(E.5) of Appendix E. We only have to specify it to the exchange of the Z boson and its Kaluza-Klein modes between the atomic electrons and the quarks inside the nucleus (see Appendix D). The result is given by

$$\begin{aligned} \mathcal{L}_{\mathrm{APV}} = & \frac{\hat{g}_Z^2}{8\, \hat{m}_Z^2 \cos^2\theta_w} \left(1 + \frac{\pi^2}{3} \frac{\hat{m}_Z^2}{M^2} \right) \\ & \left[C_{1q} \left(\overline{\Psi}_e \gamma_\mu \gamma^5 \Psi_e \right) \left(\overline{\Psi}_q \gamma^\mu \Psi_q \right) + C_{2q} \left(\overline{\Psi}_e \gamma_\mu \Psi_e \right) \left(\overline{\Psi}_q \gamma^\mu \gamma^5 \Psi_q \right) \right], \end{aligned} \qquad (6.47)$$

where we have used the same approximations leading to eq.(6.10) to sum up the contribution of all the Kaluza-Klein modes. Note, that the coefficients C_{1q} and C_{2q} are identical

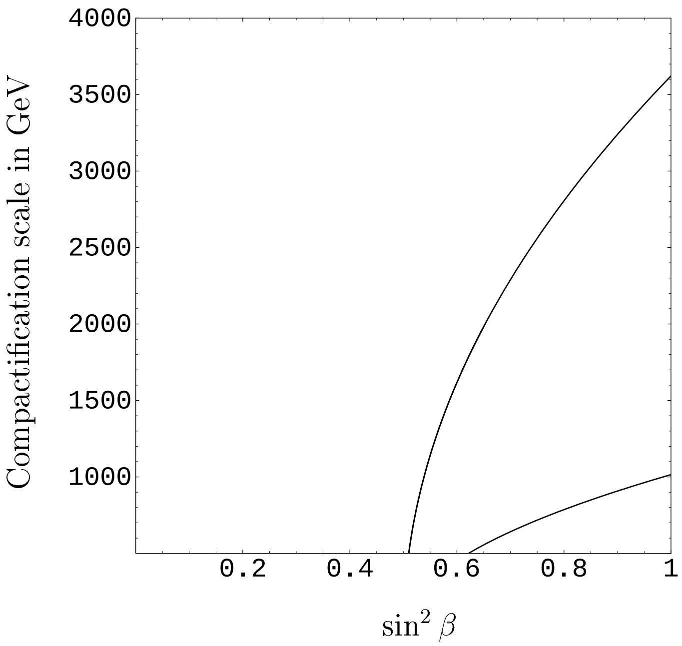


Figure 6.7: 2σ bounds on the compactification scale from the weak charge of cesium. Only the parameter space between the two curves is compatible with the data. Small $\sin\beta$ are completely excluded at the 2σ-level.

for all Kaluza-Klein modes because all the dependence of the coupling on a given Kaluza-Klein mode is absorbed in the definition of $\hat{g}_{W(n)}$. As in eq.(6.44), it is our goal to express the couplings in terms of the Fermi constant defined for the higher dimensional model in eq.(6.10). We thus have to express $\hat{g}_Z$ and $\hat{m}_Z$ in terms of $\hat{g}_W$ and $\hat{m}_W$. Using eq.(6.4) and eq.(6.6) and the tree level relation $m_W^2 = m_Z^2 \cos^2\theta_w$ one can derive

$$\frac{\hat{g}_Z^2}{16\,\hat{m}_Z^2 \cos^2\theta_w}\left(1+\frac{\pi^2}{3}\frac{\hat{m}_Z^2}{M^2}\right) = \frac{\hat{g}_W^2}{16\,\hat{m}_W^2}\left(1+\left(1-2\,\mathrm{s}^2\beta\,\mathrm{s}^2\hat{\theta}_w+\mathrm{s}^4\beta\,\mathrm{s}^2\hat{\theta}_w\right)\frac{\pi^2}{3}\frac{\hat{m}_Z^2}{M^2}\right). \tag{6.48}$$

Therefore, eq.(6.47) can be rewritten in terms of the Fermi constant (see eq.(6.10)) as

$$\begin{aligned}\mathcal{L}_{\mathrm{APV}} = &\frac{G_F}{\sqrt{2}}\left(1+\left(\mathrm{s}^2\hat{\theta}_w-2\,\mathrm{s}^2\beta\,\mathrm{s}^2\hat{\theta}_w+\mathrm{s}^4\beta\,\mathrm{s}^2\hat{\theta}_w\right)\frac{\pi^2}{3}\frac{\hat{m}_Z^2}{M^2}\right)\\ &\times\left[C_{1q}\left(\overline{\Psi}_e\gamma_\mu\gamma^5\Psi_e\right)\left(\overline{\Psi}_q\gamma^\mu\Psi_q\right)+C_{2q}\left(\overline{\Psi}_e\gamma_\mu\Psi_e\right)\left(\overline{\Psi}_q\gamma^\mu\gamma^5\Psi_q\right)\right].\end{aligned} \tag{6.49}$$

At this point it is convenient to define the quantities

$$\hat{C}_{iq} = C_{iq}\left(1+\left(1-\mathrm{s}^2\beta\right)^2\,\mathrm{s}^2\hat{\theta}_w\,\frac{\pi^2}{3}\frac{\hat{m}_Z^2}{M^2}\right), \tag{6.50}$$

where $i = 1, 2$. These are the coefficients in the effective Lagrangian describing atomic parity violation in the presence of a large extra dimension. Recalling the definition of the

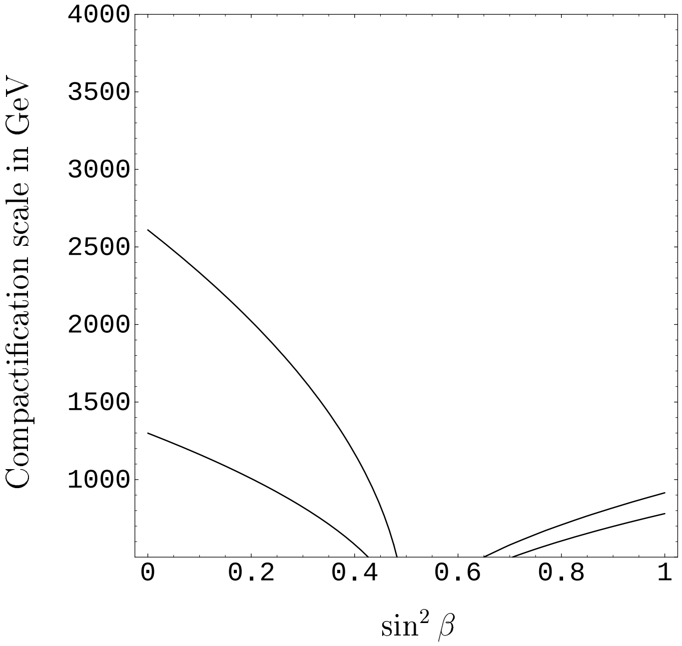


Figure 6.8: The 3 and 5σ bounds on the compactification scale (from top to bottom) from precision measurements of the weak charge of cesium.

weak charge eq.(6.46) and using the result eq.(6.50) we are left with the relation

$$Q_w = 2\big[\,(\,2P + N\,)\big(\,T_{3u} - 2\,Q_u\,\sin^2\theta_w\,\big) + (\,P + 2N\,)\big(\,T_{3d} - 2\,Q_d\,\sin^2\theta_w\,\big)\,\big] \\ \times\left(1 + \big(\,1 - \mathrm{s}^2\beta\,\big)^2\,\mathrm{s}^2\hat{\theta}_w\,\frac{\pi^2}{3}\frac{\hat{m}_Z^2}{M^2}\right). \tag{6.51}$$

To factorize out the standard model prediction we finally have to replace the angle θ_w by the effective weak mixing angle $\hat{\theta}_w$. Using eq.(6.17) a short calculation finally yields

$$Q_w = Q_w^{\mathrm{SM}}\left(1 + \left[\big(1 - \mathrm{s}^2\beta\big)^2 - 4\,P\,\big(\,Q_w^{\mathrm{SM}}\,\big)^{-1}\frac{\mathrm{c}^4\hat{\theta}_w}{\mathrm{c}2\hat{\theta}_w}\left(1 - 2\mathrm{s}^2\hat{\theta}_w - \frac{\mathrm{s}^2\hat{\theta}_w}{\mathrm{c}2\hat{\theta}_w}\mathrm{s}^4\beta\right)\right]\mathrm{s}^2\hat{\theta}_w\,\frac{\pi^2}{3}\frac{\hat{m}_Z^2}{M^2}\right), \tag{6.52}$$

where we have inserted the weak isospin and charge quantum numbers of the quarks. In the standard model, at tree level, the weak charge is given by $Q_w^{\mathrm{SM}} = (\,P - N\,) - 4\,P\,\mathrm{s}^2\hat{\theta}_w$.

Let us now extract the bounds on the compactification scale from the precision measurements of the weak charge. For cesium we have $P = 55$ and $N = 78$, the measured value and the standard model prediction are given in Appendix F. We use the radiatively corrected standard model prediction only in the prefactor and keep the tree level relations within the correction factor, as we did for all other observables we have considered, so far. The weak charge is of special interest not only because it is precisely measured in the cesium system but also because it deviates by more than two standard deviations from the standard model prediction. For large $\sin\beta$ the effects from a compactified extra dimension actually improve the agreement between theory and experiment. Agreement at the 2σ level favors large $\sin\beta$, i.e. a vacuum expectation value dominated by a brane Higgs field, and a compactification scale in the range shown in figure 6.7. For small $\sin\beta$ the deviation from the measured weak charge increases in comparison to the standard model case. At $\sin\beta = 1$, only a compactification scale between 1.0 and 3.6 TeV makes theory and data compatible at the 2σ level. Nevertheless, the significance of these considerations should not be overestimated. In section 6.3 we will analyze all the observables in a global fit and no upper bound on the compactification scale will be found.

At a higher significance level, like 3σ or 5σ, the standard model is compatible with the data and, again, we can only extract lower bounds on the compactification scale as shown in figure 6.8. Because there is still a large error in the measured weak charge (partly due to theoretical uncertainties within the calculation of the weak charge in terms of the measured transition rates), the bounds on the compactification scale at the 3σ-level are not larger than 2.6 TeV even for small $\sin\beta$. For large $\sin\beta$ the bounds are so close to the weak scale that the approximations used in the analysis become questionable.

6.3 Global Analysis

In the previous sections we investigated one observable at a time. In this section we will finally perform a global analysis [33]. Searching for the best fit values and the bounds on the compactification scale we define χ^2 as usual, i.e.

$$\chi^2 = \sum_i \frac{\left(\mathcal{O}_i^{\mathrm{EXP}} - \mathcal{O}_i^{\mathrm{HDSM}} \right)^2}{\left(\Delta\mathcal{O}_i \right)^2}, \tag{6.53}$$

where $\mathcal{O}_i^{\mathrm{EXP}}$ denotes the experimentally measured quantities, $\mathcal{O}_i^{\mathrm{HDSM}}$ the theoretical predictions of the 5D standard model and $\Delta\mathcal{O}_i$ the errors in the observables. The index i runs over all the observables included in the global analysis. As mentioned before, the errors $\Delta\mathcal{O}_i$ used in the analysis are calculated by quadratically adding the experimental error and the error in the theoretical prediction given in Appendix F. No additional errors due to the approximations concerning the tree level analysis and the radiative corrections are taken into account. The best fit values are those minimizing χ^2.

To simplify the fit procedure we will calculate the best fit compactification scale M for a fixed $\sin\beta$ and scan through the different values of $\sin\beta$. As fit parameter for the compactification scale we choose $1/M^2$ to avoid infinities if the best fit value is close to the

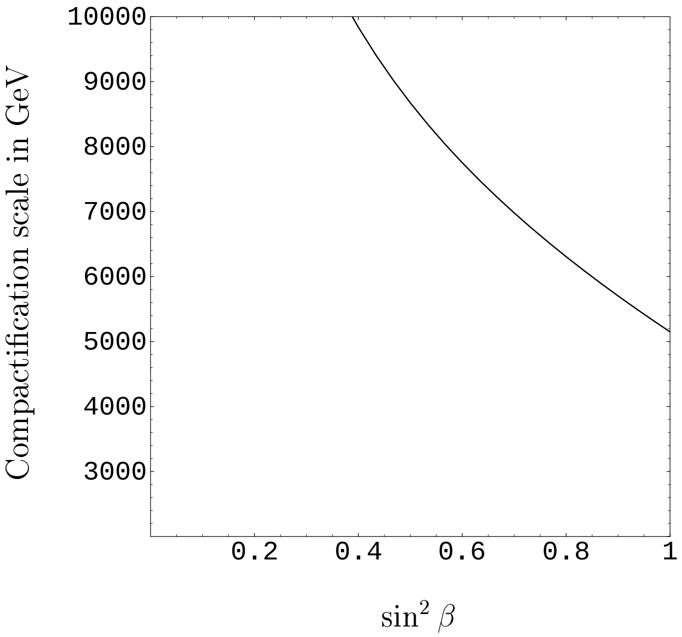


Figure 6.9: The best fit values for the compactification scale for a fixed $\sin^2\beta$ are found by minimizing χ^2 defined in eq.(6.53).

standard model case where M can become arbitrarily large. The fit procedure also allows for negative values of the compactification scale squared which are, of course, unphysical and only indicate that the corrections due to the extra dimension increase the difference between prediction and measurement. Our analysis considers the five observables which have been discussed in detail in the preceding sections. As we have already seen in the analysis of most of the observables, figure 6.9 shows that a vacuum expectation value dominated by a brane Higgs field favors small compactification scales. For $\sin^2\beta = 1$ the best fit value is only slightly above 5 TeV while for a vacuum expectation value dominated by a bulk Higgs field the best fit value is extremely large. Although extremely large, the best fit compactification scale stays positive and, thus, physical for the whole range of $\sin^2\beta$.

Comparing the best fit values of χ^2, denoted by χ^2_{best}, for different $\sin^2\beta$ we find that the data favors a vacuum expectation value dominated by a Higgs field on a brane. For $\sin^2\beta = 1$ we have $\chi^2_{\text{best}} = 8.95$ which is 0.85 below χ^2_{best} for the standard model fit where the compactification scale is set to infinity (see figure 6.11).

Even more important than the best fit values are the bounds on the compactification scale to be inferred from a global χ^2 analysis. From the general theory of χ^2 analysis [33, 34] it is known that the values of the fit variables allowed by the data lie within a given range

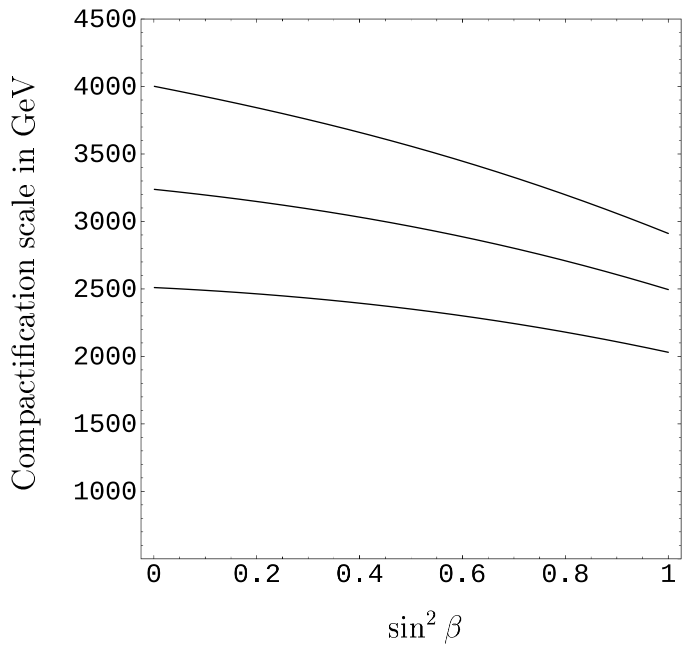


Figure 6.10: The 95 %, 3 and 5σ bounds on the compactification scale (from top to bottom) from a global χ^2 analysis.

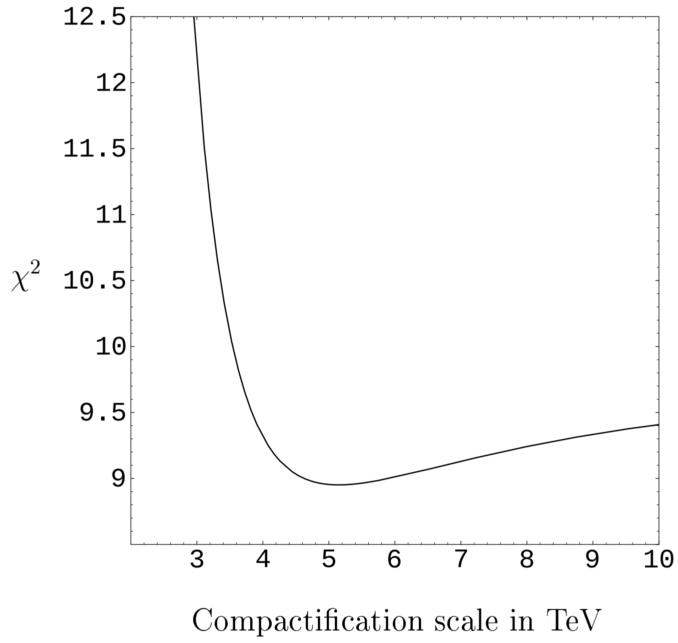


Figure 6.11: χ^2 as a function of the compactification scale at $\sin\beta = 1$

around the best fit values. The maximal deviation $\Delta\chi^2$ from χ^2_{best} for which fit parameters are accepted at a given significance level σ^2 is given by $\Delta\chi^2 = \sigma^2$.

It is common to state a 95 % confidence interval which corresponds to $\sigma = 1.96$ and the 3 and 5σ-intervals. The resulting lower bounds on the compactification scale in these three cases are shown in figure 6.10. There is no upper bound at either significance level. Figure 6.11 illuminates this by showing χ^2 as a function of the compactification scale at $\sin\beta = 1$. Although the lower bounds on the compactification scale, e.g. at the 2σ level, can be close to the minimum of χ^2 the dependence of χ^2 for larger compactification scales is so weak that an upper bound cannot be extracted, i.e. the theory is perfectly compatible to the data in the standard model limit. Therefore, the meaning of the best fit values should not be overestimated.

To summarize, we can exclude the existence of a large extra dimension with a compactification scale smaller than 2.5 GeV for the whole range of $\sin^2\beta$ at a significance level of 5σ. If we relax the significance level to 95 % compactification scales below 4.0 TeV for $\sin^2\beta = 0$, decreasing to 2.9 TeV for $\sin^2\beta = 1$, can be excluded. Although we had to correct some inconsistencies in the literature, the final bounds on the compactification scale are in the same range stated in different publications, e.g. [8, 9, 10, 12]. Only slightly larger compactification scales are perfectly compatible with today's precision measurement data. Indeed, we have seen that the best fit to the data prefers a theory with a brane vacuum expectation value and a compactification scale at roughly 5 TeV which may be even reachable at future colliders.

Chapter 7

Conclusions

This work investigates the main features of a 5D standard model, compactified on a S^1/Z_2 orbifold, in its effective four dimensional form. This specific higher dimensional standard model assumes the gauge degrees of freedom to live in the bulk while all fermionic degrees of freedom are localized on a brane. The most general Higgs sector contains two Higgs doublets to include the effects of a bulk and a brane Higgs.

We have derived a complete effective four dimensional theory by integrating out the extra dimension. The typical Kaluza-Klein towers for the bulk degrees of freedom are encountered. Concerning gauge invariance, we state the gauge transformation properties of the Kaluza-Klein modes in considered models. The particle content of the higher dimensional theory has been derived by investigating the model in specific gauges. We have recovered the unitary gauge introduced earlier in the literature [8, 9, 10, 12]. Considering more general gauges, there has recently been some confusion about the right choice of the gauge fixing function and its consequences [13, 14]. We have presented both a standard and a generalized R_ξ-gauge before integrating out the extra dimension and clarified their consequences in terms of the corresponding propagators. Especially the generalized R_ξ-gauge which had not been explored in the literature in its five dimensional form, yet, has turned out to be very appropriate for the higher dimensional standard model. Moreover, unitary gauge has been recovered as a limit of the generalized R_ξ-gauge.

We have particularly focused on the different features of Higgs fields living in the bulk or on the brane. For a bulk Higgs, spontaneous symmetry breaking has been investigated in detail in the effective four dimensional theory. The results are equivalent to a non-vanishing expectation value of the Higgs field with respect to the full five dimensional vacuum. For a brane Higgs, the mass eigenstates of the Kaluza-Klein modes differ from the fields in a simple Fourier expansion. The phenomenologically important mass eigenvalues are given in terms of a transcendental equation which can, in principle, be solved with arbitrary accuracy, while they are only found approximately in the literature [8]. The same is true for the basis transformation between the mass and the gauge eigenstates and for the couplings of the different particles in a Kaluza-Klein tower. Moreover, the physical degrees of freedom are separated from the modes playing the role of the would-be Goldstones of the theory. The latter are explicitly stated in terms of gauge eigenstates even in the most general 2-Higgs model because they are important concerning gauge invariance. In generalized R_ξ-gauge we have used these results to show gauge invariance of fermion scattering at tree level.

Furthermore, we have analyzed the generic structure of interactions between bulk fields and interactions of a bulk field with a field localized on a brane. Again, we have not restricted our attention to unitary gauge but formulated the interactions in generalized R_ξ-gauge. The gauge boson self-interactions within the effective theory are derived for an arbitrary non-Abelian gauge group and the corresponding Feynman rules are given. Moreover, the gauge boson fermion interactions in the 5D standard model have been explicitly derived including the Feynman rules. Most of the Higgs sector interactions depend strongly on the Higgs potential and are not stated in full generality. Nevertheless, the presented formalism should leave it as an easy exercise to derive the corresponding Feynman rules in any given model.

Finally, we have investigated the phenomenological consequences of a higher dimensional standard model. The analysis of five precisely measured observables showed that a compactification scale larger than 4 TeV is compatible with the existing data in any case. For a vacuum expectation value dominated by a brane Higgs, the data excludes only compactification scales below 2.9 TeV at the 95 % confidence level. The best fit value for the compactification scale is found to be slightly larger than 5 TeV in a model dominated by a brane Higgs and rapidly increases for an increasing vacuum expectation value of a bulk Higgs. However, because the standard model is in perfect agreement with the data, an upper bound on the compactification scale cannot be established. Hence, a next generation linear collider probably will not be able to resonantly produce higher Kaluza-Klein modes, but if there is a large extra dimension with a compactification scale between 3 and 5 TeV it will certainly leave its fingerprints in future high energy experiments.

While we have tried to investigate the specific model considered in the thesis as completely as possible, there is of course a lot of work left to be done. In particular, the phenomenological implications reach far beyond the five observables taken into account. For example, the implications for cross sections at increasing collider energies are only mentioned in a simple 5D-QED example and not worked out for the full standard model. Moreover, the model under consideration could be slightly changed. What consequences are implied if not all of the three gauge groups of the standard model correspond to bulk degrees of freedom?

A model where either SU(2) or $U(1)_Y$ is localized on a brane could be of special interest because the Kaluza-Klein modes of the photon and the Z boson would not be pure bulk or brane fields any more. This possibility has not been investigated in detail in the literature [10], so far.

Finally, we hope that future experiments as well as theoretical considerations will work together to allow for a more precise answer to all the questions concerning extra dimensions, e.g.: In how many dimensions do we live?

Appendix A

An Alternative Gauge Boson Field Expansion

In section 2.1 we expanded the gauge boson fields living in the five dimensional bulk into a Fourier series. Although this is always possible, it only *a posteriori* turned out to be a good choice because the expansion coefficients, i.e. the Kaluza-Klein fields, can be identified with the physical particle excitations. In section 4.1.2, it turned out that this situation is spoiled by a vacuum expectation value of a brane field. The expansion coefficients are no longer equivalent to the mass eigenstates in an effective 4D theory. In this Appendix we will show the reasons why this is the case.

For notational convenience, let us first investigate a theory in five dimensions containing a single scalar field ϕ. In a free theory the corresponding Lagrangian is simply given by

$$\mathcal{L} \,=\, \frac{1}{2}\,\partial_\mu\phi\,\partial^\mu\phi \,-\, \frac{1}{2}\,(\,\partial_4\phi\,)^2\,. \tag{A.1}$$

One can expand $\phi(x,y)$ into an arbitrary system $f_{(n)}(y)$ of orthonormal, continuous functions with respect to S^1/Z_2. Moreover, we assume that the functions have a definite Z_2-parity. Integrating over the extra dimension, the 4D effective Lagrangian is given by

$$\mathcal{L}^{\rm 4D} = \frac{1}{2}\sum_{n,m=0}^{\infty}\partial_\mu\phi_{(n)}\,\partial^\mu\phi_{(m)}\int_0^{2\pi R}dy\,f_{(n)}f_{(m)} - \frac{1}{2}\sum_{n,m=0}^{\infty}\phi_{(n)}\phi_{(m)}\int_0^{2\pi R}dy\,\partial_4 f_{(n)}\,\partial_4 f_{(m)}\,. \tag{A.2}$$

While the integral in the first term, due to the orthonormality of the $f_{(n)}$, equals $\delta_{n,m}$, that is not necessarily true for the second term. Partial integration yields instead

$$\int_0^{2\pi R}dy\,\partial_4 f_{(n)}\,\partial_4 f_{(m)} \;=\; -\int_0^{2\pi R}dy\,f_{(n)}\,\partial_4^2 f_{(m)}\,, \tag{A.3}$$

where the surface terms vanish because the $f_{(n)}$ are continuous on the considered orbifold. To avoid the appearance of mixing terms after integrating over the extra dimension it is necessary and sufficient that all the $f_{(n)}$ satisfy the relation

$$- \partial_4^2 f_{(n)} \; = \; \hat{m}_{(n)}^2 f_{(n)} \, , \tag{A.4}$$

where $\hat{m}_{(n)}^2$ is a proportionality constant of dimension mass squared which can be identified with the mass squared of the n^{th} Kaluza-Klein excitation in the effective 4D theory. This well-known differential equation is solved by the Fourier modes. That is the fundamental reason why it is so convenient to work with the Fourier expansion. In section 2.3 we only had to rescale the Fourier modes because we did not properly normalize the Fourier modes in the first place.

If a brane field, coupling to the fields in the Lagrangian, acquires a vacuum expectation value as in section 4.1.2, eq.(A.1) changes to

$$\mathcal{L} \; = \; \frac{1}{2} \, \partial_\mu \phi \, \partial^\mu \phi \; - \; \frac{1}{2} \, (\, \partial_4 \phi \,)^2 \; - \; \delta(y) \, \frac{1}{2} \, m_5^2 \, \phi^2 \, , \tag{A.5}$$

where m_5 denotes the five dimensional symmetry breaking mass due to the vacuum expectation value. The same analysis as before leads to an altered differential equation for the expansion modes given by

$$- \partial_4^2 f_{(n)} \; + \; \delta(y) \, m_5^2 \, f_{(n)} \; = \; \hat{m}_{(n)}^2 f_{(n)} \, . \tag{A.6}$$

Before we investigate the solution of the differential equation (A.6) we will show that the simple scalar example, discussed so far, is completely equivalent to the original problem encountered in sections 2.1 or 4.1.2. The kinetic terms of the gauge boson field in the gauge-fixed Lagrangian of the Abelian Higgs model are given by

$$\mathcal{L} \; = \; -\frac{1}{4} \, F_{\mu\nu} F^{\mu\nu} \; + \; \frac{1}{2} \partial_4 A_\mu \, \partial_4 A^\mu \; + \; \frac{1}{2} \delta(y) m_5^2 A_\mu A^\mu \; - \; \frac{1}{2\xi} (\, \partial_\mu A^\mu \,)^2 \, . \tag{A.7}$$

Eq.(A.7) follows from eq.(4.33) and the gauge fixing terms eq.(4.59). The first and the last term in eq.(A.7) do not result in mixing after integrating over the extra dimension, no matter which orthonormal system of functions is chosen. The remaining terms are analogous with those in eq.(A.5). We only have to replace the scalar field ϕ by the vector field A_μ. The Lorentz structure of the fields to be expanded is completely irrelevant for the following analysis. Therefore, the differential equation (A.6) for the expansion functions can be derived again.

The differential equation (A.6) is formally equivalent to the stationary, one-dimensional Schrödinger equation for a particle moving in a δ-like potential. It is well-known that the even solutions (we are searching for) are no longer simple Fourier modes. Nevertheless, solving this differential equation is a textbook problem of introductory quantum mechanics

[35]. We only have to keep in mind that the extra dimension has the special topology of S^1. We have to solve the differential equation within the interval $[0, 2\pi R]$ where the points 0 and $2\pi R$ are identified. Using the usual ansatz (solving the differential equation for $y \neq 0$)

$$f_{(n)} \;=\; A_{(n)} \cos\big(\, k_{(n)} y \,\big) \;+\; B_{(n)} \sin\big(\, k_{(n)} y \,\big) \tag{A.8}$$

for the n^{th} solution of the complete set, the continuity condition on the solution at $y = 0$ is translated into

$$f_{(n)}(0) = f_{(n)}(2\pi R)\,. \tag{A.9}$$

The condition on the derivative of the $f_{(n)}$ also has to be rederived carefully for the S^1 topology. Integrating the differential equation yields

$$\int_{-\epsilon}^{\epsilon} dy\, \partial_4^2 f_{(n)} \;=\; \partial_4 f_{(n)}|_{\epsilon} - \partial_4 f_{(n)}|_{-\epsilon = 2\pi R - \epsilon} \;=\; m_5^2 f_{(n)}(0) \;-\; \int_{-\epsilon}^{\epsilon} dy\, \hat{m}_{(n)}^2 f_{(n)}\,, \tag{A.10}$$

where we identify $-y$ with $2\pi R - y$. In the limit $\epsilon \to 0$ we finally arrive at the second condition on $A_{(n)}$, $B_{(n)}$ and $k_{(n)}$, namely

$$\partial_4 f_{(n)}|_0 - \partial_4 f_{(n)}|_{2\pi R} \;=\; m_5^2 f_{(n)}(0)\,. \tag{A.11}$$

Together with the normalization condition on the $f_{(n)}$, eq.(A.9) and eq.(A.11) are sufficient to calculate $A_{(n)}$, $B_{(n)}$ and $k_{(n)}$ and, thus, to find solutions for the $f_{(n)}$. The different solutions are automatically orthogonal as is well-known from introductory quantum mechanics. After some tedious but simple algebra and concentrating on the even solutions we finally find a transcendental equation for $k_{(n)}$ given by

$$2k_{(n)} \;=\; m_5^2 \cot\big(\, \pi k_{(n)} R \,\big)\,. \tag{A.12}$$

Using $m_5^2 = 2\pi R m^2$, where m denotes the effective four dimensional symmetry breaking mass, and identifying the different solutions for $k_{(n)}$ with the mass eigenvalues $\hat{m}_{(n)}$ we recover eq.(4.42) which was quite differently derived in section 4.1.2. In terms of these mass eigenvalues $\hat{m}_{(n)}$ the n^{th} solution of the differential equation (A.6) is given by

$$f_{(n)} \;=\; \frac{1}{\sqrt{\pi R}} \left(1 + \pi^2 R^2 \frac{m^4}{\hat{m}_{(n)}^2} + \frac{m^2}{\hat{m}_{(n)}^2} \right)^{-1/2} \big[\cos\big(\, \hat{m}_{(n)} y \,\big) + \tan\big(\, \pi R \hat{m}_{(n)} \,\big) \sin\big(\, \hat{m}_{(n)} y \,\big) \big]\,. \tag{A.13}$$

Although it is not obvious, $f_{(n)}$ is an even function if we, again, identify $-y$ with $2\pi R - y$.

The explicit knowledge of the physically relevant expansion functions whose coefficient fields can be identified with the physical Kaluza-Klein excitations is also a powerful tool if we investigate the interactions with fermions living on a brane. Before integrating out the extra dimension the coupling to some fermionic current J^μ (see eq.(4.53)) is given by

$$\mathcal{L}_{\text{int}} \;=\; \delta(y)\, e_5\, J^\mu \sum_{n=0}^{\infty} \hat{A}_\mu^{(n)} f_{(n)}\,, \tag{A.14}$$

where the hat denotes that the fields can be identified with mass eigenstates. After integrating out the extra dimension we can write the result as in eq.(4.54). Using $e = e_5/\sqrt{2\pi R}$ and the explicit form of the $f_{(n)}$ in eq.(A.13) we immediately recover the result for $\hat{e}_{(n)}$ found in eq.(4.56).

To summarize, the basis transformation for the vector fields in section 4.1.2 is not necessary if the appropriate expansion is used right from the beginning. The transformation matrix relating the physical basis set and the Fourier expansion modes can be calculated via simple integrals over the expansion functions as known from wave mechanics. Hence, it is possible to cross-check all the results in a completely independent way making both methods extremely valuable.

Appendix B

Gauge Transformation Properties

In this appendix we derive the gauge transformation properties of the Kaluza-Klein modes in non-Abelian gauge theories starting from eq.(3.14), i.e.

$$\begin{aligned}
A^a_\mu(x,y) &= \sum_{n=0}^{\infty} \frac{1}{\sqrt{2\pi R}}\, A^a_{(n)\mu} \cos\Big(\frac{n}{R}y\Big) \\
&\rightarrow \sum_{n=0}^{\infty} \frac{1}{\sqrt{2\pi R}} \left[A^a_{(n)\mu} + \partial_\mu \Theta^a_{(n)} - g_5 f^{abc} A^c_{(n)\mu} \sum_{m=0}^{\infty} \frac{\Theta^b_{(m)}}{\sqrt{2\pi R}} \cos\Big(\frac{m}{R}y\Big) \right] \cos\Big(\frac{n}{R}y\Big)\,, \\
A^a_4(x,y) &= \sum_{n=1}^{\infty} \frac{1}{\sqrt{2\pi R}}\, A^a_{(n)4} \sin\Big(\frac{n}{R}y\Big) \\
&\rightarrow \sum_{n=1}^{\infty} \frac{1}{\sqrt{2\pi R}} \left[A^a_{(n)4} - \frac{n}{R}\Theta^a_{(n)} - g_5 f^{abc} A^c_{(n)4} \sum_{m=0}^{\infty} \frac{\Theta^b_{(m)}}{\sqrt{2\pi R}} \cos\Big(\frac{m}{R}y\Big) \right] \sin\Big(\frac{n}{R}y\Big)\,.
\end{aligned} \tag{B.1}$$

To identify the gauge transformation properties of the different modes we have to reformulate eq.(B.1) as

$$A^a_\mu(x,y) \rightarrow \sum_{n=0}^{\infty} \frac{A^{a\,\prime}_{(n)\mu}}{\sqrt{2\pi R}} \cos\Big(\frac{n}{R}y\Big) \quad \text{and} \quad A^a_4(x,y) \rightarrow \sum_{n=1}^{\infty} \frac{A^{a\,\prime}_{(n)4}}{\sqrt{2\pi R}} \sin\Big(\frac{n}{R}y\Big)\,, \tag{B.2}$$

where whether $A^{a\,\prime}_{(n)\mu}$ nor $A^{a\,\prime}_{(n)4}$ are to depend on the extra dimension. Let us investigate the vector and scalar modes separately and begin with the vector modes. The non-trivial term (neglecting prefactors) due to the non-Abelian structure of the gauge group is given by

$$\Delta_\mathrm{v} = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} A^c_{(n)\mu}\, \Theta^b_{(m)} \cos\Big(\frac{m}{R}y\Big) \cos\Big(\frac{n}{R}y\Big)\,. \tag{B.3}$$

We can use the trigonometric identities eq.(2.20) to obtain

$$\Delta_\mathrm{v} = \frac{1}{2} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} A^c_{(n)\mu}\, \Theta^b_{(m)} \cos\Big(\frac{m+n}{R}y\Big) + \frac{1}{2} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} A^c_{(n)\mu}\, \Theta^b_{(m)} \cos\Big(\frac{m-n}{R}y\Big)\,, \tag{B.4}$$

where we have interchanged the infinite sums. Substituting $k = m + n$ in the first term and $l = m - n$ in the second term on the right hand side of eq.(B.4) yields

$$\Delta_{\mathrm{v}} = \frac{1}{2} \sum_{m=0}^{\infty} \sum_{k=m}^{\infty} A^c_{(k-m)\mu}\, \Theta^b_{(m)} \cos\Big(\frac{k}{R} y\Big) + \frac{1}{2} \sum_{m=0}^{\infty} \sum_{l=-\infty}^{m} A^c_{(m-l)\mu}\, \Theta^b_{(m)} \cos\Big(\frac{l}{R} y\Big) . \tag{B.5}$$

The second term can be recasted into the form

$$\begin{aligned} &\frac{1}{2} \sum_{m=0}^{\infty} \sum_{l=-\infty}^{m} A^c_{(m-l)\mu}\, \Theta^b_{(m)} \cos\Big(\frac{l}{R} y\Big) = \\ &\quad = \frac{1}{2} \sum_{m=0}^{\infty} \sum_{l=1}^{m} A^c_{(m-l)\mu}\, \Theta^b_{(m)} \cos\Big(\frac{l}{R} y\Big) + \frac{1}{2} \sum_{m=0}^{\infty} \sum_{u=0}^{\infty} A^c_{(m+u)\mu}\, \Theta^b_{(m)} \cos\Big(\frac{u}{R} y\Big) , \end{aligned} \tag{B.6}$$

where we have substituted $u = -l$. Plugging eq.(B.6) into eq.(B.5) and renaming summation indices according to $k \to n$, $l \to n$ and $u \to n$ one obtains

$$\begin{aligned} \Delta_{\mathrm{v}} = {} & \frac{1}{2} \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} A^c_{(|n-m|)\mu}\, \Theta^b_{(m)} \cos\Big(\frac{n}{R} y\Big) + \frac{1}{2} \sum_{m=0}^{\infty} A^c_{(0)\mu}\, \Theta^b_{(m)} \cos\Big(\frac{m}{R} y\Big) \\ & + \frac{1}{2} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} A^c_{(m+n)\mu}\, \Theta^b_{(m)} \cos\Big(\frac{n}{R} y\Big) . \end{aligned} \tag{B.7}$$

In order to extract the gauge transformation properties of the Kaluza-Klein modes it is convenient to rewrite eq.(B.7) as

$$\begin{aligned} \Delta_{\mathrm{v}} = {} & \frac{1}{2} \sum_{n=1}^{\infty} \sum_{m=0}^{\infty} (1 + \delta_{m,n})\, A^c_{(|n-m|)\mu}\, \Theta^b_{(m)} \cos\Big(\frac{n}{R} y\Big) \\ & + \frac{1}{2} \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} (1 + \delta_{m,0}\, \delta_{n,0})\, A^c_{(m+n)\mu}\, \Theta^b_{(m)} \cos\Big(\frac{n}{R} y\Big) , \end{aligned} \tag{B.8}$$

where we have again interchanged the sums. Plugging eq.(B.8) back into eq.(B.1) we can immediately read off the gauge transformation properties of the different modes. We finally obtain

$$\begin{aligned} A^a_{(0)\mu} &\to A^a_{(0)\mu} + \partial_\mu \Theta^a_{(0)} - \frac{1}{2} \frac{g_5}{\sqrt{2\pi R}} f^{abc} \sum_{m=0}^{\infty} \Theta^b_{(m)} \Big((1 + \delta_{m,0})\, A^c_{(m)\mu} \Big) , \\ A^a_{(n)\mu} &\to A^a_{(n)\mu} + \partial_\mu \Theta^a_{(n)} - \frac{1}{2} \frac{g_5}{\sqrt{2\pi R}} f^{abc} \sum_{m=0}^{\infty} \Theta^b_{(m)} \Big((1 + \delta_{m,n})\, A^c_{(|m-n|)\mu} + A^c_{(m+n)\mu} \Big) . \end{aligned} \tag{B.9}$$

The analysis for the scalar Kaluza-Klein modes is quite analogous. Here the non-trivial term is given by

$$\Delta_{\mathrm{s}} = \sum_{n=1}^{\infty} \sum_{m=0}^{\infty} A^c_{(n)4}\, \Theta^b_{(m)} \cos\Big(\frac{m}{R} y\Big) \sin\Big(\frac{n}{R} y\Big) . \tag{B.10}$$

Along the lines of the preceding discussion it is easy to show the analogue of eq.(B.5), i.e.

$$\Delta_{\rm s} = \frac{1}{2}\sum_{m=0}^{\infty}\sum_{k=m+1}^{\infty} A^c_{(k-m)\mu}\,\Theta^b_{(m)}\,\sin\Big(\frac{k}{R}y\Big) - \frac{1}{2}\sum_{m=0}^{\infty}\sum_{l=-\infty}^{m-1} A^c_{(m-l)\mu}\,\Theta^b_{(m)}\,\sin\Big(\frac{l}{R}y\Big)\,. \quad \text{(B.11)}$$

Following the same steps leading to eq.(B.8) we obtain

$$\Delta_{\rm s} = \frac{1}{2}\sum_{n=1}^{\infty}\sum_{m=0}^{\infty} \mathrm{sign}(n-m)\,A^c_{(|n-m|)\mu}\,\Theta^b_{(m)}\,\sin\Big(\frac{n}{R}y\Big) + \frac{1}{2}\sum_{n=1}^{\infty}\sum_{m=0}^{\infty} A^c_{(m+n)\mu}\,\Theta^b_{(m)}\,\sin\Big(\frac{n}{R}y\Big)\,, \quad \text{(B.12)}$$

where $\mathrm{sign}(n-m) = 1$ for $n > m$, $\mathrm{sign}(n-m) = 0$ for $n = m$ and $\mathrm{sign}(n-m) = -1$ for $n < m$. Hence, the gauge transformation properties of the scalar Kaluza-Klein modes are given by $(n \geq 1)$

$$A^a_{(n)4} \to A^a_{(n)4} - \frac{n}{R}\Theta^a_{(n)} - \frac{1}{2}\frac{g_5}{\sqrt{2\pi R}}\,f^{abc}\sum_{m=0}^{\infty}\Theta^b_{(m)}\,\Big(\mathrm{sign}(n-m)\,A^c_{(|m-n|)4} + A^c_{(m+n)4}\Big)\,. \quad \text{(B.13)}$$

Note, that the fields have to be rescaled according to eq.(2.24). The gauge transformation properties of the rescaled field are given in eq.(3.15).

Appendix C

4D Lagrangian of the 5D Abelian Higgs Model

In this appendix we state the complete effective four dimensional Lagrangian of the five dimensional Abelian Higgs model in R_ξ-gauge.

C.1 Higgs Field in the Bulk

Considering a Higgs field in the bulk, we take

$$\mathcal{L} = -\frac{1}{4} F^{MN} F_{MN} + |D_M \Phi(x,y)|^2 - V(\Phi(x,y)) - \frac{1}{2\xi}(\partial^\mu A_\mu - \xi(\partial_4 A_4 + ev\chi))^2 , \tag{C.1}$$

where

$$\begin{aligned} V(\Phi) &= -\lambda_5 v_5^2 |\Phi|^2 + \lambda_5 \left(|\Phi|^2 \right)^2 , \\ D_M &= \partial_M + i e_5 A_M \end{aligned} \tag{C.2}$$

and e_5 is the five dimensional gauge coupling constant. The complex scalar field is parameterized by $\Phi = 1/\sqrt{2}(v_5 + h + i\chi)$, where v_5 is the vacuum expectation value of Φ. In terms of Kaluza-Klein modes the fields

$$\begin{aligned} G_{(n)} &= \left((n/R)^2 + (ev)^2 \right)^{-1/2} \left(\left(\frac{n}{R} \right) A_{(n)4} + (ev)\chi_{(n)} \right), \\ a_{(n)} &= \left((n/R)^2 + (ev)^2 \right)^{-1/2} \left((ev) A_{(n)4} - \left(\frac{n}{R} \right) \chi_{(n)} \right), \end{aligned} \tag{C.3}$$

i.e. the Goldstone and physical scalar modes, form the physically relevant basis (for $n \geq 1$). Nevertheless, we will display the interactions contained in the Lagrangian in terms of the fields $\chi_{(n)}$ and $A_{(n)4}$ for notational convenience. Note, that for actual calculations the Feynman rules have to be evaluated in terms of $G_{(n)}$ and $a_{(n)}$ by a simple basis transformation.

Moreover, the four dimensional constants $\lambda = \lambda_5/(2\pi R)$ and $v = \sqrt{2\pi R}v_5$ are used. The effective four dimensional Lagrangian corresponding to the theory described by eq.(C.1) consists of the following terms:

Free Lagrangian for the gauge fields

$$\begin{aligned}\mathcal{L}^A_{\rm kin} &= -\frac{1}{4}F_{(0)\mu\nu}\,F^{\mu\nu}_{(0)} - \frac{1}{2\xi}\left(\partial_\mu A^\mu_{(0)}\right)^2 \\ &+\sum_{n=1}^{\infty}\left[-\frac{1}{4}F_{(n)\mu\nu}\,F^{\mu\nu}_{(n)} + \frac{1}{2}m^2_{A(n)}A_{(n)\mu}A^\mu_{(n)} - \frac{1}{2\xi}\left(\partial_\mu A^\mu_{(n)}\right)^2\right],\end{aligned} \tag{C.4}$$

where $m^2_{A(n)} = (\,n/R\,)^2 + (\,ev\,)^2$ denotes the gauge boson mass squared.

Free Lagrangian for the Higgs field

$$\mathcal{L}^h_{\rm kin} = \sum_{n=0}^{\infty}\left[\frac{1}{2}\left(\partial_\mu h_{(n)}\right)^2 - \frac{1}{2}\left(\left(n/R\right)^2 + 2\lambda v^2\right)h^2_{(n)}\right] \tag{C.5}$$

Free Lagrangian for the additional physical scalar

$$\mathcal{L}^a_{\rm kin} = \sum_{n=1}^{\infty}\left[\frac{1}{2}\left(\partial_\mu a_{(n)}\right)^2 - \frac{1}{2}m^2_{A(n)}\,a^2_{(n)}\right] \tag{C.6}$$

Free Lagrangian for the unphysical Goldstone boson field

$$\mathcal{L}^G_{\rm kin} = \frac{1}{2}\left(\partial_\mu\,\chi_{(0)}\right)^2 - \frac{1}{2}\xi\,(\,ev\,)^2\,\chi^2_{(0)} + \sum_{n=1}^{\infty}\left[\frac{1}{2}\left(\partial_\mu G_{(n)}\right)^2 - \frac{1}{2}\xi\,m^2_{A(n)}\,G^2_{(n)}\right] \tag{C.7}$$

Self-interactions of the Higgs degrees of freedom

$$\begin{aligned}\mathcal{L}_{\rm s} = &\;\frac{\lambda}{4}\sum_{k,l,m,n=0}^{\infty}\left(h_{(k)}h_{(l)}h_{(m)}h_{(n)} + \chi_{(k)}\chi_{(l)}\chi_{(m)}\chi_{(n)}\right)c(k,l,m,n) \\ &+\frac{\lambda}{2}\sum_{k,l,m,n=0}^{\infty}h_{(k)}h_{(l)}\chi_{(m)}\chi_{(n)}\,c(k,l,m,n) \\ &+\lambda v\sum_{k,l,m=0}^{\infty}\left(h_{(k)}h_{(l)}h_{(m)} + h_{(k)}\chi_{(l)}\chi_{(m)}\right)c(k,l,m)\,,\end{aligned} \tag{C.8}$$

where

$$\begin{aligned}c(k,l,m) &= \left(1/\sqrt{2}\right)^{\delta_{k,0}+\delta_{l,0}+\delta_{m,0}+1}\delta_{k,l,m}\quad, \\ c(k,l,m,n) &= \left(1/\sqrt{2}\right)^{\delta_{k,0}+\delta_{l,0}+\delta_{m,0}+\delta_{n,0}+2}\delta_{k,l,m,n}\end{aligned} \tag{C.9}$$

and $\delta_{k,l,m}$ and $\delta_{k,l,m,n}$ are given by eq.(3.27) and its direct generalization.

Interactions of the gauge boson fields with the Higgs degrees of freedom

$$
\begin{aligned}
\mathcal{L}_{\mathrm{GC}} =& e \sum_{k,l,m=0}^{\infty} \Big(A^{\mu}_{(k)} h_{(l)} \big(\partial_\mu \chi_{(m)} \big) - A^{\mu}_{(k)} \chi_{(l)} \big(\partial_\mu h_{(m)} \big) \Big) c(k,l,m) \\
&+ e \sum_{\substack{k,m=1\\ l=0}}^{\infty} \big((m/R) A_{4(k)} h_{(l)} \big(\partial_\mu \chi_{(m)} \big) - (m/R) A_{4(k)} \chi_{(l)} \big(\partial_\mu h_{(m)} \big) \big) \tilde{c}(k,l,m) \\
&+ \frac{e^2}{2} \sum_{\substack{k,l=0\\ m,n=0}}^{\infty} A^{\mu}_{(k)} A_{(l)\mu} h_{(m)} h_{(n)} c(k,l,m,n) - \frac{e^2}{2} \sum_{\substack{k,l=0\\ m,n=1}}^{\infty} A_{(k)4} A_{(l)4} h_{(m)} h_{(n)} \tilde{c}(k,l,m,n) \\
&+ \frac{e^2}{2} \sum_{\substack{k,l=0\\ m,n=0}}^{\infty} A^{\mu}_{(k)} A_{(l)\mu} \chi_{(m)} \chi_{(n)} c(k,l,m,n) - \frac{e^2}{2} \sum_{\substack{k,l=0\\ m,n=1}}^{\infty} A_{(k)4} A_{(l)4} \chi_{(m)} \chi_{(n)} \tilde{c}(k,l,m,n) \\
&+ e^2 v \sum_{k,l,m=0}^{\infty} A^{\mu}_{(k)} A_{(l)\mu} h_{(m)} c(k,l,m) - e^2 v \sum_{\substack{k,l=0\\ m=1}}^{\infty} A_{(k)4} A_{(l)4} h_{(m)} \tilde{c}(k,l,m) \,,
\end{aligned}
\tag{C.10}
$$

where

$$
\begin{aligned}
\tilde{c}(k,l,m) &= \Big(1/\sqrt{2} \Big)^{\delta_{l,0}+1} \tilde{\delta}_{k,l,m} \,, \\
\tilde{c}(k,l,m,n) &= \Big(1/\sqrt{2} \Big)^{\delta_{k,0}+\delta_{l,0}+1} \tilde{\delta}_{k,l,m,n} \,,
\end{aligned}
\tag{C.11}
$$

$\tilde{\delta}_{k,l,m}$ and $\tilde{\delta}_{k,l,m,n}$ are given by eq.(3.31) and eq.(3.40), respectively, and $e = e_5/\sqrt{2\pi R}$.

C.2 Higgs Field on a Brane

Considering a Higgs field on a brane, we take

$$
\mathcal{L} = -\frac{1}{4} F^{MN} F_{MN} + \delta(y) \, |D_\mu \Phi(x)|^2 - \delta(y) \, V(\Phi(x)) - \frac{1}{2\xi} (\partial^\mu A_\mu - \xi (\partial_4 A_4 + \delta(y) \chi))^2 \,,
\tag{C.12}
$$

where

$$
\begin{aligned}
V(\Phi) &= -\lambda v^2 |\Phi|^2 + \lambda \left(|\Phi|^2 \right)^2 , \\
D_\mu &= \partial_\mu + i \, e \, A_\mu \,.
\end{aligned}
\tag{C.13}
$$

The complex scalar field is parameterized by $\Phi = 1/\sqrt{2}(v + h + i\chi)$, where v is the vacuum expectation value of the brane field Φ. The Kaluza-Klein mass eigenstates of the vector boson field are given by $(n \geq 0)$

$$
\hat{A}^{\mu}_{(n)} = \left(1 + \pi^2 m^2 R^2 + \frac{\hat{m}^2_{A(n)}}{m^2} \right)^{-\frac{1}{2}} \sum_{j=0}^{\infty} \frac{-2 \, \hat{m}_{A(n)} \, m}{\hat{m}^2_{A(n)} - (j/R)^2} \left(\frac{1}{\sqrt{2}} \right)^{\delta_{j,0}} A^{\mu}_{(j)}
\tag{C.14}
$$

and the would-be Goldstone modes of the theory are given by ($n \geq 0$)

$$G_{(n)} = \left(1 + \pi^2 m^2 R^2 + \frac{\hat{m}^2_{A(n)}}{m^2} \right)^{-\frac{1}{2}} \left(\sqrt{2}\,\chi + \sum_{j=1}^{\infty} \frac{-2\,(j/R)\,m}{\hat{m}^2_{A(n)} - (j/R)^2} A_{(j)4} \right), \tag{C.15}$$

where $\hat{m}_{A(n)}$, again, denotes the physical mass of the mass eigenstate $\hat{A}^{\mu}_{(n)}$. It is given by the n^{th} solution of the transcendental equation $\hat{m}_{A(n)} = \pi e^2 v^2 R \cot(\pi R\, \hat{m}_{A(n)})$. However, we will display the interactions in the Lagrangian in terms of the fields χ, $A_{(n)4}$ and the gauge eigenstates $A^{\mu}_{(n)}$ for notational convenience. Note, that for actual calculation the Feynman rules have to be evaluated in terms of $G_{(n)}$ and $\hat{A}^{\mu}_{(n)}$ by a basis transformation. The effective four dimensional Lagrangian corresponding to the theory described by eq.(C.12) consists of the following terms:

Free Lagrangian for the gauge fields

$$\begin{aligned} \mathcal{L}^{A}_{\text{kin}} &= -\frac{1}{4} F_{(0)\mu\nu} F^{\mu\nu}_{(0)} - \frac{1}{2\xi} \left(\partial_\mu \hat{A}^{\mu}_{(0)} \right)^2 \\ &\quad + \sum_{n=1}^{\infty} \left[-\frac{1}{4} F_{(n)\mu\nu} F^{\mu\nu}_{(n)} + \frac{1}{2} \hat{m}^2_{A(n)} \hat{A}_{(n)\mu} \hat{A}^{\mu}_{(n)} - \frac{1}{2\xi} \left(\partial_\mu \hat{A}^{\mu}_{(n)} \right)^2 \right] \end{aligned} \tag{C.16}$$

being formally identical to eq.(C.4).

Free Lagrangian for the Higgs field

$$\mathcal{L}^{h}_{\text{kin}} = \frac{1}{2} \left(\partial_\mu h \right)^2 - \frac{1}{2} \left(2\lambda v^2 \right) h^2 \tag{C.17}$$

Free Lagrangian for the unphysical Goldstone boson field

$$\mathcal{L}^{G}_{\text{kin}} = \sum_{n=0}^{\infty} \left[\frac{1}{2} \left(\partial_\mu G_{(n)} \right)^2 - \frac{1}{2} \xi\, \hat{m}^2_{A(n)} G^2_{(n)} \right] \tag{C.18}$$

Self-interactions of the Higgs degrees of freedom

$$\mathcal{L}_{\text{s}} = \frac{\lambda}{4} \left(h^4 + \chi^4 \right) + \frac{\lambda}{2} h^2 \chi^2 + \lambda v \left(h^3 + h \chi^2 \right) \tag{C.19}$$

Interactions of the gauge boson fields with the Higgs degrees of freedom

$$\begin{aligned} \mathcal{L}_{\text{GC}} = \sqrt{2}\, e \sum_{k=0}^{\infty} & \left(A^{\mu}_{(k)} h \left(\partial_\mu \chi \right) - A^{\mu}_{(k)} \chi \left(\partial_\mu h \right) \right) \left(\frac{1}{\sqrt{2}} \right)^{\delta_{k,0}} \\ & + e^2 \sum_{k,l=0}^{\infty} A^{\mu}_{(k)} A_{(l)\mu} h^2 \left(\frac{1}{\sqrt{2}} \right)^{\delta_{k,0}+\delta_{l,0}} \\ & + e^2 \sum_{k,l=0}^{\infty} A^{\mu}_{(k)} A_{(l)\mu} \chi^2 \left(\frac{1}{\sqrt{2}} \right)^{\delta_{k,0}+\delta_{l,0}} \\ & + 2\, e^2 v \sum_{k,l=0}^{\infty} A^{\mu}_{(k)} A_{(l)\mu} h \left(\frac{1}{\sqrt{2}} \right)^{\delta_{k,0}+\delta_{l,0}} \end{aligned} \tag{C.20}$$

Appendix D

The GSW-Theory in five Dimensions

This appendix summarizes the 5D standard model in its most general form as discussed in section 4.2.3, where one Higgs doublet lives in the bulk and one Higgs doublet is localized on a brane. All fermions live on the same brane at $y = 0$ as one of the Higgs fields. The gauge and the Higgs sector have been introduced in section 4.2.3, the fermion and Yukawa sector in section 5.2.

The kinetic terms of the gauge bosons and the corresponding unphysical would-be Goldstone bosons in generalized R_ξ-gauge are listed below. The corresponding propagators are shown in figure D.1.

Free Lagrangian for the W bosons

$$\mathcal{L}_{\text{kin}}^{\text{W}} = \sum_{n=0}^{\infty} \left[-\frac{1}{2}\, \hat{F}^{+}_{(n)\mu\nu} \hat{F}^{-\,\mu\nu}_{(n)} + \hat{m}^2_{W(n)}\, \hat{W}^{+}_{(n)\mu} \hat{W}^{-\mu}_{(n)} - \frac{1}{\xi} \left(\partial_\mu \hat{W}^{+\mu}_{(n)} \right) \left(\partial_\mu \hat{W}^{-\mu}_{(n)} \right) \right], \tag{D.1}$$

where $\hat{F}^{\pm\,\mu\nu}_{(n)} = \partial^\mu \hat{W}^{\pm\,\nu}_{(n)} - \partial^\nu \hat{W}^{\pm\,\mu}_{(n)}$ and $\hat{m}_{W(n)}$ denotes the physical mass of the n^{th} Kaluza-Klein mass eigenstate of the W boson. It is given by $\hat{m}^2_{W(n)} = \tilde{m}^2_{W(n)} + m^2_W \cos^2\beta$, where $\tilde{m}^2_{W(n)}$ is the n^{th} solution of the transcendental equation $\tilde{m}^2_{W(n)} = \pi m^2_W \sin^2\!\beta\, R \cot(\pi R\, \tilde{m}^2_{W(n)})$ and $m_W = gv/2$.

Free Lagrangian for the Z bosons

$$\mathcal{L}_{\text{kin}}^{\text{Z}} = \sum_{n=0}^{\infty} \left[-\frac{1}{4}\, \hat{F}_{(n)\mu\nu} \hat{F}^{\mu\nu}_{(n)} + \hat{m}^2_{Z(n)}\, \hat{Z}_{(n)\mu} \hat{Z}^{\mu}_{(n)} - \frac{1}{2\xi} \left(\partial_\mu \hat{Z}^{0\,\mu}_{(n)} \right)^2 \right], \tag{D.2}$$

where $\hat{F}^{\mu\nu}_{(n)} = \partial^\mu \hat{Z}^{\nu}_{(n)} - \partial^\nu \hat{Z}^{\mu}_{(n)}$ and $\hat{m}_{Z(n)}$ denotes the physical mass of the n^{th} Kaluza-Klein mass eigenstate of the Z boson. It is given by the same equation as the physical W-boson masses, where $m_W = gv/2$ has to be substituted by $m_Z = gv/(2\cos\theta_w)$.

Free Lagrangian for the photon

$$\mathcal{L}_{\rm kin}^{A} = -\frac{1}{4}\,F_{(0)\mu\nu}F_{(0)}^{\mu\nu} - \frac{1}{2\xi}\left(\partial_\mu A_{(0)}^{\mu}\right)^2 + \sum_{n=1}^{\infty}\left[-\frac{1}{4}\,F_{(n)\mu\nu}F_{(n)}^{\mu\nu} + \left(\frac{n}{R}\right)^2 A_{(n)\mu}A_{(n)}^{\mu} - \frac{1}{2\xi}\left(\partial_\mu A_{(n)}^{\mu}\right)^2\right], \tag{D.3}$$

where $F_{(n)}^{\mu\nu} = \partial^\mu A_{(n)}^{\nu} - \partial^\nu A_{(n)}^{\mu}$.

Free Lagrangian for the unphysical, neutral would-be Goldstone modes

$$\mathcal{L}_{\rm kin}^{G^3} = \sum_{n=1}^{\infty}\left[\frac{1}{2}\left(\partial_\mu G_{(n)}^{3}\right)^2 - \frac{\xi}{2}\,\hat{m}_{Z(n)}^2\,G_{(n)}^{3\,2}\right] + \sum_{n=1}^{\infty}\left[\frac{1}{2}\left(\partial_\mu A_{(n)4}\right)^2 - \frac{\xi}{2}\left(\frac{n}{R}\right)^2 A_{(n)4}^2\right] \tag{D.4}$$

Free Lagrangian for the unphysical charged would-be Goldstone modes

$$\mathcal{L}_{\rm kin}^{G^\pm} = \sum_{n=1}^{\infty}\left[\left(\partial_\mu\, G_{(n)}^{+}\right)\left(\partial^\mu\, G_{(n)}^{-}\right) - \xi\,\hat{m}_{W(n)}^2\,G_{(n)}^{+}\,G_{(n)}^{-}\right] \tag{D.5}$$

The Higgs sector

The physical, massive scalars in the Higgs sector crucially depend on the free parameters in the Higgs potential. Starting from the most general Higgs sector Lagrangian

$$\mathcal{L}^{\rm H} = (\,D_M\,\Phi_1\,)^\dagger\left(D^M\,\Phi_1\right) + \delta(y)(\,D_\mu\,\Phi_2\,)^\dagger\,(\,D^\mu\,\Phi_2\,) - V(\Phi_1,\Phi_2) \tag{D.6}$$

it is easy but tedious to derive all the kinetic terms and the interactions involving physical, massive scalars. All the interactions are direct generalizations of the ones shown in Appendix C for the Abelian model.

Gauge boson self-interactions

Considering the gauge boson self-interactions we can use the analysis of chapter 3 by specifying the structure constants $f^{abc} = \epsilon^{abc}$ for SU(2). Let us now consider fermions and their interactions.

Kinetic terms of the fermions

$$\mathcal{L}_{\rm kin}^{\rm F}(x) = \overline{\Psi}_f(x)\,(\,i\,\gamma^\mu\,\partial_\mu - m_f\,)\,\Psi_f(x)\,, \tag{D.7}$$

where f denotes one of the six leptons e, ν_e, μ, ν_μ, τ and ν_τ or one of the six quarks u, d, c, s, t and b. The propagators are identical to the standard model ones.

Fermion interactions with the gauge bosons

Below we will list all the interaction terms between the vector bosons of the theory with the standard model fermions:

$$\mathcal{L}_{\rm int}(x,y) \;=\; \sum_{n=0}^{\infty} \hat{g}_{W(n)} \Big(\hat{W}^{+}_{(n)\mu}\, J^{+\mu}_{W} \;+\; \hat{W}^{-}_{(n)\mu}\, J^{-\mu}_{W} \Big) \;+\; \hat{g}_{Z(n)}\, \hat{Z}_{\mu}\, J^{\mu}_{Z} \;+\; \hat{e}_{(n)}\, \hat{A}_{\mu}\, J^{\mu}_{\rm EM}\,, \tag{D.8}$$

where

$$\begin{aligned} J^{+\mu}_{W} \;&=\; \frac{1}{2\sqrt{2}} \left[\bar{l}_{\nu}\, \gamma^{\mu} \left(1-\gamma^{5}\right) l_{e} \;+\; \overline{q}_{u}\, \gamma^{\mu} \left(1-\gamma^{5}\right) q_{d}\; V_{q_u q_d} \right], \\ J^{-\mu}_{W} \;&=\; \frac{1}{2\sqrt{2}} \left[\bar{l}_{e}\, \gamma^{\mu} \left(1-\gamma^{5}\right) l_{\nu} \;+\; \overline{q}_{d}\, \gamma^{\mu} \left(1-\gamma^{5}\right) q_{u}\; V_{q_u q_d} \right], \\ J^{\mu}_{Z} \;&=\; \frac{1}{4\cos\theta_w} \left[\overline{f}\, \gamma^{\mu} \left(\left[2\, T_{3\,f} - 4\, Q_f \sin^2\theta_w \right] \;-\; 2\, T_{3\,f}\, \gamma^{5} \right) f \right], \\ J^{\mu}_{EM} \;&=\; \left[\overline{f}\, Q_f\, \gamma^{\mu}\, f \right] \end{aligned} \tag{D.9}$$

and we have introduced the notation $l_\nu = (\nu_e,\, \nu_\mu,\, \nu_\tau)$, $l_e = (e,\, \mu,\, \tau)$, $q_u = (u,\, c,\, t)$, $q_d = (d,\, s,\, b)$ and f denotes all 12 standard model fermions. As in the conventional standard model the weak isospins T_{3f} and the electromagnetic charges Q_f of the different fermions are given by:

	l_ν	l_e	q_u	q_d
T_{3f}	1/2	- 1/2	1/2	- 1/2
Q_f	0	- 1	2/3	- 1/3

$\hat{A}^{\mu}_{(0)}$, $\hat{Z}^{\mu}_{(0)}$ and $\hat{W}^{-\mu}_{(0)}$ correspond to the experimentally found photon, Z and W bosons, $\hat{A}^{\mu}_{(n)}$, $\hat{Z}^{\mu}_{(n)}$ and $\hat{W}^{-\mu}_{(n)}$ $(n = 1,2,3\ldots)$ are the higher Kaluza-Klein mass eigenstates of the electroweak gauge bosons. $V_{q_u q_d}$ denotes the standard model CKM-matrix due to mixing in the Yukawa sector (see below). The physical couplings $\hat{e}_{(n)}$, $\hat{g}_{Z(n)}$ and $\hat{g}_{W(n)}$ are given, in analogy with eq.(4.77), by

$$\begin{aligned} \hat{e}_{(0)} \;&=\; e\,, \qquad\qquad \hat{e}_{(n\geq 1)} \;=\; \sqrt{2}\, e\,, \\ \hat{g}_{Z(n)} \;&=\; \sqrt{2}\, g \left(1 \;+\; \frac{m_Z^2 \sin^2\beta}{\hat{m}^2_{Z(n)} - m_Z^2 \cos^2\beta} \;+\; \frac{\pi^2}{M^2} \frac{m_Z^4 \sin^4\beta}{\hat{m}^2_{Z(n)} - m_Z^2 \cos^2\beta} \right)^{-\frac{1}{2}}, \\ \hat{g}_{W(n)} \;&=\; \sqrt{2}\, g \left(1 \;+\; \frac{m_W^2 \sin^2\beta}{\hat{m}^2_{W(n)} - m_W^2 \cos^2\beta} \;+\; \frac{\pi^2}{M^2} \frac{m_W^4 \sin^4\beta}{\hat{m}^2_{W(n)} - m_W^2 \cos^2\beta} \right)^{-\frac{1}{2}}, \end{aligned} \tag{D.10}$$

where $m_Z = gv/(2\cos\theta_w)$ and $m_W = gv/2$. Approximate values for $\hat{g}_{Z(0)}$ and $\hat{g}_{W(0)}$ are given in eq.(6.5) and eq.(6.6). For low energy physics it is mostly sufficient to work in the approximation $\hat{g}_{Z(n\geq 1)} = \sqrt{2}g$ and $\hat{g}_{W(n\geq 1)} = \sqrt{2}g$. The Feynman rules corresponding to the interaction Lagrangian eq.(D.8) are shown in the mass eigenbasis in figure D.2.

Yukawa sector

We have the following gauge invariant interactions of leptons with the Higgs doublets:

$$\mathcal{L}_l^{\rm Y}(x,y) \;=\; -\,\lambda_{1,l}\,\delta(y)\,\overline{E}_L\,\Phi_1(x,y)\,e_R \;-\; \lambda_{2,l}\,\delta(y)\,\overline{E}_L\,\Phi_2(x)\,e_R \;+\; \text{h.c.}\,, \tag{D.11}$$

where $E_L(x)$ denotes the left-handed components of the doublets (ν_e, e), (ν_μ, μ), (ν_τ, τ) and e_R the right-handed components of e, μ, τ, respectively. The Higgs doublets $\Phi_1(x,y)$ and $\Phi_2(x)$ are given in eq.(4.131). The couplings $\lambda_{1,l}$ and $\lambda_{2,l}$ are different for each family.

For the quarks we have

$$\mathcal{L}_q^{\rm Y}(x,y) = -\lambda_{1,u}\,\delta(y)\,\overline{Q}_L\,\Phi_1(x,y)\,d_R - \lambda_{1,d}\,\delta(y)\,\epsilon^{ab}\overline{Q}_{La}\,\Phi_{1b}^\dagger(x)\,u_R + (1 \leftrightarrow 2) + \text{h.c.}\,, \tag{D.12}$$

where $Q_L(x)$ denotes the left-handed components of the doublets (u,d), (c,s), (t,b), u_R the right-handed components of u, c, t and d_R the right-handed components of d, s, b, respectively. The Lagrangians in eq.(D.11) and eq.(D.12) include the fermion mass terms in eq.(D.7). The mass matrix of the quarks is already diagonalized introducing the CKM-matrix in eq.(D.9).

Fermion interactions with the would-be Goldstone modes

While the physical scalars crucially depend on the Higgs potential the unphysical would-be Goldstone modes do not.

Generalizing the proof in section 5.3 for the electroweak model one can show that the Feynman rules for the interactions of the would-be Goldstone modes with fermions are those shown in figure D.3.

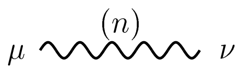
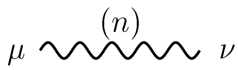
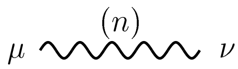
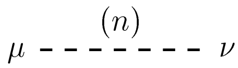
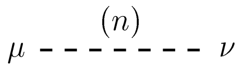
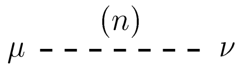
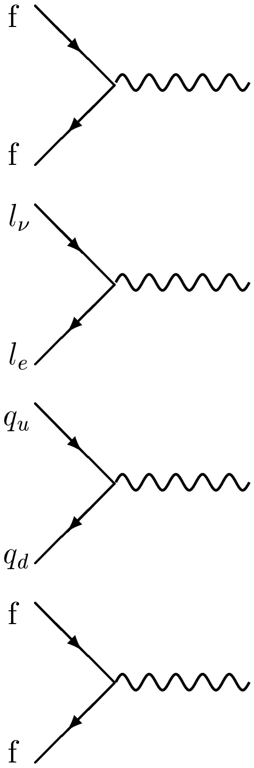
photon propagator:		$= \frac{i}{k^2-\left(\frac{n}{R}\right)^2}\left(-g^{\mu\nu} + \frac{(1-\xi)k^\mu\, k^\nu}{k^2-\xi\left(\frac{n}{R}\right)^2}\right)$
W-boson propagator:		$= \frac{i}{k^2-\hat{m}^2_{W(n)}}\left(-g^{\mu\nu} + \frac{(1-\xi)k^\mu\, k^\nu}{k^2-\xi\,\hat{m}^2_{W(n)}}\right)$
Z-boson propagator:		$= \frac{i}{k^2-\hat{m}^2_{Z(n)}}\left(-g^{\mu\nu} + \frac{(1-\xi)k^\mu\, k^\nu}{k^2-\xi\,\hat{m}^2_{Z(n)}}\right)$
A_4-propagator:		$= \frac{i}{k^2-\xi\left(\frac{n}{R}\right)^2}$
G^3-propagator:		$= \frac{i}{k^2-\xi\,\hat{m}^2_{Z(n)}}$
$G^\pm$-propagator:		$= \frac{i}{k^2-\xi\,\hat{m}^2_{W(n)}}$

Figure D.1: The propagators of the gauge bosons and the corresponding would-be Goldstone modes in the 5D standard model in generalized R_ξ-gauge.



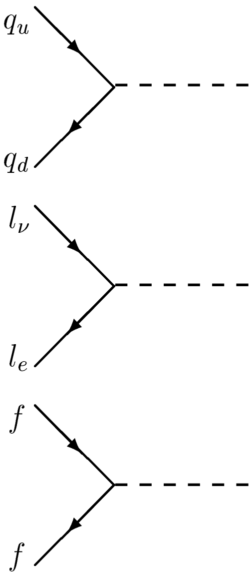
$$\hat{A}^{\mu}_{(n)} \quad = i\,\hat{e}_{(n)}\,Q_f\,\gamma^{\mu}$$

$$\hat{W}^{-\mu}_{(n)} \quad = \tfrac{i}{2\sqrt{2}}\,\hat{g}_{W(n)}\,\gamma^{\mu}\,(\,1-\gamma^5\,)$$

$$\hat{W}^{-\mu}_{(n)} \quad = \tfrac{i}{2\sqrt{2}}\,\hat{g}_{W(n)}\,\gamma^{\mu}\,(\,1-\gamma^5\,)V_{q_u q_d}$$

$$\hat{Z}^{\mu}_{(n)} \quad = \tfrac{i}{4\cos\theta_w}\,\hat{g}_{Z(n)}\,\gamma^{\mu}\,\left[\,\left(\,2T_{3f}-4Q_f\sin^2\theta_w\,\right)-2T_{3f}\gamma^5\,\right]$$

Figure D.2: Vertices for the interactions of fermions with gauge bosons in 5D electroweak theory. The abbreviations used in this figure are explained in the text following eq.(D.8).



$$\hat{G}^{-}_{(n)} \quad = \frac{-\,i}{2\sqrt{2}\,\hat{m}_{W(n)}}\,\hat{g}_{W(n)}\,[\,m_{q_d}(\,1-\gamma^5\,)-m_{q_u}(\,1+\gamma^5\,)\,]\,V_{q_u q_d}$$

$$\hat{G}^{-}_{(n)} \quad = \frac{-\,i m_{l_e}}{2\sqrt{2}\,\hat{m}_{W(n)}}\,\hat{g}_{W(n)}\,(\,1-\gamma^5\,)$$

$$\hat{G}^{3}_{(n)} \quad = \frac{m_f}{2\,\cos\theta_w\,\hat{m}_{Z(n)}}\,\hat{g}_{Z(n)}\,(\,-2T_{3f}\,)\,\gamma^5$$

Figure D.3: Vertices for the interactions of fermions with the unphysical would-be Goldstone-modes of the theory. m_f and m_{l_e} denote the corresponding fermion masses.

Appendix E

Effective Four-Fermion Interactions

In gauge theories the interactions between fermions are mediated by the exchange of vector bosons. However, if these vector bosons are heavy compared with the momentum exchanged in a given process it is common to approximate the theory by a four-fermion interaction. We start from a Lagrangian describing such a heavy gauge boson coupled to fermions, i.e.

$$\mathcal{L} \,=\, -\frac{1}{4}\,F_{\mu\nu}\,F^{\mu\nu} \,+\, \frac{1}{2}m^2 A_\mu A^\mu \,+\, \overline{\Psi}\,(\,i\partial_\mu\gamma^\mu \,-\, m_f\,)\,\Psi \,+\, \overline{\Psi}\,\gamma^\mu\big(\,g_V + g_A\gamma^5\,\big)A_\mu\Psi\,, \quad \text{(E.1)}$$

where m denotes the gauge boson mass and m_f a possible fermion mass. If the exchanged momentum carried by the gauge boson is indeed negligible in comparison to the gauge boson mass it is a good approximation to neglect the momentum of the gauge boson in any gauge boson propagator. On the Lagrangian level this is equivalent to neglecting the kinetic term of the exchanged particle which is responsible for the momentum dependence of the propagator. Setting $F_{\mu\nu} = 0$ makes A^μ be an auxiliary field which can be integrated out using its equation of motion

$$m^2 A^\mu \,=\, -\,\overline{\Psi}\,\gamma^\mu\big(\,g_V + g_A\gamma^5\,\big)\Psi\,. \quad \text{(E.2)}$$

Inserting eq.(E.2) into the Lagrangian eq.(E.1) we obtain the effective four-fermion Lagrangian

$$\mathcal{L}_{\mathrm{4F}} \,=\, \overline{\Psi}\,(\,i\partial_\mu\gamma^\mu \,-\, m_f\,)\,\Psi \,-\, \frac{1}{2m^2}\,\big(\,\overline{\Psi}\,\gamma^\mu\big(\,g_V + g_A\gamma^5\,\big)\Psi\,\big)\,\big(\,\overline{\Psi}\,\gamma_\mu\big(\,g_V + g_A\gamma^5\,\big)\Psi\,\big)\,. \quad \text{(E.3)}$$

So far, we investigated a theory with only one fermion species but the generalization to a wider class of theories with several different fermions is obvious.

Within the 5D standard model there is a whole tower of extremely massive vector bosons corresponding to each of the three standard model gauge bosons. In almost all actual calculations it is a very good approximation to integrate out at least the heavy Kaluza-Klein mass eigenstates. The effective four-fermion Lagrangian for each of the three gauge bosons Kaluza-Klein towers is given by

$$\mathcal{L}_{4\mathrm{F}} = -\sum_{n=1}^{\infty} \frac{1}{2\hat{m}^2_{(n)}} \Big(\overline{\Psi}_i \, \gamma^\mu \Big(\hat{g}^{ij}_{(n)V} + \hat{g}^{ij}_{(n)A} \gamma^5 \Big) \Psi_j \Big) \left(\overline{\Psi}_k \, \gamma_\mu \left(\hat{g}^{kl}_{(n)V} + \hat{g}^{kl}_{(n)A} \gamma^5 \right) \Psi_l \right), \tag{E.4}$$

where $\hat{m}_{(n)}$ denotes the mass of the n^{th} Kaluza-Klein mass eigenstate, i, j, k and l are labels for the standard model fermions and $\hat{g}^{ij}_{(n)V,A}$ denotes the corresponding couplings. If the zero mode of the Kaluza-Klein tower, i.e. the standard model Z or W boson can also be integrated out, as for example in the derivation of the muon lifetime, the effective Lagrangian is given by

$$\begin{aligned} \mathcal{L}_{4\mathrm{F}} = &-\frac{1}{2\,\hat{m}^2_{Z,W}} \left(\overline{\Psi}_i \gamma^\mu \left(\hat{g}^{ij}_V + \hat{g}^{ij}_A \gamma^5 \right) \Psi_j \right) \left(\overline{\Psi}_k \gamma_\mu \left(\hat{g}^{kl}_V + \hat{g}^{kl}_A \gamma^5 \right) \Psi_l \right) \\ &-\sum_{n=1}^{\infty} \frac{1}{2\,\hat{m}^2_{Z,W(n)}} \Big(\overline{\Psi}_i \gamma^\mu \Big(\hat{g}^{ij}_{(n)V} + \hat{g}^{ij}_{(n)A} \gamma^5 \Big) \Psi_j \Big) \left(\overline{\Psi}_k \gamma_\mu \left(\hat{g}^{kl}_{(n)V} + \hat{g}^{kl}_{(n)A} \gamma^5 \right) \Psi_l \right). \end{aligned} \tag{E.5}$$

For the special case of the muon decay the relevant part of the effective Lagrangian is given by $(i,j) = (\mu, \nu_\mu)$, $(k,l) = (e, \nu_e)$ and $(i,j) = (e, \nu_e)$, $(k,l) = (\mu, \nu_\mu)$. Inserting $\hat{g}^{\mu\nu_\mu}_{WV} = -\hat{g}^{\mu\nu_\mu}_{WA} = \hat{g}^{e\nu_e}_{WV} = -\hat{g}^{e\nu_e}_{WA} = \hat{g}_W/2\sqrt{2}$ and the equivalent relation for the couplings of the higher Kaluza-Klein mass eigenstates we finally arrive at

$$\mathcal{L}_{4\mathrm{F}} = -\left(\frac{\hat{g}_W}{8\,\hat{m}^2_W} + \sum_{n=1}^{\infty} \frac{\hat{g}_{W(n)}}{8\,\hat{m}^2_{Z(n)}} \right) \left(\overline{\Psi}_\mu \gamma^\mu \left(1 - \gamma^5 \right) \Psi_{\nu_{mu}} \right) \left(\overline{\Psi}_e \gamma_\mu \left(1 - \gamma^5 \right) \Psi_{\nu_e} \right). \tag{E.6}$$

Recalling the definition of the Fermi constant in the context of an effective four-fermion Lagrangian [30]

$$\mathcal{L}_{4\mathrm{F}} = -\frac{G_F}{\sqrt{2}} \left(\overline{\Psi}_\mu \gamma^\mu \left(1 - \gamma^5 \right) \Psi_{\nu_\mu} \right) \left(\overline{\Psi}_e \gamma_\mu \left(1 - \gamma^5 \right) \Psi_{\nu_e} \right) \tag{E.7}$$

leads directly to the relation used in eq.(6.9).

Appendix F

Input Parameters, Observables and Standard Model Predictions

This Appendix lists the numerical values of the input parameters and observables as well as their standard model predictions used in chapter 6 to constrain the parameters of the 5D standard model. As input parameters we use the best measured observables, namely the Fermi constant G_F, measured in muon decay, the fine structure constant α determined from the quantum Hall effect and the Z-boson mass M_Z [31]:

$$\begin{aligned} G_F &= 1.16637(1) \times 10^{-5}\,\mathrm{GeV}^{-2}\,, \\ \alpha &= 1/137.0359895(61)\,, \\ M_Z &= 91.1872(21)\,\mathrm{GeV}\,, \end{aligned} \tag{F.1}$$

where the numbers in parenthesis are the 1σ-errors.

From these input parameters one can predict the observables in the standard model framework. These calculations have been done and their results are also listed by the Particle Data Group along with the measured values of the observables [31]. Table F.1 lists the observables taken into account in our analysis in chapter 6.

Furthermore, one can define the weak mixing angle $\hat{\theta}_w$ by

$$G_F = \frac{\pi\alpha}{\sqrt{2}\sin^2\hat{\theta}_w\,\cos^2\hat{\theta}_w\,M_Z^2}\,. \tag{F.2}$$

Including the running of the fine structure, that is $\alpha(M_Z) = 1/128.92(3)$ at the Z pole, the input parameters result in

$$\sin^2\hat{\theta}_w = 0.23105(8)\,. \tag{F.3}$$

This is the numerical value for $\sin^2\hat{\theta}_w$ used in chapter 6.

Observable	Value	Standard Model Prediction
M_W	80.448(62) GeV	80.378(20) GeV
Γ_Z(had)	1.7439(20) GeV	1.7422(15) GeV
$\Gamma_Z(l^+l^-)$	83.96(9) MeV	84.00(3) MeV
$\Gamma_Z(\nu\overline{\nu})$	498.8(1.5) MeV	501.65(15) MeV
Q_W(Cs)	- 72.06(44)	-73.09(03)

Table F.1: Precision measurements and the corresponding standard model predictions. M_W is the mass of the W boson, Γ_Z(had) the hadronic width of the Z boson, $\Gamma_Z(\mathrm{l}^+\mathrm{l}^-)$ the leptonic width, $\Gamma_Z(\nu\overline{\nu})$ the invisible width and Q_W(Cs) the weak charge for cesium which parameterizes the atomic parity violation. For a detailed discussion see chapter 6.

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Erklärung

Hiermit erkläre ich, dass ich die vorliegende Arbeit selbständig verfasst und keine anderen als die angegebenen Hilfsmittel verwendet habe.

Würzburg, den 18.06.01

Alexander Mück