Mass Factorization of SUSY QCD Processes in Dimensional Reduction

Diplomarbeit
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In dieser Arbeit wird die Massenfaktorisierung zweier für die Detektion von Supersymmetrie (SUSY) an Hadron-Collidern wichtigen partonischen SUSY QCD Prozesse, $qq \to \widetilde{t}\,\overline{\widetilde{t}}$ und $gg \to \widetilde{t}\,\overline{\widetilde{t}}$, in dimensionaler Reduktion (DRED) gezeigt. Dieses Regularisierungsschema ist dadurch ausgezeichnet, dass es SUSY erhält. Vor fast 20 Jahren fanden Beenakker et al. [1] bei ähnlichen Prozessen in DRED nicht faktorisierbare Terme, welche diese Regularisierungsmethode in Frage stellten. Letztes Jahr schlugen Signer und Stöckinger eine Lösung für dieses sogenannte Faktorisierungsproblem vor [2], in der sie das 4-komponentige Gluon in einen D-komponentigen Vektor und ein Skalar aufteilten und diese beiden als einzelne Partonen behandelten. In der vorliegenden Arbeit wird demonstriert, wie mit der herkömmlichen Methode vermeintlich nicht faktorisierbare Terme auftreten. Analog zu der Betrachtungsweise von Signer und Stöckinger werden diese dann umgeschrieben und gezeigt, dass das erwünschte Ergebnis auch für die betrachteten Prozesse erzielt wird.

In this thesis we demonstrate the mass factorization for the partonic SUSY QCD processes $qq \to \widetilde{t}\,\overline{\widetilde{t}}$ and $gg \to \widetilde{t}\,\overline{\widetilde{t}}$ which are important as evidence of supersymmetry (SUSY) at hadron colliders. The calculation is performed in dimensional reduction (DRED) which preserves SUSY. Almost 20 years ago, Beenakker et al. [1] found that non-factorizable terms in DRED calculations appear, which rendered this regularisation scheme questionable. Last year, Signer and Stöckinger [2] proposed a solution for this so-called factorization problem. They considered the 4 component gluon to be the combination of a D component vector and a scalar which both are partons of their own. The present work demonstrates how seemingly non-factorizable terms appear with the conventional method. Following the approach of Signer and Stöckinger we rewrite these terms and demonstrate how the desired results are achieved for the processes investigated in this thesis.

Contents

1 Introduction and Overview

The standard model (SM) of particle physics describes the interactions of fundamental particles based on a $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge theory. The first factor describes the strong interaction mediated by gauge bosons called gluons. The subgroup $SU(2)_L \times U(1)_Y$ describes weak isospin and (weak) hypercharge. It is responsible for the electroweak force which is mediated by electroweak gauge bosons $(\gamma, W^{\pm}, Z)$. Additionally the SM provides mass for some particles by spontaneous electroweak symmetry breaking through the Higgs mechanism.
Besides massive neutrinos, all predictions made by the standard model have been confirmed impressively by experimental data. Despite the achievements up to the present day, this model is incomplete and open questions remain such as the implementation of gravitation or the nature of dark matter. Beyond this, several phenomena, e.g. the electric charge quantization, can be accommodated but not explained.
A prominent unsolved problem in the SM is the large hierarchy between the Higgs mass and the Planck mass. This is one of the fine-tuning problems. Consider the self-energy in next-to-leading order (NLO) of a fermion, e.g. an electron as in fig. 1.1(1). In this case, we can absorb all divergences which are due to the loop integral in fig. 1.1(1) by expressing the renormalized mass as

$$m_e = m_0 \left(1 + \frac{3\alpha}{4\pi} \log \frac{\Lambda^2}{m_0^2}\right) + \ldots$$

which is only logarithmically divergent in the cut-off parameter Λ. m_0 is the bare electron mass. This implies that quantum electrodynamics (QED) includes divergences which are innocuous as long as the parameter Λ is identified with the maximum validitiy of the theory, e.g. the Planck mass ($M_{Pl} = 1/\sqrt{G} \propto 1.2 \cdot 10^{19}$GeV). This keeps the ratio m_e/m_0 at about 1.17 and the smallness of the electron mass $m_e \ll M_{Pl}$ is therefore natural.
If we look at the NLO corrections to the Higgs boson in fig. 1.1(2) we have a similar situation. The propagator of each fermion contributes with $1/k$, so that the total loop integral is divergent with

$$\int^{\Lambda} d^4k \frac{1}{k^2} = \int^{\Lambda} dk\, k \propto \Lambda^2$$

where Λ is again the cut-off parameter which should be chosen as the Planck scale if the theory is to be valid up to this scale. Unlike in the electron self energy, we have a quadratically divergent part of the loop integral.

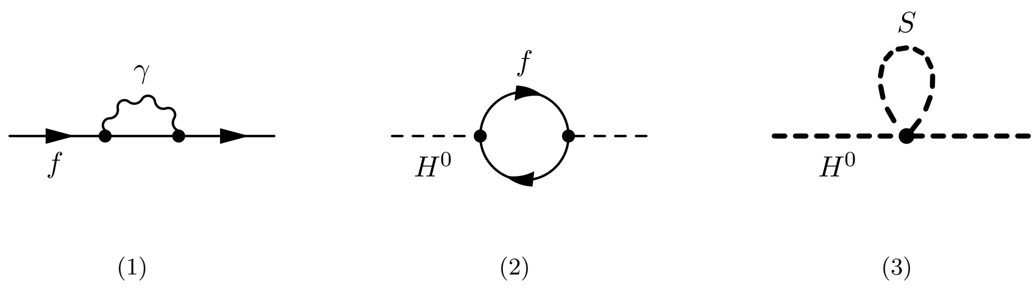


Figure 1.1: Self energy of a fermion (1) and the Higgs boson in the SM (2) and in SUSY (3).

If we consider only a fermion in the loop, the Higgs mass evaluates to

$$m_H^2 = m_{H,0}^2 - \frac{|\lambda_f|^2}{8\pi^2}\Lambda^2 + \mathcal{O}\left(\log\frac{\Lambda^2}{m_f^2}\right) + \ldots \tag{1.1}$$

where λ_f denotes the dominant fermion-Higgs boson coupling. Thus the contributions of the correction terms to the Higgs mass increase quadratically with the cut-off parameter Λ.
The crucial point is that, if we choose the Planck scale as the cut-off parameter, Λ^2 is of the order of $10^{38}\,\mathrm{GeV}^2$. Since the physical Higgs mass is expected to be of the order of 100 GeV, the bare squared Higgs mass must be precisely set close to the Planck scale to an accuracy of about 32 digits since small variations of the bare Higgs mass would change the physical Higgs mass drastically.
This discussion demonstrates that the SM without extension can only be valid up to energies far below the Planck scale without fine-tuning. An elegant solution to avoid these problems uses the fact that boson loops and fermion loops have a relative sign.
Consider a scalar coupling with a Higgs boson as in fig. 1.1(3). The contribution of this graph to the Higgs mass is divergent with $\lambda_s \cdot \Lambda^2$ where λ_s is the 4-scalar coupling. Such a coupling with $\lambda_s = |\lambda_f|^2$ would compensate the entire Λ^2 divergence in eq. 1.1. This can also be applied to higher order processes which are beyond the scope of this discussion.
The supersymmetric version of the SM extends the particle spectrum in exactly this way. SM fermions and their corresponding spin-0 partners (sfermions) form a chiral supermultiplet and the couplings of sfermions and fermions to the Higgs boson are such that $\lambda_s = |\lambda_f|^2$, thus providing a solution to the hierarchy problem. The corresponding fermionic supersymmetric partners of the SM gauge bosons are called gauginos and both together form the gauge supermultiplet. For further reading we refer to [3].
Since no superpartners of the SM particles have been observed so far, SUSY must be softly broken in such a way that the masses of the SM particles and their SUSY partners differ. Soft breaking means that masses and couplings may be modified only in ways that do not reintroduce the quadratic divergences in eq. 1.1. For example, the selectron, the superpartner to the electron, is required to be heavier than 73 GeV at a confidence level of 95% [4] and therefore has to receive a SUSY breaking mass.

Since the Large Hadron Collider (LHC) at CERN will commence operation in the next year, the predictions of SUSY and other alternative theories beyond the SM can be tested up to an unprecedented energy reaching several TeV. For the exclusion or confirmation of alternative theories, precision calculations have to be performed. These "next to leading order" (NLO) or "next to next to leading order" (NNLO) calculations imply loop calculations which lead to infinities. Several regularization schemes have been introduced over the years to ensure a proper treatment of such divergences.
In QED for example, the Pauli Villars regularization scheme introduces additional heavy particles in order to remove ultraviolet (UV) divergences. After performing the calculation, the mass of these particles is taken to infinity. This regularization scheme does only work in abelian gauge theories as it breaks non-abelian gauge invariance. An improvement which preserves non-abelian gauge invariance is dimensional regularization (DREG) [5]. Here the space time is not taken as 4- but D-dimensional in order to render loop integrals finite.
As can be shown, this regularization scheme breaks SUSY and requires the inclusion of correction terms which restore it [6, 7, 8, 9]. However, Siegel [10] and Capper et al. [11] found a regularization scheme preserving supersymmetry, namely dimensional reduction (DRED) where all momenta are kept in D dimensions while vector fields retain 4 components. Thus the integrals are regularized and finite, and SUSY is preserved [6, 11, 7, 8, 9, 12, 13].
Apart from UV and infrared (IR), another kind of divergence appears if a particle is emitted collinear to an initial, effectively massless particle. In our case we consider collinear gluonic initial state radiation. The squared matrix element has a universal structure and the divergence can always be extracted and absorbed into a redefinition of the parton distribution function.

Almost 20 years ago Beenakker et al. [1] tried to show mass factorization for processes calculated in dimensional reduction, but found seemingly non-factorizable terms which violate the factorization theorem which are absent in DREG. Thus the problem rendered DRED questionable [6].
Last year, Signer and Stöckinger [2] were able to rewrite the seemingly non-factorizable terms and showed that the process can be factorized by reinterpreting the 4-dimensional gluon as two particles.
In this work, two partonic SUSY quantum chromodynamics (SUSY QCD) processes are derived and factorized in DRED. In one of the two processes, the factorization problem appears, and it will be demonstrated that this process also factorizes in DRED by applying the technique of [2]. The crucial point of the solution is that in DRED the 4-dimensional vector boson can be written as a composition of a D-dimensional vector boson and a scalar. The seemingly non-factorizable terms can be rewritten as a linear combination of processes where these two components are treated as two distinct partons.
Since the processes in this work factorize with the method proposed by Signer and Stoeckinger, their solution of the factorization problem is corroborated.

This thesis is organized as follows:
In chapter 2 we discuss deep inelastic scattering, mass factorization and the Dokshitzer-Gribov-

Lipatov-Altarelli-Parisi (DGLAP) equations. In addition, collinear factorization for a particular example is introduced and calculated. In chapter 3 we give an overview and introduction to SUSY and discuss the need of dimensional reduction in MSSM processes.
The result for the mass factorization in DRED of the most important processes for stop production at LHC

$$q\overline{q} \to g\,\widetilde{t}\,\overline{\widetilde{t}} \tag{1.2a}$$

$$gg \to g\,\widetilde{t}\,\overline{\widetilde{t}} \tag{1.2b}$$

is given and discussed in chapter 4. Though the process in eq. 1.2a factorizes in DRED as expected, the process in eq. 1.2b seems to violate the factorization theorem. From this we motivate the separation of the 4-dimensional gluon into two partons and calculate the Feynman rules and splitting functions for this new scalar.
In chapter 5 we calculate both processes in eq. 1.2 in LO in dimensional reduction followed by the NLO calculation of the processes and the factorization in 4 dimensions and dimensional reduction in chapter 6. We demonstrate that they factorize as desired when the method proposed in [2] is applied.
After the detailed demonstration of the calculation we discuss the result in chapter 7, give criteria for the appearance of the factorization problem and argue its absence in the massless limit.
In chapter 8 the calculations and results are summarized.

2 QCD, DIS and the Factorization Theorem

Due to confinement no free quarks exist in nature but only color neutral bound states called hadrons which include baryons (quark triplets) and mesons (quark pairs). This makes direct quark collision experiments impossible and therefore hadron collisions have to be used to investigate quark scattering.
In this work we present the mass factorization of two SUSY QCD processes. For the derivation of these we treat the initial partons as free particles with well defined momenta. Since we investigate processes which are probed at the LHC, the scattering particles are protons. In this chapter we will give an overview of the calculation with partons, its connection with the hadronic cross section, and we discuss mass factorization.

2.1 Deep Inelastic Scattering

At low energies a proton can be described as a spin 1/2 particle with electric charge +1 and mass of about one GeV. High energy interactions (≥ 10 GeV) unveil a substructure of the proton as a bound state of quarks and gluons. When a proton is probed at these high energies, it seems to be constituted by gluons and one down and two up quarks which define the quantum numbers of the proton and are called valence quarks.
For even higher energies, the gluons split into quark antiquark pairs, and quarks radiate gluons so that new particles are present in the parton. However, the net quark content defined as $\sum_i q_i - \overline{q}_i$ (i runs over the quark flavors) stays the same and still corresponds to the valence quarks. These particles, which do not alter the quantum number of the hadron, are called sea-quarks. Scattering processes at energies which probe this partonic structure of hadrons are referred to as deep inelastic scattering processes (DIS).
Due to asymptotic freedom, short distance cross sections can be calculated perturbatively [14] [15]. In the following we demonstrate how a hadronic cross section is constituted by a partonic cross section and the corresponding parton distribution functions (PDFs), which give the probability to find a parton with a certain fraction of the total momentum inside the hadron. We consider an experiment in which two protons with momenta P_1 and P_2 are collided as in fig. 2.1. All constituents carry a fraction of the total hadron momentum

$$\begin{aligned} p_1 &= x_1 P_1 \\ p_2 &= x_2 P_2 \end{aligned}$$

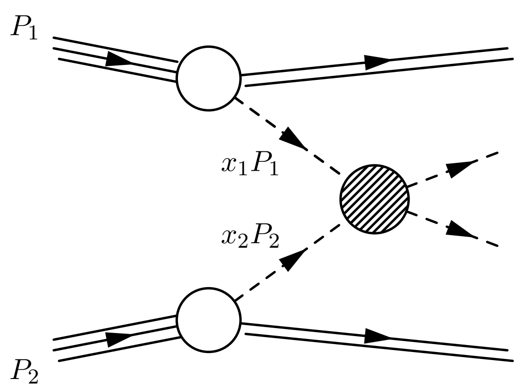


Figure 2.1: Deep inelastic scattering of protons. At energies above 10 GeV the substructure can be tested and the proton collision can be described by a collision of two quarks or gluons, each carrying a fraction $x_i\, P_i$ of the proton momentum P_i. The arrows stand for the flow of momenta.

where x_i is between 0 and 1. The cross section for hard scattering processes with initial hadron momenta P_1 and P_2 can be written as

$$\begin{aligned}&\sigma\left(p(P_1) + p(P_2) \to X + Y\right) = \\ &\int_0^1 dx_1 \int_0^1 dx_2 \sum_{i,j} f_i(x_1, \mu) f_j(x_2, \mu) \cdot \hat{\sigma}\left(q_i(x_1 P_1) + q_j(x_2 P_2) \to Y\right)\end{aligned} \tag{2.1}$$

The momenta are defined as they appear in fig. 2.1. Consequently, the cross section for two hadrons is the result of the cross section between quarks or gluons as the constituents of the incoming hadrons. The functions $f_i(x_j, \mu)$ which give the probability of finding the constituent with the fraction x_j of the momentum, namely the quark and gluon distribution inside the hadron, are known as the parton distribution functions (PDFs). They are defined at a factorization scale μ, which is a parameter that separates the long (hadronic) and short (partonic) distance physics.

Equation 2.1 implies, that the long distance hadronic cross section can be expressed in terms of the short distance cross section by separating long distance pieces (e.g. collinear gluon radiation) by absorbing them into the PDFs. This separation into the partonic short distance cross section and the PDF (eq. 2.1) is referred to as the factorization theorem and was proven by Collins and Soper to all orders of perturbation theory [16].
The short distance cross section is independent from the details or type of the incoming hadron and can be calculated perturbatively. This implies that we can use ordinary QCD Feynman rules for the short distant part of the calculation of hadron scattering. For a detailed description of

DIS we refer to [17].
The PDF $f_i(x_j, \mu)$ can not be calculated from perturbative QCD, since the strongly interacting physics of bound hadrons enters the calculation. This is why the PDFs have to be obtained by fitting the parton model predictions to experimental data. In the next sections, we will discuss that the PDFs depend on the factorization scale μ and sketch the derivation of the DGLAP equations which give the evolution of the PDFs with this energy scale μ^2.

2.2 Collinear Divergences

Radiative corrections to QCD processes lead to divergences which can either be removed by renormalization or canceled with soft virtual corrections at NLO. Another divergence appearing in real corrections NLO originates from collinearly (but not necessarily soft) radiated gluons. We demonstrate how this divergence arises and how it can be treated. Our approach is similar to [18].
As an example we consider the emission of a gluon g by an (almost) massless down type quark with momentum q in a scattering process

$$W^+ + d \to u + g \tag{2.2}$$

where the momenta of the initial and final particles are denoted as in fig. 2.2. For scattering processes at high energies where $E \gg m$, the initial particle emitting the gluon can be treated as massless.
The gluon is radiated collinear to an initial particle with a small transverse momentum $k_\perp \ll \mu$. The propagator of the massless parton emitting the gluon (highlighted in fig. 2.2) goes onshell for $k_\perp \to 0$ and a divergence appears. After factorization, this divergence can be absorbed into PDFs. To be more specific, consider the process eq. 2.2 shown in fig. 2.2. The Feynman amplitude for this process is

$$\mathcal{M}_{W^+ + d \to g + u} = \overline{v}(p+q-k) \cdot i\, e\, \gamma_\nu \frac{1-\gamma^5}{2} \frac{V_{ij}}{\sqrt{2s}} \cdot \epsilon^{*\,\nu}(q) \frac{i(\not{p} - \not{k})}{(p-k)^2} \cdot i\, g\, \gamma_\mu\, t^a\, \epsilon_a^{*\,\mu}(k)\, u(p) \tag{2.3}$$

For high energy scattering we neglect the mass of the incoming particle m. We introduce the momentum p'

$$(p-k) = p' \tag{2.4}$$

The so-called mass singularity appears as the pole $1/(p-k)^2$ which diverges when the emitted gluon is made collinear to the incoming quark by setting $(p-k)^2 = 0$. We first parametrize the gluon momentum

$$k^\mu = zp^\mu + k_\perp^\mu - \frac{k_\perp^2\, n^\mu}{2z\ p \cdot n} \tag{2.5a}$$

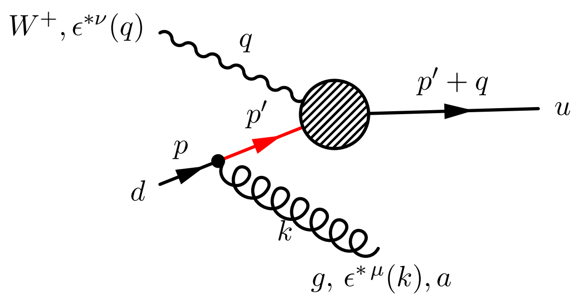


Figure 2.2: Graph for the collinear gluon emission in our example. A charged vector boson merges with a down type quark to produce an up type quark. Before the actual collision the down type quark radiates a collinear gluon.

$$p'^{\mu} \quad = \quad (1-z)p^{\mu} - k_{\perp}^{\mu} + \frac{k_{\perp}^2\, n^{\mu}}{2z\; p\cdot n} \tag{2.5b}$$

as sketched in fig. 2.3. The parametrization in eq. 2.5a is chosen as in [2] and [19]. As we see from eqs. 2.5, momentum is conserved, but the down type quark with momentum p' is offshell with $p'^2 = k_{\perp}^2/z$. Since the incoming quark is onshell ($p^2 = 0$), this parametrization guarantees that the gluon is also onshell [1]. The variable z determines the energy of the radiated gluon and in the limit $z \to 0$ it is infinitely soft and an IR divergence appears. Since the collinear divergence is only a function of the direction of the momentum, the energy of the gluon is arbitrary and z enters the calculation. For $k_{\perp} \to 0$ the collinear limit is reached which implies that the transverse momentum $k_{\perp}$ determines the acollinearity of the gluon. The parametrization is constructed with the auxiliary vector n (fig. 2.3) to ensure that the gluon and the initial quark are onshell. Since n is lightlike and perpendicular to $k_{\perp}$ and p we can use the following identities

$$\begin{aligned} n^2 &= 0 \\ p\cdot n &= 0 \\ k_{\perp}\cdot n &= 0 \end{aligned} \tag{2.6}$$

Now we square the Feynman amplitude (eq. 2.3) and obtain

[1] There is another method to perform the collinear limit which will be used in section 4.4. There, we do not conserve momentum, but keep all particles onshell. In this example here, momentum conservation is guaranteed since we have a hadronic process after the splitting of the quark and the gluon. This allows us to treat the exchanged down type quark as a propagating offshell particle. Both methods are discussed in [20].

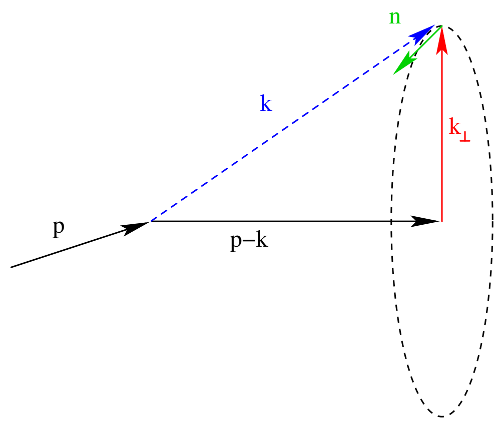


Figure 2.3: Illustration of the vectors n and $k_\perp$ parametrizing the collinear limit of the particles with the momenta p and k. n is an auxiliary lightlike vector which is perpendicular to $k_\perp$.

$$|\mathcal{M}|^2_{W^++d\to g+u} = \widetilde{\mathcal{M}}^\dagger(\not p - \not k)\gamma_\mu u(p)\overline{u}(p)\gamma_{\mu'}(\not p - \not k)\widetilde{\mathcal{M}}\frac{\epsilon_a^{*\,\mu}(k)\epsilon_{a'}^{\mu'}(k)}{(p-k)^4}C_s(r)\delta^{aa'}$$

Here $\widetilde{\mathcal{M}}^\dagger$ stands for the expression

$$\widetilde{\mathcal{M}}^\dagger = \overline{u}(p'+q)\cdot e\,\gamma_\nu\frac{1-\gamma^5}{2}\frac{V_{ij}}{\sqrt{2}s}\cdot\epsilon^{*\,\nu}(q)$$

and will later give the amplitude in eq. 2.10. Since the incoming down type quark is onshell with $p^2 = 0$ we can use eq. A.13 for massless quarks

$$|\mathcal{M}|^2_{W^++d\to g+u} = \widetilde{\mathcal{M}}^\dagger(\not p - \not k)\gamma_\mu \not p\,\gamma_{\mu'}(\not p - \not k)\widetilde{\mathcal{M}}\frac{\epsilon_a^{*\,\mu}(k)\epsilon_{a'}^{\mu'}(k)}{(p-k)^4}C_2(r)\delta^{aa'} \tag{2.7}$$

We use the polarization sum for massless gauge bosons

$$\sum_{s=1,2}\epsilon_s^{*\,\mu}(k)\epsilon_s^{\mu'}(k) = -g^{\mu\mu'} + \frac{k^\mu n^{\mu'} + k^{\mu'}n^\mu}{k\cdot n} \tag{2.8}$$

First, we insert the metric tensor from eq. 2.8 into eq. 2.7, denoting this part with A:

$$\begin{aligned} A &= \widetilde{\mathcal{M}}^\dagger(\not p - \not k)\gamma^\mu \not p\,\gamma^{\mu'}(\not p - \not k)\widetilde{\mathcal{M}}(-g_{\mu\mu'}) \\ &= -\widetilde{\mathcal{M}}^\dagger(\not p - \not k)\gamma^\mu \not p\,\gamma_\mu(\not p - \not k)\widetilde{\mathcal{M}} \end{aligned}$$

$$\begin{aligned} A &\overset{1}{=} 2\widetilde{\mathcal{M}}^\dagger(\not{p}-\not{k})\,\not{p}\,(\not{p}-\not{k})\widetilde{\mathcal{M}} \\ &\overset{2}{=} 2\mathcal{M}^\dagger\,\not{k}\,\not{p}\,\not{k}\,\widetilde{\mathcal{M}} \\ &\overset{1}{=} 4\,\widetilde{\mathcal{M}}^\dagger\not{k}\widetilde{\mathcal{M}}\;p\cdot k \end{aligned}$$

Now we can express the momentum k as

$$k = \frac{z}{1-z}p'$$

which follows from the parametrization of the momenta in eq. 2.5. Obviously, $\widetilde{\mathcal{M}}$ contains no poles of the form $1/(p-k)$ and therefore we can neglect all terms containing $k_\perp$ in this substitution. Now we substitute p' and k using eq. 2.5 and obtain

$$\begin{aligned} A &= 4\widetilde{\mathcal{M}}^\dagger\not{p}'\widetilde{\mathcal{M}}\frac{z}{1-z}\cdot\frac{-k_\perp^2}{2z} \\ &= -2|\widetilde{\mathcal{M}}|^2_{W^++d\to u}\frac{k_\perp^2}{(1-z)} \end{aligned} \tag{2.9}$$

where $|\widetilde{\mathcal{M}}|^2_{W^++d\to u}$ is defined with

$$\widetilde{\mathcal{M}}_{W^++d\to u} = \widetilde{\mathcal{M}}^\dagger\not{p}'\widetilde{\mathcal{M}} \tag{2.10}$$

and represents the scattering process $W^+ + d \to u$.
Now we insert $\frac{k_\mu n_{\mu'}}{k\cdot n}$ from eq. 2.8 into eq. 2.7

$$\begin{aligned} B &= \widetilde{\mathcal{M}}^\dagger(\not{p}-\not{k})\gamma^\mu\not{p}\gamma^{\mu'}(\not{p}-\not{k})\widetilde{\mathcal{M}}\left(\frac{k_\mu n_{\mu'}}{k\cdot n}\right) \\ &= \widetilde{\mathcal{M}}^\dagger(\not{p}-\not{k})\,\not{k}\,\not{p}\,\not{n}(\not{p}-\not{k})\,\widetilde{\mathcal{M}}\frac{1}{k\cdot n} \\ &\overset{3\ 4}{\doteq} \widetilde{\mathcal{M}}^\dagger\,\not{p}\,\not{k}\,\not{p}\,\not{n}\,\not{p}\,\widetilde{\mathcal{M}}\frac{1-z}{k\cdot n} \\ &\overset{5}{=} 4\widetilde{\mathcal{M}}^\dagger\not{p}\widetilde{\mathcal{M}}\;n\cdot p\;k\cdot p\frac{1-z}{k\cdot n} \\ &\overset{6}{=} 2|\widetilde{\mathcal{M}}|^2_{W^+d\to u}\cdot\frac{-k_\perp^2}{z^2} \end{aligned} \tag{2.11}$$

[1]with $\not{p}\,\not{k}\,\not{p} = p_\mu\,k_\nu\,p_\rho\,(\gamma^\mu\gamma^\nu\gamma^\rho) = p_\mu\,k_\nu\,p_\rho\,(\gamma^\mu(\gamma^\nu\gamma^\rho+\gamma^\rho\gamma^\nu)-\gamma^\mu\gamma^\rho\gamma^\nu) = p_\mu\,k_\nu\,p_\rho\,(\gamma^\mu 2g^{\nu\rho}-\gamma^\mu\gamma^\rho\gamma^\nu) = \not{p}\,2k\cdot p - \not{p}\not{p}\not{k}$.
[2]with $\not{p}\not{p} = 0$ since $p^2 = 0$

The third part of eq. 2.8 gives the same result as eq. 2.11. Now we add both squared amplitudes given in eq. 2.9 and eq. 2.11

$$
\begin{aligned}
|\mathcal{M}|^2_{W^++d\to g+u} = \frac{1}{(p-k)^4}\,(A+2B)\cdot C_2(r) \overset{\langle k||p\rangle^7}{=} \\
= \frac{-2\,z^2}{k_\perp^4}|\widetilde{\mathcal{M}}|^2_{W^+d\to u}\cdot k_\perp^2\cdot\left(\frac{2}{z^2}+\frac{1}{(1-z)}\right)\cdot C_2(r) \\
= -2|\widetilde{\mathcal{M}}|^2_{W^+d\to u}\cdot\frac{z^2}{(1-z)k_\perp^2}\cdot\frac{z^2-2z+2}{z}\cdot C_2(r) \quad (2.12)
\end{aligned}
$$

The last factor $C_2(r)\cdot(z^2-2z+2)/z$ of eq. 2.12 can be identified with the gluon-quark splitting function. We notice that the squared amplitude $|\mathcal{M}_{W^++d\to u+g}|^2$ is proportional to the matrix element $|\mathcal{M}_{W^++d\to u}|^2$ in the collinear limit. The factorization of this process into the divergence, the splitting function and the LO cross section is called mass factorization.
The cross section from the process in fig. 2.2 is then

$$
d\sigma_{W^++d\to u+g} \quad \propto \quad \int \frac{d^3k}{(2\pi)^3 2E_k}\cdot\frac{z^2}{(1-z)k_\perp^2}\cdot\frac{z^2-2z+2}{z}\,(1-z)\,d\sigma_{W^++d\to u}
$$

where the phase space integral of the other three particles is contained in $d\sigma_{W^++d\to u}$. The factor $(1-z)$ corrects the flux factor for the cross section $d\sigma_{W^++d\to u}$, since the incoming momentum for this cross section is $(1-z)p$ rather than p.
The integration of the gluon phase space can be decomposed into

$$
\frac{d^3k}{(2\pi)^3 2E_k} \quad = \quad \frac{dz\;d\phi\;dk_\perp^2}{4(2\pi)^3 z} + \mathcal{O}(k_\perp^2)
$$

Since the cross section is independent of the angle ϕ, the integration over $d\phi$ yields a factor 2π and we find

$$
\begin{aligned}
d\sigma_{W^++d\to u+g} \propto \int\frac{dk_\perp^2}{k_\perp^2}\int_0^1 dz\frac{z^2-2z+2}{z}d\sigma_{W^++d\to u} \\
\propto \log\left(k_\perp^2\right)\Big|_{k^2_{\perp,min}}^{k^2_{\perp,max}}\int_0^1 dz\left(\frac{z^2-2z+2}{z}d\sigma_{W^++d\to u}\right) \quad (2.13)
\end{aligned}
$$

[3]the first $p-k$ is set to p since $\not{k}\not{k}=0$ as $k^2=0$

[4]the second $p-k$ is rewritten using eq. 2.5 and substituted with $p-k=(1-z)p$, since $n\cdot k_\perp=0$ and $n^2=0$.

[5]we use $\not{p}\not{k}\not{p}\not{n}\not{p} = p_\mu\,k_\nu\,p_\rho\,n_\sigma\,p_\tau\,(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma^\tau) = p_\mu\,k_\nu\,p_\rho\,n_\sigma\,p_\tau\,(\gamma^\mu\gamma^\nu\gamma^\rho(\gamma^\sigma\gamma^\tau+\gamma^\tau\gamma^\sigma)-\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\tau\gamma^\sigma) = p_\mu\,k_\nu\,p_\rho\,n_\sigma\,p_\tau\,(\gamma^\mu\gamma^\nu\gamma^\rho 2g^{\sigma\tau}-\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\tau\gamma^\sigma) = p_\mu\,k_\nu\,p_\rho\,n_\sigma\,p_\tau\,(2g^{\sigma\tau}(\gamma^\mu(\gamma^\nu\gamma^\rho+\gamma^\rho\gamma^\nu)-\gamma^\mu\gamma^\rho\gamma^\nu)) = p_\mu\,k_\nu\,p_\rho\,n_\sigma\,p_\tau\,(2g^{\sigma\tau}(\gamma^\mu 2g^{\nu\rho}-\gamma^\mu\gamma^\rho\gamma^\nu)) = 4\not{p}\,n\cdot p\;k\cdot p$

[6]$k\cdot p=-k_\perp^2/2/z$, $k\cdot n = z\,n\cdot p$ and $\widetilde{\mathcal{M}}^\dagger\not{p}\widetilde{\mathcal{M}} = \widetilde{\mathcal{M}}^\dagger\not{p}'\widetilde{\mathcal{M}}/(1-z) = |\widetilde{\mathcal{M}}|^2_{W^+d\to u}/(1-z)$

[7]Denoting that the particle with momentum k is collinear to the particle with momentum p.

The upper limit of the transverse phase space integration is determined by Q, the energy scale of the process. Since we set all particle masses to zero, the lower limit is zero and represents the collinear singularity. If the particle mass is finite, the lower limit is of the order of this mass and the singularity is regularized. The collinear singularity is then parametrized with the mass of the emitting particle.
We finally find for the cross section

$$d\sigma_{q+q_i\to q_j+q_f} \quad \propto \quad \log\left(\frac{Q^2}{m^2}\right)\int_0^1 dz\ P_{ji}(z)\ d\sigma_{q+q_i\to q_f} \tag{2.14}$$

The factor $P_{ji}(z)$ is called splitting function and depends on the vertex structure of the radiated and emitting particles. For different particles, e.g. quarks, gluons, or electrons radiating gluons or photons we need different splitting functions which depend on the particular vertex structure. We will calculate the splitting functions needed for this work in chapter 4.4.

These results for the cross section given in eq. 2.14 have a physical interpretation which is related to the hadron picture described in section 2.1. The cross section with a collinear particle (e.g. gluon or photon) in the final state factorizes into the splitting function $P_{ji}(z)$, the pole and the LO cross section. After having radiated collinearly, the particle enters the short distance cross section with momentum fraction $(1-z)$, while the collinear divergence is absorbed into the PDFs.

Summarizing these results, mass factorization predicts that a NLO cross section factorizes into a collinear pole, the corresponding LO process and a splitting function. In terms of the squared amplitude $|\mathcal{M}|^2$ this can be expressed as

$$|\mathcal{M}|^2_{p_a+p_b\to g+p_c+p_d} \quad \stackrel{\langle k_2||k_3\rangle}{=} |\mathcal{M}|^2_{p_a+p_b\to p_c+p_d}\cdot\tfrac{1}{t_1(1-z)}\cdot P_{ji}(z) \tag{2.15}$$

where $1/t_1(1-z)$ is the mass singularity[2] arising from the propagator and $P_{ji}(z)$ is the splitting function. The Mandelstam variable t_1 is defined in eq. A.4. The pole enters the parton distribution functions and changes the probability of finding a parton inside another parton. Each parton can therefore absorb or emit any amount of other collinear partons. These effects lead to a dependence of the PDFs on the energy scale Q^2 in shape and magnitude.

Heuristically, mass factorization can be explained by the fact that collinearly emitted particles can not be detected in a measurement since they are in the direction of the beam line. This implies that the real NLO $2\to 3$ process with a collinear final particle can not be distinguished experimentally from the LO $2\to 2$ process.

[2]The factor 1/(1-z) is always factored out in the following calculations. This factor corrects the flux for the differential cross section.

2.3 Multiple Splitting and Evolution Equations

In the last section we saw that collinear splittings with effectively massless partons lead to divergences which can be extracted by factorization. The following section discusses multiple splittings which can lead to large contributions in QCD cross sections. To take them into account, we extract the multiple splittings by factorization and sum them.
We introduce a probability for a collinear splitting of a parton depending on the energy of the process and sketch how a change in the energy leads to different splitting probabilities. This consideration leads to integro-differential equations which are known as the DGLAP equations (Dokshitzer, Gribov, Lipatov, Altarelli and Parisi) describing the evolution of parton distribution functions.
Consider a process as in fig. 2.4 where two collinear gluons with momenta k_1 and k_2 are emitted from a quark. We now have two collinear singularities giving a large contribution to the cross section. Let the first collinear gluon have a smaller $k_\perp$ as the second one

$$|k_{1,\perp}| \ll |k_{2,\perp}|$$

The double divergence is obtained analogous to section 2.2

$$\int_{m^2}^{Q^2} \frac{d\,k_{2,\perp}^2}{k_{2,\perp}^2} \int_{m^2}^{|k_{2,\perp}|^2} \frac{d\,k_{1,\perp}^2}{k_{1,\perp}^2} \quad \propto \quad \left(\log \frac{Q^2}{m^2}\right)^2$$

where the singularitiy is regularized by the mass m of the quark. For multiple splittings this can be generalized to

$$\int_{m^2}^{Q^2} \frac{d\,k_{n,\perp}^2}{k_{n,\perp}^2} \cdot \ldots \cdot \int_{m^2}^{|k_{2,\perp}|^2} \frac{d\,k_{1,\perp}^2}{k_{1,\perp}^2} \quad \propto \quad \left(\log \frac{Q^2}{m^2}\right)^n$$

The contribution of these terms leads to an integral equation which we will now derive. Consider the distribution function $f(x,Q^2)$ as the probability to find a parton j with momentum fraction x "inside" the parton i at the energy scale Q^2. We take into account that the particle i can radiate a gluon with transverse momentum $k_\perp$ with $k_\perp^2 \ll Q^2 = \mu^2$. Then, the particle i has lost energy by emitting a gluon and the probability of finding another particle j changes. This means, that the distribution function $f(x,Q^2)$ canges to $f(x,Q^2+\Delta\,Q^2)$. The differential probability for a gluon splitting where the gluon carries a fraction x of the initial momentum is

$$\frac{d\,k_\perp^2}{k_\perp^2} \cdot P(x)$$

where $P(x)$ is the splitting function also mentioned in section 2.2 which results from the vertex structure of the splitting. The fraction $d\,k_\perp^2/k_\perp^2$ is identified with the maximal transverse mo-

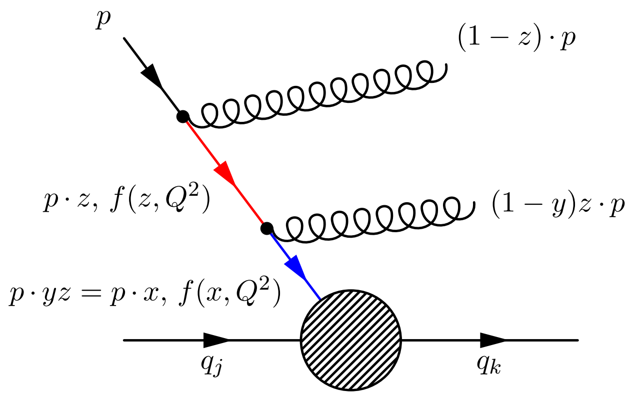


Figure 2.4: Graph for the double collinear gluon emission.

mentum dQ^2/Q^2.
For the distribution function at the energy $Q^2 + d\,Q^2$ we find with fig. 2.4

$$f(x, Q^2 + \Delta\, Q^2) \;=\; f(x,Q^2) + \int\limits_0^1 dy \int\limits_0^1 dz\, \delta(x - yz) \frac{\alpha}{2\pi} \frac{d\,Q^2}{Q^2} \cdot P(y) f(z,Q^2)$$

This term is a compound of the parton distribution for the parton with momentum fraction z at energy Q^2 and the probability for splitting off a particle with momentum fraction y. We integrate this over all possible momenta z and y. The δ-function enters the term to guarantee momentum conservation. After evaluating the integral over y we obtain

$$f(x, Q^2 + \Delta\, Q^2) \;=\; f(x,Q^2) + \frac{d\,Q^2}{Q^2} \frac{\alpha}{2\pi} \int_x^1 \frac{dz}{z} P\left(\frac{x}{z}\right) f(z,Q^2) \tag{2.16}$$

This leads to the integro-differential equation

$$\frac{d}{d(\log Q^2)} f(x,Q^2) \;=\; \frac{\alpha}{2\pi} \int_x^1 \frac{dz}{z} P\left(\frac{x}{z}\right) f(z,Q^2) \tag{2.17}$$

which describes the Q^2 dependence of the parton distribution function for an initial distribution $f(x, m^2)$.

Thus the parton distribution functions in eq. 2.1 do not only have an x dependence but also a $\mu^2 = Q^2$ dependence. In early experiments (fig. 2.6) no Q^2 dependence was observed due to the limited energy range. This phenomenon which was a supporting evidence for the naive parton model is referred to as Bjorken scaling. It was only in more recent experiments that the Q^2 dependence derived above (Bjorken scaling violation) was observed (fig. 2.5). For the large

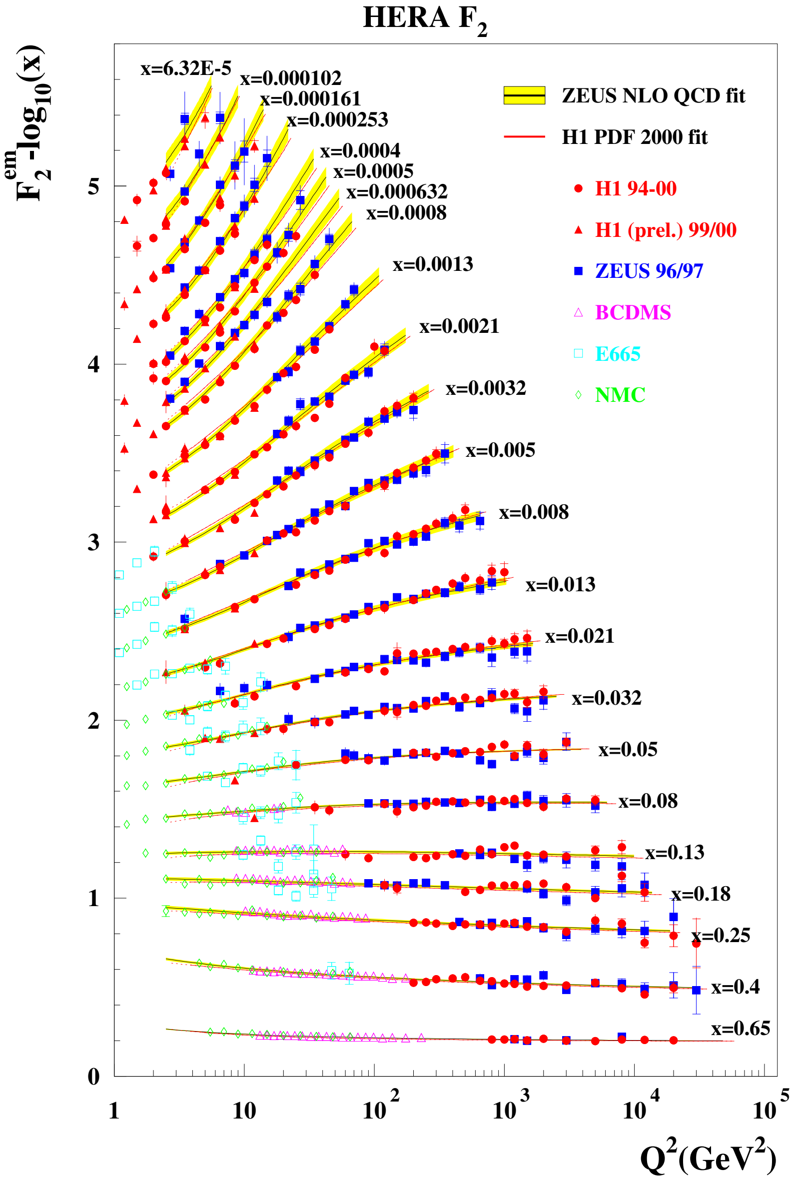


Figure 2.5: The results for the structure function F_2^{em} versus Q^2 are shown for fixed x [21].

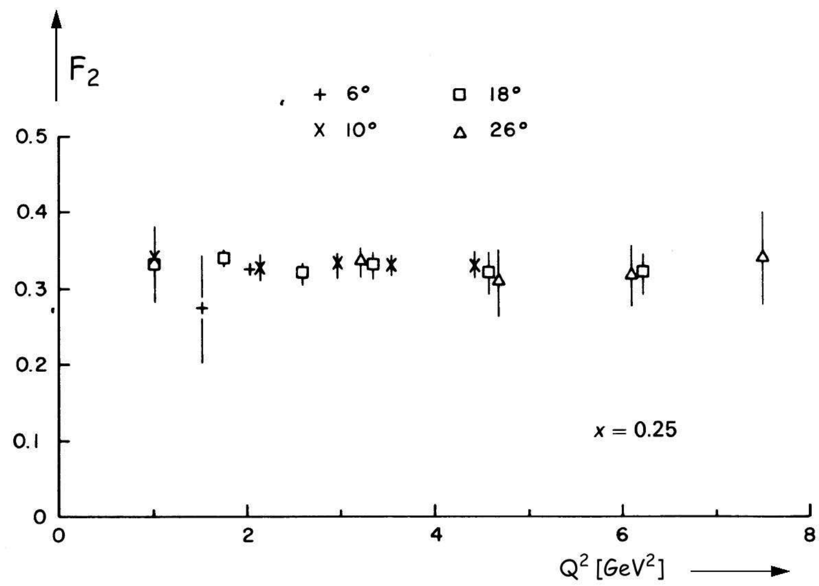


Figure 2.6: Scaling behaviour of the structure function F_2 as found in experiments at SLAC in the early 1970s (according to [22])

energy limit of the PDF one found

$$f(x, Q^2)|_{Q^2 \to \infty, x \to 0} = \infty$$
$$f(x, Q^2)|_{Q^2 \to \infty, x \to 1} = 0$$

This means, that with increasing energy Q^2 we find less quarks with high momentum fraction in the hadron, but we have a large number of partons each carrying a smaller fraction of the total momentum. This is due to the fact that at higher energies the probability for emission or absorption of gluons by partons and the creation of new quark pairs increases.
The equations describing the full Q^2 dependence of the hadron PDFs were derived by Dokshitzer, Gribov, Lipatov, Altarelli and Parisi [23] [24] [25] and are referred to as DGLAP or Altarelli-Parisi equations. They are an extension of eq. 2.17 for quarks and gluons in the context of hadron scattering taking into account gluon-gluon, gluon-quark and quark-gluon splitting.

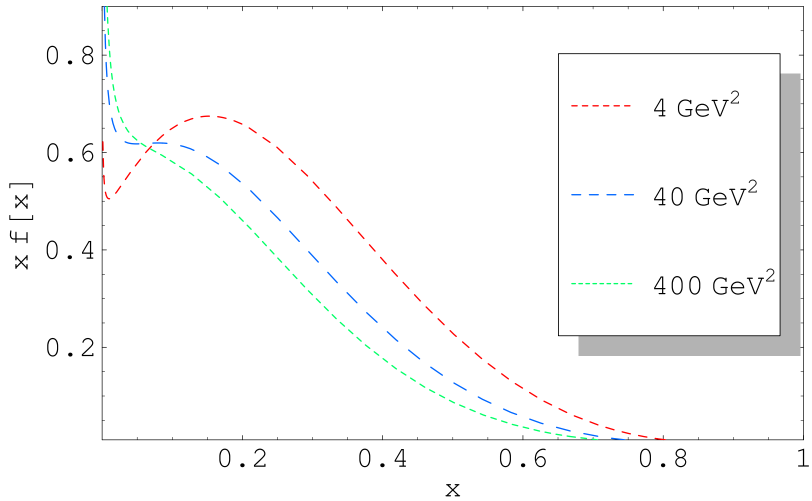


Figure 2.7: Weighted Parton distribution functions for the up quark for different scales Q^2 ($4\,\mathrm{GeV}^2$, $40\,\mathrm{GeV}^2$, $400\,\mathrm{GeV}^2$) demonstrating Bjorken scaling violation. The functions are taken from the CTEQ package [26].

$$\begin{aligned}
\frac{d\, f_g(x,Q)}{d\,\log Q} &= \frac{\alpha_s(Q^2)}{\pi}\int_x^1 \frac{d\,z}{z}\left(P_{gq}(z)\sum_q\left(f_q(\frac{x}{z},Q)+f_{\overline{q}}(\frac{x}{z},Q)\right)+P_{gg}(z)f_g(\frac{x}{z},Q)\right)\\
\frac{d\, f_q(x,Q)}{d\,\log Q} &= \frac{\alpha_s(Q^2)}{\pi}\int_x^1 \frac{d\,z}{z}\left(P_{qq}(z)f_q(\frac{x}{z},Q)+P_{qg}(z)f_g(\frac{x}{z},Q)\right)\\
\frac{d\, f_{\overline{q}}(x,Q)}{d\,\log Q} &= \frac{\alpha_s(Q^2)}{\pi}\int_x^1 \frac{d\,z}{z}\left(P_{qq}(z)f_{\overline{q}}(\frac{x}{z},Q)+P_{qg}(z)f_g(\frac{x}{z},Q)\right)
\end{aligned}$$

This evolution is shown for three values of Q^2 in fig. 2.7. To derive the Altarelli-Parisi-equations, the above discussion has to be extended by introducing a set of splitting functions $P_{ij}(z)$ giving the probability to find a parton i inside a parton j with momentum fraction z.

The parton distribution functions must be normalized in such a way that they yield the correct quantum numbers of the hadron. For protons this implies

$$\begin{aligned}
\int_0^1 dx\,(f_u(x)-f_{\overline{u}}(x)) &= 2\\
\int_0^1 dx\,(f_d(x)-f_{\overline{d}}(x)) &= 1
\end{aligned}$$

as its valence quark content is *uud*.
The total amount of momentum carried by the hadron must be the sum over the momentum fraction of each parton integrated over x

$$\int_0^1 dx\, x \left(f_u(x) + f_{\overline{u}}(x) + f_d(x) + f_{\overline{d}}(x) + f_g(x)\right) \quad = \quad 1$$

$f_i(x)$ integrated over dx from 0 to 1 gives the total number of particles of type i, while $x f_i(x)$ integrated over dx from 0 to 1 gives the fraction of the momentum carried by particles of type i. The weighted parton distributions $x\, f(x)$ for up, down, strange, charme quarks and antiquarks are plotted in fig. 2.8 for $Q^2 = 4$ GeV^2. Note that the contribution of quarks and antiquarks to the total momentum is only half of the total hadron momentum while the other half is carried by gluons.

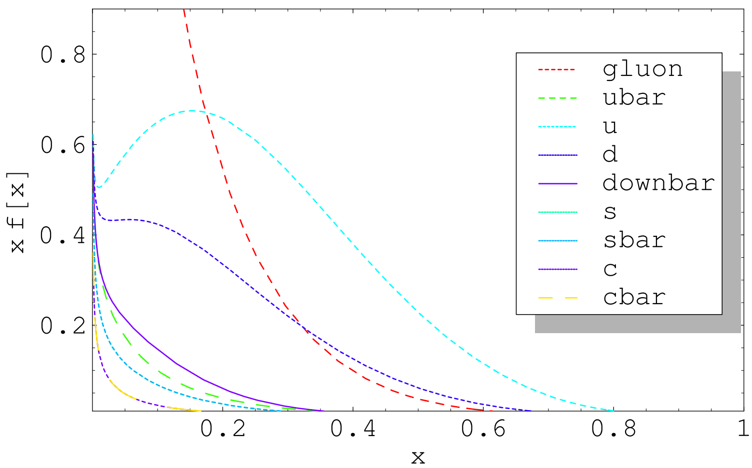


Figure 2.8: Weighted PDFs $x\, f(x)$ for gluons, quarks and antiquarks at the scale $Q^2 = 4$ GeV^2. The functions are taken from the CTEQ package [26].

3 Supersymmetry and Regularization Schemes

The simplest and most popular realistic supersymmetric model is the straightforward introduction of supersymmetric particles to the SM. Only those couplings and fields that are necessary and sufficient for the consistency of the theory are included. This model is called minimal supersymmetric standard model (MSSM). Before we introduce the MSSM we will discuss a toy model to illustrate how field theories are made supersymmetric.

3.1 A Supersymmetric Toy Model

Supersymmetry is a symmetry in particle physics which transforms fermions into bosons and vice versa and thus circumvents the Coleman-Mandula theorem. Particles which are connected by this SUSY transformation are called superpartners. In the early 1970s a toy model consisting of one chiral multiplet was constructed by Wess and Zumino [27].
We consider a minimal nonabelian supersymmetric gauge Lagrangian

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu\ a} - i\lambda^{\dagger\ a}\overline{\sigma}^\mu D_\mu\lambda^a + \frac{1}{2}D^a D^a \tag{3.1}$$

with the auxiliary field D^a and the two-component Weyl fermion λ^a. The index a runs over the adjoint representation of the gauge group ($a = 1\dots8$ for $SU(3)_C$ and $a = 1\dots3$ for $SU(2)_L$). $F^a_{\mu\nu}$ is the usual Yang-Mills field strength

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_s\ f^{abc}\ A^b_\mu\ A^c_\nu$$

and D_μ the corresponding covariant derivative in the adjoint representation

$$D_\mu\lambda^a = \partial_\mu\lambda^a + g_s\ f^{abc}\ A^b_\mu\lambda^c$$

g_s is the gauge coupling and f^{abc} are the totally antisymmetric structure constants of the gauge group.

The action $\int d^4x\, \mathcal{L}_{\text{gauge}}$ is invariant under the following SUSY transformations

$$\begin{aligned}
\delta A^a_\mu &= \frac{1}{\sqrt{2}} \left(\epsilon^\dagger \overline{\sigma}_\mu \lambda^a + \lambda^{\dagger\, a} \overline{\sigma}_\mu \epsilon \right) \\
\delta \lambda^a_\alpha &= \frac{i}{2\sqrt{2}} \left(\sigma^\mu \overline{\sigma}^\nu \epsilon \right)_\alpha F^a_{\mu\nu} + \frac{1}{\sqrt{2}} \epsilon_\alpha D^a \\
\delta D^a &= \frac{i}{\sqrt{2}} \left(\epsilon^\dagger \overline{\sigma}^\mu D_\mu \lambda^a - D_\mu \lambda^{\dagger\, a} \overline{\sigma}^\mu \epsilon \right)
\end{aligned} \tag{3.2}$$

So far, all particles in a multiplet have the same mass. Since to date no superpartners of SM particles have been found which would be inevitable if they had the same mass, a SUSY breaking mechanism must be part of any realistic model. One can for example construct the minimal supersymmetric standard model (MSSM) based on the SM in which the masses of all supersymmetric partners can be increased by SUSY breaking terms to satisfy experimental constraints.

3.2 Description of the MSSM

In the following we give a brief description of the MSSM particle content which represents the minimal number of fields necessary to formulate a supersymmetric extension of the SM.
The mechanism that gives mass to the up type fermions in the SM can not be used in the MSSM as it would be incompatible with SUSY. Therefore we need two Higgs doublets with 8 real degrees of freedom. Three of them are eaten by the massive gauge bosons and 5 remain as physical Higgs bosons. The two doublets come with 4 spin-1/2 partners called higgsinos.
The superpartners of the electroweak gauge bosons are the spin-1/2 Bino ($\widetilde{B}$) and Wino ($\widetilde{W}^{1,2,3}$). The gauginos, like the higgsinos, are not mass eigenstates and together they form 4 Majorana neutralinos and two Dirac charginos.
The SUSY partners of the left- and righthanded Quarks (q_L and q_R) are the left- and righthanded squarks ($\widetilde{q}_L$ and $\widetilde{q}_R$). They transform under $SU(2)_L \times U(1)_Y$ and also under the fundamental representation of the QCD gauge group $SU(3)_C$ like the quarks.
The SUSY partners of the leptons are the sleptons. They both transform under the fundamental representation of the $SU(2)_L \times U(1)_Y$.
The group representations of all particles are summarized in tab. 3.1. The fact that none of these supersymmetric particles have been found so far leads to the conclusion that supersymmetry must be broken and superpartners differ in mass from their SM particles. The mechanism for SUSY breaking is not yet known but we can parametrize it by introducing the most general breaking terms which nevertheless preserve the solution for the hierarchy problem as discussed in chapter 1.
The effective Lagrangian for softly broken SUSY can be written as

$$\mathcal{L} = \mathcal{L}_{\text{SUSY}} + \mathcal{L}_{\text{soft}}$$

Gauge Group	SM Field	s	Super-partner	s	Super-field
$U(1)_Y$, adj. repr.	B	1	$\widetilde{B}$	1/2	V_1
$SU(2)_L$, adj. repr.	$W^{1,2,3}$	1	$\widetilde{W}^{1,2,3}$	1/2	V_2
$SU(2)_L \times U(1)_Y$, fund. repr.	$(\nu, e)_L$	1/2	$(\widetilde{\nu}, \widetilde{e})_L$	0	L
$SU(3)_C \times SU(2)_L \times U(1)_Y$, fund. repr.	$(u, d)_L$	1/2	$(\widetilde{u}, \widetilde{d})_L$	0	Q
$U(1)_Y$, fund. repr.	e_R	1/2	$\widetilde{e}_R$	0	E^c,
$SU(3)_C \times U(1)_Y$, fund. repr.	u_R, d_R	1/2	$\widetilde{u}_R,\ \widetilde{d}_R$	0	$U^c,\ D^c$
$SU(3)_C$, adj. repr.	g	1	$\widetilde{g}$	1/2	V_3
$SU(2)_L \times U(1)$, fund. repr.	$(\phi^+, h^d + i\phi_3)$	0	$(\widetilde{\phi}^+, \widetilde{h}^d)$	1/2	H_1
Extended Higgs sector in MSSM $SU(2)_L \times U(1)$, fund. repr.	$(h^u + iA, H^-)$	0	$(\widetilde{h}^u, \widetilde{H}^-)$	1/2	H_2

Table 3.1: Standard model particles and their corresponding superpartners with spin, superfield and gauge group.

The explicit form of the most general soft SUSY breaking terms is

$$\mathcal{L}_{\text{soft}} = -\left(\frac{1}{2}M_a\lambda^a\lambda^a + \frac{1}{6}a^{ijk}\phi_i\phi_j\phi_k + \frac{1}{2}b^{ij}\phi_i\phi_j + t^i\phi_i\right) + c.c. - (m^2)^i_j\phi^{j*}\phi_i \tag{3.3}$$

All other possible mass terms are redundant and can be absorbed into a redefinition of the superpotential and the coefficients in eq. 3.3. M_a stands for the different gaugino masses, a^{ijk} are the three scalar and b^{ij} the two scalar couplings. The tadpole coupling is represented by t^i, the scalar mass terms by ${m^2}^i_j$.

The SUSY Lagrangian can contain couplings between two SM and one SUSY particle. This coupling leads to undesired effects like quick proton decay. However, the current bound of proton decay is $> 10^{31}$ to 10^{33} years [4]. So, in the MSSM these coupling have to be forbidden or removed by enforcing a new discrete symmetry, which is called R parity. This number is defined for each particle as

$$R = (-1)^{3(B-L)+2S}$$

where B denotes the baryon number, L the lepton number and S stands for the spin of the particle. This number gives $+1$ for SM particles and -1 for SUSY partners. Terms in the Lagrangian are only permitted if the product of the R parities gives $+1$. In other words, R parity forbids couplings where an odd number of SUSY lines meet. This new parity implies that there is a lightest stable SUSY particle (LSP) with parity −1. It turns out that the LSP can also act as a good dark matter candidate, but for this it is required to be electrically neutral and weakly interacting. All other SUSY particles decay into an uneven number of LSPs. Furthermore, this implies that only even numbers of SUSY particles can be produced at colliders.

3.3 Regularization and SUSY

An important technical point is the choice of the renormalization and regularization method in SUSY. In this section we will discuss that the most popular regularization scheme, dimensional regularization (DREG), breaks SUSY. Then we will review an extension of DREG that preserves it.

Higher order calculations such as real and virtual NLO calculations involve divergent terms which need to be regularized to be manageable. After regularizing, the expressions have to be renormalized, which means consistently subtracting infinite terms according to a special renormalization scheme. The result must be independent from the regularization scheme since it is a physical observable. There are two different types of divergences which have to be regularized, namely UV and IR divergences.
For amplitudes including one or more loops we have to integrate over the loop momenta. Schematically, a loop integral can be of the form

$$\int^{\Lambda} d^4k \frac{1}{k^4} \int^{\Lambda} d^4k \frac{1}{k^4} \propto \int^{\Lambda} dk \frac{k^3}{k^4} \propto \int^{\Lambda} dk \frac{1}{k} \propto \log \Lambda \tag{3.4}$$

For a large momentum k the integral diverges logarithmically, which is a UV divergence. For small momentum $k \to 0$ the integral diverges as well which is known as an IR divergence. It appears if massless particles propagating in the loop become soft. This divergence cancels with another originating from the emission of soft massless particles in the same order of perturbation theory, called bremsstrahlung. Meanwhile, the calculation can be performed by giving a small mass to the massless particle which is set to zero in the end.
Managing UV divergences usually requires regularization and subsequent renormalization.
There are different regularization schemes which are adapted to particular applications. A simple scheme is the introduction of a cut-off parameter Λ as the maximum limit in eq. 3.4, so that the result depends on the logarithm of Λ. This method breaks Lorentz and gauge invariance.
A solution that was used for early QED calculations is the Pauli Villars regularization where fictitious heavy particles are introduced to ensure proper cancellation of divergences. After performing the calculation, the mass of the fictitious particle is taken to infinity. This method works only in abelian gauge theories as it breaks nonabelian gauge invariance. For further reading we refer to [28].
Another popular regularization method that manifestly preserves gauge invariance and therefore can be used in QCD and electroweak theory has its origin in power counting. The degree of divergence can be estimated by counting the mass dimensions. This means that a loop integral

Dimension	Vector	Spinor
2	0	$8 \cdot 2$
3	$8 \cdot 1$	$8 \cdot 2$
4	$8 \cdot 2$	$8 \cdot 2$
5	$8 \cdot 3$	$8 \cdot 4$

Table 3.2: Degrees of freedom in integer dimensions for the $SU(3)$ gauge group. The degrees of freedom for vectors and spinors depend differently on the dimension of space time.

can be made finite by evaluating it in different spacetime dimensions

$$\int \frac{d^4 k}{(2\pi)^4} \longrightarrow \int \frac{d^D k}{(2\pi)^4}$$

This was the idea of t'Hooft and Veltman in 1972 [5]. The approach is the following: All Feynman diagrams are calculated as a function of the spacetime dimension D and the amplitudes are analytically continued to non-integer values of D. For sufficiently small D, all integrals will converge, and the final result will be well defined for $D \to 4$.
This popular method is known as dimensional regularization (DREG) and has no important shortcomings in SM calculations. However, using this method in the MSSM leads to serious problems.
DREG explicitly breaks SUSY since the number of degrees of freedom of gauge bosons and gauginos do not match for $D \neq 4$. Let's look at the Lagrangian from eq. 3.1 to illustrate this issue

$$\mathcal{L}_{\text{gauge}} = \underbrace{-\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu\ a}}_{(1)} - \underbrace{i\lambda^{\dagger\ a} \overline{\sigma}^\mu D_\mu \lambda^a}_{(2)} + \underbrace{\frac{1}{2} D^a D^a}_{(3)}$$

The onshell degrees of freedom for the field strength (1) are $8 \cdot (D-2)$. The 8 has its origin in the dimension of the gauge group $SU(3)$ which is $(N^2 - 1) = 8$. Offshell there would be D degress of freedom, but subtracting two degrees for the momentum direction and the energy, we get $(D-2)$. The fermion (2) has $8 \cdot 2$ degress of freedom onshell as it is a Weyl spinor. The auxiliary field (3) has obviously no degree of freedom onshell since it has no kinetic term. We see now that in 4 dimensions the number of the gauge field degress of freedom (1) is the same as for its superpartner (2). Otherwise, the SUSY transformation eq. 3.2 would lose degrees of freedom.
For $D \neq 4$ dimensions, the degrees of freedom are apparently not the same. In tab. 3.2 one can see how the degrees of freedom for a vector and a spinor change with dimension. This violation of SUSY can be corrected in practical calculations by adding supersymmetry restoring counterterms, whose existence is always guaranteed by the renormalizability of supersymmetric gauge theories. Nevertheless, these counterterms pose practical complications and are difficult

to calculate and to implement [6, 7, 8, 9].
To avoid these complications, we can use another method to regularize divergences in SUSY namely regularization by dimensional reduction (DRED) [10] [11]. This regularization scheme leaves the vector fields 4-dimensional to maintain the match with the gaugino degrees of freedom. In contrast, the momenta are calculated in D dimensions as in DREG. This ensures that the loop integrals converge while presevering SUSY at the same time.

For practical reasons we can split the four dimensional vector field A^μ in DRED into a D dimensional vector field and a field which is effectively a scalar in D dimensions.

$$A^\mu A_\mu = A^\sigma A_\sigma + A^i A_i \qquad \mu \in 4,\ \sigma \in D,\ i \in 4 - D$$

This component A_i of the gluon behaves like a particle of its own, and is called ϵ scalar. As for the other particles we can calculate Feynman rules for this scalar, which will be done in section 4.3. We can therefore calculate different amplitudes, one with D-dimensional gluons and one with the ϵ scalar. However, before calculating the hadronic cross section by convolution with the PDFs, all processes corresponding to a physical one have to be summed up. This implies that there is no PDF for ϵ scalars. Keeping this in mind, we thus call the ϵ scalar a parton of its own.
This method (calculating the vector bosons in D dimensions and adding a new particle to cover the remaining $(4 - D)$ degress of freedom) is equivalent to the somewhat awkward approach of leaving the vector boson in 4 dimensions while the momenta are calculated in D dimensions.
As mentioned at the beginning, Beenakker et al. [1] found non-factorizable terms in a massive QCD process calculated in DRED. In the next chapter we will consider non-factorizable terms appearing in the real NLO QCD process $gg \to g\,\widetilde{t}\,\widetilde{\bar{t}}$ which are similar to the terms found in [1].

4 The Factorization Problem in DRED and its Remedy

Beenakker et al. found a problem concerning the factorization theorem when using DRED known as the factorization problem [1]. The real NLO corrections for the process $gg \to t\bar{t}$ were calculated using DRED and DREG. For the collinear limit they found non-factorizable terms which could not be absorbed as a constant contribution to the PDFs. In this chapter we find similar non-factorizable terms in the real NLO corrections to $gg \to \tilde{t}\tilde{\bar{t}}$ and we will demonstrate how this problem can be solved analogous to [2].
For this purpose we now only state the results while the derivation of the NLO amplitude and the factorization is demonstrated in detail in chapter 6.

4.1 Real NLO Corrections and non-Factorizable Terms

We consider hadroproduction of stops via gluon and quark fusion which are essential processes relevant for an experimental confirmation of SUSY and for the determination of SUSY parameters. While the process involving quarks factorizes in DRED as expected, we also find non-factorizable terms for the stop production via gluon fusion, a process similar to the one for which factorization has been found to fail before [1]. We solve the problem for this process following the idea of Signer and Stöckinger, who applied their solution [2] to the problem in [1]. Using their approach, the factorization problem does not seem to be a shortcoming of DRED any longer.
We will now investigate how the processes for stop production behave in DRED and in 4 dimensions. The factorization of both processes in 4 dimensions can be performed as expected, for the calculation see sections 6.1.1 and 6.2.1:

$$\frac{1}{8}\sum_{\text{pols}} |\mathcal{M}|^2_{q\bar{q}\to G\tilde{t}\tilde{\bar{t}}} \overset{\langle k_2 || k_3\rangle}{=} \alpha_s \pi \cdot \underbrace{\frac{1}{(1-z)t_1}}_{\text{pole}} |\mathcal{M}|^2_{q\bar{q}\to\tilde{t}\tilde{\bar{t}}} \cdot \underbrace{C_2(r)\frac{(z-2)z+2}{z}}_{P_{Gq}(z)}$$

and

$$\frac{1}{8}\sum_{\text{pols}} |\mathcal{M}|^2_{GG\to G\tilde{t}\tilde{\bar{t}}} \overset{\langle k_2 || k_3\rangle}{=} \alpha_s \pi \cdot \underbrace{\frac{1}{(1-z)t_1}}_{\text{pole}} \cdot |\mathcal{M}|^2_{GG\to\tilde{t}\tilde{\bar{t}}} \underbrace{\frac{2N((z-1)z+1)^2}{(z-1)z}}_{P_{GG}(z)}$$

As we see, the processes factorize to the LO result, the corresponding splitting function and the collinear divergence.
The result we obtain for $q\bar{q} \to G\tilde{t}\bar{\tilde{t}}$ in DRED is

$$\frac{1}{8}\sum_{\text{pols}} |\mathcal{M}|^2_{q\bar{q}\to G\tilde{t}\bar{\tilde{t}}} \overset{\langle k_2 || k_3\rangle}{=} \alpha_s\pi \underbrace{\frac{1}{(1-z)t_1}}_{\text{pole}} |\mathcal{M}|^2_{q\bar{q}\to\tilde{t}\bar{\tilde{t}}} \cdot \underbrace{C_2(r)\frac{(z-2)z+2}{z}}_{P_{GG}(z)} \tag{4.1}$$

which is the same as in 4 dimensions. The argument why this process factorizes in DRED without any further treatment is given in section 6.1.2.

Let us now consider $GG \to G\tilde{t}\bar{\tilde{t}}$ in DRED. After taking the collinear limit we obtain

$$\begin{aligned}
&\frac{1}{8}\sum_{\text{pols}} |\mathcal{M}|^2_{GG\to G\tilde{t}\bar{\tilde{t}}} \overset{\langle k_2 || k_3\rangle}{=} \\
&= \frac{\alpha_s^3\pi^3}{t_1(1-z)}\times \\
&\left(\frac{512\left(4T^2-UT+4U^2\right)\left(4(T+U)^2m^4+4TU(T+U)m^2+(D-2)T^2U^2\right)((z-1)z+1)^2}{T^2U^2(T+U)^2(1-z)z}\right. \\
&+(1-n_{RS})\left(\frac{512(D-4)\left(4T^2-UT+4U^2\right)\left(4(T+U)^2m^4+4TU(T+U)m^2+(D-2)T^2U^2\right)z}{(D-2)T^2U^2(T+U)^2(z-1)}\right. \\
&\left.\left.\qquad+\frac{512(D-4)\left(4T^2-UT+4U^2\right)(z-1)\left(z^2+1\right)}{(T+U)^2z}\right)\right) \\
&= \frac{\alpha_s\pi}{t_1\,(1-z)}\left(|\mathcal{M}|^2_{gg\to\tilde{t}\bar{\tilde{t}}}\cdot 2N\frac{((z-1)z+1)^2}{(1-z)z}+\right. \\
&\left.+\,(1\,-\,n_{RS})\left(|\mathcal{M}|^2_{gg\to\tilde{t}\bar{\tilde{t}}}\frac{(4-D)}{(D-2)}\frac{2Nz}{(1-z)}+|\mathcal{M}|^2_{gg\to\tilde{t}\bar{\tilde{t}}}\Big|_{m=0}\cdot\frac{(4-D)}{(D-2)}\cdot\frac{2N(1-z)(z^2+1)}{z}\right)\right)
\end{aligned} \tag{4.2}$$

The factor n_{RS} is 1 for DREG and 0 for DRED. Only the first term seems to fullfill the factorization theorem while the others seem to violate it. Due to their prefactors, they vanish in DREG and the problem is only present in DRED.
For the derivation in DRED we would expect a change in the splitting function or the prefactor, but not in the structure of the LO result. Similar to the non-factorizable result in [2] the problem vanishes in the massless limit. This is shown for all massless SUSY $2 \to 3$ processes by [30].
In the next section we will present the solution of the factorization problem proposed by [2] and calculate the Feynman rules and splitting functions that spring from it. For a motivation and discussion of the origin of the seemingly non-factoriable results we refer to chapter 7.

4.2 Remedy: The ϵ Scalar

There is a simple argument why factorization works in DREG but not in DRED. In DREG we calculate both particles and momenta in D dimensions and thus reduce QCD to D dimensions consistently. As we saw, DRED constrains the momenta to D dimensions, but leaves the tensor fields in 4 dimensions. This does not lead to a D-dimensional QCD but to 4-dimensional QCD dimensionally reduced to D dimensions which is obviously not equivalent to QCD in any $D < 4$ due to the additional degrees of freedom.
From now on we write capital G for gluons in 4 dimensions and lower case g for gluons in D dimensions. The ϵ scalars are denoted with ϕ. For metric tensors in D dimensions we write $\widetilde{g}^{\mu\nu}$ ($\hat{g}^{\mu\nu}$ for $4-D$). All momenta at any vertex are conventionally defined incoming.
We split the 4 dimensional vector field A^μ in a D and a $(4-D)$ component

$$A^\mu A_\mu = A^\sigma A_\sigma + A^i A_i \tag{4.3}$$

with $\sigma \in D$ and $i \in 4-D$. We can interpret this as a splitting of the vector field into two particles, one with D components (let this be the regular boson reduced to D dimensions) and one with $(4-D)$ components. This $(4-D)$ component particle transforms in D dimensions as a scalar and is referred to as the ϵ scalar. From this point of view, QCD is reduced to D dimensions in DRED but contains a new particle with $(4-D)$ components.
The important point from this separation of the 4 dimensional gluon eq. 4.3 into two distinct physical particles is that the process $GG \to \widetilde{t}\,\widetilde{\bar{t}}$ also separates into two distinct processes

$$\begin{aligned} gg &\to \widetilde{t}\,\widetilde{\bar{t}} \\ \phi\phi &\to \widetilde{t}\,\widetilde{\bar{t}} \end{aligned} \tag{4.4}$$

Together they are equivalent to the evaluation of the process $GG \to \widetilde{t}\,\widetilde{\bar{t}}$ in DRED. Mixed initial states $\phi g \to \widetilde{t}\,\widetilde{\bar{t}}$ do not contribute to this particular process due to its topologies.
This implies that we can distinguish the ϵ scalar and gluon in the initial state and that we can treat the corresponding amplitudes separately. With these definitions, the real NLO corrections take the form

$$|\mathcal{M}_{GG\to G\widetilde{t}\widetilde{\bar{t}}}|^2 = |\mathcal{M}_{gg\to g\widetilde{t}\widetilde{\bar{t}}}|^2 + |\mathcal{M}_{\phi\phi\to g\widetilde{t}\widetilde{\bar{t}}}|^2 + |\mathcal{M}_{\phi g\to \phi\widetilde{t}\widetilde{\bar{t}}}|^2 + |\mathcal{M}_{g\phi\to \phi\widetilde{t}\widetilde{\bar{t}}}|^2$$

and are expected to factorize separately into one of the two LO processes in eq. 4.4. Indeed we find this behaviour in chapter 6. We can therefore conclude that the proposition in [2] provides a solution for the factorization problem found in eq. 4.2 as well.

In the next section we calculate the QCD Feynman rules and splitting functions for the ϵ scalar which are later needed in chapters 5 and 6.

4.3 Feynman Rules for the ϵ Scalar

To perform the actual calculations in DRED, we will first derive the Feynman rules from the QCD Lagrangian. We start with the 4-dimensional Lagrangian for QCD dimensionally reduced to D dimensions, and split the fields A_μ into their D and $(4-D)$ component parts. We directly obtain the Lagrangian in terms of the the D dimensional gluon and the ϵ scalar. Starting from this we derive the Feynman rules for these particles.
Let us consider the QCD Lagrangian coupled to a complex scalar (which will be identified with a stop)

$$\mathcal{L}_{\text{QCD,S}} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu\,a} + (D_\mu\phi)^\dagger(D^\mu\phi) - m^2\phi^2 + \overline{\psi}(i\not{D} - m_f)\psi \tag{4.5}$$

We insert the covariant derivative and the field strength F

$$\begin{aligned} F^a_{\mu\nu} &= \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_s f^{abc} A^b_\mu A^c_\nu \\ D_\mu &= \partial_\mu - i g_s A^a_\mu t^a \end{aligned}$$

where f^{abc} are the structure constants of the gauge group. We substitute the field strength into the pure gauge part $-\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu\,a}$

$$\begin{aligned} -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu\,a} &= -\frac{1}{4}(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_s f^{abc}A^b_\mu A^c_\nu)(\partial^\mu A^{\nu\,a} - \partial^\nu A^{\mu\,a} + g_s f^{aef} A^{\mu\,e} A^{\nu f}) \\ &= -\frac{1}{4}(\partial_\mu A^a_\nu \partial_\mu A^{\nu,a} - \partial_\mu A^a_\nu \partial^\nu A^{\mu,a} + \underbrace{\partial_\mu A^a_\nu g_s f^{aef} A^{\mu\,e} A^{\nu\,f}}_{*} \\ &\quad - \partial_\nu A^a_\mu \partial_\mu A^{\nu,a} + \partial_\nu A^a_\mu \partial^\nu A^{\mu,a} - \underbrace{\partial_\nu A^a_\mu g_s f^{aef} A^{\mu\,e} A^{\nu\,f}}_{*} + \underbrace{g_s f^{abc} A^b_\mu A^c_\nu \partial^\mu A^{\nu\,a}}_{*} \\ &\quad - \underbrace{g_s f^{abc} A^b_\mu A^c_\nu \partial^\nu A^{\mu\,a}}_{*} + \underbrace{g_s^2 f^{abc} A^b_\mu A^c_\nu f^{aef} A^{\mu\,e} A^{\nu\,f}}_{*}) \end{aligned} \tag{4.6}$$

The parts from which we evaluate the Feynman rules for the vertices used here are marked with $*$. We now split the 4 dimensional vector fields with indices $\mu\,,\nu$ into D dimensions (marked by $\rho\,,\sigma$) and $(4-D)$ dimensions (marked by i, j):

$$\begin{aligned} A_\mu A^\mu &\to A_\rho A^\rho + A_j A^j \\ \partial_\mu A^\mu &\to \partial_\rho A^\rho + \underbrace{\partial_j A^j}_{0} \end{aligned} \tag{4.7}$$

The last term with the derivative in $(4-D)$ dimensions does not exist as the momenta only have components in D dimensions.

We introduce relations for the two metric tensors

$$\begin{aligned} g_{\mu\nu} &= \hat{g}_{\mu\nu} + \tilde{g}_{\mu\nu} \\ \hat{g}_{ij}\hat{g}^{ij} &= 4 - D \\ \tilde{g}_{\rho\sigma}\tilde{g}^{\rho\sigma} &= D \end{aligned} \tag{4.8}$$

where the tilde (hat) denotes D $(4-D)$ dimensions. Note that D will not be an integer number in the final result.

To obtain the Feynman rules, the standard procedure is applied. The Lagrangian is multiplied with i and Fourier transformed by exchanging the derivative ∂_μ with the corresponding momentum k^μ

$$\partial_\mu \quad \rightarrow \quad -ik_\mu \tag{4.9}$$

Then the expression has to be symmetrized in identical fields and finally all fields are erased which corresponds to taking the functional derivative.

The VVV and $V\phi\phi$ Vertices

The three vector vertex is calculated from the terms $*$ containing three vector fields. Inserting eq. 4.9 we obtain

$$\begin{aligned} ig_s f^{abc} A^b_\mu A^c_\nu \; \partial^\mu A^{\nu\, a} \quad &\rightarrow \quad g_s f^{abc} A^b_\mu A^c_\nu \; k^{\mu\,(a)} A^{\nu\, a} \\ -ig_s f^{abc} A^b_\mu A^c_\nu \; \partial^\nu A^{\mu\, a} \quad &\rightarrow \quad -g_s f^{abc} A^b_\mu A^c_\nu \; k^{\nu\,(a)} A^{\mu\, a} \end{aligned} \tag{4.10}$$

which yields[1]

$$-\frac{1}{2}\, g_s\, A^{b\mu} A^{c\nu} f^{abc} \left(k^{(a)}_\mu A^a_\nu - k^{(a)}_\nu A^a_\mu \right)$$

Now the color and Lorentz indices have to be exchanged symmetrically. For clarity we redefine the particles and momenta as shown in fig. 4.1(1). The Feynman rule for the three vector vertex is

$$g_s f^{abc} \left(\tilde{g}^{\mu\nu}(k-p)^\rho + \tilde{g}^{\nu\rho}(p-q)^\mu + \tilde{g}^{\rho\mu}(q-k)^\nu \right) \tag{4.11}$$

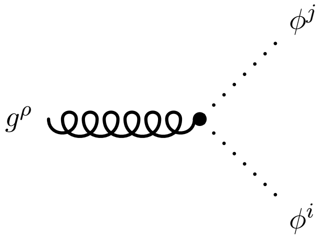
For convenience all Feynman rules are summarized in section A.2. Analogous to this approach we derive the vector-two ϵ scalar vertex. In eq. 4.10 we insert eq. 4.7 and choose the parts corresponding to the ϵ scalar

$$g_s f^{abc} \left(k^{(a)}_\rho A^{\rho b} A^c_i A^{ia} - A^b_i A^{ia} k^{(a)}_\rho A^{\rho c} \right)$$

[1] $k^{(a)}$ denotes the momentum corresponding to the particle with color index a.

Figure 4.1: Denotation of the momenta, color indices and Lorentz indices. The arrows mark the direction of the momentum.

After symmetrizing in a and c we find the Feynman rule for the vector-two ϵ scalar vertex



$$g_s\,(q-k)^{\rho}\,\hat{g}^{ij}\,f^{abc} \tag{4.12}$$

The same result is achieved by removing all terms in eq. 4.11 with metric tensors mixing D and $(4-D)$-dimensional indices.

The $VVVV$, $VV\phi\phi$ and $\phi\phi\phi\phi$ Vertices

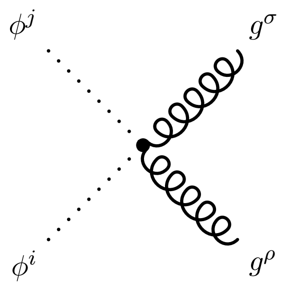
The 4-vector vertex is calculated from the part of the Lagrangian

$$g_s^2 f^{abc} A_{\mu}^{b} A_{\nu}^{c} f^{aef} A^{\mu\,e} A^{\nu\,f}$$

by decomposing the expression $A_{\mu}A^{\mu}$ with eqs. 4.7. For the D-dimensional component we obtain the result

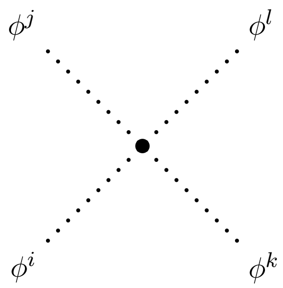
$$\begin{aligned} -ig_s^2 \big(& f^{abe} f^{cde} \left(\widetilde{g}^{\mu\rho}\widetilde{g}^{\nu\sigma} - \widetilde{g}^{\mu\sigma}\widetilde{g}^{\nu\rho}\right) \\ & + f^{ace} f^{bde} \left(\widetilde{g}^{\mu\nu}\widetilde{g}^{\rho\sigma} - \widetilde{g}^{\mu\sigma}\widetilde{g}^{\nu\rho}\right) \\ & + f^{ade} f^{bce} \left(\widetilde{g}^{\mu\nu}\widetilde{g}^{\rho\sigma} - \widetilde{g}^{\mu\rho}\widetilde{g}^{\nu\sigma}\right)\big) \end{aligned} \tag{4.13}$$

Analogous we evaluate the 2-vector 2-ϵ scalar coupling



$$-ig_s^2 f^{abe} f^{cde} \hat{g}^{ij} \widetilde{g}^{\rho\sigma} \tag{4.14}$$

Additionally there is a four scalar vertex including only ϵ scalars:



$$\begin{aligned} -ig_s^2 \big(& f^{abe} f^{cde} \left(\hat{g}^{ij}\hat{g}^{kl} - \hat{g}^{jk}\hat{g}^{il}\right) \\ & + f^{ace} f^{bde} \left(\hat{g}^{jl}\hat{g}^{ik} - \hat{g}^{jk}\hat{g}^{li}\right) \\ & + f^{ade} f^{bce} \left(\hat{g}^{jl}\hat{g}^{ik} - \hat{g}^{ij}\hat{g}^{lk}\right)\big) \end{aligned} \tag{4.15}$$

The FFV and $FF\phi$ Vertices

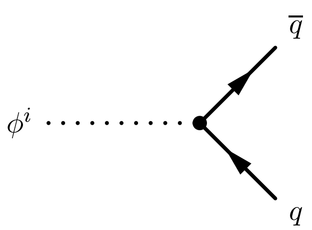
The vertex for the coupling of two fermions and a vector boson is

$$ig_s \widetilde{\gamma}^\rho t^a$$

where γ^ρ is a Dirac matrix. The γ matrices are defined such that

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \tag{4.16a}$$
$$\{\widetilde{\gamma}^\rho, \widetilde{\gamma}^\sigma\} = 2\widetilde{g}^{\rho\sigma} \tag{4.16b}$$
$$\{\hat{\gamma}^i, \hat{\gamma}^j\} = 2\hat{g}^{ij} \tag{4.16c}$$
$$\{\hat{\gamma}^i, \widetilde{\gamma}^\rho\} = 0 \tag{4.16d}$$

The ϵ scalar two-fermion vertex is derived as



$$ig_s \hat{\gamma}^i t^a \tag{4.17}$$

The trace identities for these γ matrices can be found in A.1.

The $VV\widetilde{F}\widetilde{F}$ and $\widetilde{F}\widetilde{F}\phi\phi$ Vertices

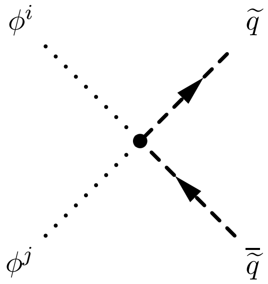
The procedure to obtain the Feynman rules is the same as before. First, the 4-dimensional fields are split in 4 and $(4-D)$ components, then we symmetrize and Fourier transform $i\mathcal{L}$ and erase all external fields. The couplings of vector bosons and squarks are derived from

$$(D_\mu\phi)^\dagger(D^\mu\phi) \quad = \quad \left(\left(\partial_\mu - ig_s A_\mu^b t^b\right)\phi\right)^\dagger \left(\partial^\mu - ig_s A^{\mu\,a} t^a\right)\phi$$

which is part of the Lagrangian in eq. 4.5. After symmetrizing color and Lorentz indices, the Feynman rule for the vector-two scalar (squarks) coupling is

$$ig_s^2\{t^a, t^c\}\tilde{g}^{\mu\nu} \tag{4.18}$$

For the ϵ scalar the procedure is analogous and we obtain



$$ig_s^2\{t^a, t^b\}\hat{g}^{ij} \tag{4.19}$$

The Feynman rules for ϵ scalars, vector bosons and fermions which are used in this work are summarized the appendix A.2.

4.4 Derivation of the Splitting Functions in DRED

Mass factorization predicts that matrix elements in the collinear limit $\langle k_2|\,|k_3\rangle$ factorize

$$|\mathcal{M}|^2_{p_1+p_2\to g+p_3+p_4} \overset{\langle k_2||k_3\rangle}{=} \alpha_s\pi|\mathcal{M}|^2_{p_a+p_b\to p_c+p_d}\cdot\tfrac{1}{t_1(1-z)}\cdot P_{ij}(z) \tag{4.20}$$

where $P_{ij}(z)$ is the splitting function we introduced in chapter 6. This function is independent of the energy and only a function of color factors, the parameter z and the dimension D. The parameter z is the fraction of the momentum that is carried away by the radiated particle.

The splitting functions can be derived from common three particle vertices by setting two particles collinear. We derive the splitting function using the definition

$$P_{ij}(z) = \frac{z(1-z)}{2k_\perp^2} \sum_{\text{pols}} |\mathcal{M}_{i->jf}|^2 \tag{4.21}$$

where j is the collinear particle and i the inital particle radiating j [23]. These functions are known for standard splittings such as

$$\begin{aligned} q &\rightarrow \; q\,G \\ G &\rightarrow GG \end{aligned} \tag{4.22}$$

and can be found in [23] for 4 dimensions.
Since we treat the ϵ scalars from the last section as partons of their own, new splitting functions have to be evaluated

$$\begin{aligned} q &\rightarrow \phi\, q \\ g &\rightarrow \phi\, \phi \\ \phi &\rightarrow \phi\, g \\ \phi &\rightarrow g\, \phi \end{aligned} \tag{4.23}$$

We derive these six splitting functions in the following in D dimensions to confirm the results in chapter 6. The momenta for the following vertices are defined incoming for the initial particles and outbound for the particles in the final state. For simplicity the coupling constant g_s is omitted in this section.
As mentioned in section 2.2 there are at least two ways to derive splitting functions. In section 2.2 we guarantee momentum conservation by the parametrization of the collinear momentum with eqs. 2.5 and the momenta as they appear in fig. 4.2. Thus the particle with momentum $k_c = k_a - k_b$ is offshell with $k_\perp^2/z$. This approach is similar to [20]. The crucial point is that for offshell particles, we can not use the polarization sum as in eq. A.13. Therefore the particle which enters the partonic process is kept offshell but is seen as a propagating particle (which is acceptable since they are always offshell). This propagator can then be absorbed into the partonic process as done in section 2.2.
To avoid the use of a partonic process, we violate momentum conservation in order to keep all three particles onshell. This has the advantage that we can use the polarization sum eq. A.13. A comparison, e.g. with the derivation in 2.2 shows that both approaches lead to the same results.

Splitting Function for $q \rightarrow gq$

Consider a quark a with momentum k_a which emits a gluon b with momentum k_b. We are interested in the vertex structure of this process where the quark a emits the gluon collinearly

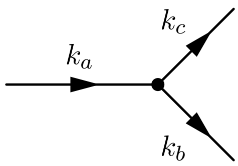


Figure 4.2: Naming conventions for the collinear particles. The arrows denote the direction of the momenta.

with momentum fraction z. The three momenta k_a, k_b and k_c are substituted with

$$\begin{aligned} k_a^\mu &\to p^\mu \\ k_b^\mu &\to z \cdot p^\mu + k_\perp^\mu - \frac{k_\perp^2 n^\mu}{2zp \cdot n} \\ k_c^\mu &\to (1-z) \cdot p^\mu - k_\perp^\mu - \frac{k_\perp^2 n^\mu}{2(1-z)p \cdot n} \end{aligned} \tag{4.24}$$

where n is a lightlike auxiliary vector with $n \cdot n = n \cdot k_\perp = 0$ and $p \cdot k_\perp = 0$ as defined in section 2.2. For the collinear limit $k_\perp \to 0$, momentum conservation is again restored. We evaluate the splitting functions for $m_q = 0$ since we assume the quarks to be effectively massless. The amplitude for this process is

$$\mathcal{M} \quad = \quad \overline{u}(k_c)\, ig\, \widetilde{\gamma}^\mu\, t^a\; u(k_a)\, \epsilon_\mu^*$$

After squaring and evaluating the polarization sum (eq. A.13) for massless quarks we obtain

$$|\mathcal{M}|^2 = \frac{1}{2} \sum_{\text{pols}} \mathbf{Tr}\, [\not{k}_c \widetilde{\gamma}_\mu \not{k}_a \widetilde{\gamma}_\nu] \cdot \widetilde{\epsilon}^{*\mu}(k_b) \widetilde{\epsilon}^\nu(k_b) \cdot \mathbf{Tr}\, [t^a t^a] \tag{4.25}$$

where the factor 1/2 is due to the averaging over the initial quark spin. The color trace $\mathbf{Tr}\,[t^a t^a]$ gives a factor $C_2(r) = 8/6$. As there appears a massless gauge boson, the following polarization sum is used

$$\widetilde{\epsilon}^{*\mu}(k_b) \widetilde{\epsilon}^\nu(k_b) = -\widetilde{g}^{\mu\nu} + \frac{k_b^\mu n^\nu + k_b^\nu n^\mu}{k \cdot n} - \frac{n^2 k_b^\mu k_b^\nu}{(k_b \cdot n)^2} \tag{4.26}$$

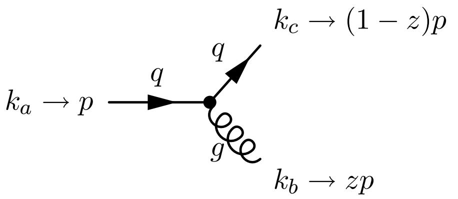
The momenta are substituted using eq. 4.24 and we obtain

$$|\mathcal{M}|^2 \quad = \quad C_2(r) \frac{1}{2} \frac{2\, k_\perp^2}{z(1-z)} \frac{(z((D-2)z-4)+4)}{z}$$

which gives for $D = 4$

$$|\mathcal{M}|^2 = C_2(r) \cdot \frac{z^2 - 2z + 2}{(z-1)z^2} \cdot \frac{2\,k_\perp^2}{z(1-z)}$$

where $(z(1-z))/(2k_\perp^2)$ represents the collinear pole of the process. The splitting function is a function of z, the dimension and a color factor



$$\begin{aligned} &P_{gq} = \\ &C_2(r)\tfrac{1}{2} \cdot \tfrac{(z((D-2)z-4)+4)}{z} \stackrel{D\to 4}{=} \\ &P_{Gq} = C_2(r) \cdot \tfrac{1+(1-z)^2}{z} \end{aligned} \tag{4.27}$$

Splitting Function for $q \to \phi q$

For the calculation in dimensional reduction we need the splitting function for $q \to \phi q$. The function is calculated in the same way as in the last section with respect to the ϵ scalar. The amplitude for this process is the same as eq. 4.25 except for the $(4-D)$ dimensional Lorentz index of the Dirac matrices and the $(4-D)$ dimensional polarization vectors. Analogous to eq. 4.25 one derives

$$|\mathcal{M}|^2 = \frac{1}{2} \sum_{\text{pols}} \mathbf{Tr}\left[\not{k}_c \hat{\gamma}_\mu \not{k}_a \hat{\gamma}_\nu\right] \cdot \hat{\epsilon}^{*\mu}(k_b) \hat{\epsilon}^\nu(k_b) \cdot \mathbf{Tr}\left[t^a t^a\right]$$

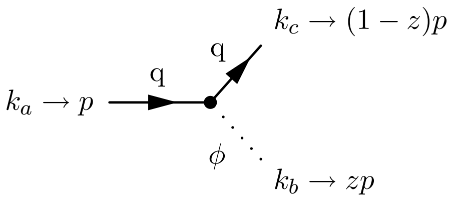
The polarization sum of the ϵ scalars has the simple form

$$\hat{\epsilon}^{*\mu}(k_b)\hat{\epsilon}^\nu(k_b) = -\hat{g}^{\mu\nu} \tag{4.28}$$

Since the momenta are still in D dimensions, they do not appear in this polarization sum as in eq. 4.26. Due to these changes we derive

$$\begin{aligned} |\mathcal{M}|^2 &= \frac{1}{2} \sum_{\text{pols}} C_s(r) \cdot \frac{2(4-D)}{(z-1)} \cdot k_\perp^2 \\ &= 2C_2(r) \frac{1}{2} \frac{2\,k_\perp^2}{z(1-z)} \cdot z(4-D) \end{aligned}$$

with eq. 4.21 the last expression yields the splitting function



$$P_{\phi q} = \tfrac{1}{2} C_2(r)(4-D)z \tag{4.29}$$

$P_{\phi q}$ and P_{gq} together give the 4 dimensional splitting function P_{Gq}.

Splitting Function for $g \to gg$

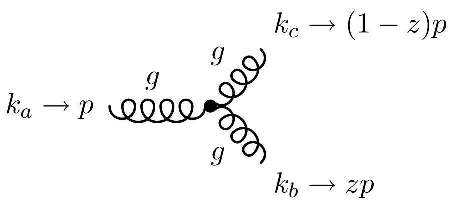
In order to show the factorization of the process $g\,g \to g\tilde{t}\tilde{\bar{t}}$ we need the splitting function for the splitting of a gluon to two gluons $g \to gg$. The derivation is analogous to that of the last splitting function and is just sketched here.
First the amplitude for the $g \to gg$ vertex is derived from the Feynman diagram in eq. 4.30

$$\begin{aligned}\mathcal{M} \;=\; & f^{abc}\Big(\big(-k_b^{c_1} - k_a^{c_1}\big)\,\widetilde{g}^{a_1 b_1} + \Big(k_a^{b_1} + k_c^{b_1}\Big)\,\widetilde{g}^{a_1 c_1} + \big(-k_c^{a}1 + k_b^{a_1}\big)\,\widetilde{g}^{b_1 c_1}\Big) \times \\ & \widetilde{\epsilon}(k_a)_{a_1}\widetilde{\epsilon}(k_b)_{b_1}\widetilde{\epsilon}(k_c)_{c_1}\end{aligned}$$

The collinear particle carries the momentum k_b. For clarity, the Lorentz indices were named analogous to the particle numbering. The momenta are substituted with eq. 4.24 in order to take the collinear limit. The polarization sum is performed with eq. A.6 in D dimensions. After taking the collinear limit and contracting the Lorentz indices the squared amplitude becomes

$$\frac{1}{(D-2)}\sum_{\text{pols}} |\mathcal{M}|^2 \;=\; \frac{2}{(D-2)}\frac{(D-2)((z-1)z+1)^2}{(1-z)z} \cdot C_2(G)\frac{2\,k_\perp^2}{z(1-z)}$$

We now have to divide by a factor of $(D-2)$ for the possible polarization states of the gluon in D dimensions. Setting $C_2(G) = N$ we obtain the splitting function:



$$P_{GG} = P_{gg} = 2\frac{((z-1)z+1)^2}{(1-z)z} \cdot N \tag{4.30}$$

Due to the symmetric appearance of the momenta in the three gluon vertex we have three possibilities for the substitution of two vector bosons by two ϵ scalars.

Splitting Function for $g \to \phi\phi$

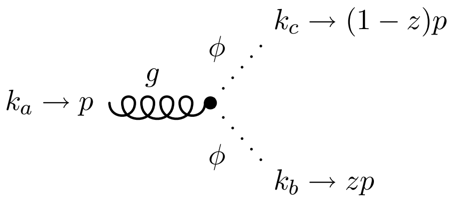
The amplitude for a gluon "decaying" into ϵ scalars is

$$\mathcal{M} \quad = \quad g_s f^{abc}\,(k_b - k_c)^{a_1} \cdot g^{b_1 c_1} \widetilde{\epsilon}^*_{a_1}(k_a)\,\hat{\epsilon}^*_{b_1}(k_b)\,\hat{\epsilon}^*_{c_1}(k_c)$$

The polarization sum for the two ϵ scalars is evaluated with eq. 4.28, the polarization sum for the gluon is derived with eq. 4.26. After contracting all Lorentz indices and taking the collinear limit, the squared amplitude evaluates to

$$\frac{1}{(D-2)} \sum_{\text{pols}} |\mathcal{M}|^2 \quad = \quad \frac{2k_\perp^2}{z(1-z)} \underbrace{2\,N \frac{(4-D)}{(D-2)} \cdot z(1-z)}_{P_{\phi G}} \tag{4.31}$$

We divide by a factor of $(D-2)$ to average over the initial polarizations of the gluon



$$P_{\phi g} = 2\,z(1-z)\tfrac{(4-D)}{(D-2)} \cdot N \tag{4.32}$$

Note that this splitting function is symmetric under an exchange of z and $(1-z)$, so it is also symmetric under an exchange of the two ϵ scalars. Furthermore, it vanishes in 4 dimensions.

Splitting function for $\phi \to g\phi$

We now derive the splitting function for the splitting $\phi \to g\phi$ with the gluon being the collinear particle. First we set up the amplitude for this process

$$\mathcal{M} = g f^{abc}(k_a + k_c)^{b_1} \widetilde{g}^{a_1 b_1} \cdot \hat{\epsilon}^*_{a_1}(k_a) \hat{\epsilon}^*_{c_1}(k_c) \widetilde{\epsilon}^*_{b_1}(k_b)$$

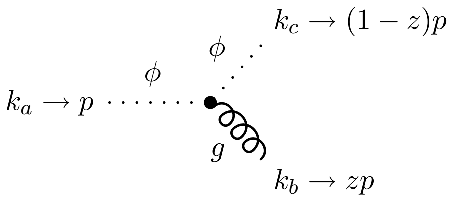
After performing the polarization sums eq. 4.28 for the ϵ scalars and eq. 4.26 for the gluon, we find

$$|\mathcal{M}|^2 = g^2 C_2(G)\Bigg(- \frac{(4-D) g_s n^2 (k_a \cdot k_b + k_b \cdot k_c)^2}{(k_b \cdot n)^2} - \frac{2 g_s (4-D)(-(k_a \cdot k_b) k_a \cdot n - k_b \cdot k_c\; k_a \cdot n + k_a \cdot k_c\; k_b \cdot n - k_a \cdot k_b\; k_c \cdot n - k_b \cdot k_c k_c \cdot n)}{k_b \cdot n}\Bigg)$$

The collinear limit is taken with the substitution in eq. 4.24. After evaluating the color sum one finds

$$\frac{1}{4-D}\sum_{\text{pols}}|\mathcal{M}|^2 \quad = \quad g^2\frac{2k_\perp^2}{(1-z)z}\,C_2(G)\frac{(1-z)}{z}$$

As the incoming particle is an ϵ scalar we have to divide by a factor of $(4-D)$. The splitting function is then



$$P_{g\phi} = 2\tfrac{(1-z)}{z}\cdot N \tag{4.33}$$

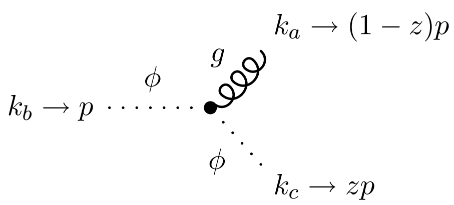
The next splitting function can be derived from this one by an exchange of of z and $(1-z)$.

Splitting Function for $\phi \to \phi g$

The last splitting function which is important for this work is the emission of a gluon by an ϵ scalar as before but with the collinear particle being the ϵ scalar.
The approach is analogous to the calculation above and the result is symmetric in an exchange of the the momenta k_b and k_c. After replacing the momenta as in eq. 4.24, the squared amplitude is

$$\frac{1}{(4-D)}\sum_{\text{pols}}|\mathcal{M}|^2 \quad = \quad g^2\;\frac{1}{k_\perp^2(1-z)z}C_2(G)\frac{z}{(1-z)}$$

The factor $(4-D)$ cancels and we find for the splitting function



$$P_{\phi\phi} = 2\tfrac{z}{(1-z)}\cdot N \tag{4.34}$$

Note that only for the splitting of $g \to \phi\phi$ the splitting function has a dimensional dependence and vanishes for $D \to 4$. The others still give a contribution in 4 dimensions.

As we have now derived the splitting functions, we know what to expect after the factorization of both processes investigated in this work (eqs. 1.2) and we will be able to see if the interpretation of the ϵ scalar as a parton of its own solves the factorization problem for these cases. We will start with the calculation of the LO processes in DRED and then proceed to the real NLO processes and their factorization.

5 The $2 \to 2$ Case

As an introduction into DRED we calculate both processes eq. 1.2 in LO and show how the ϵ scalars are implemented in the calculation. Both results are needed in chapter 6 since there we will calculate real NLO corrections to these processes and perform the mass factorization.

5.1 Stop Production via Quark Fusion: $q\overline{q} \to \tilde{t}\tilde{\overline{t}}$

In the following we calculate the process $q\overline{q} \to \tilde{t}\tilde{\overline{t}}$ with massless initial quarks. In difference to the process discussed next, no ϵ scalars appear in the LO process and the only contributing Feynman diagram is shown in fig. 5.1. We are only interested in QCD couplings and neglect electroweak couplings.

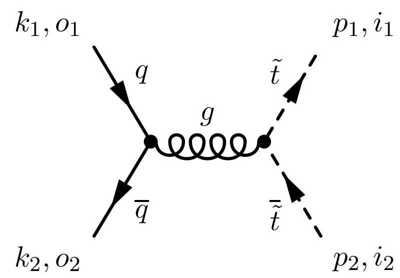


Figure 5.1: Feynman diagram for $q\overline{q} \to \tilde{t}\tilde{\overline{t}}$. k_i, p_i denote the momenta, o_i and i_i the color indices.

The amplitude for this process is

$$\mathcal{M}_{q\overline{q}\to\tilde{t}\tilde{\overline{t}}} = 2\alpha_s\pi \frac{\overline{v}(k_2)\not{p}_1 u(k_1)}{(k_1 \cdot k_2)} \left(\delta_{i_1 o_1}\, \delta_{i_2 o_2} - \frac{\delta_{i_1 i_2}\, \delta_{o_1 o_2}}{3} \right)$$

The color trace and the polarization sum for fermions is evaluated with the polarization sums in eq. A.13 for massless fermions. The Mandelstam variables are chosen according to eq. A.2. The squared amplitude is

$$|\mathcal{M}|^2_{q\overline{q}\to\tilde{t}\tilde{\overline{t}}} = \alpha_s^2\pi^2 \frac{256\left((T+U)m^2 + TU\right)}{(T+U)^2} \tag{5.1}$$

where m denotes the stop mass. In difference to the example in section 5.2 this squared amplitude lacks any dimensional dependence, which is due to the absence of external vector bosons.

The result is equal to [29][1]. The result from eq. 5.1 is not yet averaged over polarization states and colors. After averaging we get

$$\frac{1}{9\cdot 4}|\mathcal{M}|^2_{q\bar{q}\to\widetilde{t}\,\bar{\widetilde{t}}} = \alpha_s^2\pi^2\frac{1}{9}\frac{64\left((T+U)m^2+TU\right)}{(T+U)^2}$$

This result is given for completeness, but in further calculations we will always refer to the unaveraged result in eq. 5.1.

5.2 Stop Production via Gluon Fusion: $GG \to \widetilde{t}\,\bar{\widetilde{t}}$

Now we look at the stop production via gluon fusion in LO. To simplify the calculation, we perform it with an unphysical polarization sum and subtract the unphysical states via external BRST ghosts.

The Feynman rules needed have been derived in chapter 4.3 and are summarized in the appendix A.2. The Feynman diagrams for the LO $gg \to \widetilde{t}\,\bar{\widetilde{t}}$ are shown in fig. 5.2, those for the ghost processes in fig. 5.3.

We calculate the amplitudes for the D-dimensional gluon g and the ϵ scalar (ϕ) separately. Therefore we consider the process $gg \to \widetilde{t}\,\bar{\widetilde{t}}$ and $\phi\phi \to \widetilde{t}\,\bar{\widetilde{t}}$ [2]. Since the calculation is performed in DRED with particles and momenta in D dimensions, we have to derive the process where the gluon is substituted by the ϵ scalar as shown in fig. 5.4. The color indices and momenta are defined as they appear in fig. 5.2(1). The amplitudes for the $g\,g \to \widetilde{t}\,\bar{\widetilde{t}}$ are

$$\begin{aligned}
\mathcal{M}_1 &= \left(t^a\,t^b - t^b\,t^a\right)_{ij}\frac{2\alpha_s\pi}{(k_1\cdot k_2)}\times \\
&\Big(\epsilon^a(k_1)\cdot k_2\;\;\epsilon^b(k_2)\cdot(k_2) + 2\epsilon^a(k_1)\cdot p_1(2\epsilon^b(k_2)\cdot k_1 + \epsilon^b(k_2)\cdot k_2) - \\
&-4\epsilon^a(k_1)\cdot k_2\;\;\epsilon^b(k_2)\cdot p_1 - (\epsilon^a(k_1)\cdot k_1)(\epsilon^b(k_2)\cdot k_1 + 2\epsilon^b(k_2)\cdot p_1) - \\
&-2\epsilon^a(k_1)\cdot\epsilon^b(k_2)\;\;k_1\cdot p_1 + 2\epsilon^a(k_1)\cdot\epsilon^b(k_2)\;\;k_2\cdot p_1\Big) \\
\mathcal{M}_2 &= \frac{2\alpha_s\pi\,(t^a\,t^b)_{ij}}{(k_1\cdot p_1)}\cdot((\epsilon^a(k_1)\cdot k_1) - 2(\epsilon^a(k_1)\cdot p_1))\times \\
&(2(\epsilon^b(k_2)\cdot k_1) + (\epsilon^b(k_2)\cdot k_2) - 2(\epsilon^b(k_2),p_1)) \\
\mathcal{M}_3 &= 4\alpha_s\pi\left(t^a\,t^b + t^b t^a\right)_{ij}\left(\epsilon^a(k_1)\cdot\epsilon^b(k_2)\right)
\end{aligned}$$

[1] There is a relative factor of two since we take only one type of stop into account.

[2] LO processes with a gluon and an ϵ scalar are also possible, but are not present in this process.

$$\mathcal{M}_4 = \frac{2\alpha_s\pi\,(t^b\,t^a)_{ij}}{(k_1\cdot p_2)}\times$$
$$(\epsilon^a(k_1)\cdot k_1 + 2\epsilon^a(k_1)\cdot k_2 - 2(\epsilon^a(k_1)\cdot p_1)(\epsilon^b(k_2)\cdot k_2 - 2(\epsilon^b(k_2)\cdot p_1))$$

The polarization sum is evaluated with eq. A.7 in D dimensions. After evaluating the color trace, we derive the squared matrix element

$$|\mathcal{M}|^2_{g\,g\to\tilde{t}\bar{\tilde{t}},\text{unphys.}} = \alpha_s^2\pi^2\left(\frac{4096m^4}{3U^2} + \frac{32\left(128m^4+128Tm^2+32DT^2-55T^2\right)}{3T^2}\right.$$
$$\left.+\frac{1024\left(4m^2T-m^4\right)}{3TU} - \frac{384\left(8m^2+2DT-3T\right)}{T+U} + \frac{384\left(2DT^2-3T^2\right)}{(T+U)^2}\right)$$

where m ist the stop mass. Due to the use of the unphysical polarization sum, BRST ghosts have to be subtracted after squaring both amplitudes. The Feynman graphs for the BRST ghosts are shown in fig. 5.3 and the corresponding amplitudes are

$$\mathcal{M}_{1,\eta\,\overline{\eta}\to\tilde{t}\bar{\tilde{t}}} = \frac{2\alpha_s\pi\left(t^at^b-t^bt^a\right)_{ij}(k_1\cdot k_2-2k_1\cdot p_1)}{k_1\cdot k_2}$$
$$\mathcal{M}_{2,\overline{\eta}\,\eta\to\tilde{t}\bar{\tilde{t}}} = \frac{2\alpha_s\pi\left(t^at^b-t^bt^a\right)_{ij}(k_1\cdot k_2-2k_2\cdot p_1)}{k_1\cdot k_2}$$

After evaluating the color trace, the squared amplitude is

$$|\mathcal{M}|^2_{\eta\overline{\eta}\to\tilde{t}\bar{\tilde{t}}} = |\mathcal{M}|^2_{\overline{\eta}\eta\to\tilde{t}\bar{\tilde{t}}} = \alpha_s^2\pi^2\frac{48(T-U)^2}{(T+U)^2}$$

Now we perform the calculation for the process including the ϵ scalar. Due to the topology of vertices for the vector-scalar coupling, there is always an even number of ϵ scalars. The Feynman graphs for the processes including ϵ scalars are shown in fig. 5.4.
The amplitudes are calculated with the help of the Feynman rules from section 4.3:

$$\mathcal{M}_{1,\phi} = -\frac{4\alpha_s\pi\left(t^at^b-t^bt^a\right)_{ij}\hat{\epsilon}^a(k_1)\cdot\hat{\epsilon}^b(k_2)(k_1\cdot p_1-k_2\cdot p_1)}{k_1\cdot k_2}$$
$$\mathcal{M}_{2,\phi} = 4\alpha_s\pi\left(t^at^b+t^bt^a\right)_{ij}\hat{\epsilon}^a(k_1)\cdot\hat{\epsilon}^b(k_2)$$

The polarization sum for the ϵ scalars is evaluated with eq. A.8. The polarization vectors $\hat{\epsilon}(k_1)$ and $\hat{\epsilon}(k_2)$ are $(4-D)$-dimensional and therefore give a $(4-D)$-dimensional metric tensor $\hat{g}^{\mu\nu}$.

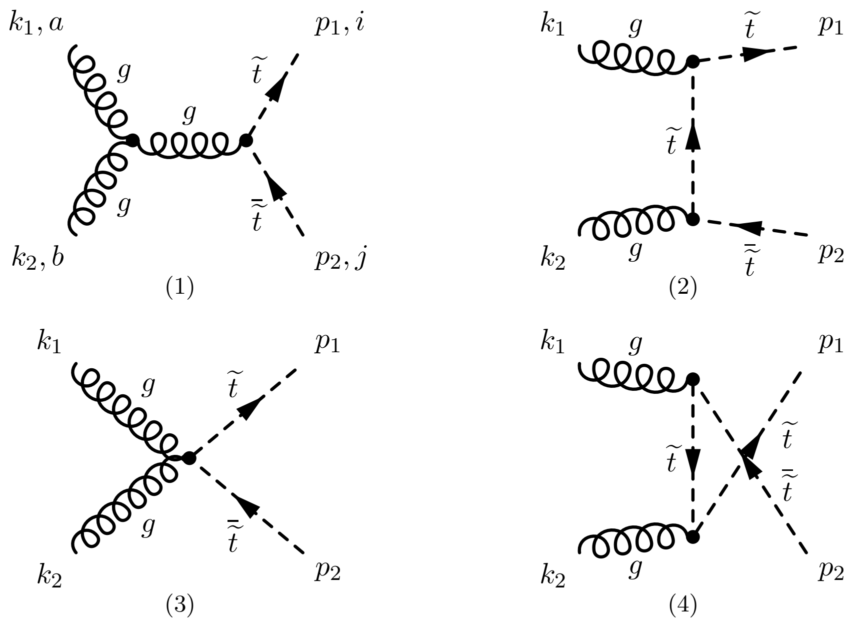


Figure 5.2: Feynman diagrams for $gg \to \widetilde{t}\,\overline{\widetilde{t}}$. The amplitudes are named analogous to the numbering of the graphs. The first graph also gives the notation for the color indices of the particles. The charge flow is indicated by the direction of the arrow at the scalar lines.

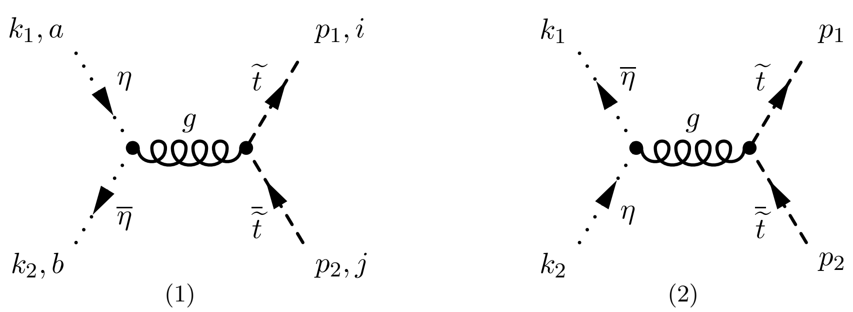


Figure 5.3: Feynman diagrams for the BRST ghosts $\eta\overline{\eta} \to \widetilde{t}\,\overline{\widetilde{t}}$ and $\overline{\eta}\eta \to \widetilde{t}\,\overline{\widetilde{t}}$.

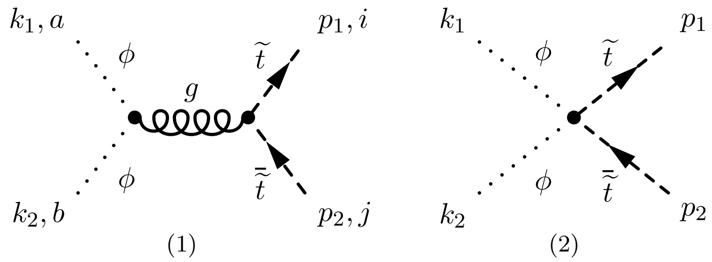


Figure 5.4: Feynman diagrams for $\phi\phi \to \tilde{t}\bar{\tilde{t}}$ at LO.

After squaring the amplitudes we obtain terms of the form giving a dimensional dependence

$$\begin{aligned}|\mathcal{M}_{1,\phi} + \mathcal{M}_{2,\phi}|^2 &\propto \hat{\epsilon}(k_1)^\mu \hat{\epsilon}(k_2)^\mu \cdot \hat{\epsilon}^*_\nu(k_1)\hat{\epsilon}^*_\nu(k_2) \\ &\propto \hat{g}^{\mu\nu}\hat{g}_{\mu\nu} = (4 - D)\end{aligned}$$

After calculating the color trace and sum over the polarization states, the squared amplitudes for the ϵ scalars are

$$\sum_{\text{pols}} |\mathcal{M}|^2_{\phi\phi\to\tilde{t}\bar{\tilde{t}}} = \alpha_s^2\pi^2 \cdot \frac{256(4-D)\left(4T^2 - UT + 4U^2\right)}{3(T+U)^2} \tag{5.2}$$

Since the ϵ scalars are formally seen as independent particles (see chapter 4), we need this result to show the NLO factorization in chapter 6. For $D \to 4$ this matrix element vanishes as expected.
Now we can sum up the process including gluons and ghosts and derive the full matrix element for $gg \to \tilde{t}\bar{\tilde{t}}$ in D dimensions which is equal to the DREG result

$$\begin{aligned}\sum_{\text{pols}} |\mathcal{M}|^2_{g\,g,\text{ phys.}} &= \sum_{\text{pols}} \left(|\mathcal{M}|^2_{g\,g,\text{ unphys.}} - |\mathcal{M}|^2_{\eta\bar{\eta}} - |\mathcal{M}|^2_{\bar{\eta}\eta}\right) \\ &= \alpha_s^2\pi^2 \frac{256\left(4T^2 - UT + 4U^2\right)\left(4(T+U)^2m^4 + 4TU(T+U)m^2 + (D-2)T^2U^2\right)}{3T^2U^2(T+U)^2}\end{aligned} \tag{5.3}$$

To get the complete matrix element for the process $gg \to \tilde{t}\bar{\tilde{t}}$ in DRED, the squared amplitude including the ϵ scalars is added

$$\begin{aligned}\frac{1}{2}\sum_{\text{pols}} |\mathcal{M}|^2_{GG\to\tilde{t}\bar{\tilde{t}}} &= \frac{1}{2}\left(\sum_{\text{pols}} |\mathcal{M}|^2_{g\,g\to\tilde{t}\bar{\tilde{t}},\text{ unphys.}} + |\mathcal{M}|^2_{\phi\phi\to\tilde{t}\bar{\tilde{t}}} - 2|\mathcal{M}|^2_{\eta\bar{\eta}\to\tilde{t}\bar{\tilde{t}}}\right) \\ &= \frac{1}{2}\alpha_s^2\pi^2 \frac{512\left(4T^2 - UT + 4U^2\right)\left(2(T+U)^2m^4 + 2TU(T+U)m^2 + T^2U^2\right)}{3T^2U^2(T+U)^2}\end{aligned} \tag{5.4}$$

The factor 1/2 is due to identical initial particles. Immediately the D dependence of the physical process with gluons and ϵ scalars in the initial state vanishes and the result is equal to the 4-dimensional result without regularization.

This result is in agreement with [29][3]. After color and polarization averaging we find

$$\frac{1}{2}\frac{1}{4}\frac{1}{64}\sum_{\text{pols}} |\mathcal{M}|^2_{GG\to\tilde{t}\bar{\tilde{t}}} = \alpha_s^2\pi^2\frac{\left(4T^2 - UT + 4U^2\right)\left(2(T+U)^2m^4 + 2TU(T+U)m^2 + T^2U^2\right)}{3T^2U^2(T+U)^2}$$

[3]There is again a relative factor of two since we take only one stop into account.

6 Factorization of the Real NLO Processes in DRED

The two most important channels for stop production at hadron colliders are

$$
\begin{aligned}
q\overline{q} &\rightarrow \tilde{t}\tilde{\bar{t}} \\
gg &\rightarrow \tilde{t}\tilde{\bar{t}}
\end{aligned}
\tag{6.1}
$$

In this chapter we will demonstrate that the real squared NLO amplitudes factorize in the collinear limit in DRED if the method proposed in [2] is applied.
After deriving the squared amplitude for the two LO processes in chapter 5 and calculating the splitting functions for gluon-quark and gluon-gluon splitting and their corresponding splitting functions involving ϵ scalars in section 4.3 and 4.4, we now have the technical background to explicitly calculate the collinear factorization for both processes in DRED.
Both amplitudes are first factorized in 4 dimensions in order to show how the radiated gluon is made collinear and how to approach factorization. Then both calculations are performed in DRED. In the following we denote the 4 component gluon with capital G and the D-dimensional gluon with lower case g. Without loss of generality we always consider the particle with momentum k_3 collinear to that with k_2. If the momentum k_3 is chosen collinear to k_1 the approach is the same with respect to the pole $1/u_1$.

6.1 Stop Production via Quark Fusion: $q\overline{q} \rightarrow G\tilde{t}\tilde{\bar{t}}$

We calculate the real NLO correction for stop production via quark fusion. Since $m_u, m_d \ll m_{\tilde{t}}$ we consider the initial quarks to be massless. As discussed in chapter 2, massless particles in NLO corrections lead to singularities if the emitted particle is collinear to the emitting particle. This singularity has to be handled by absorbing it into the PDFs. After separating the singularity we are left with the LO process. Thus we expect that the squared matrix element of the real NLO correction in the collinear limit should have the form

$$
|\mathcal{M}|^2_{2\rightarrow 3} \overset{\langle k_2 || k_3 \rangle}{=} \alpha_s \pi \frac{1}{t_1(1-z)} |\mathcal{M}|^2_{2\rightarrow 2} \cdot P(z) \tag{6.2}
$$

where P(z) denotes the splitting function for the corresponding process derived in 4.4. The singularity is represented by $1/t_1$. The Mandelstam variable t_1 is defined in eq. A.4.
We will first perform the calculation $q\overline{q} \rightarrow G\tilde{t}\tilde{\bar{t}}$ in 4 dimensions and demonstrate in detail how the collinear limit is taken and how the factorization is obtained. After this, the factorization is

performed in DRED.

6.1.1 Mass Factorization in 4 Dimensions

The contributing diagrams for $q\overline{q} \to G\widetilde{t}\widetilde{\overline{t}}$ are shown in fig. 6.1 where we also defined the notation (fig. 6.1(1)). The highlighted propagator in fig. 6.1(1) produces the factor $1/t_1$ which diverges if the gluon is collinear to the quark with momentum k_2.

First we derive the amplitudes for each graph, then square them. Afterwards we perform the factorization in detail and demonstrate that the result fullfills the factorization theorem in 4 dimensions.
The individual amplitudes corresponding to the Feynman graphs in fig. 6.1 are

$$
\begin{aligned}
\mathcal{M}_1 &= ig_s^3 \frac{\overline{v}(k_2) t^a \gamma^\alpha (\not{k}_2 - \not{k}_3) t^b \gamma^\beta u(k_1) g^{\beta\gamma} (-k_1 - k_2 + k_3 + 2p_1)^\gamma t^b_{i_2 i_1}}{(k_2 - k_3)^2 (k_1 + k_2 - k_3)^2} \epsilon^{*a}_\alpha(k_3) \\
\mathcal{M}_2 &= ig_s^3 \frac{\overline{v}(k_2) \gamma^\beta t^b u(k_1)\ g^{\beta\gamma} (k_1 + k_2 - p_1)^\gamma\ (2k_1 + 2k_2 - k_3 - 2p_1)^\alpha\ (t^a t^b)_{i_2 i_1}}{(k_1 + k_2)^2\ ((k_1 + k_2 - p_1)^2 - m^2)} \epsilon^{*a}_\alpha(k_3) \\
\mathcal{M}_3 &= ig_s^3 \frac{\overline{v}(k_2) t^b \gamma^\alpha (\not{k}_1 - \not{k}_3) t^a \gamma^\beta u(k_1) g^{\beta\gamma} (-k_1 - k_2 + k_3 + 2p_1)^\gamma t^b_{i_2 i_1}}{(k_1 - k_3)^2 (k_1 + k_2 - k_3)^2} \epsilon^{*a}_\alpha(k_3) \\
\mathcal{M}_4 &= -ig_s^3 \frac{\overline{v}(k_2) \gamma^\beta t^b u(k_1)\ g^{\beta\gamma} (-k_1 - k_2 + 2k_3 + 2p_1)^\gamma\ (2p_1 + k_3)^\alpha\ (t^b t^a)_{i_2 i_1}}{(k_1 + k_2)^2\ ((k_3 + p_1)^2 - m^2)} \epsilon^{*a}_\alpha(k_3) \\
\mathcal{M}_5 &= g_s^3 \frac{\overline{v}(k_2) \gamma^\delta t^b u(k_1)\ g^{\delta\beta}\ g^{\gamma\rho} (-k_1 - k_2 + k_3 + 2p_1)^\rho}{(k_1 + k_2)^2} \cdot f^{abc} \epsilon^{a*}_\alpha(k_3) \\
&\quad \cdot \frac{\left[g^{\alpha\beta} (-k_3 - k_1 - k_2)^\gamma + g^{\beta\gamma} (2k_1 + 2k_2 - k_3)^\alpha + g^{\gamma\alpha} (-k_1 - k_2 + 2k_3)^\beta \right] t^c_{i_2 i_1}}{(k_1 + k_2 - k_3)^2} \\
\mathcal{M}_6 &= -ig_s^3 \frac{\overline{v}(k_2) t^b \gamma^\beta u(k_1)\ g^{\beta\alpha}}{(k_1 + k_2)^2} \epsilon^{*a}_\alpha(k_3)\ \{t^a, t^b\}_{i_2 i_1}
\end{aligned}
\tag{6.3}
$$

After squaring the amplitudes, the Dirac traces and the polarization sums are evaluated with eq. A.7 and eq. A.13 for massless fermions. Since there is only one external gluon, the Ward identity ensures that we can use the unphysical polarization sum eq. A.7. The result is lenghty and therefore only given in the appendix, eq. C.1.
We set the emitted gluon collinear to the incoming quark with momentum k_2. The pole arising

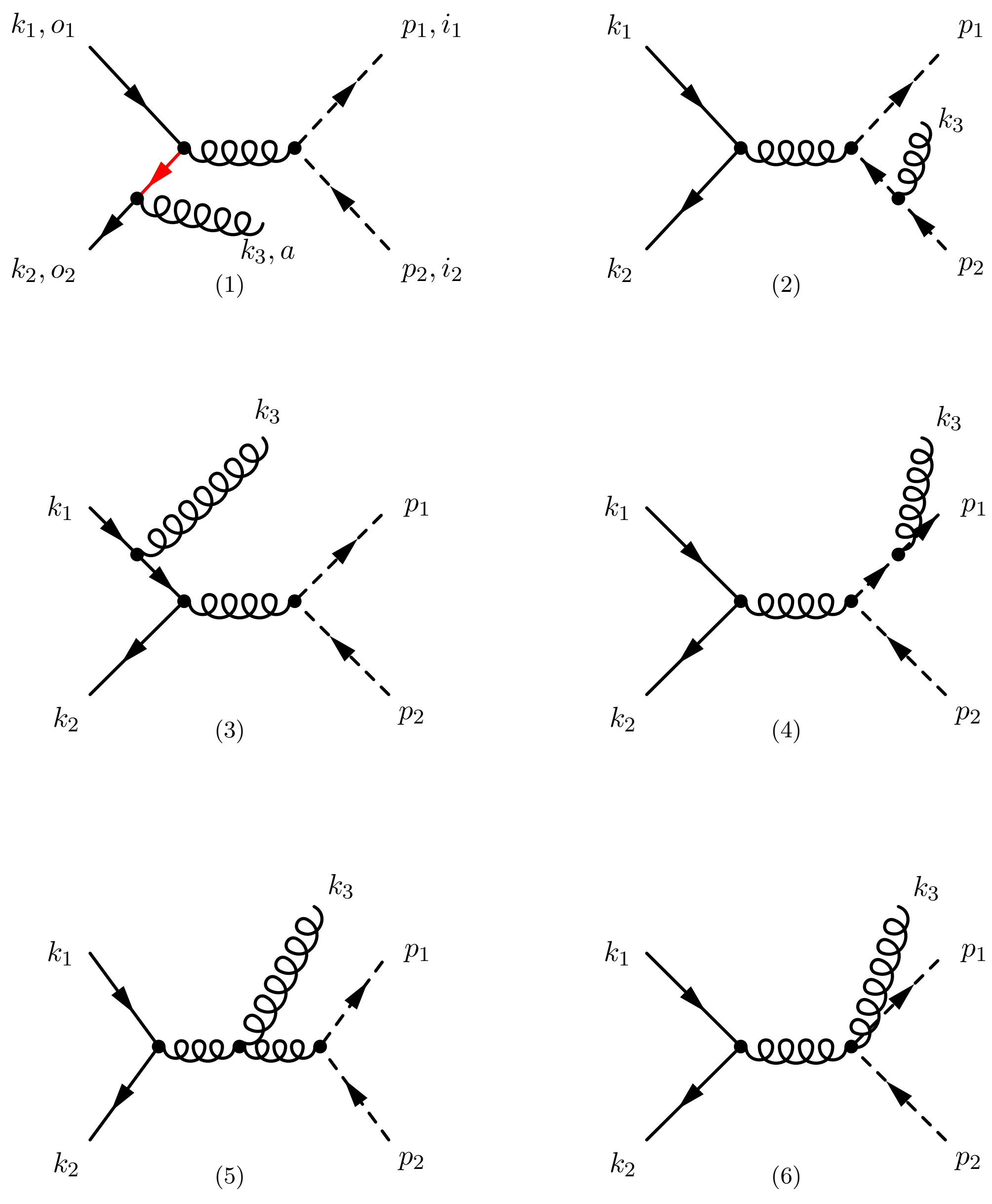


Figure 6.1: Feynman diagrams for the process $q\overline{q} \to G\widetilde{t}\overline{\widetilde{t}}$. $k_1 \ldots k_3$ and p_1, p_2 denote the momenta of the particles. In this calculation, p_2 is always substituted by the other four momenta via momentum conservation. The numbering corresponds to that of the amplitudes.

from this momentum configuration appears in the squared amplitude as indicated (eq. 6.2). Based on the squared amplitude (eq. C.1) we will now give the detailed prescription how to perform the factorization:

- To simplify the expression, the pole leading to the collinear divergence is extracted by multpliying the full expression eq. C.1 with $(1-z)(k_2-k_3)^2 = (1-z)t_1$.

- Since in these remaining terms no further pole $1/t_1$ appears, we can set $k_2 \cdot k_3 \rightarrow 0$.

- Now we implement the collinear limit for the gluon and the quarks by expressing the momentum of the emitted gluon in terms of the radiating quark momentum with

$$k_3^\mu \rightarrow z k_2^\mu + \delta k_\perp^\mu - \delta^2 \frac{(k_\perp)^2 \, n^\mu}{2z \; k_2 \cdot n} \tag{6.4}$$

  For $z = 0$ the gluon is infinitely soft while for $z = 1$ it carries the total momentum of the incoming quark. Both $k_\perp$ and n are needed to construct the collinear direction and guarantee momentum conservation. They are chosen as in eq. 2.5. A factor δ is introduced for each $k_\perp$ so that the expression can later be expanded in δ around 0. This approach ensures that the order in $k_\perp$ of each term is identified correctly.

- The squared amplitude (eq. C.1) with the substitution eq. 6.4 is expanded in δ around 0 up to $\mathcal{O}(\delta^0)$. We have to check if terms depending on δ^{-1} or δ^{-2} appear. This is not the case in this example but will be of importance later (section 6.2).

- The remaining expression depends on k_1, k_2, p_1 and z

$$|\mathcal{M}|^2_{q\overline{q}\rightarrow G\widetilde{t}\widetilde{\overline{t}}} \overset{\langle k_2 || k_3 \rangle}{=} \frac{\alpha_s^3 \pi^3}{t_1(1-z)} \Bigg(-\left(m^2 k_1\cdot k_2 - 2k_1\cdot p_1 k_2\cdot p_1\right) \times \frac{128\left((z-1)\left(\left(z^2-11z+2\right)k_1\cdot k_2 + 9(z-2)(k_1\cdot p_1 - (z-1)k_2\cdot p_1)\right)\right)}{3\left(z k_1\cdot k_2(k_1\cdot p_1 - (z-1)k_2\cdot p_1)^2\right)} + O\left(\delta^1\right) \Bigg)$$

- To compare this result with the LO result in eq. 5.1, we introduce kinetic variables defined in eq. A.2. However, since the particle with momentum $k_2' = (1-z)\cdot k_2$ enters the LO cross section, we need to redefine the Mandelstam variables according to fig. 6.2

$$k_1\cdot k_2 = \frac{S}{2(1-z)} \qquad k_1\cdot p_1 = -\frac{T}{2} \qquad k_2\cdot p_1 = -\frac{U}{2(1-z)} \tag{6.5}$$

  The relation $S+T+U=0$ can still be used after this redefinition[1].

[1] This relation is always correct after taking the collinear limit. Had we used this relation before the expansion around δ, we would have neglected that the particle with momentum p' is offshell implying a dependence of $S+T+U \propto \delta, \delta^2 \neq 0$. This relation will be derived in eq. 6.13.

After going through all these steps we finally obtain

$$\frac{1}{8}\sum_{\text{pols}}|\mathcal{M}|^2_{q\bar{q}\to G\,\tilde{t}\bar{\tilde{t}}} \stackrel{\langle k_2||k_3\rangle}{=} \alpha_s\pi \underbrace{\frac{1}{(1-z)(k_2-k_3)^2}}_{\text{pole}} \underbrace{256\alpha_s^2\pi^2\frac{(T+U)m^2+T\cdot U}{(T+U)^2}}_{|\mathcal{M}|^2_{q\bar{q}\to\tilde{t}\bar{\tilde{t}}}} \cdot \underbrace{C_2(r)\frac{(z-2)z+2}{z}}_{P_{Gq}(z)}$$

$$= \alpha_s\pi\frac{1}{(1-z)t_1}|\mathcal{M}|^2_{q\bar{q}\to\tilde{t}\bar{\tilde{t}}}\cdot P_{Gq}(z) \qquad (6.6)$$

with $C_2(r) = 8/6$ and the pole $1/((1-z)t_1) = z/((z-1)\,k_\perp^2)$. We average over the collinear gluon and thus muliply with a factor of $1/8$. $P_{Gq}(z)$ is the gluon-quark splitting function derived in 4.27. $|\mathcal{M}|^2_{q\bar{q}\to\tilde{t}\bar{\tilde{t}}}$ is the LO squared amplitude which is derived in section 5.1.

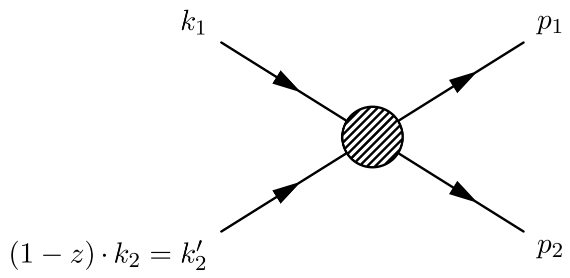


Figure 6.2: Redefinition of Mandelstam variables. After emitting a collinear gluon, the remaining quark carries the momentum fraction $(1-z)\cdot k_2$ which enters the definition of kinematical variables.

6.1.2 Mass Factorization in DRED

In the last section, mass factorization was shown for the process $q\bar{q}\to G\,\tilde{t}\bar{\tilde{t}}$ in 4 dimensions. Now the same process will be calculated in the DRED scheme and in the end we will see that the process factorizes as expected. A subtlety here is the treatment of the ϵ scalar in the NLO process.

This process is a good example to learn how to factorize in DRED. The approach is the following: First the amplitudes for the $q\bar{q}\to g\,\tilde{t}\bar{\tilde{t}}$ process in D dimensions with D-dimensional momenta and fields are calculated. The desired result is a product of the LO result, the D-dependent splitting function for the gluon-quark splitting and the pole.

In the second part of this section we calculate the amplitudes for $q\bar{q}\to\phi\,\tilde{t}\bar{\tilde{t}}$ and show that they factorize separately, as if the ϵ scalar was a parton of its own.

Factorization of the Squared Amplitude of $qq \to g\,\widetilde{t}\,\overline{\widetilde{t}}$

The Feynman diagrams and the amplitudes are the same as in fig. 6.1 and eq. 6.3 for D dimensions with D-dimensional momenta and polarization vectors. The calculation is straightforward and can be performed as in the last section, the only difference being that the momenta, the metric tensor and polarization vectors are D-dimensional.
We simplify the calculation and consider only the squared amplitudes which include the pole $1/t_1$. We therefore take the following matrix elements into account[2]:

$$|\mathcal{M}|^2_{\text{contr.}} = \mathcal{M}_1\mathcal{M}_1^* + 2\mathcal{M}_1\mathcal{M}_2^* + 2\mathcal{M}_1\mathcal{M}_3^* + 2\mathcal{M}_1\mathcal{M}_4^* + 2\mathcal{M}_1\mathcal{M}_5^* + 2\mathcal{M}_1\mathcal{M}_6^*$$

All other combinations such as $\mathcal{M}_2\mathcal{M}_3^*$ do not contain a pole $1/t_1$. We use the unphysical polarization sum (eq. A.7). The squared amplitude $|\mathcal{M}|^2_{\text{contr.}}$ is shown in eq. C.2.
Now the collinear limit $\langle k_2|\,|k_3\rangle$ is taken as described in section 6.1.1. We obtain for the result after averaging over the polarization states and the possible colors for the emitted gluon

$$\begin{aligned}
\frac{1}{8}\sum_{\text{pols}}|\mathcal{M}|^2_{qq\to g\widetilde{t}\,\overline{\widetilde{t}}} &\overset{\langle k_2||k_3\rangle}{=} \alpha_s^3\pi^3\frac{1}{t_1(1-z)}\frac{256\left((T+U)m^2+TU\right)}{(T+U)^2}\cdot\frac{C_2(r)\cdot(z((D-2)z-4)+4)}{2z} \\
&= \alpha_s^3\pi^3\frac{256\left((T+U)m^2+TU\right)}{(T+U)^2}\cdot P(z)_{gq}\cdot\frac{1}{t_1(1-z)} \\
&= \alpha_s\pi\cdot\frac{1}{t_1(1-z)}|\mathcal{M}|^2_{q\overline{q}\to\widetilde{t}\,\overline{\widetilde{t}}}\cdot P(z)_{gq} \qquad (6.7)
\end{aligned}$$

with the splitting function P_{gq} for the gluon-quark splitting as in eq. 4.27 and the LO amplitude in eq. 5.1. This result is equal to the result in DREG. Nevertheless, in DRED the ϵ scalar still contributes.

Factorization of the Squared Amplitude $qq \to \phi\,\widetilde{t}\,\overline{\widetilde{t}}$

The Feynman diagrams including the ϵ scalars are shown in fig. 6.3. The main difference to the previous calculations concerns the polarization sum and the Dirac algebra. For example, the Dirac trace

$$\mathbf{Tr}\left[\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\right] = 4\left(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho}\right)$$

for $i, j \in 4-D$ and $\nu, \sigma \in D$ reduces to

$$\mathbf{Tr}\left[\hat{\gamma}^i\widetilde{\gamma}^\nu\hat{\gamma}^j\widetilde{\gamma}^\sigma\right] = -4\left(\hat{g}^{ij}\widetilde{g}^{\nu\sigma}\right)$$

[2] Note that here $\mathcal{M}_i\mathcal{M}_j^* = \mathcal{M}_j\mathcal{M}_i^*$.

The individual amplitudes for the graphs in fig. 6.3 are

$$\begin{aligned}
\mathcal{M}_1 &= ig_s^3 \frac{\overline{v}(k_2)\hat{\gamma}^j t^a\,(\not{k}_2 - \not{k}_3)\,\widetilde{\gamma}^\beta t^b u(k_1)\widetilde{g}^{\beta\gamma}\,(-k_1-k_2+k_3+2p_1)^\gamma\, t^b_{i_2 i_1}}{(k_2-k_3)^2(k_1+k_2-k_3)^2}\hat{\epsilon}(k_3)^{*a}_j \\
\mathcal{M}_2 &= -g_s^3 \frac{\overline{v}(k_2)t^b\hat{\gamma}^i u(k_1)\,(-k_3-k_1-k_2)^\alpha\,\hat{g}^{ij} f^{abc}\widetilde{g}^{\alpha\beta}\,(-k_1-k_2+k_3+2p_1)^\beta\, t^c_{i_2 i_1}}{(k_1+k_2)^2(k_1+k_2-k_3)^2}\hat{\epsilon}(k_3)^{*a}_j \\
\mathcal{M}_3 &= ig_s^3 \frac{\overline{v}(k_2)\widetilde{\gamma}^\beta t^b\,(\not{k}_1 - \not{k}_3)\,\hat{\gamma}^j t^a u(k_1) g^{\beta\gamma}\,(-k_1-k_2+k_3+2p_1)^\gamma\, t^b_{i_2 i_1}}{(k_1-k_3)^2(k_1+k_2-k_3)^2}\hat{\epsilon}(k_3)^{*a}_j \\
\mathcal{M}_4 &= ig_s^3 \frac{\overline{v}(k_2)\hat{\gamma}^\beta t^b u(k_1)\hat{g}^{\alpha\beta}}{(k_1+k_2)^2}\{t^a, t^b\}_{i_2 i_1}\hat{\epsilon}(k_3)^{*a}_j
\end{aligned}$$

where the hatted objects such as $\hat{g}^{\mu\nu}$ or $\hat{\epsilon}(k_1)$ are in $(4-D)$ dimensions and the tilded $\widetilde{g}^{\mu\nu}$ in D dimensions. The total result is shown in eq. C.3 with the Mandelstam definition from eq. A.4. After taking the collinear limit as in eq. 6.4 and applying the procedure as in section 6.1.1, the squared amplitude in the collinear limit is

$$\begin{aligned}
\frac{1}{8}\sum_{\text{pols}} |\mathcal{M}|^2_{qq\to\phi\tilde{t}\bar{\tilde{t}}} &\overset{\langle k_2 || k_3\rangle}{=} \alpha_s^3\pi^3 \frac{256\left((T+U)m^2+TU\right)}{(T+U)^2}\frac{1}{t_1(1-z)}\cdot\frac{C_2(r)\cdot z(4-D)}{2} \\
&= \alpha_s^3\pi^3 \frac{256\left((T+U)m^2+TU\right)}{(T+U)^2}\cdot\frac{1}{t_1(1-z)}\cdot P(z)_{\phi q} \\
&= \alpha_s\pi\frac{1}{t_1(1-z)}|\mathcal{M}|^2_{q\overline{q}\to\bar{\tilde{t}}\,\tilde{t}}\cdot P(z)_{\phi q} \qquad (6.8)
\end{aligned}$$

with the splitting function $P_{\phi q}$ as in eq. 4.29.

Adding Both Amplitudes

We see that both processes factorize independently with different splitting functions. However, the same LO squared amplitude appears in both cases. This enables us to add both processes. The splitting functions for the D and $(4-D)$ components then combine to give the 4 dimensional splitting function making this result equal to the unregularized one. This also implies that the D dependence cancels without taking the limit $D \to 4$:

$$\begin{aligned}
\frac{1}{8}\sum_{\text{pols}} |\mathcal{M}|^2_{q\overline{q}\to G\tilde{t}\bar{\tilde{t}},\text{DRED}} &\overset{\langle k_2 || k_3\rangle}{=} \alpha_s^3\pi^3 \frac{256\left((T+U)m^2+TU\right)}{(T+U)^2}\cdot\frac{1}{t_1(1-z)}\cdot(P_{\phi q}+P_{gq}) \\
&= \alpha_s^3\pi^3 \frac{256\left((T+U)m^2+TU\right)}{(T+U)^2}\cdot\frac{1}{t_1(1-z)}\cdot\frac{((z-2)z+2)}{z}C_2(r)
\end{aligned}$$

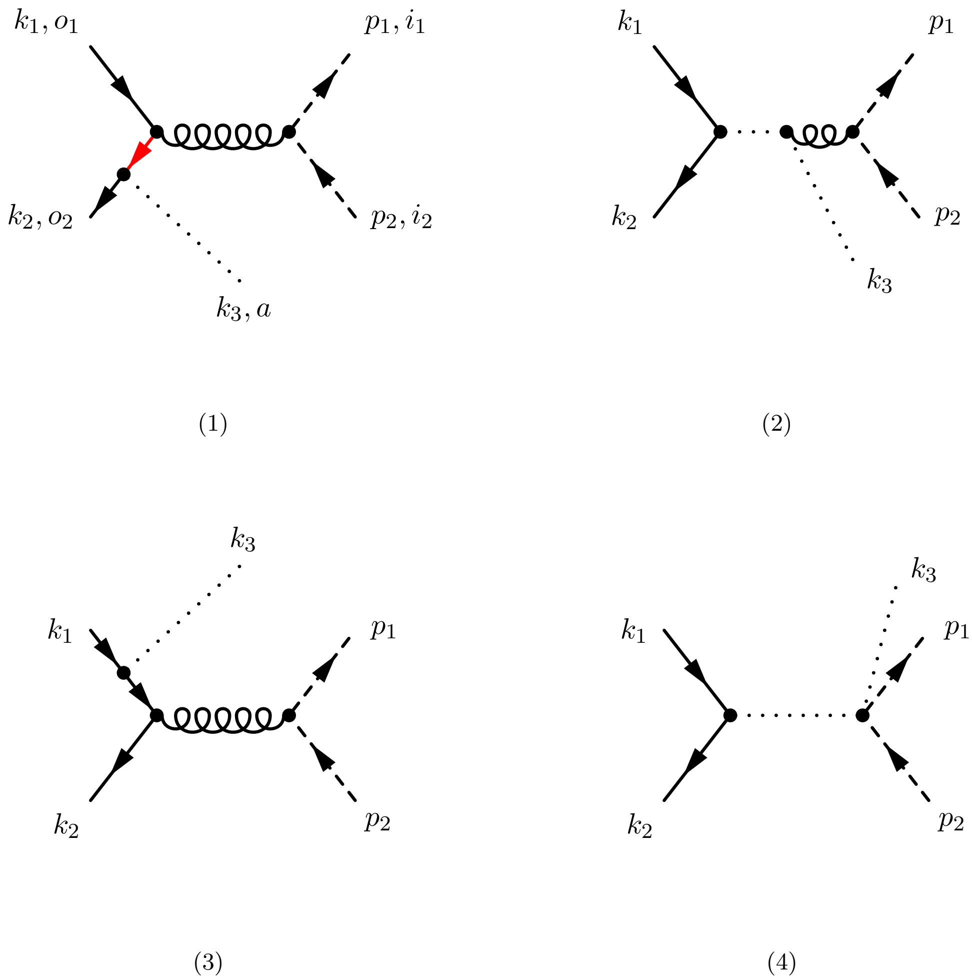


Figure 6.3: Feynman diagrams for the process $q\overline{q} \to \phi\,\widetilde{t}\,\overline{\widetilde{t}}$.

$$
\begin{aligned}
&= \alpha_s \pi \frac{1}{t_1(1-z)} |\mathcal{M}|^2_{q\overline{q}\to\widetilde{t}\,\overline{\widetilde{t}}} \cdot P_{Gq}(z) \qquad (6.9) \\
&= \frac{1}{8} \sum_{\text{pols}} |\mathcal{M}|^2_{q\overline{q}\to\widetilde{t}\,\overline{\widetilde{t}},\,4\text{ dim.}}
\end{aligned}
$$

This calculation also reproduces our previous result in 4 dimensions.

6.2 Stop Production via Gluon Fusion $GG \to G\tilde{t}\bar{\tilde{t}}$

We will now proceed with a more complex calculation with two gluons as incoming particles. The calculation is first performed in 4 dimensions and it will be demonstrated how to treat the double divergences which appear in this example in difference to the previous one. As we have pointed out in section 4.1, non-factorizable terms appear if DRED is applied naively in the context of mass factorization. Using the method outlined in section 6.1.2 one finds that there is a sensible interpretation of these terms which restores the factorization theorem.

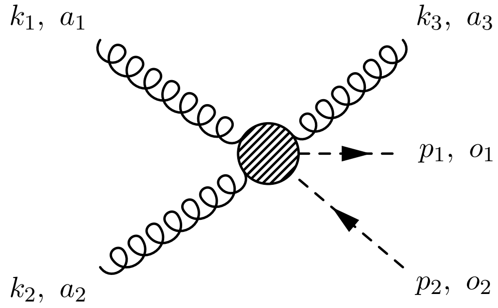


Figure 6.4: Feynman diagrams for the process $GG \to G\tilde{t}\bar{\tilde{t}}$. The momenta are k_i and p_i, the color indices are a_i and o_i. The 25 graphs contributing to this process are shown in fig. D.3.

6.2.1 Mass Factorization in 4 Dimensions

We consider the process $GG \to G\tilde{t}\bar{\tilde{t}}$ as sketched in fig. 6.4. The calculation is performed in Feynman gauge with the unphysical polarization sum (eq. A.7). The physical process is obtained by subtracting the corresponding squared amplitudes involving external BRST ghosts (fig. 6.5). Consequently we cannot use identities such as $\epsilon(k_i)\cdot k_i = 0$. The 25 Feynman amplitudes for fig. D.3 are given in appendix D.2.
The squared amplitude is

$$|\mathcal{M}|^2_{GG\to G\tilde{t}\bar{\tilde{t}},\ \text{phys.}} = \left|\sum_{i=1}^{25}\mathcal{M}_{i,\text{unphys.}}\right|^2 - \left|\sum_{i=1}^{6}\mathcal{M}_{i,\ \text{ghosts}}\right|^2 \qquad (6.10)$$

We perform the collinear limit for the gluon and ghost processes separately. After taking the collinear limit, all squared amplitudes are added together analogous to eq. 6.10 and the physical process is obtained.

The explicit result of the squared amplitude $GG \to G\tilde{t}\bar{\tilde{t}}$ is lengthy. Therefore we elucidate with the help of a short example term taken from the full squared amplitude, what type of

expressions arise and how to manage them.
The difference and difficulty in this calculation in comparison to the process calculated in section 6.1 is that not only poles of order $1/(k_2 - k_3)^2 = 1/t_1$ appear, but also terms of order $1/(k_2 - k_3)^4 = 1/t_1^2$. One of the divergences is treated as in the last chapter but the remaining one has to be dealt with separately. The key to remove it is to average over the azimuthal direction of $k_\perp$ which is unobservable in the collinear limit.
We set the gluon with momentum k_3 collinear to the one with momentum k_2 by using the parametrization in eq. 6.4. The collinear divergence originates from the diagrams 5, 17, 18 and 19 in fig. D.3 where the propagator goes onshell as $k_\perp \to 0$.
Our approach is the following: First the squared amplitudes are separated into parts depending to $1/t_1$ and $1/t_1^2$. The other terms are ignored since they do not give a contribution to the result. The terms are very lengthy and are not shown. Before the collinear limit is performed, both terms are combined.
To show how the amplitude is factorized, we pick a convenient term from the full result containing a $1/t_1^2$ divergence and go through the calculation step by step. It is demonstrated how one of the two divergences vanishes after performing the azimuthal average. For this discussion we express the pole $1/t_1^2$ as $1/(k_2 \cdot k_3)^2$, omitting all inessential factors for simplicity.
We investigate

$$f\,(k_i) \quad = \quad \frac{(k_1 \cdot k_4\ k_2 \cdot k_3 - k_1 \cdot k_3\ k_2 \cdot k_4 + k_1 \cdot k_2 k_3 \cdot k_4)^2}{(k_2 \cdot k_3)^2}$$

One pole is extracted analogous to eq. 6.2. Therefore the term is multiplied with $k_2 \cdot k_3$ and expanded to make the structure more obvious

$$\begin{aligned} f\,(k_i) \cdot (k_2 \cdot k_3) \quad = \quad & k_2 \cdot k_3 (k_1 \cdot k_4)^2 - 2k_1 \cdot k_3\, k_2 \cdot k_4\, k_1 \cdot k_4 + 2k_1 \cdot k_2\, k_3 \cdot k_4 k_1 \cdot k_4 \\ & + \frac{(k_1 \cdot k_3)^2 (k_2 \cdot k_4)^2}{k_2 \cdot k_3} + \frac{(k_1 \cdot k_2)^2 (k_3 \cdot k_4)^2}{k_2 \cdot k_3} - \frac{2k_1 \cdot k_2\, k_1 \cdot k_3\, k_2 \cdot k_4\, k_3 \cdot k_4}{k_2 \cdot k_3} \end{aligned}$$

Here terms of order $(k_2 \cdot k_3)^1$,$(k_2 \cdot k_3)^0$ and $(k_2 \cdot k_3)^{-1}$ appear. In order to explain the vanishing of these poles the collinear limit is parametrized using eq. 6.4. Since poles of the structure $(k_\perp^\mu k_\perp^\nu)/k_\perp^4$ appear, introducing the variable δ helps to determine the order of the singularity. Here the gluons with k_2 and k_3 are onshell and we can conserve momentum by keeping the gluon with momentum $k_2 - k_3$ offshell with $(k_2 - k_3)^2 \propto k_\perp^2$. Now we can introduce the modified

Mandelstam variables as in eq. 6.5

$$
\begin{aligned}
&f(k_i)\cdot(k_2\cdot k_3) = \\
&= -\frac{T^2 k_\perp^2\delta^2}{8z} - \frac{TU\left(-\frac{k_1\cdot n k_\perp^2\delta^2}{2zk_2\cdot n} + k_1\cdot k_\perp\delta + \frac{Sz}{2(1-z)}\right)}{2(1-z)} - \frac{S\,T\left(-\frac{k_4\cdot n k_\perp^2\delta^2}{2zk_2\cdot n} + k_4\cdot k_\perp\delta - \frac{Uz}{2(1-z)}\right)}{2(1-z)} \\
&\quad - \frac{U^2 z\left(-\frac{k_1\cdot n k_\perp^2\delta^2}{2zk_2\cdot n} + k_1\cdot k_\perp\delta + \frac{Sz}{2(1-z)}\right)^2}{2(1-z)^2 k_\perp^2\delta^2} - \frac{S^2 z\left(-\frac{k_4\cdot n k_\perp^2\delta^2}{2zk_2\cdot n} + k_4\cdot k_\perp\delta - \frac{Uz}{2(1-z)}\right)^2}{2(1-z)^2 k_\perp^2\delta^2} \\
&\quad - \frac{SUz\left(-\frac{k_1\cdot n k_\perp^2\delta^2}{2zk_2\cdot n} + k_1\cdot k_\perp\delta + \frac{Sz}{2(1-z)}\right)\left(-\frac{k_4\cdot n k_\perp^2\delta^2}{2zk_2\cdot n} + k_4\cdot k_\perp\delta - \frac{Uz}{2(1-z)}\right)}{(1-z)^2 k_\perp^2\delta^2}
\end{aligned} \tag{6.11}
$$

As we see, there are still remaining terms proportional to $1/k_\perp^2$ which have to cancel. So far, we have not taken advantage of the redundancy in the Mandelstam variables. For the kinematic variables defined as in eq. A.2 the equation

$$S + T + U \;=\; 0 \tag{6.12}$$

holds. However, this relation is only true if all involved particles are onshell. Since the gluon with momentum $k_2 - k_3$ entering the partonic process $gg \to \tilde{t}\,\bar{\tilde{t}}$ is slightly offshell with $k_\perp^2$ we expect that this relation is also only true up to an order of $k_\perp^2$. For further simplification of eq. 6.11 we need a new relation analogous to eq. 6.12.
After a short calculation, $(S+T+U)$ can be expressed in terms of $\delta^2 k_\perp^2$ and $\delta\, k_\perp$. We find

$$
\begin{aligned}
&S+T+U = \\
&\quad = \left((1-z)k_2 - k_4 + k_1\right)^2 - m^2 \\
&\quad \overset{1}{=} \left(\underbrace{k_1 + k_2 - k_4 - k_3}_{=k_5} + k_\perp\delta - k_\perp^2\delta^2\frac{n}{2zk_2\cdot n}\right)^2 - m^2 \\
&\quad = k_\perp^2 + 2\delta\left(k_1k_\perp - k_4k_\perp\right) - 2\delta k_3k_\perp - \frac{k_\perp^2}{zk_2\cdot n}\left(k_1\cdot n + k_2\cdot n + k_2\cdot n - k_3\cdot n - k_4\cdot n\right) \\
&\quad = 2\delta\left(k_1k_\perp - k_4k_\perp\right) - \frac{k_\perp^2\delta^2}{z}\left(\frac{k_1\cdot n}{k2\cdot n} + 1 - \frac{k_4\cdot n}{k_2\cdot n}\right)
\end{aligned}
$$

We have used that $n^2 = n\cdot k_\perp = k_2\cdot k_\perp = 0$ and $k_5 = k_1 + k_2 - k_3 - k_4$. We now substitute

$$S \;=\; -T - U + 2\delta\left(k_1k_\perp - k_4k_\perp\right) - \frac{k_\perp^2\delta^2}{z}\left(\frac{k_1\cdot n}{k_2\cdot n} + 1 - \frac{k_4\cdot n}{k_2\cdot n}\right) \tag{6.13}$$

[1]We replace zk_2 with 6.4 solved for $zk_2 = k_3 - k_\perp\delta + k_\perp^2\delta^2 n/(2zk_2\cdot n)$

in eq. 6.11. Terms of order $k_\perp^0$, $k_\perp^{-1}$ and $k_\perp^{-2}$ remain.
The part proportional to $(k_1 k_\perp - k_4 k_\perp)/(\delta k_\perp^2)$ vanishes as $k_\perp$ is integrated over the azimuthal angle ϕ from 0 to 2π. This can be inferred from the following argument: The phase space integral over terms of this type is

$$\int \frac{d\,\phi}{(2\pi)} \frac{k_\perp^\mu}{k_\perp^2} \;=\; a\,k_2^\mu + b\,n^\mu$$

The coefficients a and b are determined by contracting the term with $k_{2\,\mu}$ and n_μ

$$\begin{aligned} k_{2\,\mu}: \quad \int \frac{d\,\phi}{(2\pi)} \frac{k_\perp \cdot k_2}{k_\perp^2} &= a\,k_2 \cdot k_2 + b\,n \cdot k_2 \\ n_\mu: \quad \int \frac{d\,\phi}{(2\pi)} \frac{k_\perp \cdot n}{k_\perp^2} &= a\,k_2 \cdot n + b\,n \cdot n \end{aligned}$$

Since the initial gluon is onshell and massless, we can use $k_2^2 = 0$ and $k_\perp \cdot k_2$ is zero per definition. So the coefficients a and b are both 0 and we can set terms depending on $k_\perp^\mu / k_\perp^2$ to zero

$$\int \frac{d\phi}{2\pi} \frac{k_\perp^\mu}{k_\perp^2} \;=\; 0 \tag{6.14}$$

Now the total expression (eq. 6.11) is expanded in δ around 0 up to constant terms

$$\begin{aligned} f(k_i, z, k_\perp)\cdot(k_2 \cdot k_3) \;=\; & -\frac{z(k_1 \cdot k_\perp)^2 U^2}{2(z-1)^2 k_\perp^2} - \frac{z(k_4 \cdot k_\perp)^2 U^2}{2(z-1)^2 k_\perp^2} + \frac{z k_1 \cdot k_\perp k_4 \cdot k_\perp U^2}{(z-1)^2 k_\perp^2} \\ & -\frac{T z (k_4 \cdot k_\perp)^2 U}{(z-1)^2 k_\perp^2} + \frac{T z k_1 \cdot k_\perp k_4 \cdot k_\perp U}{(z-1)^2 k_\perp^2} - \frac{T^2 z (k_4 \cdot k_\perp)^2}{2(z-1)^2 k_\perp^2} \\ & + \mathcal{O}(\delta^1) \end{aligned} \tag{6.15}$$

Higher orders of $k_\perp$ vanish for $k_\perp \to 0$ and are neglected. We see that no terms including δ^{-1} and δ^{-2} remain.
However, there are still terms depending on $k_\perp$ of the form $(k \cdot k_\perp\, p \cdot k_\perp)/(k_\perp)^2$. They do not represent a singularity since both numerator and denominator are of the same order in δ and $k_\perp$. Thus these terms contribute to the result with a constant term. The idea is that the transverse direction $k_\perp$ is unobservable in the collinear limit and has to be azimuthally averaged over in the phase space integral. The average over the direction of $k_\perp$ is

$$\int \frac{d\,\phi}{2\pi} \frac{k_\perp^\mu k_\perp^\nu}{k_\perp^2} = a\,g^{\mu\nu} + b\,k_2^\mu k_2^\nu + c\,k_2^\mu n^\nu + d\,k_2^\nu n^\mu + e\,n^\mu n^\nu \tag{6.16}$$

the coefficients a, b, c, d and e are determined by multiplying independent terms which do not depent on $k_\perp$.

From contracting with $g^{\mu\nu}$ we find (for D dimensions)

$$\begin{aligned}
\int \frac{d\phi}{2\pi}\frac{k_\perp^2}{k_\perp^2} &= aD + \underbrace{bk_2^2}_{=0} + c\,k_2\cdot n + d\,k_2\cdot n + \underbrace{en^2}_{=0} \\
\Rightarrow 1 &= a\;D + (c+d)\;k_2\cdot n \qquad (6.17a)
\end{aligned}$$

Contracting with $k_2^\mu n^\nu$:

$$\begin{aligned}
\int \frac{d\phi}{2\pi}\underbrace{\frac{k_\perp\cdot n\,k_2\cdot k_\perp}{k_\perp^2}}_{=0} &= a\,k_2\cdot n + \underbrace{b\,k_2^2 k_2\cdot n}_{=0} + c\,(k_2\cdot n)^2 + \underbrace{d\,k_2\cdot n\,k_2\cdot k_2}_{=0} + \underbrace{e\,n^2\,k_2\cdot n}_{=0} \\
0 &= a\,k_2\cdot n + d\,(k_2\cdot n)^2 \\
\Rightarrow d &= -\frac{a}{k_2\cdot n} \qquad (6.17b)
\end{aligned}$$

Contracting with $k_2^\nu n^\mu$:

$$\begin{aligned}
\int \frac{d\phi}{2\pi}\underbrace{\frac{k_\perp\cdot n\,k_2\cdot k_\perp}{k_\perp^2}}_{=0} &= ak_2\cdot n + \underbrace{b\,k_2^2\,k_2\cdot n}_{=0} + \underbrace{c\,k_2\cdot n\,n\cdot n}_{=0} + d\,(k_2\cdot n)^2 + \underbrace{e\,n^2\,k_2\cdot n}_{=0} \\
0 &= k_2\cdot n\,a + c\,(k_2\cdot n)^2 \\
\Rightarrow c &= -\frac{a}{k_2\cdot n} \qquad (6.17c)
\end{aligned}$$

With eq. 6.17a, eq. 6.17b and eq. 6.17c we obtain

$$\begin{aligned}
1 &= D\;a + (c+d)(k_2\cdot n) \\
1 &= D\;a - \left(\frac{2a}{k_2\cdot n}\right) k_2\cdot n \\
\Rightarrow a &= \frac{1}{D-2} \\
\Rightarrow c = d &= -\frac{1}{(D-2)k_2\cdot n}
\end{aligned}$$

The other coefficients such as b and e are zero, which can be seen after multpliying with $n^\mu n^\nu$ and $k_2^\mu k_2^\nu$. We finally obtain the formula for performing the azimuthal average

$$\int \frac{d\phi}{2\pi}\frac{k_\perp^\mu k_\perp^\nu}{k_\perp^2} = \frac{1}{D-2}\left(g^{\mu\nu} - \frac{k_2^\mu n^\nu + k_2^\nu n^\mu}{k_2\cdot n}\right) \qquad (6.18)$$

It will be relevant for the discussion of the factorization problem that the averaging procedure introduces a new D dependence in DRED.
It is now applied to eq. 6.15 (for $D = 4$) and we obtain

$$f(T,U)\cdot(k_2\cdot k_3) \quad = \quad -\frac{(T+U)\left((T+U)m^2+TU\right)z}{4(z-1)^2} + \mathcal{O}(\delta^1)$$

We learn from these considerations that divergences which appear to be of order $1/k_\perp^4$ are, after carefully expanding them in δ, actually of the form $k_\perp^\mu k_\perp^\nu/k_\perp^4$ and reduce to $1/k_\perp^2$ after performing the average.
Repeating this procedure for the entire result, we obtain for the squared amplitude

$$\begin{aligned}
|\mathcal{M}|^2_{GG\to G\tilde{t}\bar{\tilde{t}},\text{unphys.}} \overset{\langle k_2|\,|k_3\rangle}{=} \; & \alpha^3\pi^3\cdot\frac{1}{T^2U^2(T+U)^2}\cdot\frac{1}{(1-z)t_1}\times \\
\Big(& -64z^2\left(1024\,T^4m^4+1024U^4m^4+1792\,T\;U^3\,m^4+1536\,T^2U^2+1792\,T^3\,Um^4\right. \\
& +1024\,T\,U^4m^2+768\,T^2\,U^3\,m^2+768\,T^3\,U^2m^2+1024\,T^4Um^2+539\,T^2U^4 \\
& \left.-182\,T^3\;U^3\;+539\,T^4U^2\right) \\
& +32z\left(2048\,T^4m^4+2048U^4m^4+3584\,T\;U^3\,m^4+3072\,T^2U^2m^4+3584\,T^3\,Um^4\right. \\
& +2048\,T\,U^4m^2+1536\,T^2\,U^3\,m^2+1536\,T^3\,U^2m^2+2048\,T^4Um^2 \\
& \left.+1051\,T^2U^4-310\,T^3\;U^3\;+1051\,T^4U^2\right)-288\left(512\,T^4m^4+512U^4m^4+896\,T\;U^3\,m^4\right. \\
& +768\,T^2U^2m^4+896\,T^3\,Um^4+255\,T^2U^4-62\,T^3\;U^3\;+255\,T^4U^2 \\
& \left.+512\,T\,U^4m^2+384\,T^2\,U^3\,m^2+384\,T^3\,U^2m^2+512\,T^4Um^2\right) \\
& -\frac{128}{(z-1)}\left(640\,T^4m^4+640U^4m^4+1120\,T\;\;U^3\;m^4+960T^2U^2m^4+1120\,T^3\,Um^4\right. \\
& +640\,T\,U^4m^2+480\,T^2\,U^3\,m^2+480\,T^3\,U^2m^2+640\,T^4Um^2+311\,T^2U^4 \\
& \left.-62\,T^3\;U^3\;+311\,T^4U^2\right) \\
& +\frac{64}{z}\left(1024\,T^4m^4+1024U^4m^4+1792\,T\;U^3\,m^4+1536\,T^2U^2m^4+1792\,T^3\,Um^4\right. \\
& +1024\,T\,U^4m^2+768\,T^2\,U^3\,m^2+768\,T^3\,U^2m^2+1024\,T^4Um^2+539\,T^2U^4 \\
& \left.-182\,T^3\;U^3\;+539\,T^4U^2\right)\Big)
\end{aligned} \tag{6.19}$$

The Feynman diagrams for the processes including external BRST ghosts are sketched in fig. 6.5. All 36 graphs are plotted in the appendix in figs. D.1 and D.2. Since the amplitudes are lengthy, they are also given in the appendix (section D.2).
The collinear limit is taken as sketched before and the squared amplitudes for the ghost graphs

are

$$|\mathcal{M}|^2_{\eta\overline{\eta}\to G\,\widetilde{t}\,\overline{\widetilde{t}}} \overset{\langle k_2||k_3\rangle}{=} -\frac{\alpha_s^3\pi^3}{(1-z)t_1}\frac{288(T-U)^2(z-3)}{(T+U)^2 z}+\mathcal{O}(\delta^1)$$

$$|\mathcal{M}|^2_{\overline{\eta}\eta\to G\,\widetilde{t}\,\overline{\widetilde{t}}} \overset{\langle k_2||k_3\rangle}{=} -\frac{\alpha_s^3\pi^3}{(1-z)t_1}\frac{288(T-U)^2(z-1)(z+3)}{(T+U)^2 z}+\mathcal{O}(\delta^1)$$

$$|\mathcal{M}|^2_{\eta G\to \eta\,\widetilde{t}\,\overline{\widetilde{t}}} \overset{\langle k_2||k_3\rangle}{=} -\frac{\alpha_s^3\pi^3}{(1-z)t_1}\frac{288(T-U)^2 z(3z-1)}{(T+U)^2}+\mathcal{O}(\delta^1)$$

$$|\mathcal{M}|^2_{\overline{\eta} G\to \overline{\eta}\,\widetilde{t}\,\overline{\widetilde{t}}} \overset{\langle k_2||k_3\rangle}{=} -\frac{\alpha_s^3\pi^3}{(1-z)t_1}\frac{288(T-U)^2(z-1)(3z+1)^{2}}{(T+U)^2}+\mathcal{O}(\delta^1)$$

$$\begin{aligned}|\mathcal{M}|^2_{G\eta\to \eta\,\widetilde{t}\,\overline{\widetilde{t}}} \overset{\langle k_2||k_3\rangle}{=}\; & \alpha_s^3\pi^3\frac{1}{T^2U^2(T+U)^2(z-1)}32z\left(-64(T+U)^2\left(4T^2-UT+4U^2\right)m^4\right.\\ & -64\,T\,U(T+U)\left(4T^2-UT+4U^2\right)m^2+\,T^2U^2\left(9(T-U)^2z\right.\\ & \left.\left.-7\left(17T^2-2UT+17U^2\right)\right)\right)+\mathcal{O}(\delta^1)\end{aligned}$$

$$\begin{aligned}|\mathcal{M}|^2_{G\overline{\eta}\to \overline{\eta}\,\widetilde{t}\,\overline{\widetilde{t}}} \overset{\langle k_2||k_3\rangle}{=}\; & \frac{\alpha_s^3\pi^3}{(1-z)t_1}\frac{1}{T^2U^2(T+U)^2(z-1)}\left(288\,T^2(T-U)^2U^2\right.\\ & -32\left(64(T+U)^2\left(4T^2-UT+4U^2\right)m^4\right.\\ & +64\,T\,U(T+U)\left(4T^2-UT+4U^2\right)m^2\\ & \left.\left.+7\,T^2U^2\left(17T^2-2UT+17U^2\right)\right)z\right)+\mathcal{O}(\delta^1)\end{aligned}$$

All amplitudes including BRST ghosts are subtracted from the unphysical squared amplitude in eq. 6.19

$$\begin{aligned}\frac{1}{16}|\mathcal{M}|^2_{G\,G\to G\,\widetilde{t}\,\overline{\widetilde{t}},\text{phys.}} = \frac{1}{16}\Big(&|\mathcal{M}|^2_{G\,G\to G\,\widetilde{t}\,\overline{\widetilde{t}},unphysical} - |\mathcal{M}|^2_{\overline{\eta}G\to\overline{\eta}\,\widetilde{t}\,\overline{\widetilde{t}}} - |\mathcal{M}|^2_{\overline{\eta}\eta\to G\,\widetilde{t}\,\overline{\widetilde{t}}}\\ & -|\mathcal{M}|^2_{\eta\overline{\eta}\to G\,\widetilde{t}\,\overline{\widetilde{t}}} - |\mathcal{M}|^2_{\eta G\to\eta\,\widetilde{t}\,\overline{\widetilde{t}}} - |\mathcal{M}|^2_{G\overline{\eta}\to\overline{\eta}\,\widetilde{t}\,\overline{\widetilde{t}}} - |\mathcal{M}|^2_{G\eta\to\eta\,\widetilde{t}\,\overline{\widetilde{t}}}\Big)\end{aligned}$$

$$\begin{aligned}\overset{\langle k_2||k_3\rangle}{=} & \frac{\alpha_s^3\pi^3}{(1-z)t_1}\frac{256\left(4T^2-UT+4U^2\right)\left(2(T+U)^2m^4+2\,T\,U(T+U)m^2+\,T^2U^2\right)}{3\,T^2U^2(T+U)^2}\times\\ & \frac{2((z-1)z+1)^2\,N}{(1-z)z}\\ & = \pi\,\alpha_s\frac{1}{t_1(1-z)}|\mathcal{M}|^2_{g\;g\to\widetilde{t}\,\overline{\widetilde{t}}}\cdot P_{GG}(z) \qquad (6.20)\end{aligned}$$

with the correct splitting function from eq. 4.30 in 4 dimensions and the LO matrix element as in eq. 5.4. We divide by 8 since we want to average over the collinear gluon. This demonstrates the factorization theorem in 4 dimensions for this particular process. In the next section we will

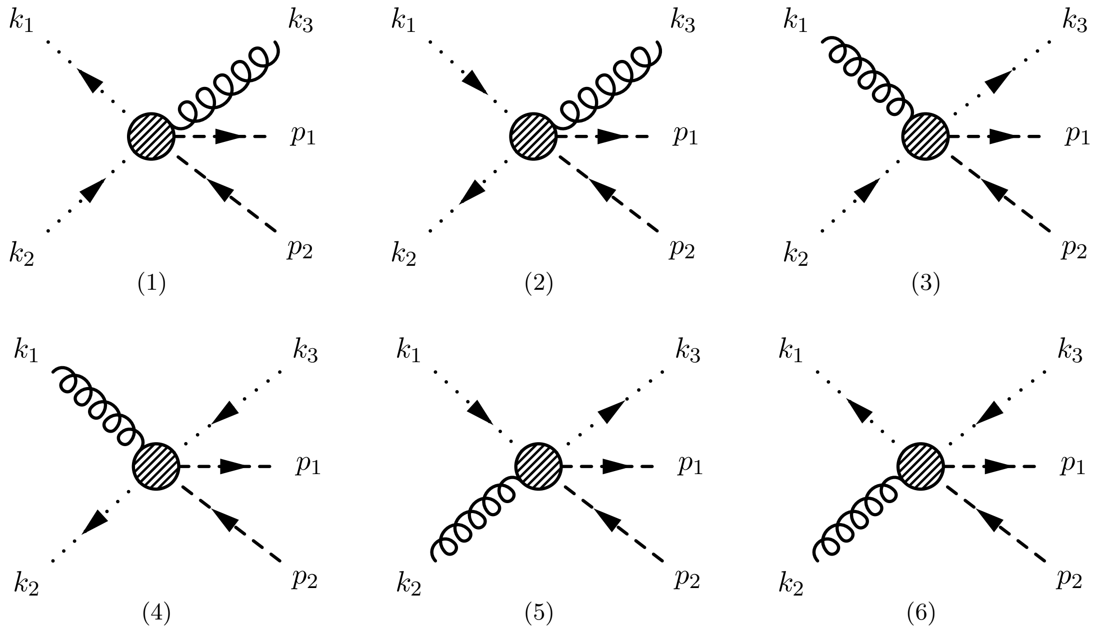


Figure 6.5: Schematic diagrams for the ghost processes corresponding to $GG \to G\tilde{t}\bar{\tilde{t}}$. All 36 graphs can be found in figs. D.1 and D.2.

apply the same techniques to this NLO process in DRED.

6.2.2 Mass Factorization in DRED

After introducing all the important tools in the last sections, we are now ready to perform the factorization in DRED. There are two possibilities for the calculation of this amplitude, the one introduced in the last sections with the ϵ scalar and a second method where the vector fields are kept 4-dimensional. Both lead to the same result.
As discussed in chapter 4 the factorization problem found by Beenakker et al. [1] for the process $G\,G \to G\,t\,\widetilde{t}$ shows up when processes are calculated with the second method. We illustrated in chapter 4 that this treatment leads to non-factorizable terms in the collinear limit. We also found that these terms vanish for $m = 0$ and are absent in DREG.

As discussed in chapter 4 we have two different partons, a gluon g and an ϵ scalar ϕ forming different partonic processes. Thus the following processes have to be considered

$$g\,g \to g\,\widetilde{t}\,\widetilde{\bar{t}} \tag{6.21a}$$
$$g\,\phi \to \phi\,\widetilde{t}\,\widetilde{\bar{t}} \tag{6.21b}$$
$$\phi\,g \to \phi\,\widetilde{t}\,\widetilde{\bar{t}} \tag{6.21c}$$
$$\phi\,\phi \to g\,\widetilde{t}\,\widetilde{\bar{t}} \tag{6.21d}$$

After factorizing these processes separately we also need different splitting functions

$$g\,g \to g \tag{6.22a}$$
$$g\,\phi \to \phi \tag{6.22b}$$
$$\phi\,g \to \phi \tag{6.22c}$$
$$\phi\,\phi \to g \tag{6.22d}$$

which we have already calculated in chapter 4.

Factorization of the D-dimensional Gluon Part (eq. 6.21a)

The Feynman diagrams for the gluonic processes are sketched in figs. 6.4 and D.3. Since the expressions for the Feynman amplitudes are the same as in 4 dimensions, we refer to section D.2.
For the Feynman diagrams including the BRST ghosts we refer to figs. 6.5, D.1 and D.2. Note that there is no coupling of ϵ scalars and BRST ghosts.
Since `FormCalc` calculates the polarization sum in 4 dimensions, we only use it to square the amplitudes and to calculate the color traces. The result then still includes the D-dimensional polarization vectors $\widetilde{\epsilon}(k)$ which are calculated with the unphysical polarization sum in eq. A.7. This leads to a dimensional dependence of the squared amplitude. Due to the length, the result is not given here.
We parametrize the momenta k_2 and k_3 as in eq. 6.4 but with the index μ living in D dimensions. We substitute the scalar products of the momenta with the modified Mandelstam variables from eq. 6.5 and use the relation in eq. 6.13 to eliminate S. This leads to terms depending on

$1/(\delta\, k_\perp)^2$, $1/(\delta k_\perp)^4$, T, U and D. Now the collinear limit is taken by expanding these terms in δ around 0 to the zeroth order

$$
\begin{aligned}
|\mathcal{M}|^2_{g\,g\to g\,\widetilde{t}\,\widetilde{\bar{t}}} \overset{\langle k_2||k_3\rangle}{=} & -\frac{\alpha_s^3\pi^3}{(1-z)t_1}\frac{32}{T^2U^2(T+U)^2(z-1)zk_\perp^2}\times \\
\Bigg(& 256(2D-3)m^2U^2\left(4T^2-UT+4U^2\right)(k_1\cdot k_\perp)^2z^2 \\
& + 256(2D-3)m^2(T+U)^2\left(4T^2-UT+4U^2\right)(k_4\cdot k_\perp)^2z^2 \\
& - 512(2D-3)m^2U(T+U)\left(4T^2-UT+4U^2\right)(k_1\cdot k_\perp)(k_4\cdot k_\perp)z^2 \\
& + \left(512(T+U)^2\left(4T^2-UT+4U^2\right)(z-1)^2\left(z^2+1\right)m^4\right. \\
& + 512TU(T+U)\left(4T^2-UT+4U^2\right)(z-1)^2\left(z^2+1\right)m^2 \\
& + T^2U^2\left(\left(512D((z-1)z+1)^2+z(z((1967-970z)z-2798)+1967)-970\right)T^2\right. \\
& - 2U\left(64D((z-1)z+1)^2+z(z((175-74z)z-334)+175)-74\right)T \\
& \left.\left.+U^2\left(512D((z-1)z+1)^2+z(z((1967-970z)z-2798)+1967)-970\right)\right)\right)k_\perp^2\Bigg) + \mathcal{O}(\delta^1)
\end{aligned}
$$

There are still terms depending on $k_\perp^\mu/k_\perp^2$ and $(k_\perp^\mu k_\perp^\nu)/k_\perp^2$. The former terms do not contribute due to the vanishing phase space integral deduced in eq. 6.14. The latter terms are averaged azimuthally with eq. 6.18 (this is where another dimensional dependence of $1/(D-2)$ enters the calculation). The result simplifies to

$$
\begin{aligned}
|\mathcal{M}|^2_{g\,g\to g\,\widetilde{t}\,\widetilde{\bar{t}},\text{unphys.}} \overset{\langle k_2||k_3\rangle}{=} & -\frac{\alpha_s^3\pi^3}{(1-z)t_1}\frac{32}{(D-2)T^2U^2(T+U)^2(z-1)z}\times \\
\Big(& 256(T+U)^2\left(4T^2-UT+4U^2\right)\left(2D((z-1)z+1)^2+z(z(-4(z-2)z-11)+8)-4\right)m^4 \\
& + 256TU(T+U)\left(4T^2-UT+4U^2\right)\left(2D((z-1)z+1)^2+z(z(-4(z-2)z-11)+8)-4\right)m^2 \\
& + (D-2)T^2U^2\left(\left(512D((z-1)z+1)^2+z(z((1967-970z)z-2798)+1967)-970\right)T^2\right. \\
& - 2U\left(64D((z-1)z+1)^2+z(z((175-74z)z-334)+175)-74\right)T \\
& \left.+U^2\left(512D((z-1)z+1)^2+z(z((1967-970z)z-2798)+1967)-970\right)\right)\Big)
\end{aligned}
\tag{6.23}
$$

There are six different ghost processes (fig. 6.5) each having 6 different topologies (figs. D.1 and D.2)

$$
\begin{array}{lll}
\overline{\eta}\eta \to g\,\widetilde{t}\,\widetilde{\bar{t}}; & \eta\overline{\eta} \to g\,\widetilde{t}\,\widetilde{\bar{t}}; & g\eta \to \eta\,\widetilde{t}\,\widetilde{\bar{t}}; \\
g\,\overline{\eta} \to \overline{\eta}\,\widetilde{t}\,\widetilde{\bar{t}}; & g\eta \to \eta\,\widetilde{t}\,\widetilde{\bar{t}}; & g\overline{\eta} \to \overline{\eta}\,\widetilde{t}\,\widetilde{\bar{t}};
\end{array}
\tag{6.24}
$$

These amplitudes are identical to the ones given in section D.2 but with the momenta living in D dimensions. Then the collinear limit can be taken analogous to the last process, and after expanding in δ around 0 we obtain

$$|\mathcal{M}|^2_{\overline{\eta}\eta\to g\tilde{t}\bar{\tilde{t}}} \overset{\langle k_2||k_3\rangle}{=} -\frac{\alpha_s^3\pi^3}{(1-z)t_1}\frac{288(T-U)^2(z-1)(z+3)}{(T+U)^2 z} + \mathcal{O}(\delta^1)$$

$$|\mathcal{M}|^2_{\eta\overline{\eta}\to g\tilde{t}\bar{\tilde{t}}} \overset{\langle k_2||k_3\rangle}{=} -\frac{\alpha_s^3\pi^3}{(1-z)t_1}\frac{288(T-U)^2(z-3)}{(T+U)^2 z} + \mathcal{O}(\delta^1)$$

$$|\mathcal{M}|^2_{g\eta\to\eta\tilde{t}\bar{\tilde{t}}} \overset{\langle k_2||k_3\rangle}{=} -\frac{\alpha_s^3\pi^3}{(1-z)t_1}\frac{288(T-U)^2 z(3z-1)}{(T+U)^2} + \mathcal{O}(\delta^1)$$

$$|\mathcal{M}|^2_{g\overline{\eta}\to\overline{\eta}\tilde{t}\bar{\tilde{t}}} \overset{\langle k_2||k_3\rangle}{=} -\frac{\alpha_s^3\pi^3}{(1-z)t_1}\frac{288(T-U)^2(1-z)(3z+1)}{(T+U)^2} + \mathcal{O}(\delta^1)$$

$$\begin{aligned}|\mathcal{M}|^2_{g\eta\to\eta\tilde{t}\bar{\tilde{t}}} \overset{\langle k_2||k_3\rangle}{=} & -\frac{\alpha_s^3\pi^3}{(1-z)t_1}\frac{32z}{(D-2)T^2U^2(T+U)^2(z-1)}\times \\ & \Big(-128(T+U)^2\left(4T^2-UT+4U^2\right)m^4 \\ & -128TU(T+U)\left(4T^2-UT+4U^2\right)m^2 \\ & +(D-2)T^2U^2\left(9(T-U)^2 z\ -7\left(17T^2-2UT+17U^2\right)\right)\Big) + \mathcal{O}(\delta^1)\end{aligned}$$

$$\begin{aligned}|\mathcal{M}|^2_{g\,\overline{\eta}\to\widetilde{\eta}\tilde{t}\bar{\tilde{t}}} \overset{\langle k_2||k_3\rangle}{=} & -\frac{\alpha_s^3\pi^3}{(1-z)t_1}\frac{1}{(D-2)T^2U^2(T+U)^2(z-1)}\left(288(D-2)T^2(T-U)^2U^2\right. \\ & -32\left(128(T+U)^2\left(4T^2-UT+4U^2\right)m^4\right. \\ & +128TU(T+U)\left(4T^2-UT+4U^2\right)m^2 \\ & \left.\left.+7(D-2)T^2U^2\left(17T^2-2UT+17U^2\right)\right)z\right) + \mathcal{O}(\delta^1)\end{aligned}$$

Subtracting the ghost processes as the unphysical polarization states from the D-dimensional gluon process, we find (neglecting higher orders δ)

$$\begin{aligned}&\frac{1}{8}|\mathcal{M}|^2_{gg\to g\tilde{t}\bar{\tilde{t}}} \overset{\langle k_2||k_3\rangle}{=} \\ &\overset{\langle k_2||k_3\rangle}{=} \frac{\alpha_s^3\pi^3}{t_1(1-z)}\cdot\frac{N\cdot 2((1-z)z+1)^2}{(z-1)z}\times \\ &\quad \frac{256\left(4T^2-UT+4U^2\right)\left(4(T+U)^2m^4+4TU(T+U)m^2+(D-2)T^2U^2\right)}{3T^2U^2(T+U)^2} \\ &= \alpha_s\pi\frac{1}{t_1(1-z)}|\mathcal{M}|^2_{gg\to\tilde{t}\bar{\tilde{t}}}\cdot P_{gg}(z) \qquad (6.25)\end{aligned}$$

This is equal to the result in DRED. Obviously, this process factorizes as expected. This result is also part of the non-factorizable result in eq. 4.2. The squared amplitude $|\mathcal{M}|^2_{g\,g\to\tilde{t}\bar{\tilde{t}}}$ was calculated in section 5.2 (eq. 5.3). The gluon-gluon splitting function is the one given in eq. 4.30

as expected.
The amplitudes corresponding to the remaining $(4-D)$ components of the vector boson are calculated in the next section.

Factorization of the Processes Including Two ϵ Scalars (eqs. 6.21b, 6.21c and 6.21d)

Since we interpret the last $(4-D)$ components of the vector boson as a particle of its own, we now have to evaluate the processes shown in fig. 6.6. All contributing diagrams are plotted in figs. D.4, D.5 and D.6.

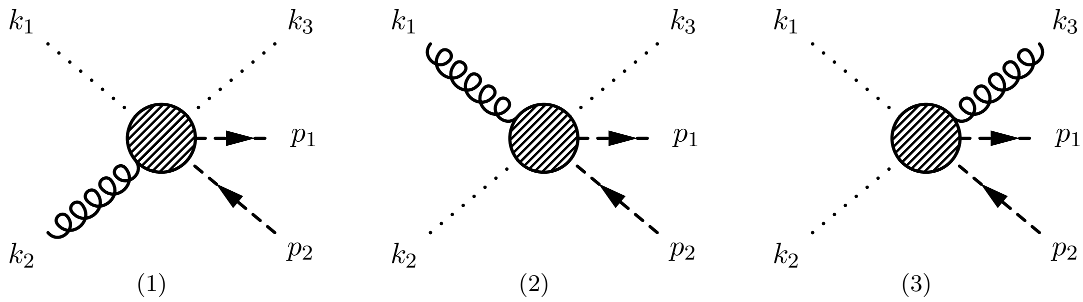


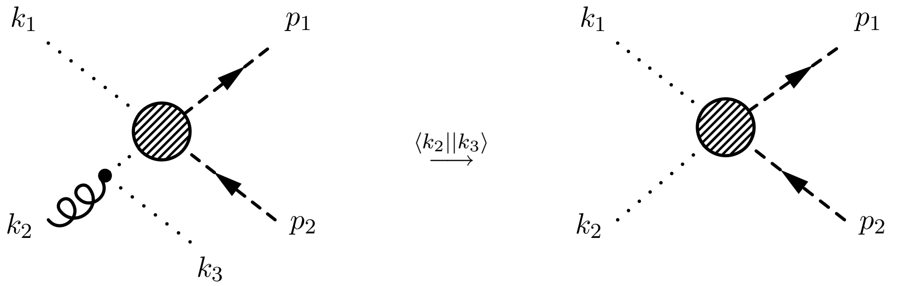
Figure 6.6: Schematic diagrams for the processes in eqs. 6.21b, 6.21c and 6.21d with a D-dimensional gluon and two ϵ scalars.

These processes factorize not only into the LO process with two inital ϵ scalars. Due to the topology of the Feynman diagrams, one of them factorizes into the LO process with two initial gluons. This is sketched in figs. 6.7(1), 6.7(2) and 6.7(3). The processes in figs. 6.7(1) and 6.7(3) factorize into the LO amplitude with ϵ scalars in the initial state, while the process in fig. 6.7(2) factorizes into the LO process with gluons in the initial state.
The Feynman amplitudes are generated with `FeynArts` (app. B.1). Since the ϵ scalar is not available in the model files of `FeynArts`, it was implemented (app. B.2). The squaring of the amplitudes and the evaluation of the color traces was done with `FeynArts`. The polarization sum was calculated by replacing the polarization vectors with their corresponding polarization sum

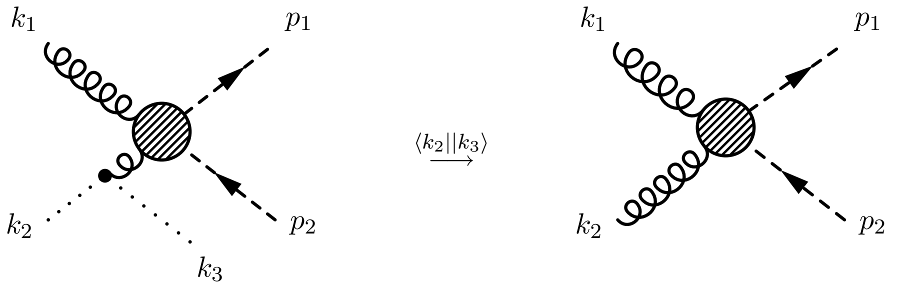
$$\text{if } k_i \text{ is the momentum of the } \epsilon \text{ scalar} \quad \begin{cases} \phi\, g \to \phi\widetilde{t}\,\widetilde{\bar{t}}: & \hat{\epsilon}(k_1)\cdot\hat{\epsilon}(k_3)\ \hat{\epsilon}^*(k_1)\cdot\hat{\epsilon}^*(k_3) \quad \to (4-D) \\ g\,\phi \to \phi\widetilde{t}\,\widetilde{\bar{t}}: & \hat{\epsilon}(k_2)\cdot\hat{\epsilon}(k_3)\ \hat{\epsilon}^*(k_2)\cdot\hat{\epsilon}^*(k_3) \quad \to (4-D) \\ \phi\,\phi \to g\widetilde{t}\,\widetilde{\bar{t}}: & \hat{\epsilon}(k_1)\cdot\hat{\epsilon}(k_2)\ \hat{\epsilon}^*(k_1)\cdot\hat{\epsilon}^*(k_2) \quad \to (4-D) \end{cases} \tag{6.26}$$

$$\text{for } k_i = \text{gluon momentum} \qquad \tilde{\epsilon}(k_i)\cdot\tilde{\epsilon}^*(k_i) \to -D \tag{6.27}$$

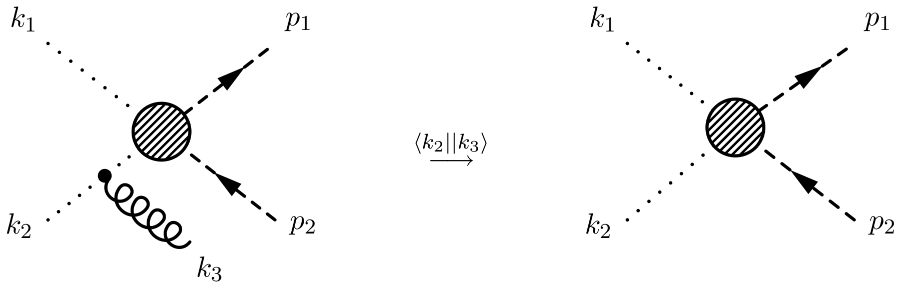
Due to the definition of the $(4-D)$-dimensional polarization vectors for the ϵ scalar they are never contracted with a momentum, so that for the ϵ scalars only structures of this form (eq. 6.26)



(1) Schematic diagrams for the process $\phi g \to \phi \tilde{t} \bar{\tilde{t}}$ with a D-dimensional gluon and ϵ scalars. This process factorizes into the $2 \to 2$ process with ϵ scalars in the initial state.



(2) Schematic diagrams for the process $g\phi \to \phi \tilde{t} \bar{\tilde{t}}$ with a D-dimensional gluon and ϵ scalars. In contrast to the other two processes involving ϵ scalars, this process factorizes into the $2 \to 2$ process with gluons in the initial state.



(3) Schematic diagrams for the process $\phi\phi \to g \tilde{t} \bar{\tilde{t}}$ with a D-dimensional gluon and ϵ scalars. This process factorizes into the $2 \to 2$ process with ϵ scalars in the initial state.

Figure 6.7: All schematic NLO diagrams with their corresponding LO processes.

appear. However, the polarization vectors of the gluons can also be contracted to momenta, e.g.

$$\tilde{\epsilon}(k_i)^\mu k_\mu \, \tilde{\epsilon}^*(k_j)^\nu p_\nu \quad \to \quad -k \cdot p$$

The squared amplitudes after performing the polarization sums and color traces are lengthy. They are shown in section D.3 where they are expressed in terms of the kinematic variables for 5 legs defined in eq. A.4. Using eq. 6.4 and eq. 6.13 we take the collinear limit and substitute S. After expanding the terms in δ around 0 to zeroth order we calculate the squared amplitudes for the process in fig. 6.6(1) (eq. 6.21c)

$$\frac{1}{8}\frac{4-D}{D-2}|M|^2_{\phi\, g\to\phi\tilde{t}\bar{\tilde{t}}} \overset{\langle k_2||k_3\rangle}{=}$$

$$= \frac{1}{8}\frac{1}{t_1(1-z)} \cdot 2z(1-z)\frac{4-D}{D-2}N \cdot \alpha_s^3\pi^3 \cdot \frac{2048\left(4T^2-UT+4U^2\right)(D-4)}{3(T+U)^2} + \mathcal{O}(\delta^1)$$

$$= \alpha_s^3\pi^3 \cdot \frac{1}{t_1(1-z)}\frac{256\left(4T^2-UT+4U^2\right)(4-D)}{3(T+U)^2} \cdot 2z(1-z)\frac{4-D}{D-2}N$$

$$= \frac{1}{t_1(1-z)}\,\alpha_s\pi\, P_{\phi g}(z)\,|\mathcal{M}|^2_{\phi\phi\to\tilde{t}\bar{\tilde{t}}} \qquad (6.28)$$

with the splitting function $P_{\phi g}(z)$ from eq. 4.32 for a gluon splitting into two ϵ scalars. For the process where the emitted particle is an ϵ scalar (fig. 6.6(2), eq. 6.21b) we find

$$\frac{1}{8}\frac{D-2}{4-D}|M|^2_{g\phi\to\phi\tilde{t}\bar{\tilde{t}}} \overset{\langle k_2||k_3\rangle}{=}$$

$$= (D-2)\frac{\alpha_s^3\pi^3}{(1-z)t_1}\frac{Nz}{(z-1)}\times$$
$$\frac{512\left(4T^2-UT+4U^2\right)\left(T^2k_\perp^2U^2+4(m^2Uk_1\cdot k_\perp-m^2(T+U)k_4\cdot k_\perp)^2\right)}{3T^2U^2(T+U)^2k_\perp^2} + \mathcal{O}(\delta^1)$$

$$\overset{1}{=} \frac{\alpha_s\pi}{(1-z)t_1}\cdot\frac{2Nz}{(1-z)}\times$$
$$\frac{256\alpha_s^2\pi^2\left(4T^2-UT+4U^2\right)\left(4(T+U)^2m^4+4TU(T+U)m^2+(D-2)T^2U^2\right)}{3T^2U^2(T+U)^2}$$

$$= \frac{1}{t_1(1-z)} \alpha_s \pi \, P(z)_{\phi\phi} \, |\mathcal{M}|^2_{gg \to \tilde{t}\bar{\tilde{t}}} \qquad (6.29)$$

with the splitting function eq. 4.34, $P(z)_{\phi\phi}$, where an ϵ scalar radiates another ϵ scalar and a gluon enters the factorized matrix element. This is the process which factorizes to the LO process with gluons in the initial state.
The process with two initial ϵ scalars and a final gluon (fig. 6.6(3), eq. 6.21d) factorizes

$$\frac{1}{8} \frac{4-D}{4-D} |M|^2_{\phi\phi \to g\tilde{t}\bar{\tilde{t}}} \overset{\langle k_2 || k_3 \rangle}{=}$$

$$= \frac{1}{(1-z)t_1} \pi^3 \alpha_s^3 \frac{256(4-D)\left(4T^2 - UT + 4U^2\right)}{3(T+U)^2} \cdot 2N \frac{(1-z)}{z} + \mathcal{O}(\delta^1)$$

$$= \alpha_s^3 \pi^3 \frac{1}{t_1(1-z)} \frac{2(1-z)N}{z} \frac{256(4-D)\left(4T^2 - UT + 4U^2\right)}{3(T+U)^2}$$

$$= \frac{1}{t_1(1-z)} \alpha_s \pi \, P(z)_{g\phi} |\mathcal{M}|^2_{\phi\,\phi \to \tilde{t}\bar{\tilde{t}}} \qquad (6.30)$$

with the splitting function in eq. 4.33 where the gluon is the collinearly radiated particle.
Each process has to be multiplied with a factor of $D-2$ $(4-D)$ for the gluon (ϵ scalar) entering the LO process and for every gluon (ϵ scalar) entering the NLO process we have to divide by a factor of $D-2$ $(4-D)$.
The LO results can be found in section 5.2 and the corresponding splitting functions in section 4.4.

These results demonstrate that all four partonic processes factorize in the desired way into the product of a splitting function, the LO process and the collinear pole, and therefore are in accordance with the mass factorization theorem.
The seemingly non-factorizable result (eq. 4.2) can be recovered by adding all the individual contributions in eqs. 6.25, 6.28, 6.29 and 6.30.

[1] The terms have to be azimuthally averaged, since the intermediate result still depends on $k_\perp$.

7 Discussion

If DRED is realized by introducing an additional $(4-D)$-component parton, new processes including external ϵ scalars arise which have to be evaluated individually. Following this idea the results from chapter 6 demonstrate that these individual processes factorize into a product of the LO process, a splitting function and the pole as desired. All problematic terms in eq. 4.2 are interpreted as partonic processes with their corresponding splitting functions.
In this section we summarize the procedure of separating the squared amplitude into processes considering dimensionally reduced gluons and their remaining $(4-D)$ components. Furthermore we motivate the individual factorization of processes in terms of the new partons, which, seen as one single physical process involving 4-dimensional gluons, fail to factorize.
We investigate why the factorization problem is absent in the massless limit and discuss, why in contrast to the second process described in this work, the first process factorizes without the need for ϵ scalars.

7.1 Interpretation of the Squared Amplitudes in DRED

In chapter 4 we tried to factorize the real NLO processes by expressing them as

$$\frac{1}{8}|\mathcal{M}|^2_{GG\to G\tilde{t}\bar{\tilde{t}}} \overset{\langle k_2||k_3\rangle}{=} \frac{1}{t_1(1-z)}\alpha_s\pi \cdot |\mathcal{M}|^2_{GG\to\tilde{t}\bar{\tilde{t}}}$$

after taking the collinear limit. However, we find non-factorizable terms (eq. 4.2).
As the first step to gain insight into the origin of this problem we split the squared amplitude with 4-component gluons into processes including D-dimensional gluons (g) and ϵ scalars (ϕ). The squared amplitude with initial 4-dimensional gluons can be split

$$\begin{aligned} |\mathcal{M}|^2_{GG\to G\tilde{t}\bar{\tilde{t}}} &= |\mathcal{M}|^2_{gg\to g\tilde{t}\bar{\tilde{t}}} + |\mathcal{M}|^2_{g\,\phi\to\phi\tilde{t}\bar{\tilde{t}}} + |\mathcal{M}|^2_{\phi\,g\to\phi\tilde{t}\bar{\tilde{t}}} + |\mathcal{M}|^2_{\phi\phi\to g\tilde{t}\bar{\tilde{t}}} \\ &= \sum_{i,j=g,\phi}\left(|\mathcal{M}|^2_{g\,i\to j\tilde{t}\bar{\tilde{t}}} + |\mathcal{M}|^2_{\phi\,i\to j\tilde{t}\bar{\tilde{t}}}\right) \end{aligned} \tag{7.1}$$

Interference terms including an ϵ scalar and a gluon vanish:

$$\sum_s \epsilon^s_\mu \epsilon^{*s}_\nu \mathcal{M}^\mu \mathcal{M}^{*\nu} = \sum_s \tilde{\epsilon}^s_\rho \tilde{\epsilon}^{*s}_\sigma \mathcal{M}^\rho \mathcal{M}^{*\sigma} + \hat{\epsilon}^s_i \hat{\epsilon}^{*s}_j \mathcal{M}^i \mathcal{M}^{*j} + (\tilde{\epsilon}^s_\rho \hat{\epsilon}^{*s}_j \mathcal{M}^\rho \mathcal{M}^{*j} + \hat{\epsilon}^s_i \tilde{\epsilon}^{*s}_\sigma \mathcal{M}^i \mathcal{M}^{*\sigma})$$

and the interference term in the parentheses are zero because

$$\sum_s \epsilon^s_\rho \epsilon^{*s}_j = -g_{\rho j} = 0$$

In the collinear limit we find

$$\begin{aligned}|\mathcal{M}|^2_{GG\to G\tilde{t}\bar{\tilde{t}}} &= \sum_{i,j,k=g,\phi} |\mathcal{M}|^2_{i\,j\to k\,\tilde{t}\bar{\tilde{t}}} \\ &\overset{\langle k_2||k_3\rangle}{=} \frac{\alpha_s \pi}{t_1(1-z)} \left(\sum_{i=g,\phi} P_{g\,i}(z) |\mathcal{M}|^2_{gg\to\tilde{t}\bar{\tilde{t}}} + \sum_{i=g,\phi} P_{\phi\,i}(z) |\mathcal{M}|^2_{\phi\phi\to\tilde{t}\bar{\tilde{t}}} \right) \end{aligned} \tag{7.2}$$

As we saw from the derivations in chapter 5, the squared amplitudes for $|\mathcal{M}|^2_{gg\to\tilde{t}\bar{\tilde{t}}}$ and $|\mathcal{M}|^2_{\phi\phi\to\tilde{t}\bar{\tilde{t}}}$ are not proportional to one another. This indicates that factorization only works in the sense that these subprocesses (eq. 7.2) factorize separately, which motivates us to consider g and ϕ as separate partons.
In DREG all contributions with external ϵ scalars are absent and therefore factorization obviously follows from eq. 7.2 as well.

7.2 Mass Dependence of the Factorization Problem

For massless squarks we found that the NLO process factorizes without problems into the correct LO process without treating the ϵ scalars as separate particles

$$\frac{1}{8}|\mathcal{M}|^2_{GG\to G\tilde{t}\bar{\tilde{t}}} \underset{m=0}{\overset{\langle k_2||k_3\rangle}{=}} \frac{\alpha_s \pi}{t_1(1-z)} \cdot \left(\frac{256\alpha_s^2\pi^2\left(4T^2-UT+4U^2\right)}{3(T+U)^2} \cdot \frac{2((z-1)z+1)^2}{(1-z)z} N \right)$$

For $m \to 0$ all terms violating the factorization theorem in eq. 4.2 vanish and it factorizes as desired. This was also noted for other processes in [1], [2] and [30].
The absence of the factorization problem can be reconstructed from the LO results for initial ϵ scalars and initial gluons. The LO process with initial ϵ scalars is

$$|\mathcal{M}|^2_{\phi\phi\to\tilde{t}\bar{\tilde{t}}} = \alpha_s^2\pi^2 \frac{256(4-D)\left(4T^2-UT+4U^2\right)}{3(T+U)^2} \tag{7.3}$$

in difference to the gluon LO result which exhibits a more complicated structure

$$\begin{aligned}|\mathcal{M}|^2_{gg\to\tilde{t}\bar{\tilde{t}}} &= \alpha_s^2\pi^2 \frac{512\left(4T^2-UT+4U^2\right)\left(4(T+U)^2m^4+4TU(T+U)m^2+(D-2)T^2U^2\right)}{3T^2U^2(T+U)^2} \\ &\overset{m=0}{=} \alpha_s^2\pi^2 \frac{256(D-2)\left(4T^2-UT+4U^2\right)}{3(T+U)^2} \end{aligned} \tag{7.4}$$

For the massless limit, both LO processes yield the same expression. Now we are able to pull the squared amplitude out of the sum in eq. 7.2, and only sums of the splitting functions remain:

$$\begin{aligned}\frac{1}{8}|\mathcal{M}|^2_{GG\to\tilde{t}\bar{\tilde{t}}} &\overset{\langle k_2||k_3\rangle}{\underset{m=0}{=}} \alpha_s^3\pi^3\frac{256(D-2)\left(4T^2-UT+4U^2\right)}{3(T+U)^2}\cdot\left(\frac{2N(D-4)z}{(D-2)(z-1)}\right.\\ &\left.+\frac{2N(D-4)(z-1)}{(D-2)z}+2N(D-4)\frac{(z-1)}{(D-2)}z+\frac{2N((z-1)z+1)^2}{(1-z)z}\right)\\ &=\alpha_s^3\pi^3\frac{256(D-2)\left(4T^2-UT+4U^2\right)}{3(T+U)^2}\left((P_{\phi\phi}+P_{\phi g})\frac{(4-D)}{(D-2)}+P_{\phi g}+P_{gg}\right)\\ &=\alpha_s^3\pi^3\frac{512\left(4T^2-UT+4U^2\right)}{3(T+U)^2}\left(\frac{2N((z-1)z+1)^2}{(1-z)z}\right)\end{aligned} \tag{7.5}$$

Even without taking the limit $D\to 4$, the last line represents the correct result for the splitting function $P_{GG}(z)$. This shows that the splitting functions eqs. 4.30, 4.32, 4.33 and 4.34 sum up to give twice the pure gluon splitting function eq. 4.30.

In [2] the reason for this behaviour in the massless limit (which is similar for the process $GG\to t\bar{t}$ investigated in [2]) is suspected to be connected with the occurence of the double pole. In their analysis, the massless and the massive cases are distinguished by the appearance of the double pole $1/t_1^2$ which has to be azimuthally averaged. In the process at hand, we also find terms with double poles $1/t_1^2$ and cope with them in chapter 6.
A closer look at the NLO squared amplitude in the collinear limit before averaging (e.g. in eq. 6.29, the important part is again given in eq. 7.6) shows that in the massless limit the terms proportional to $k_\perp^\mu k_\perp^\nu/k_\perp^2$ vanish

$$\begin{aligned}|\mathcal{M}|^2 &\propto \frac{\left(4T^2-UT+4U^2\right)\left(T^2k_\perp^2U^2+4(m^2Uk_1\cdot k_\perp-m^2(T+U)k_4\cdot k_\perp)^2\right)}{3T^2U^2(T+U)^2k_\perp^2}\\ &\overset{m=0}{=}\frac{\left(4T^2-UT+4U^2\right)}{3(T+U)^2}\end{aligned} \tag{7.6}$$

This implies that the azimuthal average over $k_\perp$ is trivial in this case. Since we have to average over $k_\perp$ for the massive case in both DRED and DREG, the appearance of the double poles has no particular connection to the appearance of the ϵ scalars and to the absence of the factorization problem.
As shown in 7.6 for $m=0$ all terms including $k_\perp^\mu k_\perp^\nu/k_\perp^2$ which would need to be averaged are zero.
In this process, the LO squared amplitudes with ϵ scalars do not introduce a new mass dependence. Furthermore the parts in eq. 7.3 and eq. 7.4 containing a prefactor of $(4-D)$ and D are equal for both amplitudes including gluons and ϵ scalars.
The terms in eq. 7.6 which do not have a dimensional factor are instead proportional to m^2. It would be interesting to understand if these mass factors are always connected to the absence of

a dimensional dependence as this would explain the absence of the factorization problem in the massless limit.

7.3 Appearance of the Factorization Problem in Specific Processes

For further calculations it would be useful to see for which kinds of processes in DRED the factorization problems appears, thus requiring us to calculate with ϵ scalars. From the calculations in this work, we can at least infer necessary conditions for the need of ϵ scalars in specific processes.

No Need for ϵ Scalars: $q\overline{q} \to G\tilde{t}\overline{\tilde{t}}$ and $q\overline{q} \to Gt\,\overline{t}$

As we saw in chapter 6, the process $q\overline{q} \to G\tilde{t}\overline{\tilde{t}}$ factorizes with or without treating the $(4-D)$ component of the gluon as a separate particle.
The LO process $q\overline{q} \to \tilde{t}\overline{\tilde{t}}$ is the same for both DRED and DREG since obviously no corresponding LO process with external ϵ scalars can be formed. This implies that the real NLO correction to this process can only factorize into this specific squared amplitude in the collinear limit, but with different splitting functions for the ϵ scalar-quark (eq. 4.29) and a gluon-quark splitting (eq. 4.27). As we see, both splitting functions add up to give the 4-dimensional quark-gluon splitting function:

$$\begin{aligned} P_{\phi q}(z) + P_{gq}(z) &= C_2(r)\left(\frac{1}{2}(4-D)z + \frac{z((D-2)z-4)+4}{z}\right) \\ &= C_2(r)\frac{1+(1-z)^2}{z} \\ &= P_{G\phi}(z) \end{aligned} \tag{7.7}$$

This is obvious from the definition of the ϵ scalar.
Since there is only one LO result the absence of the factorization problem is clear with eq. 7.2.

Now we will briefly investigate another process where the factorization problem is also absent: $q\overline{q} \to gt\,\overline{t}$. The LO Feynman diagrams are shown in fig. 7.1. Here we see, that two distinct LO processes are present, one with a gluon and one with an ϵ scalar in the S channel . So, in the collinear limit, all real NLO corrections yield one of the two processes (fig. 7.1(1) or fig. 7.1(2)). But since their kinematical structure is the same and the splitting functions add up as in eq. 7.7, the result factorizes as desired.

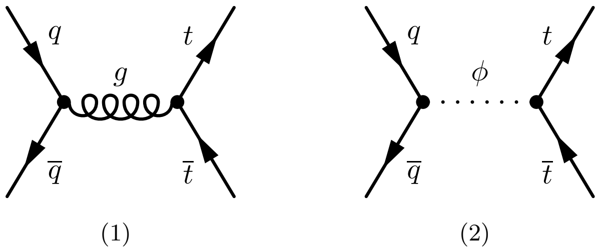


Figure 7.1: Feynman diagrams for $q\overline{q} \to t\overline{t}$.

Need for ϵ Scalars: $gg \to g\tilde{t}\tilde{\bar{t}}$ and $gg \to gt\bar{t}$

In the process

$$GG \to \tilde{t}\tilde{\bar{t}}$$

the amplitudes are of a more complicated structure. Here we have two different LO processes, one with external gluons and one with external ϵ scalars. As we have discussed before, these squared LO amplitudes differ in their kinematical structure due to the three gluon coupling and the gluon-two ϵ coupling. This and the fact that there is no two-squark ϵ scalar coupling imply these differences.
If we now calculate the real NLO corrections for both gluons and ϵ scalars, we see that the different initial NLO states lead to different LO processes:

$$gg \to g\tilde{t}\tilde{\bar{t}} \quad \overset{\text{factorizes}}{\longrightarrow} \quad gg \to \tilde{t}\tilde{\bar{t}} \tag{7.8a}$$

$$g\phi \to \phi\tilde{t}\tilde{\bar{t}} \quad \overset{\text{factorizes}}{\longrightarrow} \quad gg \to \tilde{t}\tilde{\bar{t}} \tag{7.8b}$$

$$\phi\phi \to g\tilde{t}\tilde{\bar{t}} \quad \overset{\text{factorizes}}{\longrightarrow} \quad \phi\phi \to \tilde{t}\tilde{\bar{t}} \tag{7.8c}$$

$$\phi g \to \phi\tilde{t}\tilde{\bar{t}} \quad \overset{\text{factorizes}}{\longrightarrow} \quad \phi\phi \to \tilde{t}\tilde{\bar{t}} \tag{7.8d}$$

Since the LO squared amplitudes for eq. 7.8a and eq. 7.8c differ in the kinematic structure, the sum does not factorize but the processes have to be treated separately as in eq. 7.2.
The process investigated in [1] and [2] (in fig. 7.2) shows a similar behaviour. The NLO processes can be classified by the LO processes they factorize into, which are

$$gg \to t\bar{t} \tag{7.9a}$$

$$\phi\, g \to t\bar{t} \tag{7.9b}$$

$$g\phi \to t\bar{t} \tag{7.9c}$$

$$\phi\,\phi \to t\bar{t} \tag{7.9d}$$

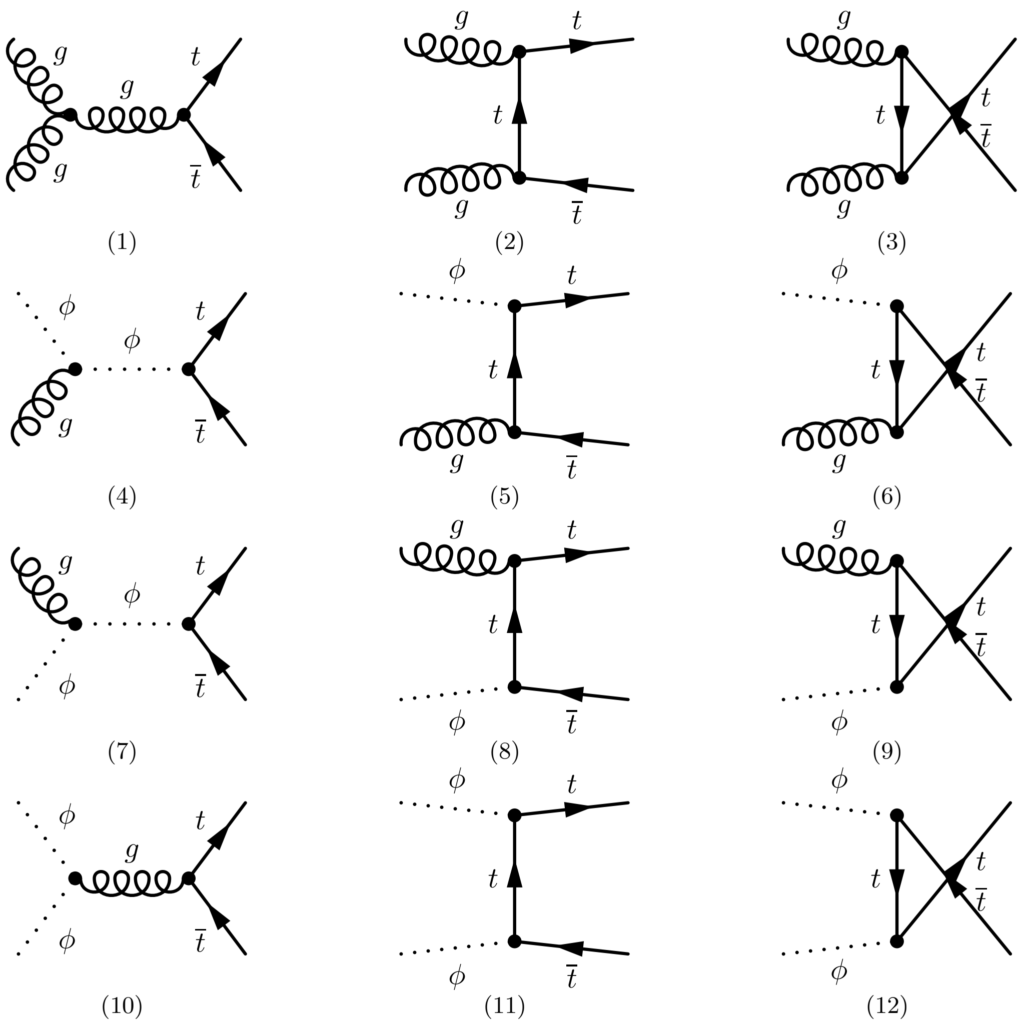


Figure 7.2: Feynman diagrams for $gg \to t\bar{t}$ in DRED.

Each is represented by one row in fig. 7.2. Each column would sum up to give the corresponding diagram for a 4 component gluon but this is spoiled since after factorization the individual diagrams come with different splitting functions.
On the other hand, since the squared LO processes differ in their kinematical structure it is not possible to pull them out of the sum in eq. 7.2 as in the case $q\overline{q} \to \widetilde{t}\,\widetilde{\overline{t}}$.

Different Squared Amplitudes for Processes Involving the ϵ Scalar and the Gluon

From the previous discussion one can conjecture that at tree level the appearance of the factorization problem is connected to external ϵ scalars in LO cross sections. In section 4.3 we found differences in the kinematic structure of some vertices originating from the same 4 component gluon coupling, summarized here:

- the VVV vertex in comparison with the $\phi\phi V$ vertex
- the $VVVV$ vertex in comparison with the $\phi\phi VV$ and the $\phi\phi\phi\phi$ vertex
- absence of the $\phi\widetilde{F}\,\widetilde{F}$ vertex
- absence of the $\phi\phi\phi$, ϕVV, $\phi\phi\phi V$ and ϕVVV vertices

The other vertices including ϵ scalars differ only in the dimension of the Lorentz indices.
So we conjecture that if external gluons appear in the LO process and thus one or more of the "critical" vertices are included, the squared LO amplitudes for the D-dimensional gluon differ from those including its $(4 - D)$ component, and that this gives rise to the factorization problem.

8 Conclusion

Almost 20 years ago, Beenakker et al. [1] found non-factorizable terms in a QCD process which was calculated in DRED, rendering this regularization scheme questionable. The problem had not shown up when the calculation was done in DREG. In [1], [2] and [30] it was found that this factorization problem vanishes in the massless limit.
Recently, Signer and Stöckinger interpreted the vector and scalar components of the gluon in DRED as separate partons, a D component gluon and the ϵ scalar [2]. The non-factorizable terms then turned out to be products of splitting functions and partonic LO processes involving ϵ scalars.
It was the aim of this work to investigate the method proposed in [2] by applying it to the SUSY QCD processes $q\overline{q} \to G\widetilde{t}\overline{\widetilde{t}}$ and $GG \to G\widetilde{t}\overline{\widetilde{t}}$. Following this idea it is shown in this work that this method can also be applied to solve the factorization problem for the processes mentioned above. The detailed approach to mass factorization in massive SUSY QCD is demonstrated and the splitting functions and Feynman rules for the ϵ scalar are derived.

We have found that for the process

$$q\overline{q} \to G\widetilde{t}\overline{\widetilde{t}} \tag{8.1}$$

the factorization problem does not appear, and we explain this with the fact that no ϵ scalars appear in the LO process. This implies that even though ϵ scalars appear in the NLO process, they disappear in the collinear limit and the result factorizes like in 4 dimensions.
For the second process investigated in this work,

$$GG \to G\widetilde{t}\overline{\widetilde{t}} \tag{8.2}$$

we find non-factorizable terms analogous to [1] and [2]. After treating the scalar as a parton of its own, all individual NLO results factorize as desired into products of the LO processes and their corresponding splitting functions. All unusual terms encountered before can be interpreted in this manner.

The factorization is demonstrated by setting the radiated particle collinear to one of the initial particles. For the process in eq. 8.2 we found double poles of the form $1/t_1^2$. An important ingredient to cope with these singularities is the integration over the unobservable azimuthal angle of the collinear particle, e.g. the gluon.
In the massless case of eq. 8.2, these double singularities vanish before being azimuthally averaged.

An essential factor for the appearance of the factorization problem is the presence of external vector particles involving vertices which change their kinematic structure after being rewritten with an ϵ scalar. If they appear we find differences in the kinematic structure of individual results (including ϵ scalars and gluons) belonging to the same physical process in DRED which give rise to the factorization problem.
These differences in the squared amplitudes are also decisive for the absence of the factorization problem in the massless limit. In this context, we also observe that there is a connection between the appearance of mass terms and the dimensional dependence which seems essential for this behaviour, namely that terms which do not have a dimensional dependence carry a mass factor. It would be desirable to explore this phenomenon further and to investigate the consistency of DRED in a precise analysis of the factorization in real NNLO.

A Conventions and Notations

A.1 Conventions and Notations

General Definitions

In this work we use the metric tensor

$$g^{\mu\nu} = \mathbf{diag}(1, -1, -1, -1)$$

where the $g^{\mu\nu}$ denotes the metric tensor in 4 dimensions.
The scalar product of two vectors is denoted with

$$k \cdot p \quad = \quad k_\mu \, p^\mu$$

unless otherwise noted, the indices are in 4 dimensions.
The dimension of the Lorentz indices is indicated by the letters used, with greek indices for 4 or D dimensions $(\mu, \nu, \rho \ldots)$ and with latin indices for $4 - D$ dimensions $(i, j, k \ldots)$. The corresponding objects (such as polarization vectors, metric tensor, Dirac matrices) are tilded $\widetilde{x}$ (D dim.) or hatted $\hat{x}$ ($4 - D$ dim.).
The trace of the metric tensor in arbitrary dimensions is defined as

$$g^\mu_\mu = 4 \qquad\qquad \widetilde{g}^\rho_\rho = D \qquad\qquad \hat{g}^i_i = 4 - D \tag{A.1}$$

Mandelstam Variables

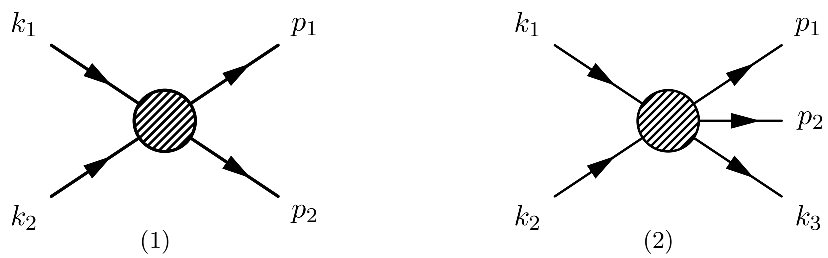


Figure A.1: Sketched processes with denotation for the momenta.

The kinetic variables are needed for 4 and for 5 external legs in this work. The notation is displayed in fig. A.1.
Since in this work only massive final state particles appear in $2 \to 2$ processes, the Mandelstam

variables for four legs (fig. A.1(1)) are defined as

$$\begin{aligned} k_1 \cdot k_2 = \tfrac{S}{2} &\iff (k_1 + k_2)^2 = S \\ k_1 \cdot p_1 = -\tfrac{T}{2} &\iff (k_1 - p_1)^2 - m^2 = T \\ k_2 \cdot p_1 = -\tfrac{U}{2} &\iff (k_2 - p_1)^2 - m^2 = U \end{aligned} \tag{A.2}$$

where m is the mass of the final state particle. The advantage of this definition is that we can use the relation

$$S + T + U = 0 \tag{A.3}$$

Analogous, we define the Mandelstam variables for 5 external legs (fig. A.1(2)), where again only two of the final state particles with momenta p_1 and p_2 are massive (with mass m). For 5 external legs we use the kinematical variables

$$\begin{aligned} s &= (k_1 + k_2)^2 \\ t_1 &= (k_2 - k_3)^2 \\ u_1 &= (k_1 - k_3)^2 \\ s_4 &= (k_3 + p_1)^2 - m^2 \\ u_6 &= (k_2 - p_1)^2 - m^2 \\ u_7 &= (k_1 - p_1)^2 - m^2 \end{aligned} \tag{A.4}$$

which satisfy

$$s + t_1 + u_1 + s_4 + u_6 + u_7 = 0 \tag{A.5}$$

Polarizations of External Particles

For massless non-Abelian gauge bosons we use the polarization sum

$$\sum_{\text{pols}} \epsilon^\mu(k) \epsilon^{*\,\nu}(k) = -g^{\mu\nu} + \frac{n^\mu k^\nu + n^\nu k^\mu}{n \cdot k} - n^2 \frac{k^\mu k^\nu}{(k \cdot n)^2} \tag{A.6}$$

where n is a arbitrary auxiliary vector which can be chosen with $n^2 = 0$.
In 4 and D dimensions we use the unphysical polarization sum

$$\sum_{\text{pols}} \epsilon^\mu(k) \epsilon^{*\,\nu}(k) = -g^{\mu\nu} \tag{A.7}$$

In the case of external massless non-Abelian gauge bosons, one must take external BRST ghosts into account.

The polarization sum for ϵ scalars is

$$\sum_{\text{pols}} \hat{\epsilon}^{\mu}(k)\hat{\epsilon}^{\nu}(k) \quad = \quad -\hat{g}^{\mu\nu} \tag{A.8}$$

where the metric tensor is in $(4-D)$ dimensions.

The polarization vectors are defined in such a way that they obey

$$\sum_{\text{pols}} \epsilon^{*}(p)_{\mu}\, \epsilon^{*\,\mu}(k)\, \epsilon(p)_{\nu}\, \epsilon^{\nu}(k) = g_{\mu\nu}g^{\mu\nu} = 4 \tag{A.9}$$

$$\sum_{\text{pols}} \hat{\epsilon}^{*}(p)_{\mu}\, \hat{\epsilon}^{*\,\mu}(k)\, \hat{\epsilon}(p)_{\nu}\, \hat{\epsilon}^{\nu}(k) = \hat{g}_{\mu\nu}\hat{g}^{\mu\nu} = 4-D \tag{A.10}$$

$$\sum_{\text{pols}} \widetilde{\epsilon}^{*}(p)_{\mu}\, \widetilde{\epsilon}^{*}(k)_{\mu}\, \widetilde{\epsilon}(p)_{\nu}\, \widetilde{\epsilon}^{\nu}(k) = \widetilde{g}_{\mu\nu}\widetilde{g}^{\mu\nu} = D \tag{A.11}$$

and further (only for physical polarizations)

$$\epsilon(p)\cdot p = 0 \qquad \hat{\epsilon}(p)\cdot p = 0 \qquad \widetilde{\epsilon}(p)\cdot p = 0 \tag{A.12}$$

The polarization sum for fermions is

$$\begin{aligned} \sum_{s} u^{s}(k)\overline{u}^{s}(k) &= \not{k} + m \\ \sum_{s} v^{s}(k)\overline{v}^{s}(k) &= \not{k} - m \end{aligned} \tag{A.13}$$

where u denotes the incoming particle, $\overline{u}$ the outgoing particle, v the outgoing antiparticle and $\overline{v}$ the incoming antiparticle.

Numerator Algebra

For the evaluation of Dirac matrices we refer to [28] and generalize the results for D- and $(4-D)$-dimensional indices

$$\begin{aligned} D \text{ dimensions}: &\quad \mathbf{Tr}\left[\widetilde{\gamma}^{\mu}\widetilde{\gamma}^{\nu}\right] = 4\widetilde{g}^{\mu\nu} \\ 4-D \text{ dimensions}: &\quad \mathbf{Tr}\left[\hat{\gamma}^{i}\hat{\gamma}^{j}\right] = 4\hat{g}^{\mu\nu} \\ \text{mixed dimensions}: &\quad \mathbf{Tr}\left[\widetilde{\gamma}^{\mu}\hat{\gamma}^{i}\right] = 0 \end{aligned} \tag{A.14}$$

and for 4 Dirac matrices

$$\begin{aligned}
D \text{ dimensions}: &\quad \mathbf{Tr}\left[\widetilde{\gamma}^{\mu}\widetilde{\gamma}^{\nu}\widetilde{\gamma}^{\rho}\widetilde{\gamma}^{\sigma}\right] = 4\Big(\widetilde{g}^{\mu\nu}\,\widetilde{g}^{\rho\sigma} - \widetilde{g}^{\mu\rho}\,\widetilde{g}^{\nu\sigma} + \widetilde{g}^{\mu\sigma}\,\widetilde{g}^{\nu\rho}\Big) \\
4-D \text{ dimensions}: &\quad \mathbf{Tr}\left[\hat{\gamma}^{i}\hat{\gamma}^{j}\hat{\gamma}^{k}\hat{\gamma}^{l}\right] = 4\left(\hat{g}^{i\,j}\,\hat{g}^{k\,l} - \hat{g}^{i\,k}\,\hat{g}^{j\,l} + \hat{g}^{i\,l}\,\hat{g}^{j\,k}\right) \qquad (A.15)\\
\text{mixed dimensions}: &\quad \mathbf{Tr}\left[\widetilde{\gamma}^{\mu}\widetilde{\gamma}^{\nu}\hat{\gamma}^{i}\hat{\gamma}^{j}\right] = 4\,g^{\mu\nu}g^{i\,j}
\end{aligned}$$

The derivation of traces involving more Dirac matrices with mixed indices is analogous.

A.2 Feynman Rules for non-Abelian Gauge Theory in DRED

As an overview over the Feynman rules used we list those which have been calculated in section 4.3 and which are known from standard literature. The standard model Feynman rules are consistent with [28], the SUSY vertices with [31] and [32].
The arrows of the propagator define the particle flow. All momenta are defined **incoming**. In this work we use Feynman Gauge ($\xi = 1$) :

$$\text{massless vector boson propagator} = \frac{-i}{k^2 + i\epsilon}\left(g^{\mu\nu} - (1-\xi)\frac{k^{\mu}k^{\nu}}{k^2}\right) \qquad (A.16)$$

Landau gauge corresponds to ($\xi = 0$).

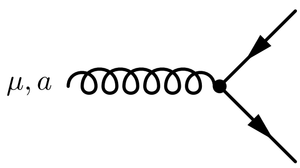
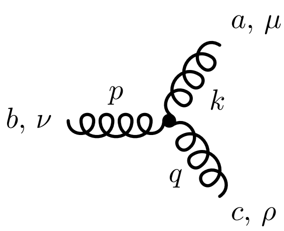
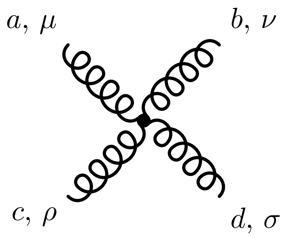
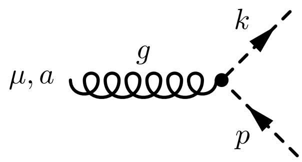
Vector 2-Fermion vertex:		$ig_s\widetilde{\gamma}^{\mu}t^a$	(A.17)
3-Vector vertex:		$g_s f^{uvw}\,(\widetilde{g}^{\mu\nu}(k-p)^{\rho}$ $+\widetilde{g}^{\nu\rho}(p-q)^{\mu}$ $+\widetilde{g}^{\rho\mu}(q-k)^{\nu})$	(A.18)
4-Vector vertex:		$-ig_s^2\left(f^{abe}f^{cde}\,(\widetilde{g}^{\mu\rho}\widetilde{g}^{\nu\sigma} - \widetilde{g}^{\mu\sigma}\widetilde{g}^{\nu\rho})\right.$ $+f^{ace}f^{bde}\,(\widetilde{g}^{\mu\nu}\widetilde{g}^{\rho\sigma} - \widetilde{g}^{\mu\sigma}\widetilde{g}^{\nu\rho})$ $\left.+\,f^{ade}f^{bce}\,(\widetilde{g}^{\mu\nu}\widetilde{g}^{\rho\sigma} - \widetilde{g}^{\mu\rho}\widetilde{g}^{\nu\sigma})\right)$	(A.19)

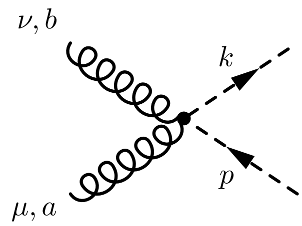
Table A.1: Feynman rules for non-Abelian gauge theory.

Vector 2-Sfermion vertex:



$$ig_s(p-k)^\mu t^a \qquad \text{(A.20)}$$

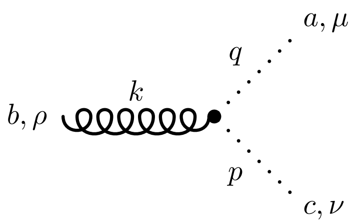
2-Vector 2-Sfermion vertex:



$$ig_s^2\ \{t^a, t^b\}\ \widetilde{g}^{\mu\nu} \qquad \text{(A.21)}$$

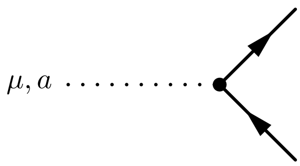
Table A.2: Feynman rules for SUSY QCD.

2-ϵ Scalar Vector vertex:



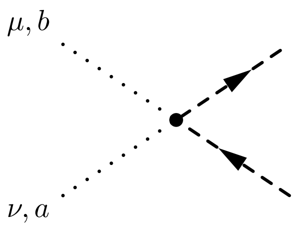
$$g_s(p-q)^\rho\ \hat{g}^{\nu\mu}\ f^{abc} \qquad \text{(A.22)}$$

2-Fermion ϵ Scalar vertex:



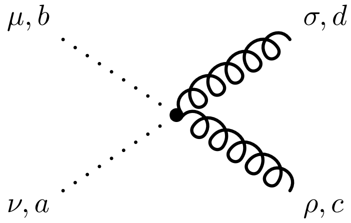
$$ig_s\hat{\gamma}^\mu t^a \qquad \text{(A.23)}$$

2-ϵ Scalar 2-Sfermion vertex:



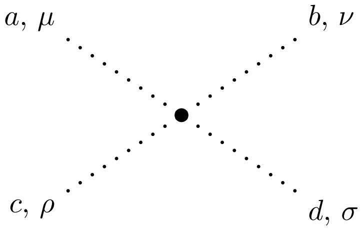
$$ig_s^2\ \{t^a, t^b\}\hat{g}^{\mu\nu} \qquad \text{(A.24)}$$

2-ϵ Scalar 2-Vector vertex:



$$-\,ig_s^2\; f^{abe}f^{cde}\,(\hat{g}^{\mu\nu}\tilde{g}^{\rho\sigma}) \tag{A.25}$$

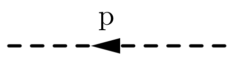
4-ϵ Scalar vertex:



$$\begin{aligned}-ig_s^2\,\big(&f^{abe}f^{cde}\,(\hat{g}^{\mu\rho}\hat{g}^{\nu\sigma}-\hat{g}^{\mu\sigma}\hat{g}^{\nu\rho})\\ &+f^{ace}f^{bde}\,(\hat{g}^{\mu\nu}\hat{g}^{\rho\sigma}-\hat{g}^{\mu\sigma}\hat{g}^{\nu\rho})\\ &+f^{ade}f^{bce}\,(\hat{g}^{\mu\nu}\hat{g}^{\rho\sigma}-\hat{g}^{\mu\rho}\hat{g}^{\nu\sigma})\big)\end{aligned} \tag{A.26}$$

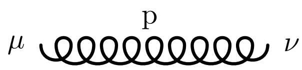
Table A.3: Feynman rules for gluon, quark, squark, fermion and ϵ scalar couplings

Sfermion Propagator:



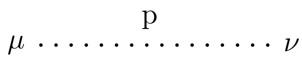
$$\frac{i}{p^2-m^2+i\epsilon} \tag{A.27}$$

Vector Boson Propagator (in Feynman Gauge):



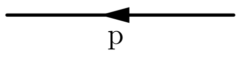
$$-\frac{i\tilde{g}^{\mu\nu}}{p^2+i\epsilon} \tag{A.28}$$

ϵ Scalar Propagator:



$$-\frac{i\hat{g}^{\mu\nu}}{p^2+i\epsilon} \tag{A.29}$$

Dirac Propagator:



$$i\frac{(\not{p}+m)}{p^2-m^2+i\epsilon} \tag{A.30}$$

Table A.4: Propagators for scalars, gluons, ϵ scalars and fermions.

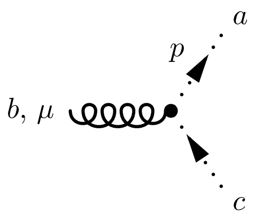
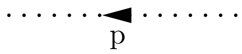
Vector 2-Ghost Vertex:		$-\,g f^{abc} p^{\mu}$	(A.31)
Ghost Propagator:		$\dfrac{i}{p^2 + i\epsilon}$	(A.32)

Table A.5: Ghost propagator and vertex

B Programs and Implementation

B.1 Programs Used in This Thesis

The following list gives an overview of the programs used in this thesis for the generation and evaluation of Feynman topologies, matrix elements and squared amplitudes.

`Mathematica`: This is a general computing environment which organizes many algorithmic, visualization and user interface capabilities within a document-like user interface paradigm. It can be extended with several packages (e.g. FeynCalc). In contrast to `Mathematica` , the packages used here are freely available.
Homepage: `http://www.wolfram.com`

`FeynCalc`: This is a `Mathematica` package for algebraic calculations in elementary particle physics. It is used for frequently occuring tasks such as contraction of Lorentz indices, color factor calculations, Dirac matrix manipulations and traces. The vectors, metric tensors and Dirac matrices can be defined in arbitrary dimensions. It was used for the LO process $q\overline{q} \to \widetilde{t}\,\overline{\widetilde{t}}$ and the NLO process $q\overline{q} \to g\,\widetilde{t}\,\overline{\widetilde{t}}$ in DRED.
Homepage: `http://www.feyncalc.org`

`FeynArts`: This a `Mathematica` package for the generation and visualization of Feynman diagrams and amplitudes [33]. It was used to generate the Feynman amplitudes for the LO process. Since the available models (such as `MSSMQCD.mod`) do not comprise an ϵ scalar, this particle with its corresponding couplings is implemented (section B.2). We use `FeynCalc`, `FeynArts` and `FormCalc` for the LO $GG \to \widetilde{t}\,\overline{\widetilde{t}}$ and for the NLO process $GG \to G\,\widetilde{t}\,\overline{\widetilde{t}}$ in 4 dimensions and in DRED. It was also used for the LO process $q\overline{q} \to \widetilde{t}\,\overline{\widetilde{t}}$ and the NLO process $q\overline{q} \to G\,\widetilde{t}\,\overline{\widetilde{t}}$ in 4 dimensions.
Homepage: `http://www.feynarts.de`

`FormCalc`: This is a `Mathematica` package which calculates and simplifies tree-level and one loop Feynman diagrams [34]. It accepts diagrams generated with `FeynArts` and returns the results for further numerical or analytical evaluation. `FormCalc` uses `Form` and `Fortran`.
Homepage: `http://www.feynarts.de/formcalc`

`FeynMF`: This is a macro package for LaTeX and METAPOST/METAFONT which generates Feynman diagrams. Aside from the graphs in section C all Feynman diagrams in this work were drawn with this program.
Homepage: `http://theorie.physik.uni-wuerzburg.de/∼ohl`

B.2 Implementation of the ϵ Scalar in FeynArts and FormCalc

The ϵ scalar was implemented in FeynArts with Feynman rules. It has to be included in the file Lorentz.gen and MSSMQCD.mod, which was for this purpose extended with DMSSMQCD.mod. The kinetic structure is given in Lorentz.gen. DMSSMQCD.mod gives the particle properties and its couplings regarding the coupling constant and the color structure.

Implementation in DMSSMQCD.mod

The particle description is

```
(*-------- epsilon-scalar:Z--------------------*)
Z[5]== {
SelfConjugate -> True,
Indices -> {Index[Gluon]},
Mass -> 0,
PropagatorLabel -> "es",
PropagatorType -> GhostDash,
PropagatorArrow -> None}
(*-------------------------------------------*)
```

Z[5] denotes the ϵ scalar for the gluon, since V[5] denotes the gluon in FeynArts. The parameter SelfConjugate set to True gives a particle which is its own antiparticle. The parameters PropagatorLabel, PropagatorType and PropagatorArrow determine the appearance of the ϵ scalar in the plot output in FeynArts. Though the ϵ scalar is a scalar, it receives Lorentz indices via Indices.
The couplings regarding color structure and coupling constants are shown in the following.

```
(* --------------------------Vector-Z-Z-----------------------------*)
C[ V[5, {g1}], Z[5, {g2}], Z[5, {g3}]]==
GS * { {SUNF[g1, g2, g3]}}
(* ----------------------------------------------------------------*)
```

This is the $V\phi\phi$ coupling as in eq. 4.12. C[a,b,c] marks the coupling of the particles a, b and c. The addition to the particle, e.g. Z[5,{g1}] denotes the particle color. SUNF[a,b,c] are the SU(N) structure constants f^{abc}.
The next vertex denotes the $\widetilde{F}\widetilde{F}\phi\phi$ vertex from eq. 4.14. GS is the strong coupling constant. S[13,{s1, j1, o1}] denotes a squark S with the sfermion type s1=1,2, the generation j1=1...3 and the color index o1=1...3.

```
(*---------------------- squark-squark-Z-Z --------------------------*)

C[ S[13,{s1, j1, o1}], -S[13, {s2, j2, o2}], Z[5,{g1}], Z[5,{g2}]] ==
I GS^2 IndexDelta[s1,s2] IndexDelta[j1, j2]*
{ {SUNT[g1, g2, o2, o1] + SUNT[g2, g1, o2, o1]}}
```

```
C[ S[14,{s1, j1, o1}], -S[14, {s2, j2, o2}], Z[5,{g1}], Z[5,{g2}]] ==
I GS^2 IndexDelta[s1, s2] IndexDelta[j1, j2]*
{ {SUNT[g1, g2, o2, o1] + SUNT[g2, g1, o2, o1]}},
(*--------------------------------------------------------------------*)
```

The following denotes the $VV\phi\phi$ vertex as in eq. 4.14. SUNF$[g_1, g_3, g_2, g_4]$ is short for $\sum_i f^{g_1 g_3 i} f^{i g_2 g_4}$.

```
(*--------------- Z-Z-Vector-Vector-----------------------------------*)

  C[ Z[5, {g1}], Z[5, {g2}], V[5, {g3}], V[5, {g4}] ] == -I GS^2 *
    { { SUNF[g1, g3, g2, g4] - SUNF[g1, g4, g3, g2]}  }
(*--- ----------------------------------------------------------------*)
```

and the $\phi\phi\phi\phi$ coupling as in eq. 4.15

```
(*--------------------Z-Z-Z-Z---------------------------------------*)
  C[ Z[5, {g1}], Z[5, {g2}], Z[5, {g3}], Z[5, {g4}] ] == -I GS^2 *
    { { SUNF[g1, g3, g2, g4] - SUNF[g1, g4, g3, g2]} ,
      { SUNF[g1, g2, g3, g4] + SUNF[g1, g4, g3, g2]},
      {-SUNF[g1, g2, g3, g4] - SUNF[g1, g3, g2, g4]} }
(*------------------------------------------------------------------*)
```

Further explanations for FeynArts and its structure can be found in [33].

Implementation in Lorentz.gen

In Lorentz.gen we first determine the index that is carried by the particle we want to implement. In our case it is

```
(*------------------------------------------------------------------*)
KinematicIndices[ Z ] = {Lorentz};
(*------------------------------------------------------------------*)
```

which means, that the ϵ particle carries a Lorentz index along a propagator line. The propagator was implemented analogous to the vector boson with

```
(*-------------------External-Propagator-----------------------------*)
  AnalyticalPropagator[External][s1 Z[j1, mom, {li2}] ] ==
    PolarizationVector[Z[j1], mom, li2],

(*-------------------Internal-Propagator-----------------------------*)
    AnalyticalPropagator[Internal][s1 Z[j1, mom, {li1} -> {li2}]] ==
    -I PropagatorDenominator[mom, Mass[Z[j1]]] *
```

```
    MetricTensor[li1, li2]
(*-----------------------------------------------------------------*)
```

FeynArts works with Internal and External Propagators. The External Propagator corresponds to the polarization vectors PolarizationVector[Z[j1], mom, li2] with momentum mom and Lorentz index li2. The Internal Propagator changes the Lorentz index li1 to li2 and comes with the same denominator and metric tensor as the gluon.
Now we just have to add the kinematical behaviour of the ϵ scalar for the vertices.

```
(*---------------------------V-Z-Z-------------------------------*)

   AnalyticalCoupling[ s1 V[j1, mom1, {li1}], s2 Z[j2, mom2, {li2}],
 s3 Z[j3, mom3, {li3}] ] ==
G[-1][s1 V[j1], s2 Z[j2], s3 Z[j3]].
      {MetricTensor[li2, li3] FourVector[mom3-mom2, li1]},

(*-----------------------Z-Z-squark-squark-------------------------*)

   AnalyticalCoupling[s1 S[j1, mom1], s2 S[j2, mom2],s3 Z[j3, mom3,
 {li3}], s4 Z[j4, mom4, {li4}]]
   == G[1][ s1 S[j1], s2 S[j2],s3 Z[j3], s4 Z[j4]].
   {MetricTensor[li3, li4]},

(*--------------------------Z-Z-V-V-------------------------------*)

   AnalyticalCoupling[ s1 Z[j1, mom1, {li1}], s2 Z[j2, mom2, {li2}],
      s3 V[j3, mom3, {li3}], s4 V[j4, mom4, {li4}] ] ==
    G[1][s1 Z[j1], s2 Z[j2], s3 V[j3], s4 V[j4]] .
      { MetricTensor[li1, li2] MetricTensor[li3, li4] }

(*--------------v-----------Z-Z-Z-Z-------------------------------*)
   AnalyticalCoupling[ s1 Z[j1, mom1, {li1}], s2 Z[j2, mom2, {li2}],
      s3 V[j3, mom3, {li3}], s4 V[j4, mom4, {li4}] ] ==
    G[1][s1 Z[j1], s2 Z[j2], s3 V[j3], s4 V[j4]] .
      { MetricTensor[li1, li2] MetricTensor[li3, li4] ,
        MetricTensor[li1, li3] MetricTensor[li2, li4],
        MetricTensor[li1, li4] MetricTensor[li3, li2] }
(*--------------------------------------------------------------*)
```

The ϵ scalars are implemented using the same Lorentz indices as the vector bosons and the distinction is only made in the polarization sum. This is possible as long as no loops appear in the calculation.

C Appendix to Section 6.1: $q\,\overline{q} \to G\,\widetilde{t}\,\widetilde{\overline{t}}$

C.1 Squared Amplitudes

Squared Amplitudes for $q\,\overline{q} \to G\,\widetilde{t}\,\widetilde{\overline{t}}$ in 4 Dimensions

$$
\begin{aligned}
&|\mathcal{M}|^2_{q\overline{q}\to G\widetilde{t}\widetilde{\overline{t}},\,\mathrm{phys}} = 256\cdot(3s^2 s_4^2 t_1 u_1 (s+t_1+u_1)^2(s_4+t_1+u_1)^2)^{-1} \times \\
&\big(4st_1u_1(s+t_1+u_1)^2\big(9s_4^2+9(t_1+u_1)s_4+4(t_1+u_1)^2\big)\,m^4 \\
&+\big(2s_4(s_4+t_1+u_1)\big(s_4^2+8t_1s_4-6u_1s_4+7t_1^2+6t_1u_1\big)\,s^4 \\
&+\big((29t_1+11u_1)s_4^4+\big(93t_1^2+94u_1t_1+u_1^2+18(t_1-u_1)u_6\big)\,s_4^3 \\
&+\big(99t_1^3+(213u_1+32u_6)t_1^2+2u_1(43u_1+23u_6)t_1-2u_1^2(5u_1+11u_6)\big)\,s_4^2 \\
&+(t_1+u_1)\big(35t_1^3+(111u_1+14u_6)t_1^2+2u_1(21u_1+23u_6)t_1-4u_1^2u_6\big)\,s_4 \\
&+16t_1u_1(t_1+u_1)^2(t_1+u_6)\big)\,s^3 \\
&+\big(\big(28t_1^2+81u_1t_1+u_1^2\big)\,s_4^4 \\
&+\big(84t_1^3+(302u_1+27u_6)t_1^2+u_1(169u_1+36u_6)t_1-u_1^2(5u_1+27u_6)\big)\,s_4^3 \\
&+\big(84t_1^4+(427u_1+48u_6)t_1^3+u_1(458u_1+215u_6)t_1^2 \\
&+u_1\big(125u_1^2+98u_6u_1+36u_6^2\big)\,t_1 \\
&-3u_1^3(2u_1+11u_6)\big)\,s_4^2+(t_1+u_1)\big(28t_1^4+(226u_1+21u_6)t_1^3 \\
&+2u_1(105u_1+92u_6)t_1^2+u_1\big(44u_1^2+89u_6u_1+36u_6^2\big)\,t_1-6u_1^3u_6\big)\,s_4 \\
&+16t_1u_1(t_1+u_1)^2(t_1+u_6)(3t_1+2u_1+u_6)\big)\,s^2 \\
&+\big(9t_1\big(t_1^2+8u_1t_1+3u_1^2\big)\,s_4^4+\big(25t_1^4+(254u_1+9u_6)t_1^3 \\
&+u_1(245u_1+117u_6)t_1^2+u_1^2(52u_1+27u_6)t_1-9u_1^3u_6\big)\,s_4^3 \\
&+(t_1+u_1)\big(23t_1^4+(313u_1+16u_6)t_1^3+u_1(225u_1+281u_6)t_1^2 \\
&+u_1\big(39u_1^2+74u_6u_1+72u_6^2\big)\,t_1-11u_1^3u_6\big)\,s_4^2 \\
&+(t_1+u_1)^2\big(7t_1^4+(188u_1+7u_6)t_1^3+19u_1(5u_1+12u_6)t_1^2 \\
&+u_1\big(14u_1^2+47u_6u_1+72u_6^2\big)\,t_1-2u_1^3u_6\big)\,s_4 \\
&+16t_1u_1(t_1+u_1)^3(t_1+u_6)(3t_1+u_1+2u_6)\big)\,s \\
&+4t_1u_1\big(9s_4^2+9(t_1+u_1)s_4+4(t_1+u_1)^2\big)\big(s_4t_1+(t_1+u_1)(t_1+u_6)\big)^2\big)\,m^2 \\
&+ss_4(s+t_1+u_1)(s_4+t_1+u_1)\big(2u_6^2+(2s+2s_4+3t_1+u_1)u_6+(s_4+t_1)(s \\
&+s_4+t_1+u_1)\big)\big(9t_1s_4^2+\big(16t_1^2+7u_1t_1+9u_6t_1-9u_1u_6\big)\,s_4+s\big(s_4^2+8t_1s_4 \\
&-6u_1s_4+7t_1^2+6t_1u_1\big)+(t_1+u_1)\big(7t_1^2+6u_1t_1+7u_6t_1-2u_1u_6\big)\big)\big) \qquad \text{(C.1)}
\end{aligned}
$$

Squared Amplitudes for $q\,\overline{q} \to g\,\widetilde{t}\,\widetilde{\overline{t}}$

$$
\begin{aligned}
&|\mathcal{M}|^2_{q\overline{q}\to g\widetilde{t}\widetilde{\overline{t}}} = -(3s_4 s t_1 u_1 (s_4+t_1+u_1)(s+t_1+u_1)^2)^{-1} \times \\
&\big(4\,\big(35u_1t_1^5 + 91u_1^2t_1^4 - Ds_4st_1^4 - 2s_4st_1^4 - 28m^2u_1t_1^4 + 4Ds_4u_1t_1^4 + 82s_4u_1t_1^4 + 70su_1t_1^4 \\
&+119u_1u_6t_1^4 + 77u_1^3t_1^3 - Ds_4s^2t_1^3 - 6s_4s^2t_1^3 - 28m^2u_1^2t_1^3 + 12Ds_4u_1^2t_1^3 + 133s_4u_1^2t_1^3 + 126su_1^2t_1^3 \\
&+140u_1u_6^2t_1^3 - 4Ds_4^2st_1^3 + 26Ds_4^2u_1t_1^3 + 21s_4^2u_1t_1^3 - 64s_4m^2u_1t_1^3 \\
&+35s^2u_1t_1^3 + 2Ds_4su_1t_1^3 + 154s_4su_1t_1^3 + 287u_1^2u_6t_1^3 - Ds_4su_6t_1^3 \\
&-10s_4su_6t_1^3 - 28m^2u_1u_6t_1^3 + 4Ds_4u_1u_6t_1^3 + 259s_4u_1u_6t_1^3 + 196su_1u_6^2t_1^2 \\
&+203su_1u_6t_1^3 + 21u_1^4t_1^2 - 4s_4s^3t_1^2 + 28m^2u_1^3t_1^2 + 12Ds_4u_1^3t_1^2 \\
&+43s_4u_1^3t_1^2 + 56su_1^3t_1^2 + 56u_1u_6^3t_1^2 - 4Ds_4^2s^2t_1^2 - 8s_4^2s^2t_1^2 - 8s_4m^2s^2t_1^2 + 52Ds_4^2u_1^2t_1^2 - 40s_4^2u_1^2t_1^2 \\
&+35s^2u_1^2t_1^2 + 56m^2su_1^2t_1^2 + 7Ds_4su_1^2t_1^2 + 173s_4su_1^2t_1^2 + 308u_1^2u_6^2t_1^2 - 8s_4su_6^2t_1^2 + 256s_4u_1u_6^2t_1^2 \\
&-5Ds_4^3st_1^2 + 6s_4^3st_1^2 + 40Ds_4^3u_1t_1^2 - 62s_4^3u_1t_1^2 - 36s_4^2m^2u_1t_1^2 + 28m^2s^2u_1t_1^2 \\
&+3Ds_4s^2u_1t_1^2 + 66s_4s^2u_1t_1^2 + 14Ds_4^2su_1t_1^2 + 74s_4^2su_1t_1^2 - 4Ds_4m^2su_1t_1^2 \\
&-20s_4m^2su_1t_1^2 + 217u_1^3u_6t_1^2 - 20s_4s^2u_6t_1^2 - 28m^2u_1^2u_6t_1^2 + 12Ds_4u_1^2u_6t_1^2 \\
&+400s_4u_1^2u_6t_1^2 + 336su_1^2u_6t_1^2 - 3Ds_4^2su_6t_1^2 - 14s_4^2su_6t_1^2 + 22Ds_4^2u_1u_6t_1^2 \\
&+138s_4^2u_1u_6t_1^2 - 36s_4m^2u_1u_6t_1^2 + 84s^2u_1u_6t_1^2 + 28m^2su_1u_6t_1^2 + 2Ds_4su_1u_6t_1^2 \\
&+361s_4su_1u_6t_1^2 + 28m^2u_1^4t_1 + 4Ds_4u_1^4t_1 - 8s_4u_1^4t_1 - 8s_4^2s^3t_1 - 8s_4m^2s^3t_1 \\
&+26Ds_4^2u_1^3t_1 - 52s_4^2u_1^3t_1 + 64s_4m^2u_1^3t_1 + 56m^2su_1^3t_1 + 4Ds_4su_1^3t_1 + 31s_4su_1^3t_1 + 112u_1^2u_6^3t_1 \\
&+72s_4u_1u_6^3t_1 + 56su_1u_6^3t_1 - 5Ds_4^3s^2t_1 + 2s_4^3s^2t_1 - 8s_4^2m^2s^2t_1 + 40Ds_4^3u_1^2t_1 \\
&-80s_4^3u_1^2t_1 + 36s_4^2m^2u_1^2t_1 + 28m^2s^2u_1^2t_1 + 4Ds_4s^2u_1^2t_1 + 35s_4s^2u_1^2t_1 + 30Ds_4^2su_1^2t_1 - 5s_4^2su_1^2t_1 \\
&+12Ds_4m^2su_1^2t_1 - 24s_4m^2su_1^2t_1 + 196u_1^3u_6^2t_1 - 8s_4s^2u_6^2t_1 + 348s_4u_1^2u_6^2t_1 \\
&+280su_1^2u_6^2t_1 - 8s_4^2su_6^2t_1 + 108s_4^2u_1u_6^2t_1 + 56s^2u_1u_6^2t_1 + 292s_4su_1u_6^2t_1 - 2Ds_4^4st_1 \\
&+4s_4^4st_1 + 18Ds_4^4u_1t_1 - 36s_4^4u_1t_1 + 4s_4s^3u_1t_1 + 13Ds_4^2s^2u_1t_1 + 7s_4^2s^2u_1t_1 \\
&-8s_4m^2s^2u_1t_1 + 10Ds_4^3su_1t_1 - 6s_4^3su_1t_1 - 4Ds_4^2m^2su_1t_1 \\
&-20s_4^2m^2su_1t_1 + 49u_1^4u_6t_1 - 8s_4s^3u_6t_1 + 28m^2u_1^3u_6t_1 \\
&+12Ds_4u_1^3u_6t_1 + 133s_4u_1^3u_6t_1 + 133su_1^3u_6t_1 \\
&-28s_4^2s^2u_6t_1 + 44Ds_4^2u_1^2u_6t_1 + 58s_4^2u_1^2u_6t_1 \\
&+84s^2u_1^2u_6t_1 + 112m^2su_1^2u_6t_1 + 7Ds_4su_1^2u_6t_1 + 321s_4su_1^2u_6t_1 \\
&-2Ds_4^3su_6t_1 - 4s_4^3su_6t_1 + 18Ds_4^3u_1u_6t_1 + 56m^2s^2u_1u_6t_1 \\
&+128s_4s^2u_1u_6t_1 + 19Ds_4^2su_1u_6t_1 + 110s_4^2su_1u_6t_1 + 36s_4m^2su_1u_6t_1 -
\end{aligned}
$$

$$
\begin{aligned}
&-4s_4^3s^3 - 8s_4^2m^2s^3 + 12Ds_4^2su_1^3 - 24s_4^2su_1^3 + 16Ds_4m^2su_1^3 - 60s_4m^2su_1^3 + 56u_1^3u_6^3\\
&+72s_4u_1^2u_6^3 + 56su_1^2u_6^3 + 72s_4su_1u_6^3 - 2Ds_4^4s^2 + 4s_4^4s^2 + 12Ds_4^2s^2u_1^2 - 20s_4^2s^2u_1^2\\
&-20s_4m^2s^2u_1^2 + 10Ds_4^3su_1^2 - 20s_4^3su_1^2 + 16Ds_4^2m^2su_1^2 - 68s_4^2m^2su_1^2\\
&+28u_1^4u_6^2 + 92s_4u_1^3u_6^2 + 84su_1^3u_6^2 - 8s_4^2s^2u_6^2 + 72s_4^2u_1^2u_6^2 + 56s^2u_1^2u_6^2\\
&+172s_4su_1^2u_6^2 + 80s_4s^2u_1u_6^2 + 64s_4^2su_1u_6^2 + 4s_4^2s^3u_1 + 8s_4m^2s^3u_1 + 10Ds_4^3s^2u_1\\
&-24s_4^3s^2u_1 - 44s_4^2m^2s^2u_1 - 2Ds_4^4su_1 + 4s_4^4su_1 + 28m^2u_1^4u_6 + 4Ds_4u_1^4u_6\\
&-8s_4u_1^4u_6 - 8s_4^2s^3u_6 + 22Ds_4^2u_1^3u_6 - 44s_4^2u_1^3u_6 + 36s_4m^2u_1^3u_6 + 84m^2su_1^3u_6\\
&+4Ds_4su_1^3u_6 + 24s_4su_1^3u_6 - 8s_4^3s^2u_6 + 18Ds_4^3u_1^2u_6 - 36s_4^3u_1^2u_6 + 56m^2s^2u_1^2u_6\\
&+40s_4s^2u_1^2u_6 + 22Ds_4^2su_1^2u_6 - 4s_4^2su_1^2u_6 + 108s_4m^2su_1^2u_6 + 8s_4s^3u_1u_6\\
&+32s_4^2s^2u_1u_6 + 72s_4m^2s^2u_1u_6 + 18Ds_4^3su_1u_6 - 44s_4^3su_1u_6\big)\big)
\end{aligned}
\tag{C.2}
$$

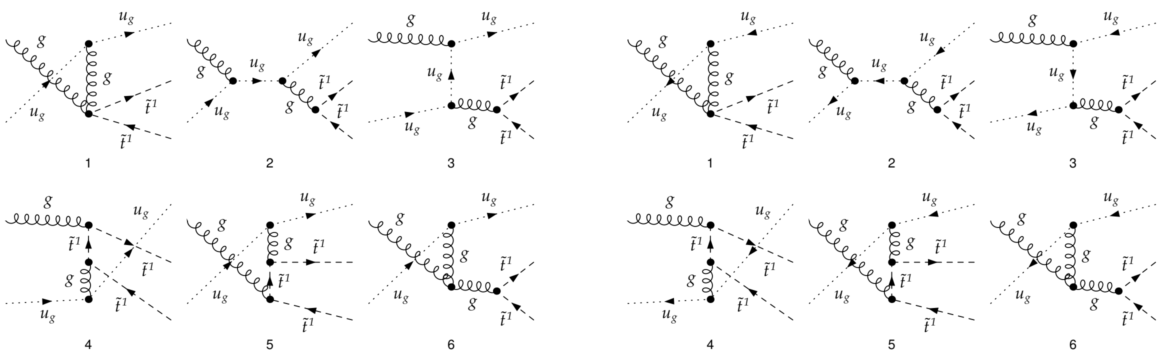
Squared Amplitudes for $q\,\overline{q} \to \phi\,\widetilde{t}\,\overline{\widetilde{t}}$

$$
\begin{aligned}
&|\mathcal{M}|^2_{q\overline{q}\to\phi\widetilde{t}\overline{\widetilde{t}}} = (3st_1u_1(s+t_1+u_1)^2)^{-1} \times\\
&4(4-D)\big(st_1^3 - 4u_1t_1^3 + s^2t_1^2 - 8u_1^2t_1^2 + 3s_4st_1^2 - 22s_4u_1t_1^2 - 3su_1t_1^2\\
&+su_6t_1^2 - 4u_1u_6t_1^2 - 4u_1^3t_1 + 3s_4s^2t_1 - 22s_4u_1^2t_1 - 4su_1^2t_1 + 2s_4^2st_1 - 18s_4^2u_1t_1 -\\
&4s^2u_1t_1 + 4m^2su_1t_1 - 14s_4su_1t_1 - 8u_1^2u_6t_1 + 2s_4su_6t_1\\
&-18s_4u_1u_6t_1 - 3su_1u_6t_1 + 2s_4^2s^2 - 16m^2su_1^2 - 12s_4su_1^2 - 12s_4s^2u_1 + 2s_4^2su_1\\
&-4u_1^3u_6 - 18s_4u_1^2u_6 - 4su_1^2u_6 - 18s_4su_1u_6\big)
\end{aligned}
\tag{C.3}
$$

The Mandelstam variables for 5 external legs are defined in eq. A.4.

D Appendix to Section 6.2: $GG \to G\tilde{t}\bar{\tilde{t}}$

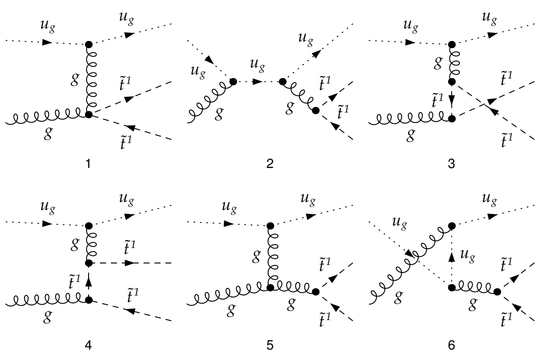
D.1 Feynman Diagrams



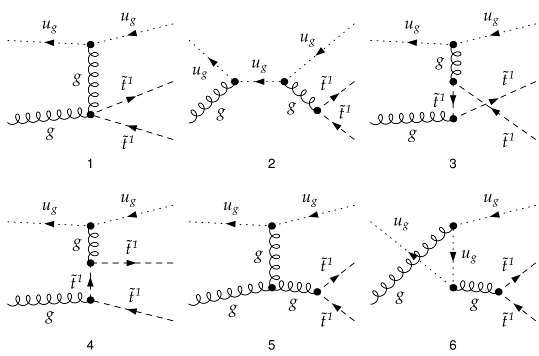
(1) $G\eta \to \eta\tilde{t}\bar{\tilde{t}}$

(2) $G\overline{\eta} \to \overline{\eta}\tilde{t}\bar{\tilde{t}}$

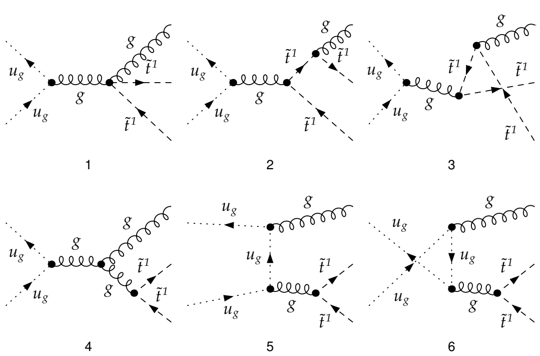
Figure D.1: Feynman Diagrams involving BRST ghosts for $GG \to G\tilde{t}\bar{\tilde{t}}$



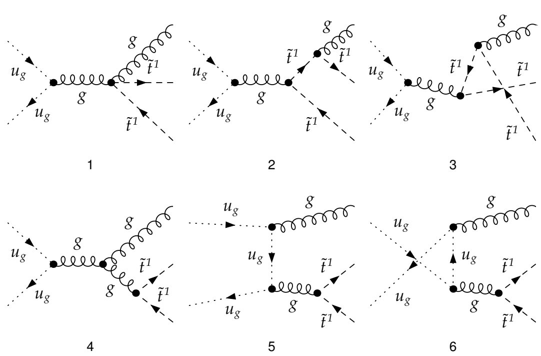
(3) $\eta G \to \eta \tilde{t}\bar{\tilde{t}}$



(4) $\overline{\eta} G \to \overline{\eta} \tilde{t}\bar{\tilde{t}}$



(5) $\overline{\eta}\eta \to G \tilde{t}\bar{\tilde{t}}$



(6) $\eta\overline{\eta} \to G \tilde{t}\bar{\tilde{t}}$

Figure D.2: Feynman Diagrams involving BRST ghosts $GG \to G \tilde{t}\bar{\tilde{t}}$

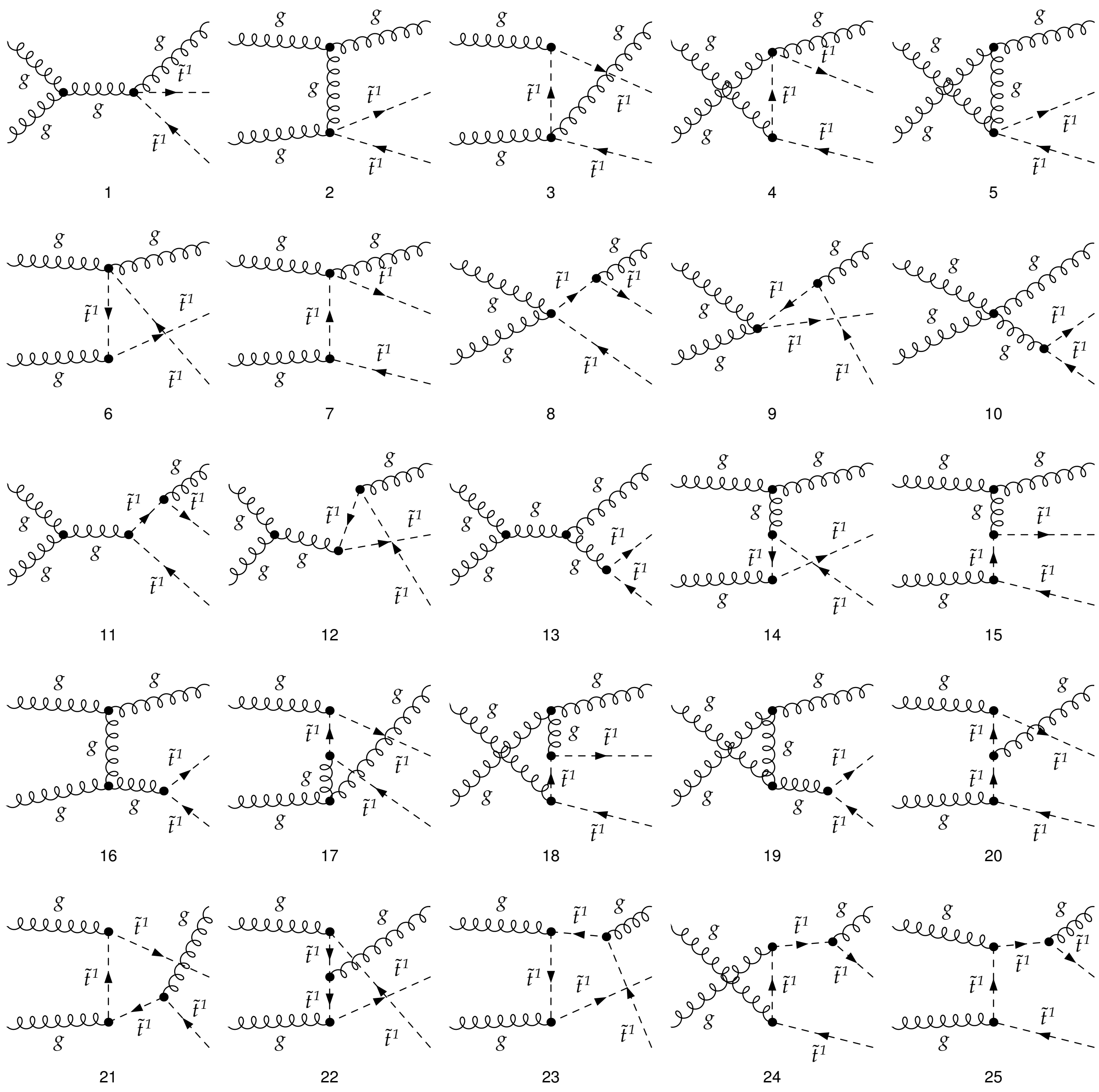


Figure D.3: All 25 graphs for the process $GG \to G\widetilde{t}\widetilde{\bar{t}}$. The graphs with number 5, 17, 18, 19 contain the propagator which diverges for $k_2 = k_3$. The momenta are denoted as in fig. 6.4.

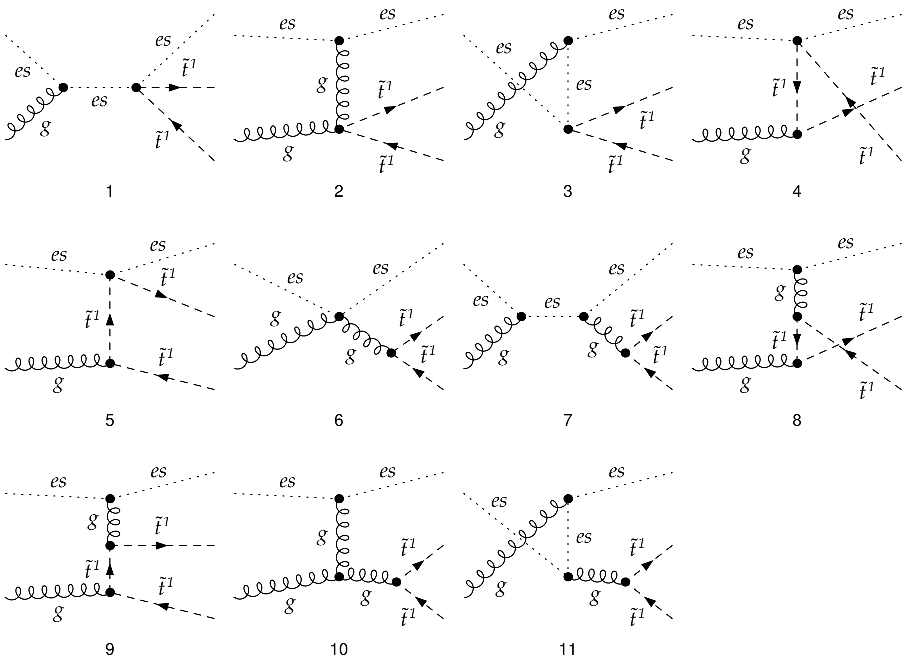


Figure D.4: $\phi\, g \to \phi\, \tilde{t}\bar{\tilde{t}}$

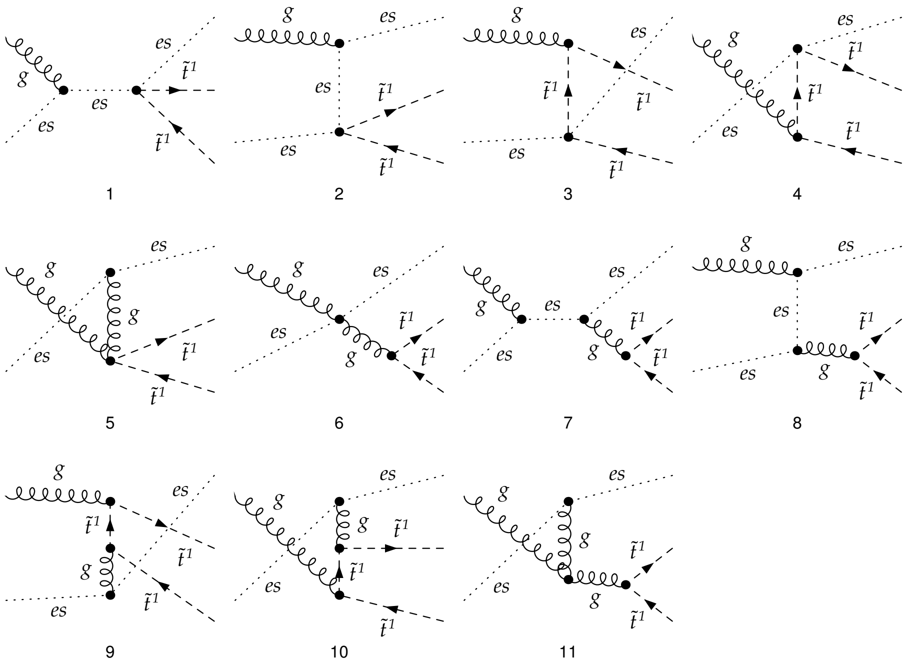


Figure D.5: $g\,\phi \to \phi\,\widetilde{t}\,\widetilde{\bar{t}}$

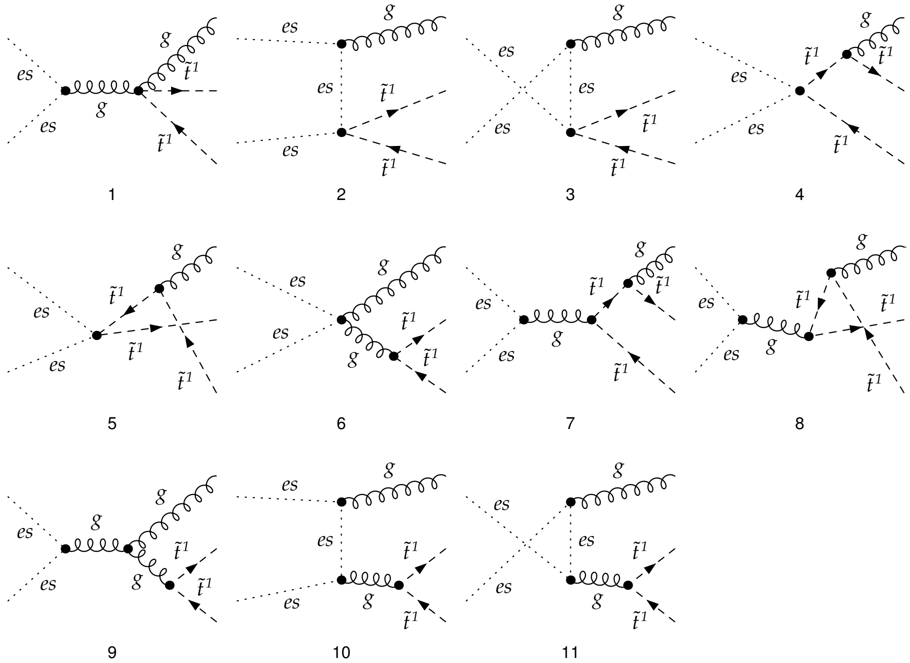


Figure D.6: $\phi\,\phi \rightarrow g\,\tilde{t}\,\bar{\tilde{t}}$

D.2 Feynman Amplitudes

The momenta and Lorentz/color indices for the following amplitudes appear as in fig. D.7.

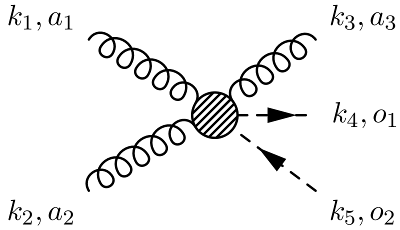


Figure D.7: Denotation of the momenta, Lorentz and color indices.

Amplitudes for $GG \to G\tilde{t}\bar{\tilde{t}}$

$$\begin{aligned}
\mathcal{M}_1 \;&=\; \frac{2\alpha_s g_s \pi}{k_1 \cdot k_2} \times \\
&((t^{a_1}t^{a_2}t^{a_3})_{o_1 o_2} - (t^{a_2}t^{a_1}t^{a_3})_{o1o2} + (t^{a_3}t^{a_1}t^{a_2})_{o_1 o_2} - (t^{a_3}t^{a_2}t^{a_1})_{o_1 o_2}) \\
&((2k_1 \cdot \epsilon^*(k_3) - k_3 \cdot \epsilon^*(k_3))\epsilon(k_1) \cdot \epsilon(k_2) - (k_1 \cdot \epsilon(k_2) + k_3 \cdot \epsilon(k_2))\epsilon(k_1) \cdot \epsilon^*(k_3) \\
&-(k_1 \cdot \epsilon(k_1) - 2k_3 \cdot \epsilon(k_1))\epsilon(k_2) \cdot \epsilon^*(k_3))
\end{aligned}$$

$$\begin{aligned}
\mathcal{M}_2 \;&=\; \frac{2\alpha_s g_s \pi}{k_1 \cdot k_3} \times \\
&((t^{a_1}t^{a_3}t^{a_2})_{o_1 o_2} + (t^{a_2}t^{a_1}t^{a_3})_{o_1 o_2} - (t^{a_2}t^{a_3}t^{a_1})_{o_1 o_2} - (t^{a_3}t^{a_1}t^{a_2})_{o_1 o_2}) \\
&((2k_1 \cdot \epsilon^*(k_3) - k_3 \cdot \epsilon^*(k_3))\epsilon(k_1) \cdot \epsilon(k_2) - (k_1 \cdot \epsilon(k_2) + k_3 \cdot \epsilon(k_2))\epsilon(k_1) \cdot \epsilon^*(k_3) \\
&-(k_1 \cdot \epsilon(k_1) - 2k_3 \cdot \epsilon(k_1))\epsilon(k_2) \cdot \epsilon^*(k_3))
\end{aligned}$$

$$\begin{aligned}
\mathcal{M}_3 \;&=\; \frac{2\alpha_s g_s \pi}{k_1 \cdot k_4} \times \\
&((t^{a_1}t^{a_2}t^{a_3})_{o_1 o_2})) + (t^{a_1}t^{a_3}t^{a_2})_{o_1 o_2})))(k_1 \cdot \epsilon(k_1) - 2k_4 \cdot \epsilon(k_1))\epsilon(k_2) \cdot \epsilon^*(k_3)
\end{aligned}$$

$$\begin{aligned}
\mathcal{M}_4 \;&=\; \frac{2\alpha_s g_s \pi}{k_1 \cdot k_5} \times \\
&(t^{a_2}t^{a_3}t^{a_1})_{o_1 o_2}) + (t^{a_3}t^{a_2}t^{a_1})_{o_1 o_2})) \\
&(k_1 \cdot \epsilon(k_1) + 2k_2 \cdot \epsilon(k_1) - 2k_3 \cdot \epsilon(k_1) - 2k_4 \cdot \epsilon(k_1))\epsilon(k_2) \cdot \epsilon^*(k_3)
\end{aligned}$$

$$\begin{aligned}
\mathcal{M}_5 \;&=\; \frac{2\alpha_s g_s \pi}{k_2 \cdot k_3} \times \\
&((t^{a_1}t^{a_2}t^{a_3})_{o_1 o_2}) - (t^{a_1}t^{a_3}t^{a_2})_{o_1 o_2}) + (t^{a_2}t^{a_3}t^{a_1})_{o_1 o_2}) - (t^{a_3}t^{a_2}t^{a_1})_{o_1 o_2})) \\
&((2k_2 \cdot \epsilon^*(k_3) - k_3 \cdot \epsilon^*(k_3))\epsilon(k_1) \cdot \epsilon(k_2) - (k_2 \cdot \epsilon(k_2) - \\
&2k_3 \cdot \epsilon(k_2))\epsilon(k_1) \cdot \epsilon^*(k_3) - (k_2 \cdot \epsilon(k_1) + k_3 \cdot \epsilon(k_1))\epsilon(k_2) \cdot \epsilon^*(k_3))
\end{aligned}$$

$$
\begin{aligned}
\mathcal{M}_6 &= \frac{2\alpha_s g_s \pi}{k_2 \cdot k_4} \times \\
&\quad ((t^{a_2}t^{a_1}t^{a_3})_{o_1o_2}) + (t^{a_2}t^{a_3}t^{a_1})_{o_1o_2})) \\
&\quad (k_2 \cdot \epsilon(k_2) - 2k_4 \cdot \epsilon(k_2))\epsilon(k_1) \cdot \epsilon^*(k_3) \\
\mathcal{M}_7 &= \frac{2\alpha_s g_s \pi}{k_2 \cdot k_5} \times \\
&\quad ((t^{a_1}t^{a_3}t^{a_2})_{o_1o_2}) + (t^{a_3}t^{a_1}t^{a_2})_{o_1o_2})) \\
&\quad (2k_1 \cdot \epsilon(k_2) + k_2 \cdot \epsilon(k_2) - 2(k_3 \cdot \epsilon(k_2) + k_4 \cdot \epsilon(k_2)))\epsilon(k_1) \cdot \epsilon^*(k_3) \\
\mathcal{M}_8 &= \frac{2\alpha_s g_s \pi}{k_3 \cdot k_4} \times \\
&\quad ((t^{a_3}t^{a_1}t^{a_2})_{o_1o_2}) + (t^{a_3}t^{a_2}t^{a_1})_{o_1o_2})) \\
&\quad (k_3 \cdot \epsilon^*(k_3) + 2k_4 \cdot \epsilon^*(k_3))\epsilon(k_1) \cdot \epsilon(k_2) \\
\mathcal{M}_9 &= -\frac{2\alpha_s g_s \pi}{k_3 \cdot k_5} \times \\
&\quad ((t^{a_1}t^{a_2}t^{a_3})_{o_1o_2}) + (t^{a_2}t^{a_1}t^{a_3})_{o_1o_2})) \\
&\quad (2k_1 \cdot \epsilon^*(k_3) + 2k_2 \cdot \epsilon^*(k_3) - k_3 \cdot \epsilon^*(k_3) - 2k_4 \cdot \epsilon^*(k_3))\epsilon(k_1) \cdot \epsilon(k_2) \\
\mathcal{M}_{11} &= \frac{\alpha_s g_s \pi}{k_1 \cdot k_2 k_3 \cdot k_4} \times \\
&\quad ((t^{a_3}t^{a_1}t^{a_2})_{o_1o_2}) - (t^{a_3}t^{a_2}t^{a_1})_{o_1o_2})) \\
&\quad (k_3 \cdot \epsilon^*(k_3) + 2k_4 \cdot \epsilon^*(k_3)) \\
&\quad (2(2k_1 \cdot \epsilon(k_2) + k_2 \cdot \epsilon(k_2))(k_3 \cdot \epsilon(k_1) + k_4 \cdot \epsilon(k_1)) \\
&\quad + k_2 \cdot \epsilon(k_1)(k_2 \cdot \epsilon(k_2) - 4(k_3 \cdot \epsilon(k_2) + k_4 \cdot \epsilon(k_2))) - k_1 \cdot \epsilon(k_1)(k_1 \cdot \epsilon(k_2) \\
&\quad + 2(k_3 \cdot \epsilon(k_2) + k_4 \cdot \epsilon(k_2))) + 2(-k_1 \cdot k_3 - k_1 \cdot k_4 + k_2 \cdot k_3 + k_2 \cdot k_4)\epsilon(k_1) \cdot \epsilon(k_2)) \\
\mathcal{M}_{12} &= \frac{\alpha_s g_s \pi}{k_1 \cdot k_2 k_3 \cdot k_5} \times \\
&\quad ((t^{a_1}t^{a_2}t^{a_3})_{o_1o_2}) - (t^{a_2}t^{a_1}t^{a_3})_{o_1o_2})) \\
&\quad (2k_1 \cdot \epsilon^*(k_3) + 2k_2 \cdot \epsilon^*(k_3) - k_3 \cdot \epsilon^*(k_3) - 2k_4 \cdot \epsilon^*(k_3)) \\
&\quad (-2(2k_1 \cdot \epsilon(k_2) + k_2 \cdot \epsilon(k_2))k_4 \cdot \epsilon(k_1) - \\
&\quad k_2 \cdot \epsilon(k_1)(k_2 \cdot \epsilon(k_2) - 4k_4 \cdot \epsilon(k_2)) \\
&\quad + k_1 \cdot \epsilon(k_1)(k_1 \cdot \epsilon(k_2) + 2k_4 \cdot \epsilon(k_2)) + 2(k_1 \cdot k_4 - k_2 \cdot k_4)\epsilon(k_1) \cdot \epsilon(k_2))
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{M}_{10} \;=\; & -\frac{2\alpha_s g_s \pi}{m^2 + k_4 \cdot k_5} \times \\
& ((t^{a_1}t^{a_2}t^{a_3})_{o_1o_2})k_1 \cdot \epsilon^*(k_3)\epsilon(k_1) \cdot \epsilon(k_2) - 2(t^{a_3}t^{a_2}t^{a_1})_{o_1o_2})k_4 \cdot \epsilon(k_1)\epsilon(k_2) \cdot \epsilon^*(k_3) \\
& -2(t^{a_1}t^{a_3}t^{a_2})_{o_1o_2})k_1 \cdot \epsilon^*(k_3)\epsilon(k_1) \cdot \epsilon(k_2) + (t^{a_2}t^{a_1}t^{a_3})_{o_1o_2})k_1 \cdot \epsilon^*(k_3)\epsilon(k_1) \cdot \epsilon(k_2) \\
& -2(t^{a_2}t^{a_3}t^{a_1})_{o_1o_2})k_1 \cdot \epsilon^*(k_3)\epsilon(k_1) \cdot \epsilon(k_2) + (t^{a_3}t^{a_1}t^{a_2})_{o_1o_2})k_1 \cdot \epsilon^*(k_3)\epsilon(k_1) \cdot \epsilon(k_2) \\
& +(t^{a_3}t^{a_2}t^{a_1})_{o_1o_2})k_1 \cdot \epsilon^*(k_3)\epsilon(k_1) \cdot \epsilon(k_2) + (t^{a_1}t^{a_2}t^{a_3})_{o_1o_2})k_2 \cdot \epsilon^*(k_3)\epsilon(k_1) \cdot \epsilon(k_2) \\
& -2(t^{a_1}t^{a_3}t^{a_2})_{o_1o_2})k_2 \cdot \epsilon^*(k_3)\epsilon(k_1) \cdot \epsilon(k_2) + (t^{a_2}t^{a_1}t^{a_3})_{o_1o_2})k_2 \cdot \epsilon^*(k_3)\epsilon(k_1) \cdot \epsilon(k_2) \\
& -2(t^{a_2}t^{a_3}t^{a_1})_{o_1o_2})k_2 \cdot \epsilon^*(k_3)\epsilon(k_1) \cdot \epsilon(k_2) + (t^{a_3}t^{a_1}t^{a_2})_{o_1o_2})k_2 \cdot \epsilon^*(k_3)\epsilon(k_1) \cdot \epsilon(k_2) \\
& +(t^{a_3}t^{a_2}t^{a_1})_{o_1o_2})k_2 \cdot \epsilon^*(k_3)\epsilon(k_1) \cdot \epsilon(k_2) - (t^{a_1}t^{a_2}t^{a_3})_{o_1o_2})k_3 \cdot \epsilon^*(k_3)\epsilon(k_1) \cdot \epsilon(k_2) \\
& +2(t^{a_1}t^{a_3}t^{a_2})_{o_1o_2})k_3 \cdot \epsilon^*(k_3)\epsilon(k_1) \cdot \epsilon(k_2) - (t^{a_2}t^{a_1}t^{a_3})_{o_1o_2})k_3 \cdot \epsilon^*(k_3)\epsilon(k_1) \cdot \epsilon(k_2) \\
& +2(t^{a_2}t^{a_3}t^{a_1})_{o_1o_2})k_3 \cdot \epsilon^*(k_3)\epsilon(k_1) \cdot \epsilon(k_2) - (t^{a_3}t^{a_1}t^{a_2})_{o_1o_2})k_3 \cdot \epsilon^*(k_3)\epsilon(k_1) \cdot \epsilon(k_2) \\
& -(t^{a_3}t^{a_2}t^{a_1})_{o_1o_2})k_3 \cdot \epsilon^*(k_3)\epsilon(k_1) \cdot \epsilon(k_2) - 2(t^{a_1}t^{a_2}t^{a_3})_{o_1o_2})k_4 \cdot \epsilon^*(k_3)\epsilon(k_1) \cdot \epsilon(k_2) \\
& +4(t^{a_1}t^{a_3}t^{a_2})_{o_1o_2})k_4 \cdot \epsilon^*(k_3)\epsilon(k_1) \cdot \epsilon(k_2) - 2(t^{a_2}t^{a_1}t^{a_3})_{o_1o_2})k_4 \cdot \epsilon^*(k_3)\epsilon(k_1) \cdot \epsilon(k_2) \\
& +4(t^{a_2}t^{a_3}t^{a_1})_{o_1o_2})k_4 \cdot \epsilon^*(k_3)\epsilon(k_1) \cdot \epsilon(k_2) - 2(t^{a_3}t^{a_1}t^{a_2})_{o_1o_2})k_4 \cdot \epsilon^*(k_3)\epsilon(k_1) \cdot \epsilon(k_2) \\
& -2(t^{a_3}t^{a_2}t^{a_1})_{o_1o_2})k_4 \cdot \epsilon^*(k_3)\epsilon(k_1) \cdot \epsilon(k_2) - 2(t^{a_1}t^{a_2}t^{a_3})_{o_1o_2})k_1 \cdot \epsilon(k_2)\epsilon(k_1) \cdot \epsilon^*(k_3) \\
& +(t^{a_1}t^{a_3}t^{a_2})_{o_1o_2})k_1 \cdot \epsilon(k_2)\epsilon(k_1) \cdot \epsilon^*(k_3) + (t^{a_2}t^{a_1}t^{a_3})_{o_1o_2})k_1 \cdot \epsilon(k_2)\epsilon(k_1) \cdot \epsilon^*(k_3) \\
& +(t^{a_2}t^{a_3}t^{a_1})_{o_1o_2})k_1 \cdot \epsilon(k_2)\epsilon(k_1) \cdot \epsilon^*(k_3) + (t^{a_3}t^{a_1}t^{a_2})_{o_1o_2})k_1 \cdot \epsilon(k_2)\epsilon(k_1) \cdot \epsilon^*(k_3) \\
& -2(t^{a_3}t^{a_2}t^{a_1})_{o_1o_2})k_1 \cdot \epsilon(k_2)\epsilon(k_1) \cdot \epsilon^*(k_3) - 2(t^{a_1}t^{a_2}t^{a_3})_{o_1o_2})k_2 \cdot \epsilon(k_2)\epsilon(k_1) \cdot \epsilon^*(k_3) \\
& +(t^{a_1}t^{a_3}t^{a_2})_{o_1o_2})k_2 \cdot \epsilon(k_2)\epsilon(k_1) \cdot \epsilon^*(k_3) + (t^{a_2}t^{a_1}t^{a_3})_{o_1o_2})k_2 \cdot \epsilon(k_2)\epsilon(k_1) \cdot \epsilon^*(k_3) \\
& +(t^{a_2}t^{a_3}t^{a_1})_{o_1o_2})k_2 \cdot \epsilon(k_2)\epsilon(k_1) \cdot \epsilon^*(k_3) + (t^{a_3}t^{a_1}t^{a_2})_{o_1o_2})k_2 \cdot \epsilon(k_2)\epsilon(k_1) \cdot \epsilon^*(k_3) \\
& -2(t^{a_3}t^{a_2}t^{a_1})_{o_1o_2})k_2 \cdot \epsilon(k_2)\epsilon(k_1) \cdot \epsilon^*(k_3) + 2(t^{a_1}t^{a_2}t^{a_3})_{o_1o_2})k_3 \cdot \epsilon(k_2)\epsilon(k_1) \cdot \epsilon^*(k_3) \\
& -(t^{a_1}t^{a_3}t^{a_2})_{o_1o_2})k_3 \cdot \epsilon(k_2)\epsilon(k_1) \cdot \epsilon^*(k_3) - (t^{a_2}t^{a_1}t^{a_3})_{o_1o_2})k_3 \cdot \epsilon(k_2)\epsilon(k_1) \cdot \epsilon^*(k_3) \\
& -(t^{a_2}t^{a_3}t^{a_1})_{o_1o_2})k_3 \cdot \epsilon(k_2)\epsilon(k_1) \cdot \epsilon^*(k_3) - (t^{a_3}t^{a_1}t^{a_2})_{o_1o_2})k_3 \cdot \epsilon(k_2)\epsilon(k_1) \cdot \epsilon^*(k_3) \\
& +2(t^{a_3}t^{a_2}t^{a_1})_{o_1o_2})k_3 \cdot \epsilon(k_2)\epsilon(k_1) \cdot \epsilon^*(k_3) + 4(t^{a_1}t^{a_2}t^{a_3})_{o_1o_2})k_4 \cdot \epsilon(k_2)\epsilon(k_1) \cdot \epsilon^*(k_3) \\
& -2(t^{a_1}t^{a_3}t^{a_2})_{o_1o_2})k_4 \cdot \epsilon(k_2)\epsilon(k_1) \cdot \epsilon^*(k_3) - 2(t^{a_2}t^{a_1}t^{a_3})_{o_1o_2})k_4 \cdot \epsilon(k_2)\epsilon(k_1) \cdot \epsilon^*(k_3) \\
& -2(t^{a_2}t^{a_3}t^{a_1})_{o_1o_2})k_4 \cdot \epsilon(k_2)\epsilon(k_1) \cdot \epsilon^*(k_3) - 2(t^{a_3}t^{a_1}t^{a_2})_{o_1o_2})k_4 \cdot \epsilon(k_2)\epsilon(k_1) \cdot \epsilon^*(k_3) \\
& +4(t^{a_3}t^{a_2}t^{a_1})_{o_1o_2})k_4 \cdot \epsilon(k_2)\epsilon(k_1) \cdot \epsilon^*(k_3) + (t^{a_1}t^{a_2}t^{a_3})_{o_1o_2})k_1 \cdot \epsilon(k_1)\epsilon(k_2) \cdot \epsilon^*(k_3) \\
& +(t^{a_1}t^{a_3}t^{a_2})_{o_1o_2})k_1 \cdot \epsilon(k_1)\epsilon(k_2) \cdot \epsilon^*(k_3) - 2(t^{a_2}t^{a_1}t^{a_3})_{o_1o_2})k_1 \cdot \epsilon(k_1)\epsilon(k_2) \cdot \epsilon^*(k_3) \\
& +(t^{a_2}t^{a_3}t^{a_1})_{o_1o_2})k_1 \cdot \epsilon(k_1)\epsilon(k_2) \cdot \epsilon^*(k_3) - 2(t^{a_3}t^{a_1}t^{a_2})_{o_1o_2})k_1 \cdot \epsilon(k_1)\epsilon(k_2) \cdot \epsilon^*(k_3) \\
& +(t^{a_3}t^{a_2}t^{a_1})_{o_1o_2})k_1 \cdot \epsilon(k_1)\epsilon(k_2) \cdot \epsilon^*(k_3) + (t^{a_1}t^{a_2}t^{a_3})_{o_1o_2})k_2 \cdot \epsilon(k_1)\epsilon(k_2) \cdot \epsilon^*(k_3) \\
& +(t^{a_1}t^{a_3}t^{a_2})_{o_1o_2})k_2 \cdot \epsilon(k_1)\epsilon(k_2) \cdot \epsilon^*(k_3) - 2(t^{a_2}t^{a_1}t^{a_3})_{o_1o_2})k_2 \cdot \epsilon(k_1)\epsilon(k_2) \cdot \epsilon^*(k_3) \\
& +(t^{a_2}t^{a_3}t^{a_1})_{o_1o_2})k_2 \cdot \epsilon(k_1)\epsilon(k_2) \cdot \epsilon^*(k_3) - 2(t^{a_3}t^{a_1}t^{a_2})_{o_1o_2})k_2 \cdot \epsilon(k_1)\epsilon(k_2) \cdot \epsilon^*(k_3) \\
& +(t^{a_3}t^{a_2}t^{a_1})_{o_1o_2})k_2 \cdot \epsilon(k_1)\epsilon(k_2) \cdot \epsilon^*(k_3) - (t^{a_1}t^{a_2}t^{a_3})_{o_1o_2})k_3 \cdot \epsilon(k_1)\epsilon(k_2) \cdot \epsilon^*(k_3) \\
& -(t^{a_1}t^{a_3}t^{a_2})_{o_1o_2})k_3 \cdot \epsilon(k_1)\epsilon(k_2) \cdot \epsilon^*(k_3) + 2(t^{a_2}t^{a_1}t^{a_3})_{o_1o_2})k_3 \cdot \epsilon(k_1)\epsilon(k_2) \cdot \epsilon^*(k_3) \\
& -(t^{a_2}t^{a_3}t^{a_1})_{o_1o_2})k_3 \cdot \epsilon(k_1)\epsilon(k_2) \cdot \epsilon^*(k_3) + 2(t^{a_3}t^{a_1}t^{a_2})_{o_1o_2})k_3 \cdot \epsilon(k_1)\epsilon(k_2) \cdot \epsilon^*(k_3) \\
& -(t^{a_3}t^{a_2}t^{a_1})_{o_1o_2})k_3 \cdot \epsilon(k_1)\epsilon(k_2) \cdot \epsilon^*(k_3) - 2(t^{a_1}t^{a_2}t^{a_3})_{o_1o_2})k_4 \cdot \epsilon(k_1)\epsilon(k_2) \cdot \epsilon^*(k_3) \\
& -2(t^{a_1}t^{a_3}t^{a_2})_{o_1o_2})k_4 \cdot \epsilon(k_1)\epsilon(k_2) \cdot \epsilon^*(k_3) + 4(t^{a_3}t^{a_1}t^{a_2})_{o_1o_2})k_4 \cdot \epsilon(k_1)\epsilon(k_2) \cdot \epsilon^*(k_3) \\
& +4(t^{a_2}t^{a_1}t^{a_3})_{o_1o_2})k_4 \cdot \epsilon(k_1)\epsilon(k_2) \cdot \epsilon^*(k_3) - 2(t^{a_2}t^{a_3}t^{a_1})_{o_1o_2})k_4 \cdot \epsilon(k_1)\epsilon(k_2) \cdot \epsilon^*(k_3))
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{M}_{13} \;=\; & \frac{\alpha_s g_s \pi}{k_1 \cdot k_2 \,(m^2 + k_4 \cdot k_5)} \times \\
& ((t^{a_1} t^{a_2} t^{a_3})_{o_1 o_2}) - (t^{a_2} t^{a_1} t^{a_3})_{o_1 o_2}) - (t^{a_3} t^{a_1} t^{a_2})_{o_1 o_2}) + (t^{a_3} t^{a_2} t^{a_1})_{o_1 o_2})) \\
& (k_1 \cdot \epsilon(k_1) k_1 \cdot \epsilon(k_2) k_1 \cdot \epsilon^*(k_3) - k_2 \cdot \epsilon(k_1) k_2 \cdot \epsilon(k_2) k_1 \cdot \epsilon^*(k_3) \\
& -8 k_1 \cdot \epsilon(k_2) k_4 \cdot \epsilon(k_1) k_1 \cdot \epsilon^*(k_3) - 4 k_2 \cdot \epsilon(k_2) k_4 \cdot \epsilon(k_1) k_1 \cdot \epsilon^*(k_3) \\
& +4 k_1 \cdot \epsilon(k_1) k_4 \cdot \epsilon(k_2) k_1 \cdot \epsilon^*(k_3) + 8 k_2 \cdot \epsilon(k_1) k_4 \cdot \epsilon(k_2) k_1 \cdot \epsilon^*(k_3) \\
& +2 k_1 \cdot k_2 \epsilon(k_1) \cdot \epsilon(k_2) k_1 \cdot \epsilon^*(k_3) + 2 k_1 \cdot k_4 \epsilon(k_1) \cdot \epsilon(k_2) k_1 \cdot \epsilon^*(k_3) \\
& -6 k_2 \cdot k_4 \epsilon(k_1) \cdot \epsilon(k_2) k_1 \cdot \epsilon^*(k_3) - 2 k_3 \cdot k_4 \epsilon(k_1) \cdot \epsilon(k_2) k_1 \cdot \epsilon^*(k_3) \\
& +k_1 \cdot \epsilon(k_1) k_1 \cdot \epsilon(k_2) k_2 \cdot \epsilon^*(k_3) - k_2 \cdot \epsilon(k_1) k_2 \cdot \epsilon(k_2) k_2 \cdot \epsilon^*(k_3) \\
& -2 k_1 \cdot \epsilon(k_2) k_3 \cdot \epsilon(k_1) k_3 \cdot \epsilon^*(k_3) - k_2 \cdot \epsilon(k_2) k_3 \cdot \epsilon(k_1) k_3 \cdot \epsilon^*(k_3) \\
& +k_1 \cdot \epsilon(k_1) k_3 \cdot \epsilon(k_2) k_3 \cdot \epsilon^*(k_3) + 2 k_2 \cdot \epsilon(k_1) k_3 \cdot \epsilon(k_2) k_3 \cdot \epsilon^*(k_3) \\
& -8 k_1 \cdot \epsilon(k_2) k_2 \cdot \epsilon^*(k_3) k_4 \cdot \epsilon(k_1) - 4 k_2 \cdot \epsilon(k_2) k_2 \cdot \epsilon^*(k_3) k_4 \cdot \epsilon(k_1) \\
& +4 k_1 \cdot \epsilon(k_2) k_3 \cdot \epsilon^*(k_3) k_4 \cdot \epsilon(k_1) + 2 k_2 \cdot \epsilon(k_2) k_3 \cdot \epsilon^*(k_3) k_4 \cdot \epsilon(k_1) \\
& +4 k_1 \cdot \epsilon(k_1) k_2 \cdot \epsilon^*(k_3) k_4 \cdot \epsilon(k_2) + 8 k_2 \cdot \epsilon(k_1) k_2 \cdot \epsilon^*(k_3) k_4 \cdot \epsilon(k_2) \\
& -2 k_1 \cdot \epsilon(k_1) k_3 \cdot \epsilon^*(k_3) k_4 \cdot \epsilon(k_2) - 4 k_2 \cdot \epsilon(k_1) k_3 \cdot \epsilon^*(k_3) k_4 \cdot \epsilon(k_2) \\
& +2 k_1 \cdot \epsilon(k_1) k_1 \cdot \epsilon(k_2) k_4 \cdot \epsilon^*(k_3) - 2 k_2 \cdot \epsilon(k_1) k_2 \cdot \epsilon(k_2) k_4 \cdot \epsilon^*(k_3) \\
& -8 k_1 \cdot \epsilon(k_2) k_3 \cdot \epsilon(k_1) k_4 \cdot \epsilon^*(k_3) - 4 k_2 \cdot \epsilon(k_2) k_3 \cdot \epsilon(k_1) k_4 \cdot \epsilon^*(k_3) \\
& +4 k_1 \cdot \epsilon(k_1) k_3 \cdot \epsilon(k_2) k_4 \cdot \epsilon^*(k_3) + 8 k_2 \cdot \epsilon(k_1) k_3 \cdot \epsilon(k_2) k_4 \cdot \epsilon^*(k_3) \\
& -2 k_1 \cdot k_2 k_2 \cdot \epsilon^*(k_3) \epsilon(k_1) \cdot \epsilon(k_2) + 6 k_1 \cdot k_4 k_2 \cdot \epsilon^*(k_3) \epsilon(k_1) \cdot \epsilon(k_2) \\
& -2 k_2 \cdot k_4 k_2 \cdot \epsilon^*(k_3) \epsilon(k_1) \cdot \epsilon(k_2) + 2 k_2 \cdot \epsilon^*(k_3) k_3 \cdot k_4 \epsilon(k_1) \cdot \epsilon(k_2) \\
& +k_1 \cdot k_3 k_3 \cdot \epsilon^*(k_3) \epsilon(k_1) \cdot \epsilon(k_2) - 2 k_1 \cdot k_4 k_3 \cdot \epsilon^*(k_3) \epsilon(k_1) \cdot \epsilon(k_2) \\
& -k_2 \cdot k_3 k_3 \cdot \epsilon^*(k_3) \epsilon(k_1) \cdot \epsilon(k_2) + 2 k_2 \cdot k_4 k_3 \cdot \epsilon^*(k_3) \epsilon(k_1) \cdot \epsilon(k_2) \\
& +4 k_1 \cdot k_3 k_4 \cdot \epsilon^*(k_3) \epsilon(k_1) \cdot \epsilon(k_2) - 4 k_2 \cdot k_3 k_4 \cdot \epsilon^*(k_3) \epsilon(k_1) \cdot \epsilon(k_2) \\
& -4 k_1 \cdot k_2 k_1 \cdot \epsilon(k_2) \epsilon(k_1) \cdot \epsilon^*(k_3) + 4 k_1 \cdot k_4 k_1 \cdot \epsilon(k_2) \epsilon(k_1) \cdot \epsilon^*(k_3) \\
& +4 k_1 \cdot \epsilon(k_2) k_2 \cdot k_4 \epsilon(k_1) \cdot \epsilon^*(k_3) - 2 k_1 \cdot k_2 k_2 \cdot \epsilon(k_2) \epsilon(k_1) \cdot \epsilon^*(k_3) \\
& +2 k_1 \cdot k_4 k_2 \cdot \epsilon(k_2) \epsilon(k_1) \cdot \epsilon^*(k_3) + 2 k_2 \cdot k_4 k_2 \cdot \epsilon(k_2) \epsilon(k_1) \cdot \epsilon^*(k_3) \\
& +4 k_1 \cdot \epsilon(k_2) k_3 \cdot k_4 \epsilon(k_1) \cdot \epsilon^*(k_3) + 2 k_2 \cdot \epsilon(k_2) k_3 \cdot k_4 \epsilon(k_1) \cdot \epsilon^*(k_3) \\
& +2 k_1 \cdot k_2 k_1 \cdot \epsilon(k_1) \epsilon(k_2) \cdot \epsilon^*(k_3) - 2 k_1 \cdot k_4 k_1 \cdot \epsilon(k_1) \epsilon(k_2) \cdot \epsilon^*(k_3) \\
& -2 k_1 \cdot \epsilon(k_1) k_2 \cdot k_4 \epsilon(k_2) \cdot \epsilon^*(k_3) + 4 k_1 \cdot k_2 k_2 \cdot \epsilon(k_1) \epsilon(k_2) \cdot \epsilon^*(k_3) \\
& -4 k_1 \cdot k_4 k_2 \cdot \epsilon(k_1) \epsilon(k_2) \cdot \epsilon^*(k_3) - 4 k_2 \cdot k_4 k_2 \cdot \epsilon(k_1) \epsilon(k_2) \cdot \epsilon^*(k_3) \\
& -2 k_1 \cdot \epsilon(k_1) k_3 \cdot k_4 \epsilon(k_2) \cdot \epsilon^*(k_3) - 4 k_2 \cdot \epsilon(k_1) k_3 \cdot k_4 \epsilon(k_2) \cdot \epsilon^*(k_3))
\end{aligned}
$$

$$\begin{aligned}
\mathcal{M}_{14} &= \frac{\alpha_s g_s \pi}{k_1 \cdot k_3 k_2 \cdot k_4} \times \\
&((t^{a_2}t^{a_1}t^{a_3})_{o_1o_2}) - (t^{a_2}t^{a_3}t^{a_1})_{o_1o_2})) \\
&(k_2 \cdot \epsilon(k_2) - 2k_4 \cdot \epsilon(k_2))(k_1 \cdot \epsilon(k_1)k_1 \cdot \epsilon^*(k_3) + 2k_3 \cdot k_4\epsilon(k_1) \cdot \epsilon^*(k_3) \\
&+4k_2 \cdot \epsilon(k_1)k_1 \cdot \epsilon^*(k_3) - 4k_4 \cdot \epsilon(k_1)k_1 \cdot \epsilon^*(k_3) - 2k_1 \cdot \epsilon(k_1)k_2 \cdot \epsilon^*(k_3) \\
&+4k_2 \cdot \epsilon^*(k_3)k_3 \cdot \epsilon(k_1) - 2k_2 \cdot \epsilon(k_1)k_3 \cdot \epsilon^*(k_3) - k_3 \cdot \epsilon(k_1)k_3 \cdot \epsilon^*(k_3) \\
&+2k_3 \cdot \epsilon^*(k_3)k_4 \cdot \epsilon(k_1) + 2k_1 \cdot \epsilon(k_1)k_4 \cdot \epsilon^*(k_3) - 4k_3 \cdot \epsilon(k_1)k_4 \cdot \epsilon^*(k_3) \\
&-2k_1 \cdot k_2\epsilon(k_1) \cdot \epsilon^*(k_3) + 2k_1 \cdot k_4\epsilon(k_1) \cdot \epsilon^*(k_3) - 2k_2 \cdot k_3\epsilon(k_1) \cdot \epsilon^*(k_3))
\end{aligned}$$

$$\begin{aligned}
\mathcal{M}_{15} &= -\frac{\alpha_s g_s \pi}{k_1 \cdot k_3 k_2 \cdot k_5} ((t^{a_1}t^{a_3}t^{a_2})_{o_1o_2} - (t^{a_3}t^{a_1}t^{a_2})_{o_1o_2}) \times \\
&(2k_1 \cdot \epsilon(k_2) + k_2 \cdot \epsilon(k_2) - 2k_3 \cdot \epsilon(k_2) - 2k_4 \cdot \epsilon(k_2)) \\
&(-k_1 \cdot \epsilon(k_1)k_1 \cdot \epsilon^*(k_3) + 4k_4 \cdot \epsilon(k_1)k_1 \cdot \epsilon^*(k_3) + k_3 \cdot \epsilon(k_1)k_3 \cdot \epsilon^*(k_3) \\
&-2k_3 \cdot \epsilon^*(k_3)k_4 \cdot \epsilon(k_1) - 2k_1 \cdot \epsilon(k_1)k_4 \cdot \epsilon^*(k_3) + 4k_3 \cdot \epsilon(k_1)k_4 \cdot \epsilon^*(k_3) \\
&-2k_1 \cdot k_4\epsilon(k_1) \cdot \epsilon^*(k_3) - 2k_3 \cdot k_4\epsilon(k_1) \cdot \epsilon^*(k_3))
\end{aligned}$$

$$\begin{aligned}
\mathcal{M}_{17} &= -\frac{\alpha_s g_s \pi}{k_1 \cdot k_4 k_2 \cdot k_3} ((t^{a_1}t^{a_2}t^{a_3})_{o_1o_2} - (t^{a_1}t^{a_3}t^{a_2})_{o_1o_2}) \times \\
&(k_1 \cdot \epsilon(k_1) - 2k_4 \cdot \epsilon(k_1)) \\
&(2k_1 \cdot \epsilon^*(k_3)k_2 \cdot \epsilon(k_2) - k_2 \cdot \epsilon^*(k_3)k_2 \cdot \epsilon(k_2) - 2k_4 \cdot \epsilon^*(k_3)k_2 \cdot \epsilon(k_2) \\
&-4k_1 \cdot \epsilon(k_2)k_2 \cdot \epsilon^*(k_3) - 4k_1 \cdot \epsilon^*(k_3)k_3 \cdot \epsilon(k_2) + 2k_1 \cdot \epsilon(k_2)k_3 \cdot \epsilon^*(k_3) \\
&+k_3 \cdot \epsilon(k_2)k_3 \cdot \epsilon^*(k_3) + 4k_2 \cdot \epsilon^*(k_3)k_4 \cdot \epsilon(k_2) - 2k_3 \cdot k_4\epsilon(k_2) \cdot \epsilon^*(k_3) \\
&-2k_3 \cdot \epsilon^*(k_3)k_4 \cdot \epsilon(k_2) + 4k_3 \cdot \epsilon(k_2)k_4 \cdot \epsilon^*(k_3) + 2k_1 \cdot k_2\epsilon(k_2) \cdot \epsilon^*(k_3) \\
&+2k_1 \cdot k_3\epsilon(k_2) \cdot \epsilon^*(k_3) - 2k_2 \cdot k_4\epsilon(k_2) \cdot \epsilon^*(k_3))
\end{aligned}$$

$$\begin{aligned}
\mathcal{M}_{18} &= -\frac{\alpha_s g_s \pi}{k_1 \cdot k_5 k_2 \cdot k_3} \times \\
&(t^{a_2}t^{a_3}t^{a_1})_{o_1o_2} - (t^{a_3}t^{a_2}t^{a_1})_{o_1o_2} \\
&(k_1 \cdot \epsilon(k_1) + 2k_2 \cdot \epsilon(k_1) - 2k_3 \cdot \epsilon(k_1) - 2k_4 \cdot \epsilon(k_1)) \\
&(-k_2 \cdot \epsilon(k_2)k_2 \cdot \epsilon^*(k_3) + 4k_4 \cdot \epsilon(k_2)k_2 \cdot \epsilon^*(k_3) \\
&+k_3 \cdot \epsilon(k_2)k_3 \cdot \epsilon^*(k_3) - 2k_3 \cdot \epsilon^*(k_3)k_4 \cdot \epsilon(k_2) - 2k_2 \cdot \epsilon(k_2)k_4 \cdot \epsilon^*(k_3) \\
&+4k_3 \cdot \epsilon(k_2)k_4 \cdot \epsilon^*(k_3) - 2k_2 \cdot k_4\epsilon(k_2) \cdot \epsilon^*(k_3) - 2k_3 \cdot k_4\epsilon(k_2) \cdot \epsilon^*(k_3))
\end{aligned}$$

$$
\begin{aligned}
\mathcal{M}_{16} \;=\; & -\frac{\alpha_s g_s \pi}{k_1 \cdot k_3\,(m^2 + k_4 \cdot k_5)} \times \\
& ((t^{a_1} t^{a_3} t^{a_2})_{o_1 o_2} - (t^{a_2} t^{a_1} t^{a_3})_{o_1 o_2} + (t^{a_2} t^{a_3} t^{a_1})_{o_1 o_2} - (t^{a_3} t^{a_1} t^{a_2})_{o_1 o_2}) \\
& (k_1 \cdot \epsilon(k_1) k_1 \cdot \epsilon(k_2) k_1 \cdot \epsilon^*(k_3) - 2 k_2 \cdot \epsilon(k_1) k_2 \cdot \epsilon(k_2) k_1 \cdot \epsilon^*(k_3) \\
& - k_1 \cdot \epsilon(k_1) k_3 \cdot \epsilon(k_2) k_1 \cdot \epsilon^*(k_3) - 8 k_1 \cdot \epsilon(k_2) k_4 \cdot \epsilon(k_1) k_1 \cdot \epsilon^*(k_3) \\
& - 4 k_2 \cdot \epsilon(k_2) k_4 \cdot \epsilon(k_1) k_1 \cdot \epsilon^*(k_3) + 8 k_3 \cdot \epsilon(k_2) k_4 \cdot \epsilon(k_1) k_1 \cdot \epsilon^*(k_3) \\
& + 2 k_1 \cdot \epsilon(k_1) k_4 \cdot \epsilon(k_2) k_1 \cdot \epsilon^*(k_3) + 8 k_2 \cdot \epsilon(k_1) k_4 \cdot \epsilon(k_2) k_1 \cdot \epsilon^*(k_3) \\
& + 4 k_1 \cdot k_3 \epsilon(k_1) \cdot \epsilon(k_2) k_1 \cdot \epsilon^*(k_3) + 4 k_1 \cdot k_4 \epsilon(k_1) \cdot \epsilon(k_2) k_1 \cdot \epsilon^*(k_3) \\
& - 4 k_2 \cdot k_4 \epsilon(k_1) \cdot \epsilon(k_2) k_1 \cdot \epsilon^*(k_3) - 4 k_3 \cdot k_4 \epsilon(k_1) \cdot \epsilon(k_2) k_1 \cdot \epsilon^*(k_3) \\
& + k_1 \cdot \epsilon(k_1) k_2 \cdot \epsilon(k_2) k_2 \cdot \epsilon^*(k_3) - 2 k_2 \cdot \epsilon(k_2) k_2 \cdot \epsilon^*(k_3) k_3 \cdot \epsilon(k_1) \\
& + k_2 \cdot \epsilon(k_1) k_2 \cdot \epsilon(k_2) k_3 \cdot \epsilon^*(k_3) - k_1 \cdot \epsilon(k_2) k_3 \cdot \epsilon(k_1) k_3 \cdot \epsilon^*(k_3) \\
& + k_3 \cdot \epsilon(k_1) k_3 \cdot \epsilon(k_2) k_3 \cdot \epsilon^*(k_3) + 4 k_1 \cdot \epsilon(k_2) k_3 \cdot \epsilon^*(k_3) k_4 \cdot \epsilon(k_1) \\
& + 2 k_2 \cdot \epsilon(k_2) k_3 \cdot \epsilon^*(k_3) k_4 \cdot \epsilon(k_1) - 4 k_3 \cdot \epsilon(k_2) k_3 \cdot \epsilon^*(k_3) k_4 \cdot \epsilon(k_1) \\
& - 4 k_1 \cdot \epsilon(k_1) k_2 \cdot \epsilon^*(k_3) k_4 \cdot \epsilon(k_2) + 8 k_2 \cdot \epsilon^*(k_3) k_3 \cdot \epsilon(k_1) k_4 \cdot \epsilon(k_2) \\
& - 4 k_2 \cdot \epsilon(k_1) k_3 \cdot \epsilon^*(k_3) k_4 \cdot \epsilon(k_2) - 2 k_3 \cdot \epsilon(k_1) k_3 \cdot \epsilon^*(k_3) k_4 \cdot \epsilon(k_2) \\
& + 4 k_1 \cdot \epsilon(k_1) k_1 \cdot \epsilon(k_2) k_4 \cdot \epsilon^*(k_3) + 2 k_1 \cdot \epsilon(k_1) k_2 \cdot \epsilon(k_2) k_4 \cdot \epsilon^*(k_3) \\
& - 8 k_1 \cdot \epsilon(k_2) k_3 \cdot \epsilon(k_1) k_4 \cdot \epsilon^*(k_3) - 4 k_2 \cdot \epsilon(k_2) k_3 \cdot \epsilon(k_1) k_4 \cdot \epsilon^*(k_3) \\
& - 4 k_1 \cdot \epsilon(k_1) k_3 \cdot \epsilon(k_2) k_4 \cdot \epsilon^*(k_3) + 8 k_3 \cdot \epsilon(k_1) k_3 \cdot \epsilon(k_2) k_4 \cdot \epsilon^*(k_3) \\
& - 2 k_1 \cdot k_3 k_3 \cdot \epsilon^*(k_3) \epsilon(k_1) \cdot \epsilon(k_2) - 2 k_1 \cdot k_4 k_3 \cdot \epsilon^*(k_3) \epsilon(k_1) \cdot \epsilon(k_2) \\
& + 2 k_2 \cdot k_4 k_3 \cdot \epsilon^*(k_3) \epsilon(k_1) \cdot \epsilon(k_2) + 2 k_3 \cdot k_4 k_3 \cdot \epsilon^*(k_3) \epsilon(k_1) \cdot \epsilon(k_2) \\
& - 2 k_1 \cdot k_3 k_1 \cdot \epsilon(k_2) \epsilon(k_1) \cdot \epsilon^*(k_3) + 2 k_1 \cdot k_4 k_1 \cdot \epsilon(k_2) \epsilon(k_1) \cdot \epsilon^*(k_3) \\
& + 2 k_1 \cdot \epsilon(k_2) k_2 \cdot k_4 \epsilon(k_1) \cdot \epsilon^*(k_3) + k_1 \cdot k_2 k_2 \cdot \epsilon(k_2) \epsilon(k_1) \cdot \epsilon^*(k_3) \\
& + 2 k_1 \cdot k_4 k_2 \cdot \epsilon(k_2) \epsilon(k_1) \cdot \epsilon^*(k_3) + k_2 \cdot k_3 k_2 \cdot \epsilon(k_2) \epsilon(k_1) \cdot \epsilon^*(k_3) \\
& + 6 k_1 \cdot \epsilon(k_2) k_3 \cdot k_4 \epsilon(k_1) \cdot \epsilon^*(k_3) + 2 k_2 \cdot \epsilon(k_2) k_3 \cdot k_4 \epsilon(k_1) \cdot \epsilon^*(k_3) \\
& - 2 k_1 \cdot k_3 k_3 \cdot \epsilon(k_2) \epsilon(k_1) \cdot \epsilon^*(k_3) - 6 k_1 \cdot k_4 k_3 \cdot \epsilon(k_2) \epsilon(k_1) \cdot \epsilon^*(k_3) \\
& + 2 k_2 \cdot k_4 k_3 \cdot \epsilon(k_2) \epsilon(k_1) \cdot \epsilon^*(k_3) - 2 k_3 \cdot k_4 k_3 \cdot \epsilon(k_2) \epsilon(k_1) \cdot \epsilon^*(k_3) \\
& - 4 k_1 \cdot k_2 k_4 \cdot \epsilon(k_2) \epsilon(k_1) \cdot \epsilon^*(k_3) - 4 k_2 \cdot k_3 k_4 \cdot \epsilon(k_2) \epsilon(k_1) \cdot \epsilon^*(k_3) \\
& - 2 k_1 \cdot k_3 k_1 \cdot \epsilon(k_1) \epsilon(k_2) \cdot \epsilon^*(k_3) - 2 k_1 \cdot k_4 k_1 \cdot \epsilon(k_1) \epsilon(k_2) \cdot \epsilon^*(k_3) \\
& + 2 k_1 \cdot \epsilon(k_1) k_2 \cdot k_4 \epsilon(k_2) \cdot \epsilon^*(k_3) + 2 k_1 \cdot \epsilon(k_1) k_3 \cdot k_4 \epsilon(k_2) \cdot \epsilon^*(k_3) \\
& + 4 k_1 \cdot k_3 k_3 \cdot \epsilon(k_1) \epsilon(k_2) \cdot \epsilon^*(k_3) + 4 k_1 \cdot k_4 k_3 \cdot \epsilon(k_1) \epsilon(k_2) \cdot \epsilon^*(k_3) \\
& - 4 k_2 \cdot k_4 k_3 \cdot \epsilon(k_1) \epsilon(k_2) \cdot \epsilon^*(k_3) - 4 k_3 \cdot k_4 k_3 \cdot \epsilon(k_1) \epsilon(k_2) \cdot \epsilon^*(k_3))
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{M}_{19} \;=\; & -\frac{\alpha_s g_s \pi}{k_2 \cdot k_3\,(m^2 + k_4 \cdot k_5)} \times \\
& ((t^{a_1}t^{a_2}t^{a_3})_{o_1o_2} - (t^{a_1}t^{a_3}t^{a_2})_{o_1o_2} - (t^{a_2}t^{a_3}t^{a_1})_{o_1o_2} + (t^{a_3}t^{a_2}t^{a_1})_{o_1o_2}) \\
& (-k_1 \cdot \epsilon(k_1)k_1 \cdot \epsilon^*(k_3)k_2 \cdot \epsilon(k_2) - k_2 \cdot \epsilon(k_1)k_2 \cdot \epsilon^*(k_3)k_2 \cdot \epsilon(k_2) \\
& +k_2 \cdot \epsilon^*(k_3)k_3 \cdot \epsilon(k_1)k_2 \cdot \epsilon(k_2) + 4k_1 \cdot \epsilon^*(k_3)k_4 \cdot \epsilon(k_1)k_2 \cdot \epsilon(k_2) \\
& -2k_2 \cdot \epsilon^*(k_3)k_4 \cdot \epsilon(k_1)k_2 \cdot \epsilon(k_2) - 2k_1 \cdot \epsilon(k_1)k_4 \cdot \epsilon^*(k_3)k_2 \cdot \epsilon(k_2) \\
& -4k_2 \cdot \epsilon(k_1)k_4 \cdot \epsilon^*(k_3)k_2 \cdot \epsilon(k_2) + 4k_3 \cdot \epsilon(k_1)k_4 \cdot \epsilon^*(k_3)k_2 \cdot \epsilon(k_2) \\
& -2k_1 \cdot k_4\epsilon(k_1) \cdot \epsilon^*(k_3)k_2 \cdot \epsilon(k_2) + 2k_2 \cdot k_3\epsilon(k_1) \cdot \epsilon^*(k_3)k_2 \cdot \epsilon(k_2) \\
& +2k_2 \cdot k_4\epsilon(k_1) \cdot \epsilon^*(k_3)k_2 \cdot \epsilon(k_2) - 2k_3 \cdot k_4\epsilon(k_1) \cdot \epsilon^*(k_3)k_2 \cdot \epsilon(k_2) \\
& +2k_1 \cdot \epsilon(k_1)k_1 \cdot \epsilon(k_2)k_2 \cdot \epsilon^*(k_3) + 2k_1 \cdot \epsilon(k_1)k_1 \cdot \epsilon^*(k_3)k_3 \cdot \epsilon(k_2) \\
& -k_1 \cdot \epsilon(k_1)k_1 \cdot \epsilon(k_2)k_3 \cdot \epsilon^*(k_3) + k_2 \cdot \epsilon(k_1)k_3 \cdot \epsilon(k_2)k_3 \cdot \epsilon^*(k_3) \\
& -k_3 \cdot \epsilon(k_1)k_3 \cdot \epsilon(k_2)k_3 \cdot \epsilon^*(k_3) - 8k_1 \cdot \epsilon(k_2)k_2 \cdot \epsilon^*(k_3)k_4 \cdot \epsilon(k_1) \\
& -8k_1 \cdot \epsilon^*(k_3)k_3 \cdot \epsilon(k_2)k_4 \cdot \epsilon(k_1) + 4k_1 \cdot \epsilon(k_2)k_3 \cdot \epsilon^*(k_3)k_4 \cdot \epsilon(k_1) \\
& +2k_3 \cdot \epsilon(k_2)k_3 \cdot \epsilon^*(k_3)k_4 \cdot \epsilon(k_1) + 4k_1 \cdot \epsilon(k_1)k_2 \cdot \epsilon^*(k_3)k_4 \cdot \epsilon(k_2) \\
& +8k_2 \cdot \epsilon(k_1)k_2 \cdot \epsilon^*(k_3)k_4 \cdot \epsilon(k_2) - 8k_2 \cdot \epsilon^*(k_3)k_3 \cdot \epsilon(k_1)k_4 \cdot \epsilon(k_2) \\
& -2k_1 \cdot \epsilon(k_1)k_3 \cdot \epsilon^*(k_3)k_4 \cdot \epsilon(k_2) - 4k_2 \cdot \epsilon(k_1)k_3 \cdot \epsilon^*(k_3)k_4 \cdot \epsilon(k_2) \\
& +4k_3 \cdot \epsilon(k_1)k_3 \cdot \epsilon^*(k_3)k_4 \cdot \epsilon(k_2) + 4k_1 \cdot \epsilon(k_1)k_3 \cdot \epsilon(k_2)k_4 \cdot \epsilon^*(k_3) \\
& +8k_2 \cdot \epsilon(k_1)k_3 \cdot \epsilon(k_2)k_4 \cdot \epsilon^*(k_3) - 8k_3 \cdot \epsilon(k_1)k_3 \cdot \epsilon(k_2)k_4 \cdot \epsilon^*(k_3) \\
& +4k_1 \cdot k_4k_2 \cdot \epsilon^*(k_3)\epsilon(k_1) \cdot \epsilon(k_2) - 4k_2 \cdot k_3k_2 \cdot \epsilon^*(k_3)\epsilon(k_1) \cdot \epsilon(k_2) \\
& -4k_2 \cdot k_4k_2 \cdot \epsilon^*(k_3)\epsilon(k_1) \cdot \epsilon(k_2) + 4k_2 \cdot \epsilon^*(k_3)k_3 \cdot k_4\epsilon(k_1) \cdot \epsilon(k_2) \\
& -2k_1 \cdot k_4k_3 \cdot \epsilon^*(k_3)\epsilon(k_1) \cdot \epsilon(k_2) + 2k_2 \cdot k_3k_3 \cdot \epsilon^*(k_3)\epsilon(k_1) \cdot \epsilon(k_2) \\
& +2k_2 \cdot k_4k_3 \cdot \epsilon^*(k_3)\epsilon(k_1) \cdot \epsilon(k_2) - 2k_3 \cdot k_4k_3 \cdot \epsilon^*(k_3)\epsilon(k_1) \cdot \epsilon(k_2) \\
& +4k_1 \cdot k_4k_3 \cdot \epsilon(k_2)\epsilon(k_1) \cdot \epsilon^*(k_3) - 4k_2 \cdot k_3k_3 \cdot \epsilon(k_2)\epsilon(k_1) \cdot \epsilon^*(k_3) \\
& -4k_2 \cdot k_4k_3 \cdot \epsilon(k_2)\epsilon(k_1) \cdot \epsilon^*(k_3) + 4k_3 \cdot k_4k_3 \cdot \epsilon(k_2)\epsilon(k_1) \cdot \epsilon^*(k_3) \\
& -k_1 \cdot k_2k_1 \cdot \epsilon(k_1)\epsilon(k_2) \cdot \epsilon^*(k_3) - k_1 \cdot k_3k_1 \cdot \epsilon(k_1)\epsilon(k_2) \cdot \epsilon^*(k_3) \\
& -2k_1 \cdot \epsilon(k_1)k_2 \cdot k_4\epsilon(k_2) \cdot \epsilon^*(k_3) - 2k_1 \cdot k_4k_2 \cdot \epsilon(k_1)\epsilon(k_2) \cdot \epsilon^*(k_3) \\
& +2k_2 \cdot k_3k_2 \cdot \epsilon(k_1)\epsilon(k_2) \cdot \epsilon^*(k_3) - 2k_2 \cdot k_4k_2 \cdot \epsilon(k_1)\epsilon(k_2) \cdot \epsilon^*(k_3) \\
& -2k_1 \cdot \epsilon(k_1)k_3 \cdot k_4\epsilon(k_2) \cdot \epsilon^*(k_3) - 6k_2 \cdot \epsilon(k_1)k_3 \cdot k_4\epsilon(k_2) \cdot \epsilon^*(k_3) \\
& -2k_1 \cdot k_4k_3 \cdot \epsilon(k_1)\epsilon(k_2) \cdot \epsilon^*(k_3) + 2k_2 \cdot k_3k_3 \cdot \epsilon(k_1)\epsilon(k_2) \cdot \epsilon^*(k_3) \\
& +6k_2 \cdot k_4k_3 \cdot \epsilon(k_1)\epsilon(k_2) \cdot \epsilon^*(k_3) + 2k_3 \cdot k_4k_3 \cdot \epsilon(k_1)\epsilon(k_2) \cdot \epsilon^*(k_3) \\
& +4k_1 \cdot k_2k_4 \cdot \epsilon(k_1)\epsilon(k_2) \cdot \epsilon^*(k_3) + 4k_1 \cdot k_3k_4 \cdot \epsilon(k_1)\epsilon(k_2) \cdot \epsilon^*(k_3))
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{M}_{20} \;=\; & \frac{\alpha_s g_s \pi}{k_1 \cdot k_4 k_2 \cdot k_5}(t^{a_1}t^{a_3}t^{a_2})_{o_1o_2} \times \\
& (k_1 \cdot \epsilon(k_1) - 2k_4 \cdot \epsilon(k_1))(2k_1 \cdot \epsilon(k_2) + k_2 \cdot \epsilon(k_2) - 2k_3 \cdot \epsilon(k_2) - 2k_4 \cdot \epsilon(k_2)) \\
& (2k_1 \cdot \epsilon^*(k_3) - k_3 \cdot \epsilon^*(k_3) - 2k_4 \cdot \epsilon^*(k_3))
\end{aligned}
$$

$$\begin{aligned}
\mathcal{M}_{21} &= -\frac{\alpha_s g_s \pi}{k_1\cdot k_4 k_3\cdot k_5}(t^{a_1}t^{a_2}t^{a_3})_{o_1o_2}\times\\
&\quad (k_1\cdot\epsilon(k_1)-2k_4\cdot\epsilon(k_1))(2k_1\cdot\epsilon(k_2)+k_2\cdot\epsilon(k_2)-2k_4\cdot\epsilon(k_2))\\
&\quad (2k_1\cdot\epsilon^*(k_3)+2k_2\cdot\epsilon^*(k_3)-k_3\cdot\epsilon^*(k_3)-2k_4\cdot\epsilon^*(k_3))\\
\mathcal{M}_{22} &= \frac{\alpha_s g_s \pi}{k_1\cdot k_5 k_2\cdot k_4}(t^{a_2}t^{a_3}t^{a_1})_{o_1o_2}\times\\
&\quad (k_1\cdot\epsilon(k_1)+2k_2\cdot\epsilon(k_1)-2k_3\cdot\epsilon(k_1)-2k_4\cdot\epsilon(k_1))(k_2\cdot\epsilon(k_2)-2k_4\cdot\epsilon(k_2))\\
&\quad (2k_2\cdot\epsilon^*(k_3)-k_3\cdot\epsilon^*(k_3)-2k_4\cdot\epsilon^*(k_3))\\
\mathcal{M}_{23} &= -\frac{\alpha_s g_s \pi}{k_2\cdot k_4 k_3\cdot k_5}(t^{a_2}t^{a_1}t^{a_3})_{o_1o_2}\times\\
&\quad (k_1\cdot\epsilon(k_1)+2k_2\cdot\epsilon(k_1)-2k_4\cdot\epsilon(k_1))(k_2\cdot\epsilon(k_2)-2k_4\cdot\epsilon(k_2))\\
&\quad (2k_1\cdot\epsilon^*(k_3)+2k_2\cdot\epsilon^*(k_3)-k_3\cdot\epsilon^*(k_3)-2k_4\cdot\epsilon^*(k_3))\\
\mathcal{M}_{24} &= \frac{\alpha_s g_s \pi}{k_1\cdot k_5 k_3\cdot k_4}(t^{a_3}t^{a_2}t^{a_1})_{o_1o_2}\times\\
&\quad (k_1\cdot\epsilon(k_1)+2k_2\cdot\epsilon(k_1)-2k_3\cdot\epsilon(k_1)-2k_4\cdot\epsilon(k_1))(k_2\cdot\epsilon(k_2)-2k_3\cdot\epsilon(k_2)-2k_4\cdot\epsilon(k_2))\\
&\quad (k_3\cdot\epsilon^*(k_3)+2k_4\cdot\epsilon^*(k_3))\\
\mathcal{M}_{25} &= \frac{\alpha_s g_s \pi}{k_2\cdot k_5 k_3\cdot k_4}(t^{a_3}t^{a_1}t^{a_2})_{o_1o_2}\times\\
&\quad (k_1\cdot\epsilon(k_1)-2k_3\cdot\epsilon(k_1)-2k_4\cdot\epsilon(k_1))(2k_1\cdot\epsilon(k_2)+k_2\cdot\epsilon(k_2)-2k_3\cdot\epsilon(k_2)-2k_4\cdot\epsilon(k_2))\\
&\quad (k_3\cdot\epsilon^*(k_3)+2k_4\cdot\epsilon^*(k_3))
\end{aligned}$$

Amplitudes for $GG \to G\tilde{t}\bar{\tilde{t}}$, Ghosts

$$
\begin{aligned}
\mathcal{M}_{\eta\bar{\eta}\to g\tilde{t}\bar{\tilde{t}}} = \alpha_s g_s \pi\, (\, & \frac{(t^{a_1}t^{a_3}t^{a_2})_{o_1o_2}}{k_1\cdot k_3 k_2\cdot k_3\,(m^2+p_1\cdot p_2)} \times \\
& ((\epsilon^*(k_3)\cdot k_1-\epsilon^*(k_3)\cdot k_3)k_2\cdot k_3(k_1\cdot k_2-k_2\cdot k_3-2k_2\cdot p_1) \\
& +\epsilon^*(k_3)\cdot k_2 k_1\cdot k_3(-k_1\cdot k_2+k_1\cdot k_3+2(k_2\cdot k_3+k_2\cdot p_1-k_3\cdot p_1))) \\
& +\frac{(t^{a_2}t^{a_3}t^{a_1})_{o_1o_2}}{k_1\cdot k_3 k_2\cdot k_3\,(m^2+p_1\cdot p_2)}((\epsilon^*(k_3)\cdot k_1-\epsilon^*(k_3)\cdot k_3)k_2\cdot k_3(k_1\cdot k_2-k_2\cdot k_3-2k_2\cdot p_1) \\
& +\epsilon^*(k_3)\cdot k_2 k_1\cdot k_3(-k_1\cdot k_2+k_1\cdot k_3+2(k_2\cdot k_3+k_2\cdot p_1-k_3\cdot p_1))) \\
& +(t^{a_2}t^{a_1}t^{a_3})_{o_1o_2}\left(-\frac{2\epsilon^*(k_3)\cdot k_2}{k_1\cdot k_2}\right. \\
& +\frac{(2\epsilon^*(k_3)\cdot k_1+2\epsilon^*(k_3)\cdot k_2-\epsilon^*(k_3)\cdot k_3-2\epsilon^*(k_3)\cdot p_1)(k_1\cdot k_2-2k_2\cdot p_1)}{k_1\cdot k_2 k_3\cdot p_2} \\
& -\frac{(\epsilon^*(k_3)\cdot k_1-\epsilon^*(k_3)\cdot k_3)(k_1\cdot k_2-k_2\cdot k_3-2k_2\cdot p_1)}{k_1\cdot k_3\,(m^2+p_1\cdot p_2)} \\
& +\frac{1}{k_1\cdot k_2\,(m^2+p_1\cdot p_2)}\,(2\epsilon^*(k_3)\cdot p_1(k_1\cdot k_2-2k_2\cdot k_3)+\epsilon^*(k_3)\cdot k_1(k_1\cdot k_2-4k_2\cdot p_1) \\
& -\epsilon^*(k_3)\cdot k_3(k_2\cdot k_3-2k_2\cdot p_1)-\epsilon^*(k_3)\cdot k_2(k_1\cdot k_2-2(k_1\cdot p_1-k_2\cdot p_1+k_3\cdot p_1)))\ ) \\
& -\frac{(t^{a_3}t^{a_2}t^{a_1})_{o_1o_2}}{k_1\cdot k_2 k_2\cdot k_3 k_3\cdot p_1\,(m^2+p_1\cdot p_2)}\left(2\epsilon^*(k_3)\cdot p_1 k_2\cdot k_3\left(-2k_2\cdot p_1\left(m^2+p_1\cdot p_2\right)\right.\right. \\
& +k_1\cdot k_2\left(m^2+k_3\cdot p_1+p_1\cdot p_2\right)-2k_2\cdot k_3\left(m^2+k_3\cdot p_1+p_1\cdot p_2\right)\big) \\
& +\epsilon^*(k_3)\cdot k_3 k_2\cdot k_3\left(k_1\cdot k_2\left(m^2+p_1\cdot p_2\right)-2k_2\cdot p_1\left(m^2-k_3\cdot p_1+p_1\cdot p_2\right)\right. \\
& -k_2\cdot k_3\left(k_3\cdot p_1+2\left(m^2+p_1\cdot p_2\right)\right)\big)+k_3\cdot p_1\left(\epsilon^*(k_3)\cdot k_1 k_2\cdot k_3(k_1\cdot k_2-4k_2\cdot p_1)\right. \\
& +\epsilon^*(k_3)\cdot k_2\left(-(k_1\cdot k_2)^2+(k_1\cdot k_3+k_2\cdot k_3+2k_2\cdot p_1-2k_3\cdot p_1)k_1\cdot k_2\right. \\
& +2k_2\cdot k_3\left(m^2+k_1\cdot p_1-k_2\cdot p_1+k_3\cdot p_1+p_1\cdot p_2\right)\big)\big)\big) \\
& +\frac{(t^{a_3}t^{a_1}t^{a_2})_{o_1o_2}}{k_1\cdot k_2 k_1\cdot k_3 k_3\cdot p_1\,(m^2+p_1\cdot p_2)}\left(2\epsilon^*(k_3)\cdot p_1 k_1\cdot k_3\left(-2k_2\cdot p_1\left(m^2+p_1\cdot p_2\right)\right.\right. \\
& +k_1\cdot k_2\left(m^2+k_3\cdot p_1+p_1\cdot p_2\right)-2k_2\cdot k_3\left(m^2+k_3\cdot p_1+p_1\cdot p_2\right)\big) \\
& +k_3\cdot p_1\left(\epsilon^*(k_3)\cdot k_1\left(-(k_1\cdot k_2)^2+(k_1\cdot k_3+k_2\cdot k_3+2k_2\cdot p_1)k_1\cdot k_2-4k_1\cdot k_3 k_2\cdot p_1\right)\right. \\
& +\epsilon^*(k_3)\cdot k_2 k_1\cdot k_3\left(2\left(m^2+k_1\cdot p_1-k_2\cdot p_1+k_3\cdot p_1+p_1\cdot p_2\right)-k_1\cdot k_2\right)\big) \\
& +\epsilon^*(k_3)\cdot k_3\left(k_3\cdot p_1(k_1\cdot k_2)^2+\left(k_1\cdot k_3\left(m^2+p_1\cdot p_2\right)-(k_2\cdot k_3+2k_2\cdot p_1)k_3\cdot p_1\right)k_1\cdot k_2\right. \\
& -k_1\cdot k_3\left(2k_2\cdot p_1\left(m^2-k_3\cdot p_1+p_1\cdot p_2\right)+k_2\cdot k_3\left(k_3\cdot p_1+2\left(m^2+p_1\cdot p_2\right)\right)\right)\big)\big) \\
& +\frac{(t^{a_1}t^{a_2}t^{a_3})_{o_1o_2}}{k_1\cdot k_2 k_2\cdot k_3 k_3\cdot p_2\,(m^2+p_1\cdot p_2)}\left(\epsilon^*(k_3)\cdot k_1 k_2\cdot k_3\left(4k_2\cdot p_1\left(m^2+k_3\cdot p_2+p_1\cdot p_2\right)\right.\right. \\
& -k_1\cdot k_2\left(k_3\cdot p_2+2\left(m^2+p_1\cdot p_2\right)\right)\big)+\epsilon^*(k_3)\cdot k_2\left(k_3\cdot p_2(k_1\cdot k_2)^2-\right.
\end{aligned}
$$

$$
\begin{aligned}
&-\left(\left((k_1\cdot k_3+2k_2\cdot p_1-2k_3\cdot p_1)k_3\cdot p_2+k_2\cdot k_3\left(k_3\cdot p_2+2\left(m^2+p_1\cdot p_2\right)\right)\right)k_1\cdot k_2\right.\\
&+2k_2\cdot k_3\left(k_3\cdot p_2\left(m^2-k_1\cdot p_1-k_3\cdot p_1+p_1\cdot p_2\right)+k_2\cdot p_1\left(k_3\cdot p_2+2\left(m^2+p_1\cdot p_2\right)\right)\right)\Big)\\
&+k_2\cdot k_3\left(\epsilon^*(k_3)\cdot k_3\left(k_2\cdot k_3k_3\cdot p_2+k_1\cdot k_2\left(m^2+p_1\cdot p_2\right)-2k_2\cdot p_1\left(m^2+k_3\cdot p_2+p_1\cdot p_2\right)\right)\right.\\
&\left.\left.+2\epsilon^*(k_3)\cdot p_1\left(k_1\cdot k_2\left(m^2-k_3\cdot p_2+p_1\cdot p_2\right)-2\left(k_2\cdot p_1\left(m^2+p_1\cdot p_2\right)-k_2\cdot k_3k_3\cdot p_2\right)\right)\right)\right)\ )
\end{aligned}
$$

$$
\begin{aligned}
&\mathcal{M}_{g\eta\to\eta\bar{\tilde{t}}\tilde{t}}=\alpha_s g_s\pi\left(\left(\frac{(t^{a_2}t^{a_1}t^{a_3})_{o_1o_2}}{k_1\cdot k_2k_1\cdot k_3\left(m^2+p_1\cdot p_2\right)}+\frac{(t^{a_3}t^{a_1}t^{a_2})_{o_1o_2}}{k_1\cdot k_2k_1\cdot k_3\left(m^2+p_1\cdot p_2\right)}\right)\times\right.\\
&((\epsilon(k_1)\cdot k_1+\epsilon(k_1)\cdot k_2)k_1\cdot k_3(k_1\cdot k_3+k_2\cdot k_3-2k_3\cdot p_1)\\
&+\epsilon(k_1)\cdot k_3k_1\cdot k_2(k_1\cdot k_2-2k_1\cdot k_3-2k_1\cdot p_1-k_2\cdot k_3+2k_3\cdot p_1))\\
&+(t^{a_3}t^{a_2}t^{a_1})_{o_1o_2}\left(-\frac{2\epsilon(k_1)\cdot k_3}{k_2\cdot k_3}\right.\\
&-\frac{(\epsilon(k_1)\cdot k_1+2\epsilon(k_1)\cdot k_2-2\epsilon(k_1)\cdot k_3-2\epsilon(k_1)\cdot p_1)(k_2\cdot k_3-2k_3\cdot p_1)}{k_1\cdot p_2k_2\cdot k_3}\\
&-\frac{(\epsilon(k_1)\cdot k_1+\epsilon(k_1)\cdot k_2)(k_1\cdot k_3+k_2\cdot k_3-2k_3\cdot p_1)}{k_1\cdot k_2\left(m^2+p_1\cdot p_2\right)}\\
&+\frac{1}{k_2\cdot k_3\left(m^2+p_1\cdot p_2\right)}(2\epsilon(k_1)\cdot p_1(2k_1\cdot k_3+k_2\cdot k_3)+\epsilon(k_1)\cdot k_2(k_2\cdot k_3-4k_3\cdot p_1)\\
&-\epsilon(k_1)\cdot k_1(k_1\cdot k_3+2k_3\cdot p_1)+\epsilon(k_1)\cdot k_3(-2k_1\cdot p_1+k_2\cdot k_3+2(k_2\cdot p_1+k_3\cdot p_1))))\\
&+\frac{(t^{a_1}t^{a_2}t^{a_3})_{o_1o_2}}{k_1\cdot k_2k_1\cdot p_1k_2\cdot k_3\left(m^2+p_1\cdot p_2\right)}\\
&\left(-2\epsilon(k_1)\cdot p_1k_1\cdot k_2\left(-2k_3\cdot p_1\left(m^2+p_1\cdot p_2\right)+2k_1\cdot k_3\left(m^2-k_1\cdot p_1+p_1\cdot p_2\right)\right.\right.\\
&\left.\left.+k_2\cdot k_3\left(m^2-k_1\cdot p_1+p_1\cdot p_2\right)\right)+k_1\cdot p_1\left(\epsilon(k_1)\cdot k_2(k_1\cdot k_2(k_2\cdot k_3-4k_3\cdot p_1)\right.\\
&-k_2\cdot k_3(k_1\cdot k_3+k_2\cdot k_3-2k_3\cdot p_1))+\epsilon(k_1)\cdot k_3k_1\cdot k_2\left(-2k_1\cdot p_1+k_2\cdot k_3\right.\\
&\left.\left.+2\left(m^2+k_2\cdot p_1+k_3\cdot p_1+p_1\cdot p_2\right)\right)\right)+\epsilon(k_1)\cdot k_1\left(k_1\cdot k_2\left(k_2\cdot k_3\left(m^2+p_1\cdot p_2\right)\right.\right.\\
&\left.-2k_3\cdot p_1\left(m^2+k_1\cdot p_1+p_1\cdot p_2\right)+k_1\cdot k_3\left(2\left(m^2+p_1\cdot p_2\right)-k_1\cdot p_1\right)\right)-\\
&k_1\cdot p_1k_2\cdot k_3(k_1\cdot k_3+k_2\cdot k_3-2k_3\cdot p_1)))\\
&+\frac{(t^{a_2}t^{a_3}t^{a_1})_{o_1o_2}}{k_1\cdot k_3k_1\cdot p_2k_2\cdot k_3\left(m^2+p_1\cdot p_2\right)}\\
&\left(k_1\cdot k_3\left(\epsilon(k_1)\cdot k_1\left(k_1\cdot k_3k_1\cdot p_2+k_2\cdot k_3\left(m^2+p_1\cdot p_2\right)-2k_3\cdot p_1\left(m^2-k_1\cdot p_2+p_1\cdot p_2\right)\right)\right.\right.\\
&-2\epsilon(k_1)\cdot p_1\left(2k_1\cdot k_3k_1\cdot p_2-2k_3\cdot p_1\left(m^2+p_1\cdot p_2\right)+k_2\cdot k_3\left(m^2+k_1\cdot p_2+p_1\cdot p_2\right)\right)\\
&\left.-\epsilon(k_1)\cdot k_2\left(4k_3\cdot p_1\left(m^2-k_1\cdot p_2+p_1\cdot p_2\right)+k_2\cdot k_3\left(k_1\cdot p_2-2\left(m^2+p_1\cdot p_2\right)\right)\right)\right)\\
&+\epsilon(k_1)\cdot k_3\left(k_1\cdot p_2k_2\cdot k_3(-k_1\cdot k_2+2k_1\cdot p_1+k_2\cdot k_3-2k_3\cdot p_1)\right.\\
&+k_1\cdot k_3\left(k_1\cdot p_2\left(2k_1\cdot p_1+k_2\cdot k_3+2\left(m^2-k_2\cdot p_1-k_3\cdot p_1+p_1\cdot p_2\right)\right)\right.\\
&\left.\left.\left.-2(k_2\cdot k_3-2k_3\cdot p_1)\left(m^2+p_1\cdot p_2\right)\right)\right)\right)-
\end{aligned}
$$

$$-\frac{(t^{a_1}t^{a_3}t^{a_2})_{o_1o_2}}{k_1\cdot k_3 k_1\cdot p_1 k_2\cdot k_3\left(m^2+p_1\cdot p_2\right)}\times$$
$$\left(-2\epsilon(k_1)\cdot p_1 k_1\cdot k_3\left(-2k_3\cdot p_1\left(m^2+p_1\cdot p_2\right)+2k_1\cdot k_3\left(m^2-k_1\cdot p_1+p_1\cdot p_2\right)\right.\right.$$
$$\left.+k_2\cdot k_3\left(m^2-k_1\cdot p_1+p_1\cdot p_2\right)\right)+\epsilon(k_1)\cdot k_1 k_1\cdot k_3\left(k_2\cdot k_3\left(m^2+p_1\cdot p_2\right)\right.$$
$$\left.-2k_3\cdot p_1\left(m^2+k_1\cdot p_1+p_1\cdot p_2\right)+k_1\cdot k_3\left(2\left(m^2+p_1\cdot p_2\right)-k_1\cdot p_1\right)\right)$$
$$+k_1\cdot p_1\left(\epsilon(k_1)\cdot k_2 k_1\cdot k_3(k_2\cdot k_3-4k_3\cdot p_1)+\epsilon(k_1)\cdot k_3\left(k_2\cdot k_3(k_1\cdot k_2-2k_1\cdot p_1-\right.\right.$$
$$\left.\left.\left.k_2\cdot k_3+2k_3\cdot p_1)+k_1\cdot k_3\left(-2k_1\cdot p_1-k_2\cdot k_3+2\left(m^2+k_2\cdot p_1+k_3\cdot p_1+p_1\cdot p_2\right)\right)\right)\right)\right)$$

$$\mathcal{M}_{g\overline{\eta}\to\overline{\eta}\tilde{t}\tilde{\bar{t}}}=\alpha_s g_s\pi\times$$
$$\left(\frac{(t^{a_2}t^{a_1}t^{a_3})_{o_1o_2}}{k_1\cdot k_2 k_1\cdot k_3\left(m^2+p_1\cdot p_2\right)}\right.$$
$$(\epsilon(k_1)\cdot k_3 k_1\cdot k_2(k_1\cdot k_2-k_2\cdot k_3-2k_2\cdot p_1)+\epsilon(k_1)\cdot k_1 k_1\cdot k_2(-k_1\cdot k_2+k_2\cdot k_3+2k_2\cdot p_1)$$
$$+\epsilon(k_1)\cdot k_2 k_1\cdot k_3(-2k_1\cdot k_2+k_1\cdot k_3+2k_1\cdot p_1+k_2\cdot k_3+2k_2\cdot p_1))$$
$$+\frac{(t^{a_3}t^{a_1}t^{a_2})_{o_1o_2}}{k_1\cdot k_2 k_1\cdot k_3\left(m^2+p_1\cdot p_2\right)}$$
$$(\epsilon(k_1)\cdot k_3 k_1\cdot k_2(k_1\cdot k_2-k_2\cdot k_3-2k_2\cdot p_1)+\epsilon(k_1)\cdot k_1 k_1\cdot k_2(-k_1\cdot k_2+k_2\cdot k_3+2k_2\cdot p_1)$$
$$+\epsilon(k_1)\cdot k_2 k_1\cdot k_3(-2k_1\cdot k_2+k_1\cdot k_3+2k_1\cdot p_1+k_2\cdot k_3+2k_2\cdot p_1))$$
$$+(t^{a_2}t^{a_3}t^{a_1})_{o_1o_2}\left(-\frac{2\epsilon(k_1)\cdot k_2}{k_2\cdot k_3}\right.$$
$$+\frac{(\epsilon(k_1)\cdot k_1+2\epsilon(k_1)\cdot k_2-2\epsilon(k_1)\cdot k_3-2\epsilon(k_1)\cdot p_1)(k_2\cdot k_3+2k_2\cdot p_1)}{k_1\cdot p_2 k_2\cdot k_3}$$
$$+\frac{(\epsilon(k_1)\cdot k_1-\epsilon(k_1)\cdot k_3)(k_1\cdot k_2-k_2\cdot k_3-2k_2\cdot p_1)}{k_1\cdot k_3\left(m^2+p_1\cdot p_2\right)}$$
$$+\frac{1}{k_2\cdot k_3\left(m^2+p_1\cdot p_2\right)}$$
$$2\epsilon(k_1)\cdot p_1(2k_1\cdot k_2-k_2\cdot k_3)-\epsilon(k_1)\cdot k_1(k_1\cdot k_2+2k_2\cdot p_1)+\epsilon(k_1)\cdot k_3(k_2\cdot k_3+4k_2\cdot p_1)$$
$$+\epsilon(k_1)\cdot k_2(-2k_1\cdot p_1+k_2\cdot k_3-2(k_2\cdot p_1+k_3\cdot p_1)))$$
$$+\frac{(t^{a_1}t^{a_3}t^{a_2})_{o_1o_2}}{k_1\cdot k_3 k_1\cdot p_1 k_2\cdot k_3\left(m^2+p_1\cdot p_2\right)}$$
$$\left(2\epsilon(k_1)\cdot p_1 k_1\cdot k_3\left(2k_2\cdot p_1\left(m^2+p_1\cdot p_2\right)-2k_1\cdot k_2\left(m^2-k_1\cdot p_1+p_1\cdot p_2\right)\right.\right.$$
$$\left.+k_2\cdot k_3\left(m^2-k_1\cdot p_1+p_1\cdot p_2\right)\right)+k_1\cdot p_1\left(\epsilon(k_1)\cdot k_3(-k_1\cdot k_2 k_2\cdot k_3+(k_1\cdot k_3+k_2\cdot k_3)k_2\cdot k_3\right.$$
$$+2(2k_1\cdot k_3+k_2\cdot k_3)k_2\cdot p_1)+\epsilon(k_1)\cdot k_2 k_1\cdot k_3\left(-2k_1\cdot p_1+k_2\cdot k_3\right.$$
$$\left.\left.+2\left(m^2-k_2\cdot p_1-k_3\cdot p_1+p_1\cdot p_2\right)\right)\right)+\epsilon(k_1)\cdot k_1\left(-k_1\cdot p_1 k_2\cdot k_3(k_2\cdot k_3+2k_2\cdot p_1)\right.$$
$$-k_1\cdot k_3\left(k_2\cdot k_3\left(m^2+p_1\cdot p_2\right)+2k_2\cdot p_1\left(m^2+k_1\cdot p_1+p_1\cdot p_2\right)\right)+k_1\cdot k_2\left(k_1\cdot p_1 k_2\cdot k_3\right.$$
$$\left.\left.\left.+k_1\cdot k_3\left(2\left(m^2+p_1\cdot p_2\right)-k_1\cdot p_1\right)\right)\right)\right)-$$

$$-\frac{(t^{a_1}t^{a_2}t^{a_3})_{o_1o_2}}{k_1\cdot k_2 k_1\cdot p_1 k_2\cdot k_3\left(m^2+p_1\cdot p_2\right)}\times$$
$$\left(2\epsilon(k_1)\cdot p_1 k_1\cdot k_2\left(2k_2\cdot p_1\left(m^2+p_1\cdot p_2\right)-2k_1\cdot k_2\left(m^2-k_1\cdot p_1+p_1\cdot p_2\right)\right.\right.$$
$$\left.+k_2\cdot k_3\left(m^2-k_1\cdot p_1+p_1\cdot p_2\right)\right)+\epsilon(k_1)\cdot k_1 k_1\cdot k_2\left(-k_2\cdot k_3\left(m^2+p_1\cdot p_2\right)\right.$$
$$\left.-2k_2\cdot p_1\left(m^2+k_1\cdot p_1+p_1\cdot p_2\right)+k_1\cdot k_2\left(2\left(m^2+p_1\cdot p_2\right)-k_1\cdot p_1\right)\right)$$
$$+k_1\cdot p_1\left(\epsilon(k_1)\cdot k_3 k_1\cdot k_2(k_2\cdot k_3+4k_2\cdot p_1)+\epsilon(k_1)\cdot k_2\left(k_2\cdot k_3(k_1\cdot k_3+2k_1\cdot p_1\right.\right.$$
$$\left.\left.\left.+k_2\cdot k_3+2k_2\cdot p_1)-k_1\cdot k_2\left(2k_1\cdot p_1+k_2\cdot k_3-2\left(m^2-k_2\cdot p_1-k_3\cdot p_1+p_1\cdot p_2\right)\right)\right)\right)\right)$$
$$+\frac{(t^{a_3}t^{a_2}t^{a_1})_{o_1o_2}}{k_1\cdot k_2 k_1\cdot p_2 k_2\cdot k_3\left(m^2+p_1\cdot p_2\right)}\times$$
$$\left(k_1\cdot k_2\left(\epsilon(k_1)\cdot k_1\left(k_1\cdot k_2 k_1\cdot p_2-k_2\cdot k_3\left(m^2+p_1\cdot p_2\right)-2k_2\cdot p_1\left(m^2-k_1\cdot p_2+p_1\cdot p_2\right)\right)\right.\right.$$
$$+2\epsilon(k_1)\cdot p_1\left(-2k_1\cdot k_2 k_1\cdot p_2+2k_2\cdot p_1\left(m^2+p_1\cdot p_2\right)+k_2\cdot k_3\left(m^2+k_1\cdot p_2+p_1\cdot p_2\right)\right)+$$
$$\left.\epsilon(k_1)\cdot k_3\left(4k_2\cdot p_1\left(m^2-k_1\cdot p_2+p_1\cdot p_2\right)+k_2\cdot k_3\left(2\left(m^2+p_1\cdot p_2\right)-k_1\cdot p_2\right)\right)\right)$$
$$+\epsilon(k_1)\cdot k_2\left(k_1\cdot k_2\left(k_1\cdot p_2\left(2k_1\cdot p_1+k_2\cdot k_3+2\left(m^2+k_2\cdot p_1+k_3\cdot p_1+p_1\cdot p_2\right)\right)\right.\right.$$
$$\left.\left.\left.-2(k_2\cdot k_3+2k_2\cdot p_1)\left(m^2+p_1\cdot p_2\right)\right)-k_1\cdot p_2 k_2\cdot k_3(k_1\cdot k_3+2k_1\cdot p_1+k_2\cdot k_3+2k_2\cdot p_1)\right)\right)\ )$$

$$\mathcal{M}_{\eta g\to\eta\tilde{t}\bar{\tilde{t}}}=\alpha_s g_s\pi\left(\frac{(t^{a_1}t^{a_2}t^{a_3})_{o_1o_2}}{k_1\cdot k_2 k_2\cdot k_3\left(m^2+p_1\cdot p_2\right)}\times\right.$$
$$(\epsilon(k_2)\cdot k_3 k_1\cdot k_2(k_1\cdot k_2-k_1\cdot k_3-2(k_2\cdot k_3+k_2\cdot p_1-k_3\cdot p_1))$$
$$+(\epsilon(k_2)\cdot k_1+\epsilon(k_2)\cdot k_2)k_2\cdot k_3(k_1\cdot k_3+k_2\cdot k_3-2k_3\cdot p_1))$$
$$+\frac{(t^{a_3}t^{a_2}t^{a_1})_{o_1o_2}}{k_1\cdot k_2 k_2\cdot k_3\left(m^2+p_1\cdot p_2\right)}$$
$$(\epsilon(k_2)\cdot k_3 k_1\cdot k_2(k_1\cdot k_2-k_1\cdot k_3-2(k_2\cdot k_3+k_2\cdot p_1-k_3\cdot p_1))$$
$$+(\epsilon(k_2)\cdot k_1+\epsilon(k_2)\cdot k_2)k_2\cdot k_3(k_1\cdot k_3+k_2\cdot k_3-2k_3\cdot p_1))$$
$$+(t^{a_1}t^{a_3}t^{a_2})_{o_1o_2}\left(\frac{2\epsilon(k_2)\cdot k_3}{k_1\cdot k_3}\right.$$
$$+\frac{(-k_1\cdot k_2+k_1\cdot k_3+2(k_2\cdot k_3+k_2\cdot p_1-k_3\cdot p_1))\epsilon(k_2)\cdot k_3}{k_2\cdot k_3\left(m^2+p_1\cdot p_2\right)}$$
$$+\frac{(2\epsilon(k_2)\cdot k_1+\epsilon(k_2)\cdot k_2-2(\epsilon(k_2)\cdot k_3+\epsilon(k_2)\cdot p_1))(k_1\cdot k_3-2k_3\cdot p_1)}{k_1\cdot k_3 k_2\cdot p_2}$$
$$-\frac{}{k_1\cdot k_3\left(m^2+p_1\cdot p_2\right)}\left(2\epsilon(k_2)\cdot p_1(k_1\cdot k_3+2k_2\cdot k_3)+\epsilon(k_2)\cdot k_1(k_1\cdot k_3-4k_3\cdot p_1)\right.$$
$$\left.\left.-\epsilon(k_2)\cdot k_2(k_2\cdot k_3+2k_3\cdot p_1)+\epsilon(k_2)\cdot k_3(k_1\cdot k_3+2(k_1\cdot p_1-k_2\cdot p_1+k_3\cdot p_1))\right)\right)$$
$$+(t^{a_3}t^{a_1}t^{a_2})_{o_1o_2}\left(-\frac{2\epsilon(k_2)\cdot k_3}{k_1\cdot k_3}\right.$$
$$-\frac{(2\epsilon(k_2)\cdot k_1+\epsilon(k_2)\cdot k_2-2(\epsilon(k_2)\cdot k_3+\epsilon(k_2)\cdot p_1))(k_1\cdot k_3-2k_3\cdot p_1)}{k_1\cdot k_3 k_2\cdot p_2}-$$

$$-\frac{(\epsilon(k_2)\cdot k_1+\epsilon(k_2)\cdot k_2)(k_1\cdot k_3+k_2\cdot k_3-2k_3\cdot p_1)}{k_1\cdot k_2\left(m^2+p_1\cdot p_2\right)}$$
$$+\frac{1}{k_1\cdot k_3\left(m^2+p_1\cdot p_2\right)}\left(2\epsilon(k_2)\cdot p_1(k_1\cdot k_3+2k_2\cdot k_3)+\epsilon(k_2)\cdot k_1(k_1\cdot k_3-4k_3\cdot p_1)\right.$$
$$\left.-\epsilon(k_2)\cdot k_2(k_2\cdot k_3+2k_3\cdot p_1)+\epsilon(k_2)\cdot k_3(k_1\cdot k_3+2(k_1\cdot p_1-k_2\cdot p_1+k_3\cdot p_1))\right))$$
$$-\frac{(t^{a_2}t^{a_3}t^{a_1})_{o_1o_2}}{k_1\cdot k_3k_2\cdot k_3k_2\cdot p_1\left(m^2+p_1\cdot p_2\right)}$$
$$\left(\epsilon(k_2)\cdot k_2k_2\cdot k_3\left(k_1\cdot k_3\left(m^2+p_1\cdot p_2\right)-2k_3\cdot p_1\left(m^2+k_2\cdot p_1+p_1\cdot p_2\right)\right.\right.$$
$$\left.+k_2\cdot k_3\left(2\left(m^2+p_1\cdot p_2\right)-k_2\cdot p_1\right)\right)-2\epsilon(k_2)\cdot p_1k_2\cdot k_3\left(k_1\cdot k_3\left(m^2-k_2\cdot p_1+p_1\cdot p_2\right)\right.$$
$$\left.+2\left(k_2\cdot k_3\left(m^2-k_2\cdot p_1+p_1\cdot p_2\right)-k_3\cdot p_1\left(m^2+p_1\cdot p_2\right)\right)\right)$$
$$+k_2\cdot p_1\left(\epsilon(k_2)\cdot k_1k_2\cdot k_3(k_1\cdot k_3-4k_3\cdot p_1)+\epsilon(k_2)\cdot k_3\left(-(k_1\cdot k_3)^2+k_1\cdot k_2k_1\cdot k_3\right.\right.$$
$$\left.\left.\left.-(k_2\cdot k_3+2k_2\cdot p_1-2k_3\cdot p_1)k_1\cdot k_3+2k_2\cdot k_3\left(m^2+k_1\cdot p_1-k_2\cdot p_1+k_3\cdot p_1+p_1\cdot p_2\right)\right)\right)\right)$$
$$+\frac{(t^{a_2}t^{a_1}t^{a_3})_{o_1o_2}}{k_1\cdot k_2k_1\cdot k_3k_2\cdot p_1\left(m^2+p_1\cdot p_2\right)}$$
$$\left(-2\epsilon(k_2)\cdot p_1k_1\cdot k_2\left(k_1\cdot k_3\left(m^2-k_2\cdot p_1+p_1\cdot p_2\right)+2\left(k_2\cdot k_3\left(m^2-k_2\cdot p_1+p_1\cdot p_2\right)\right.\right.\right.$$
$$\left.\left.\left.-k_3\cdot p_1\left(m^2+p_1\cdot p_2\right)\right)\right)+k_2\cdot p_1\left(\epsilon(k_2)\cdot k_1(k_1\cdot k_2(k_1\cdot k_3-4k_3\cdot p_1)\right.$$
$$-k_1\cdot k_3(k_1\cdot k_3+k_2\cdot k_3-2k_3\cdot p_1))+\epsilon(k_2)\cdot k_3k_1\cdot k_2\left(k_1\cdot k_3+2\left(m^2+k_1\cdot p_1\right.\right.$$
$$\left.\left.-k_2\cdot p_1+k_3\cdot p_1+p_1\cdot p_2\right)\right))+\epsilon(k_2)\cdot k_2\left(k_1\cdot k_2\left(k_1\cdot k_3\left(m^2+p_1\cdot p_2\right)\right.\right.$$
$$\left.-2k_3\cdot p_1\left(m^2+k_2\cdot p_1+p_1\cdot p_2\right)+k_2\cdot k_3\left(2\left(m^2+p_1\cdot p_2\right)-k_2\cdot p_1\right)\right)$$
$$\left.\left.-k_1\cdot k_3k_2\cdot p_1(k_1\cdot k_3+k_2\cdot k_3-2k_3\cdot p_1)\right)\right)\ )$$

$$\mathcal{M}_{\overline{\eta}g\to\overline{\eta}\tilde{t}\tilde{\bar{t}}}=\alpha_s g_s\pi\left(-\frac{(t^{a_1}t^{a_2}t^{a_3})_{o_1o_2}}{k_1\cdot k_2k_2\cdot k_3\left(m^2+p_1\cdot p_2\right)}\times\right.$$
$$(\epsilon(k_2)\cdot k_2k_1\cdot k_2(k_1\cdot k_2-k_1\cdot k_3-2k_1\cdot p_1)+\epsilon(k_2)\cdot k_3k_1\cdot k_2(-k_1\cdot k_2+k_1\cdot k_3+2k_1\cdot p_1)$$
$$-\epsilon(k_2)\cdot k_1k_2\cdot k_3(-2k_1\cdot k_2+k_1\cdot k_3+2k_1\cdot p_1+k_2\cdot k_3+2k_2\cdot p_1))$$
$$-\frac{(t^{a_3}t^{a_2}t^{a_1})_{o_1o_2}}{k_1\cdot k_2k_2\cdot k_3\left(m^2+p_1\cdot p_2\right)}(\epsilon(k_2)\cdot k_2k_1\cdot k_2(k_1\cdot k_2-k_1\cdot k_3-2k_1\cdot p_1)$$
$$+\epsilon(k_2)\cdot k_3k_1\cdot k_2(-k_1\cdot k_2+k_1\cdot k_3+2k_1\cdot p_1)$$
$$-\epsilon(k_2)\cdot k_1k_2\cdot k_3(-2k_1\cdot k_2+k_1\cdot k_3+2k_1\cdot p_1+k_2\cdot k_3+2k_2\cdot p_1))$$
$$+(t^{a_2}t^{a_3}t^{a_1})_{o_1o_2}\left(\frac{2\epsilon(k_2)\cdot k_1}{k_1\cdot k_3}-\frac{(\epsilon(k_2)\cdot k_2-2\epsilon(k_2)\cdot p_1)(-2k_1\cdot k_2+k_1\cdot k_3+2k_1\cdot p_1)}{k_1\cdot k_3k_2\cdot p_1}\right.$$
$$+\frac{1}{k_1\cdot k_3\left(m^2+p_1\cdot p_2\right)}\left(2\epsilon(k_2)\cdot p_1(2k_1\cdot k_2-k_1\cdot k_3)-\epsilon(k_2)\cdot k_2(k_1\cdot k_2+2k_1\cdot p_1)\right.$$
$$\left.+\epsilon(k_2)\cdot k_3(k_1\cdot k_3+4k_1\cdot p_1)+\epsilon(k_2)\cdot k_1(k_1\cdot k_3-2(k_1\cdot p_1+k_2\cdot p_1+k_3\cdot p_1))\right)$$
$$\left.+\frac{(\epsilon(k_2)\cdot k_2-\epsilon(k_2)\cdot k_3)(k_1\cdot k_2-k_1\cdot k_3-2k_1\cdot p_1)}{k_2\cdot k_3\left(m^2+p_1\cdot p_2\right)}\right)+$$

$$
\begin{aligned}
&+(t^{a_1}t^{a_3}t^{a_2})_{o_1o_2}\left(-\frac{2\epsilon(k_2)\cdot k_1}{k_1\cdot k_3}\right.\\
&+\frac{(2\epsilon(k_2)\cdot k_1+\epsilon(k_2)\cdot k_2-2(\epsilon(k_2)\cdot k_3+\epsilon(k_2)\cdot p_1))(k_1\cdot k_3+2k_1\cdot p_1)}{k_1\cdot k_3k_2\cdot p_2}\\
&+\frac{1}{k_1\cdot k_3\,(m^2+p_1\cdot p_2)}\,(2\epsilon(k_2)\cdot p_1(2k_1\cdot k_2-k_1\cdot k_3)-\epsilon(k_2)\cdot k_2(k_1\cdot k_2+2k_1\cdot p_1)\\
&+\epsilon(k_2)\cdot k_3(k_1\cdot k_3+4k_1\cdot p_1)+\epsilon(k_2)\cdot k_1(k_1\cdot k_3-2(k_1\cdot p_1+k_2\cdot p_1+k_3\cdot p_1)))\\
&+\left.\frac{(\epsilon(k_2)\cdot k_2-\epsilon(k_2)\cdot k_3)(k_1\cdot k_2-k_1\cdot k_3-2k_1\cdot p_1)}{k_2\cdot k_3\,(m^2+p_1\cdot p_2)}\right)\\
&-\frac{(t^{a_2}t^{a_1}t^{a_3})_{o_1o_2}}{k_1\cdot k_2k_1\cdot k_3k_2\cdot p_1\,(m^2+p_1\cdot p_2)}\,(k_2\cdot p_1\,(\epsilon(k_2)\cdot k_3k_1\cdot k_2(k_1\cdot k_3+4k_1\cdot p_1)\\
&+\epsilon(k_2)\cdot k_1\,(k_1\cdot k_3(k_1\cdot k_3+2k_1\cdot p_1+k_2\cdot k_3+2k_2\cdot p_1)\\
&+k_1\cdot k_2\left(-k_1\cdot k_3-2\left(-m^2+k_1\cdot p_1+k_2\cdot p_1+k_3\cdot p_1-p_1\cdot p_2\right)\right)))\\
&+2\epsilon(k_2)\cdot p_1k_1\cdot k_2\left(2k_1\cdot p_1\left(m^2+p_1\cdot p_2\right)-2k_1\cdot k_2\left(m^2-k_2\cdot p_1+p_1\cdot p_2\right)\right.\\
&+\left.k_1\cdot k_3\left(m^2-k_2\cdot p_1+p_1\cdot p_2\right)\right)+\epsilon(k_2)\cdot k_2k_1\cdot k_2\left(-k_1\cdot k_3\left(m^2+p_1\cdot p_2\right)\right.\\
&\left.-2k_1\cdot p_1\left(m^2+k_2\cdot p_1+p_1\cdot p_2\right)+k_1\cdot k_2\left(2\left(m^2+p_1\cdot p_2\right)-k_2\cdot p_1\right)\right))\\
&-\frac{(t^{a_3}t^{a_1}t^{a_2})_{o_1o_2}}{k_1\cdot k_2k_1\cdot k_3k_2\cdot p_2\,(m^2+p_1\cdot p_2)}\,(k_1\cdot k_2\,(\epsilon(k_2)\cdot k_2\,(-k_1\cdot k_2k_2\cdot p_2\\
&+k_1\cdot k_3\left(m^2+p_1\cdot p_2\right)+2k_1\cdot p_1\left(m^2-k_2\cdot p_2+p_1\cdot p_2\right))\\
&-2\epsilon(k_2)\cdot p_1\left(-2k_1\cdot k_2k_2\cdot p_2+2k_1\cdot p_1\left(m^2+p_1\cdot p_2\right)+k_1\cdot k_3\left(m^2+k_2\cdot p_2+p_1\cdot p_2\right)\right)\\
&+\epsilon(k_2)\cdot k_3\left(k_1\cdot k_3\left(k_2\cdot p_2-2\left(m^2+p_1\cdot p_2\right)\right)-4k_1\cdot p_1\left(m^2-k_2\cdot p_2+p_1\cdot p_2\right)\right))\\
&+\epsilon(k_2)\cdot k_1\,(k_1\cdot k_3(k_1\cdot k_3+2k_1\cdot p_1+k_2\cdot k_3+2k_2\cdot p_1)k_2\cdot p_2\\
&+k_1\cdot k_2\left(-2k_2\cdot p_2\left(m^2+k_2\cdot p_1+k_3\cdot p_1+p_1\cdot p_2\right)+k_1\cdot k_3\left(2\left(m^2+p_1\cdot p_2\right)-k_2\cdot p_2\right)\right.\\
&\left.+k_1\cdot p_1\left(4\left(m^2+p_1\cdot p_2\right)-2k_2\cdot p_2\right)\right)))\;)
\end{aligned}
$$

$$
\begin{aligned}
&\mathcal{M}_{\overline{\eta}\eta\to g\tilde{t}\tilde{\bar{t}}}=\alpha_s g_s\pi\left(\frac{(t^{a_1}t^{a_3}t^{a_2})_{o_1o_2}}{k_1\cdot k_3k_2\cdot k_3\,(m^2+p_1\cdot p_2)}\times\right.\\
&(\epsilon^*(k_3)\cdot k_3k_1\cdot k_3(k_1\cdot k_2-k_1\cdot k_3-2k_1\cdot p_1)+\epsilon^*(k_3)\cdot k_2k_1\cdot k_3(-k_1\cdot k_2+k_1\cdot k_3+2k_1\cdot p_1)\\
&+\epsilon^*(k_3)\cdot k_1k_2\cdot k_3(k_1\cdot k_2-2k_1\cdot k_3-2k_1\cdot p_1-k_2\cdot k_3+2k_3\cdot p_1))\\
&+\frac{(t^{a_2}t^{a_3}t^{a_1})_{o_1o_2}}{k_1\cdot k_3k_2\cdot k_3\,(m^2+p_1\cdot p_2)}\times\\
&(\epsilon^*(k_3)\cdot k_3k_1\cdot k_3(k_1\cdot k_2-k_1\cdot k_3-2k_1\cdot p_1)+\epsilon^*(k_3)\cdot k_2k_1\cdot k_3(-k_1\cdot k_2+k_1\cdot k_3+2k_1\cdot p_1)\\
&+\epsilon^*(k_3)\cdot k_1k_2\cdot k_3(k_1\cdot k_2-2k_1\cdot k_3-2k_1\cdot p_1-k_2\cdot k_3+2k_3\cdot p_1))+
\end{aligned}
$$

$$
\begin{aligned}
&+(t^{a_1}t^{a_2}t^{a_3})_{o_1o_2}\left(\frac{2\epsilon^*(k_3)\cdot k_1}{k_1\cdot k_2}\right.\\
&-\frac{(2\epsilon^*(k_3)\cdot k_1+2\epsilon^*(k_3)\cdot k_2-\epsilon^*(k_3)\cdot k_3-2\epsilon^*(k_3)\cdot p_1)(k_1\cdot k_2-2k_1\cdot p_1)}{k_1\cdot k_2k_3\cdot p_2}\\
&+\frac{1}{k_1\cdot k_2\left(m^2+p_1\cdot p_2\right)}\left(-2\epsilon^*(k_3)\cdot p_1(k_1\cdot k_2-2k_1\cdot k_3)-\epsilon^*(k_3)\cdot k_2(k_1\cdot k_2-4k_1\cdot p_1)\right.\\
&\left.+\epsilon^*(k_3)\cdot k_3(k_1\cdot k_3-2k_1\cdot p_1)+\epsilon^*(k_3)\cdot k_1(k_1\cdot k_2+2k_1\cdot p_1-2k_2\cdot p_1-2k_3\cdot p_1)\right)\\
&\left.+\ \frac{(\epsilon^*(k_3)\cdot k_2-\epsilon^*(k_3)\cdot k_3)(k_1\cdot k_2-k_1\cdot k_3-2k_1\cdot p_1)}{k_2\cdot k_3\left(m^2+p_1\cdot p_2\right)}\right)\\
&+\frac{(t^{a_3}t^{a_1}t^{a_2})_{o_1o_2}}{k_1\cdot k_2k_1\cdot k_3k_3\cdot p_1\left(m^2+p_1\cdot p_2\right)}\\
&\left(2\epsilon^*(k_3)\cdot p_1k_1\cdot k_3\left(-2k_1\cdot p_1\left(m^2+p_1\cdot p_2\right)+k_1\cdot k_2\left(m^2+k_3\cdot p_1+p_1\cdot p_2\right)\right.\right.\\
&\left.-2k_1\cdot k_3\left(m^2+k_3\cdot p_1+p_1\cdot p_2\right)\right)+\epsilon^*(k_3)\cdot k_3k_1\cdot k_3\left(k_1\cdot k_2\left(m^2+p_1\cdot p_2\right)\right.\\
&\left.-2k_1\cdot p_1\left(m^2-k_3\cdot p_1+p_1\cdot p_2\right)-k_1\cdot k_3\left(k_3\cdot p_1+2\left(m^2+p_1\cdot p_2\right)\right)\right)\\
&+k_3\cdot p_1\left(\epsilon^*(k_3)\cdot k_2k_1\cdot k_3(k_1\cdot k_2-4k_1\cdot p_1)+\epsilon^*(k_3)\cdot k_1\left(-(k_1\cdot k_2)^2\right.\right.\\
&\ +(k_1\cdot k_3+2k_1\cdot p_1+k_2\cdot k_3-2k_3\cdot p_1)k_1\cdot k_2+2k_1\cdot k_3\left(m^2-k_1\cdot p_1\right.\\
&\left.\left.\left.\left.\ +k_2\cdot p_1+k_3\cdot p_1+p_1\cdot p_2\right)\right)\right)\right)\\
&+\frac{(t^{a_3}t^{a_2}t^{a_1})_{o_1o_2}}{k_1\cdot k_2k_2\cdot k_3k_3\cdot p_1\left(m^2+p_1\cdot p_2\right)}\\
&\left(2\epsilon^*(k_3)\cdot p_1k_2\cdot k_3\left(2\left(k_1\cdot p_1\left(m^2+p_1\cdot p_2\right)+k_1\cdot k_3\left(m^2+k_3\cdot p_1+p_1\cdot p_2\right)\right)\right.\right.\\
&\left.-k_1\cdot k_2\left(m^2+k_3\cdot p_1+p_1\cdot p_2\right)\right)-k_3\cdot p_1\left(\epsilon^*(k_3)\cdot k_2\left(-(k_1\cdot k_2)^2\right.\right.\\
&\left.\ +(k_1\cdot k_3+2k_1\cdot p_1+k_2\cdot k_3)k_1\cdot k_2-4k_1\cdot p_1k_2\cdot k_3\right)\\
&\left.\ +\epsilon^*(k_3)\cdot k_1k_2\cdot k_3\left(2\left(m^2-k_1\cdot p_1+k_2\cdot p_1+k_3\cdot p_1+p_1\cdot p_2\right)-k_1\cdot k_2\right)\right)\\
&\ +\epsilon^*(k_3)\cdot k_3\left(-k_3\cdot p_1(k_1\cdot k_2)^2+\left((k_1\cdot k_3+2k_1\cdot p_1)k_3\cdot p_1-k_2\cdot k_3\left(m^2+p_1\cdot p_2\right)\right)k_1\cdot k_2\right.\\
&\left.\left.\ +k_2\cdot k_3\left(2k_1\cdot p_1\left(m^2-k_3\cdot p_1+p_1\cdot p_2\right)+k_1\cdot k_3\left(k_3\cdot p_1+2\left(m^2+p_1\cdot p_2\right)\right)\right)\right)\right)\\
&+\frac{(t^{a_2}t^{a_1}t^{a_3})_{o_1o_2}}{k_1\cdot k_2k_1\cdot k_3k_3\cdot p_2\left(m^2+p_1\cdot p_2\right)}\\
&\left(\epsilon^*(k_3)\cdot k_1\left(-k_3\cdot p_2(k_1\cdot k_2)^2+((2k_1\cdot p_1+k_2\cdot k_3-2k_3\cdot p_1)k_3\cdot p_2\right.\right.\\
&\ +k_1\cdot k_3\left(k_3\cdot p_2+2\left(m^2+p_1\cdot p_2\right)\right))k_1\cdot k_2-2k_1\cdot k_3\left(k_3\cdot p_2\left(m^2-k_2\cdot p_1\right.\right.\\
&\left.\left.\left.\ -k_3\cdot p_1+p_1\cdot p_2\right)+k_1\cdot p_1\left(k_3\cdot p_2+2\left(m^2+p_1\cdot p_2\right)\right)\right)\right)\\
&-k_1\cdot k_3\left(\epsilon^*(k_3)\cdot k_3\left(k_1\cdot k_3k_3\cdot p_2+k_1\cdot k_2\left(m^2+p_1\cdot p_2\right)-2k_1\cdot p_1\left(m^2+k_3\cdot p_2+p_1\cdot p_2\right)\right)\right.\\
&+\epsilon^*(k_3)\cdot k_2\left(4k_1\cdot p_1\left(m^2+k_3\cdot p_2+p_1\cdot p_2\right)-k_1\cdot k_2\left(k_3\cdot p_2+2\left(m^2+p_1\cdot p_2\right)\right)\right)\\
&\left.\left.\ +2\epsilon^*(k_3)\cdot p_1\left(k_1\cdot k_2\left(m^2-k_3\cdot p_2+p_1\cdot p_2\right)-2\left(k_1\cdot p_1\left(m^2+p_1\cdot p_2\right)-k_1\cdot k_3k_3\cdot p_2\right)\right)\right)\right)
\end{aligned}
$$

Amplitudes for $G \to G\tilde{t}\bar{\tilde{t}}$, ϵ scalars

$$\mathcal{M}_{\phi\phi\to g\tilde{t}\bar{\tilde{t}}} = \alpha_s g_s \pi \epsilon(k_1)\cdot\epsilon(k_2)\left(\frac{(t^{a_1}t^{a_2}t^{a_3})_{o_1o_2}}{k_1\cdot k_2 k_2\cdot k_3}\times\right.$$
$$(-2\epsilon^*(k_3)\cdot k_3 k_1\cdot k_2 + \epsilon^*(k_3)\cdot k_2(4k_1\cdot k_2 - 2k_2\cdot k_3) + 2\epsilon^*(k_3)\cdot k_1 k_2\cdot k_3$$
$$-\frac{2(2\epsilon^*(k_3)\cdot k_1 + 2\epsilon^*(k_3)\cdot k_2 - \epsilon^*(k_3)\cdot k_3 - 2\epsilon^*(k_3)\cdot k_4)k_2\cdot k_3(k_1\cdot k_2 - k_1\cdot k_4 + k_2\cdot k_4)}{k_3\cdot k_5}$$
$$+\frac{1}{m^2 + k_4\cdot k_5}(2k_2\cdot k_3(2\epsilon^*(k_3)\cdot k_4(k_1\cdot k_2 + k_1\cdot k_3 - k_2\cdot k_3) + \epsilon^*(k_3)\cdot k_1(k_1\cdot k_4 - 3k_2\cdot k_4 - k_3\cdot k_4))$$
$$+\epsilon^*(k_3)\cdot k_3(k_2\cdot k_3(k_1\cdot k_3 - 2k_1\cdot k_4 - k_2\cdot k_3 + 2k_2\cdot k_4) + 2k_1\cdot k_2(k_1\cdot k_4 - k_2\cdot k_4 + k_3\cdot k_4))$$
$$+\epsilon^*(k_3)\cdot k_2(2k_2\cdot k_3(3k_1\cdot k_4 - k_2\cdot k_4 + k_3\cdot k_4) - 4k_1\cdot k_2(k_1\cdot k_4 - k_2\cdot k_4 + k_3\cdot k_4))))$$
$$+\frac{(t^{a_2}t^{a_1}t^{a_3})_{o_1o_2}}{k_1\cdot k_2 k_1\cdot k_3}\left(\frac{1}{m^2 + k_4\cdot k_5}(4\epsilon^*(k_3)\cdot k_4 k_1\cdot k_3(k_1\cdot k_2 - k_1\cdot k_3 + k_2\cdot k_3)\right.$$
$$+\epsilon^*(k_3)\cdot k_3(k_1\cdot k_3(-k_1\cdot k_3 + 2k_1\cdot k_4 + k_2\cdot k_3 - 2k_2\cdot k_4)$$
$$+2k_1\cdot k_2(-k_1\cdot k_4 + k_2\cdot k_4 + k_3\cdot k_4)) + 2(\epsilon^*(k_3)\cdot k_2 k_1\cdot k_3(-3k_1\cdot k_4 + k_2\cdot k_4 - k_3\cdot k_4)$$
$$+\epsilon^*(k_3)\cdot k_1(2k_1\cdot k_2(k_1\cdot k_4 - k_2\cdot k_4 - k_3\cdot k_4) + k_1\cdot k_3(-k_1\cdot k_4 + 3k_2\cdot k_4 + k_3\cdot k_4))))$$
$$+2\left(2\epsilon^*(k_3)\cdot k_1 k_1\cdot k_2 - \epsilon^*(k_3)\cdot k_3 k_1\cdot k_2 - \epsilon^*(k_3)\cdot k_1 k_1\cdot k_3 + \epsilon^*(k_3)\cdot k_2 k_1\cdot k_3\right.$$
$$\left.\left.-\frac{(2\epsilon^*(k_3)\cdot k_1 + 2\epsilon^*(k_3)\cdot k_2 - \epsilon^*(k_3)\cdot k_3 - 2\epsilon^*(k_3)\cdot k_4)k_1\cdot k_3(k_1\cdot k_2 + k_1\cdot k_4 - k_2\cdot k_4)}{k_3\cdot k_5}\right)\right)$$
$$-\frac{2(t^{a_2}t^{a_3}t^{a_1})_{o_1o_2}}{k_1\cdot k_3 k_2\cdot k_3\left(m^2 + k_4\cdot k_5\right)}\left(-2\epsilon^*(k_3)\cdot k_2 k_1\cdot k_3\left(m^2 + k_1\cdot k_4 - k_2\cdot k_4 + k_3\cdot k_4 + k_4\cdot k_5\right)\right.$$
$$+2k_2\cdot k_3\left(2\epsilon^*(k_3)\cdot k_4 k_1\cdot k_3 + \epsilon^*(k_3)\cdot k_1\left(m^2 + k_1\cdot k_4 - k_2\cdot k_4 - k_3\cdot k_4 + k_4\cdot k_5\right)\right)$$
$$+\epsilon^*(k_3)\cdot k_3\left(k_1\cdot k_3\left(m^2 + k_1\cdot k_4 - k_2\cdot k_4 + k_3\cdot k_4 + k_4\cdot k_5\right)\right.$$
$$\left.\left.-k_2\cdot k_3\left(m^2 + k_1\cdot k_4 - k_2\cdot k_4 - k_3\cdot k_4 + k_4\cdot k_5\right)\right)\right)$$
$$+\frac{2(t^{a_1}t^{a_3}t^{a_2})_{o_1o_2}}{k_1\cdot k_3 k_2\cdot k_3\left(m^2 + k_4\cdot k_5\right)}\left(-2\epsilon^*(k_3)\cdot k_2 k_1\cdot k_3\left(m^2 - k_1\cdot k_4 + k_2\cdot k_4 - k_3\cdot k_4 + k_4\cdot k_5\right)\right.$$
$$+2k_2\cdot k_3\left(\epsilon^*(k_3)\cdot k_1\left(m^2 - k_1\cdot k_4 + k_2\cdot k_4 + k_3\cdot k_4 + k_4\cdot k_5\right) - 2\epsilon^*(k_3)\cdot k_4 k_1\cdot k_3\right)$$
$$+\epsilon^*(k_3)\cdot k_3\left(k_1\cdot k_3\left(m^2 - k_1\cdot k_4 + k_2\cdot k_4 - k_3\cdot k_4 + k_4\cdot k_5\right)\right.$$
$$\left.\left.-k_2\cdot k_3\left(m^2 - k_1\cdot k_4 + k_2\cdot k_4 + k_3\cdot k_4 + k_4\cdot k_5\right)\right)\right)$$
$$+\frac{(t^{a_3}t^{a_2}t^{a_1})_{o_1o_2}}{k_1\cdot k_2 k_2\cdot k_3 k_3\cdot k_4\left(m^2 + k_4\cdot k_5\right)}\left(\epsilon^*(k_3)\cdot k_3\left(2k_1\cdot k_2\left(k_2\cdot k_3\left(m^2 + k_4\cdot k_5\right)\right.\right.\right.$$
$$\left.\left.+k_3\cdot k_4\left(m^2 + k_1\cdot k_4 - k_2\cdot k_4 + k_3\cdot k_4 + k_4\cdot k_5\right)\right)\right.$$
$$+k_2\cdot k_3\left(-2k_2\cdot k_3 m^2 - 2k_2\cdot k_4 m^2 - k_2\cdot k_3 k_3\cdot k_4 + 2k_2\cdot k_4 k_3\cdot k_4 - 2(k_2\cdot k_3 + k_2\cdot k_4)k_4\cdot k_5+\right.$$

$$+2k_1\cdot k_4\left(m^2-k_3\cdot k_4+k_4\cdot k_5\right)+k_1\cdot k_3\left(k_3\cdot k_4+2\left(m^2+k_4\cdot k_5\right)\right)\Big)\Big)$$
$$-2\left(k_3\cdot k_4\left(\epsilon^*(k_3)\cdot k_1k_2\cdot k_3\left(m^2-k_1\cdot k_4+3k_2\cdot k_4+k_3\cdot k_4+k_4\cdot k_5\right)\right.\right.$$
$$+\epsilon^*(k_3)\cdot k_2\left(2k_1\cdot k_2\left(m^2+k_1\cdot k_4-k_2\cdot k_4+k_3\cdot k_4+k_4\cdot k_5\right)\right.$$
$$\left.\left.-k_2\cdot k_3\left(m^2+3k_1\cdot k_4-k_2\cdot k_4+k_3\cdot k_4+k_4\cdot k_5\right)\right)\right)-2\epsilon^*(k_3)\cdot k_4k_2\cdot k_3\left(k_1\cdot k_4m^2\right.$$
$$-k_2\cdot k_3m^2-k_2\cdot k_4m^2-k_2\cdot k_3k_3\cdot k_4+(k_1\cdot k_4-k_2\cdot k_3-k_2\cdot k_4)k_4\cdot k_5+$$
$$\left.\left.k_1\cdot k_2\left(m^2+k_3\cdot k_4+k_4\cdot k_5\right)+k_1\cdot k_3\left(m^2+k_3\cdot k_4+k_4\cdot k_5\right)\right)\right)\Big)$$
$$+\frac{(t^{a_3}t^{a_1}t^{a_2})_{o_1o_2}}{k_1\cdot k_2k_1\cdot k_3k_3\cdot k_4\left(m^2+k_4\cdot k_5\right)}\left(\epsilon^*(k_3)\cdot k_3\left(2k_1\cdot k_2\left(k_1\cdot k_3\left(m^2+k_4\cdot k_5\right)\right.\right.\right.$$
$$\left.+k_3\cdot k_4\left(m^2-k_1\cdot k_4+k_2\cdot k_4+k_3\cdot k_4+k_4\cdot k_5\right)\right)+k_1\cdot k(k_3)\left(2k_2\cdot k_3m^2+2k_2\cdot k_4m^2\right.$$
$$+k_2\cdot k_3k_3\cdot k_4-2k_2\cdot k_4k_3\cdot k_4+2(k_2\cdot k_3+k_2\cdot k_4)k_4\cdot k_5-2k_1\cdot k_4\left(m^2-k_3\cdot k_4+k_4\cdot k_5\right)$$
$$\left.\left.-k_1\cdot k_3\left(k_3\cdot k_4+2\left(m^2+k_4\cdot k_5\right)\right)\right)\right)-2\left(k_3\cdot k_4\left(\epsilon^*(k_3)\cdot k_2k_1\cdot k_3\left(m^2+3k_1\cdot k_4-k_2\cdot k_4\right.\right.\right.$$
$$\left.+k_3\cdot k_4+k_4\cdot k_5\right)+\epsilon^*(k_3)\cdot k_1\left(2k_1\cdot k_2\left(m^2-k_1\cdot k_4+k_2\cdot k_4+k_3\cdot k_4+k_4\cdot k_5\right)\right.$$
$$\left.\left.-k_1\cdot k_3\left(m^2-k_1\cdot k_4+3k_2\cdot k_4+k_3\cdot k_4+k_4\cdot k_5\right)\right)\right)$$
$$-2\epsilon^*(k_3)\cdot k_4k_1\cdot k_3\left(-k_1\cdot k_4m^2+k_2\cdot k_3m^2+k_2\cdot k_4m^2+k_2\cdot k_3k_3\cdot k_4\right.$$
$$-(k_1\cdot k_4-k_2\cdot k_3-k_2\cdot k_4)k_4\cdot k_5+k_1\cdot k_2\left(m^2+k_3\cdot k_4+k_4\cdot k_5\right)$$
$$\left.\left.\left.-k_1\cdot k_3\left(m^2+k_3\cdot k_4+k_4\cdot k_5\right)\right)\right)\right)\Big)$$

$$\mathcal{M}_{g\phi\to\phi\tilde{t}\bar{\tilde{t}}}=\frac{\alpha_sg_s\pi\epsilon(k_2)\cdot\epsilon^*(k_3)}{k_1\cdot k_2k_1\cdot k_3k_1\cdot k_4k_1\cdot k_5k_2\cdot k_3\left(m^2+k_4\cdot k_5\right)}\times$$
$$\left((t^{a_1}t^{a_2}t^{a_3})_{o_1o_2}k_1\cdot k_3k_1\cdot k_5\left(\epsilon(k_1)\cdot k_1\left(\left(k_1\cdot k_4-2\left(m^2+k_4\cdot k_5\right)\right)(k_1\cdot k_2)^2\right.\right.\right.$$
$$+\left(k_1\cdot k_3\left(k_1\cdot k_4-2\left(m^2+k_4\cdot k_5\right)\right)+2\left(k_2\cdot k_3\left(m^2+k_4\cdot k_5\right)\right.\right.$$
$$\left.\left.+(k_2\cdot k_4+k_3\cdot k_4)\left(m^2+k_1\cdot k_4+k_4\cdot k_5\right)\right)\right)k_1\cdot k_2-2k_1\cdot k_4k_2\cdot k_3\left(-m^2\right.$$
$$\left.+k_1\cdot k_4+k_2\cdot k_4+k_3\cdot k_4-k_4\cdot k_5)\right)$$
$$+2\left(2\epsilon(k_1)\cdot k_4k_1\cdot k_2\left(-k_2\cdot k_3m^2-k_2\cdot k_4m^2-k_3\cdot k_4m^2+k_1\cdot k_4k_2\cdot k_3\right.\right.$$
$$-(k_2\cdot k_3+k_2\cdot k_4+k_3\cdot k_4)k_4\cdot k_5+k_1\cdot k_2\left(m^2-k_1\cdot k_4+k_4\cdot k_5\right)$$
$$\left.+k_1\cdot k_3\left(m^2-k_1\cdot k_4+k_4\cdot k_5\right)\right)+k_1\cdot k_4\left(\epsilon(k_1)\cdot k_2\left(k_1\cdot k_2\left(-m^2+k_1\cdot k_4+k_2\cdot k_4\right.\right.\right.$$
$$\left.\left.+3k_3\cdot k_4-k_4\cdot k_5\right)+2k_2\cdot k_3\left(m^2-k_1\cdot k_4-k_2\cdot k_4-k_3\cdot k_4+k_4\cdot k_5\right)\right)$$
$$\left.\left.\left.-\epsilon(k_1)\cdot k_3k_1\cdot k_2\left(m^2-k_1\cdot k_4+3k_2\cdot k_4+k_3\cdot k_4+k_4\cdot k_5\right)\right)\right)\right)$$
$$+(t^{a_1}t^{a_3}t^{a_2})_{o_1o_2}k_1\cdot k_2k_1\cdot k_5\left(\epsilon(k_1)\cdot k_1\left(\left(2\left(m^2+k_4\cdot k_5\right)-k_1\cdot k_4\right)(k_1\cdot k_3)^2\right.\right.$$
$$+k_1\cdot k_2\left(2\left(m^2+k_4\cdot k_5\right)-k_1\cdot k_4\right)k_1\cdot k_3+2\left(k_2\cdot k_3\left(m^2+k_4\cdot k_5\right)\right.$$
$$\left.-(k_2\cdot k_4+k_3\cdot k_4)\left(m^2+k_1\cdot k_4+k_4\cdot k_5\right)\right)k_1\cdot k_3$$
$$\left.+2k_1\cdot k_4k_2\cdot k_3\left(-m^2+k_1\cdot k_4-k_2\cdot k_4-k_3\cdot k_4-k_4\cdot k_5\right)\right)$$
$$+2\left(2\epsilon(k_1)\cdot k_4k_1\cdot k_3\left(-k_2\cdot k_3m^2+k_2\cdot k_4m^2+k_3\cdot k_4m^2+k_1\cdot k_4k_2\cdot k_3\right.\right.$$
$$-(k_2\cdot k_3-k_2\cdot k_4-k_3\cdot k_4)k_4\cdot k_5-k_1\cdot k_2\left(m^2-k_1\cdot k_4+k_4\cdot k_5\right)$$
$$\left.-k_1\cdot k_3\left(m^2-k_1\cdot k_4+k_4\cdot k_5\right)\right)+k_1\cdot k_4\left(\epsilon(k_1)\cdot k_2k_1\cdot k_3\left(m^2-k_1\cdot k_4-\right.\right.$$

$$-k_2\cdot k_4-3k_3\cdot k_4+k_4\cdot k_5)+\epsilon(k_1)\cdot k_3\left(2k_2\cdot k_3\left(m^2-k_1\cdot k_4+k_2\cdot k_4+k_3\cdot k_4+k_4\cdot k_5\right)\right.$$
$$\left.+k_1\cdot k_3\left(m^2-k_1\cdot k_4+3k_2\cdot k_4+k_3\cdot k_4+k_4\cdot k_5\right)\right)\Big)\Big)\Big)$$
$$+k_1\cdot k_4\left(-(t^{a_2}t^{a_3}t^{a_1})_{o_1o_2}k_1\cdot k_2\left(-2\epsilon(k_1)\cdot k_1k_1\cdot k_3k_2\cdot k_3m^2+4\epsilon(k_1)\cdot k_4k_1\cdot k_3k_2\cdot k_3m^2\right.\right.$$
$$-2\epsilon(k_1)\cdot k_1k_1\cdot k_5k_2\cdot k_3m^2-2\epsilon(k_1)\cdot k_1k_1\cdot k_3k_2\cdot k_4m^2+4\epsilon(k_1)\cdot k_4k_1\cdot k_3k_2\cdot k_4m^2$$
$$-2\epsilon(k_1)\cdot k_1k_1\cdot k_3k_3\cdot k_4m^2+4\epsilon(k_1)\cdot k_4k_1\cdot k_3k_3\cdot k_4m^2+\epsilon(k_1)\cdot k_1(k_1\cdot k_3)^2k_1\cdot k_5$$
$$-4\epsilon(k_1)\cdot k_4(k_1\cdot k_3)^2k_1\cdot k_5+\epsilon(k_1)\cdot k_1k_1\cdot k_2k_1\cdot k_3k_1\cdot k_5-4\epsilon(k_1)\cdot k_4k_1\cdot k_2k_1\cdot k_3k_1\cdot k_5$$
$$-4\epsilon(k_1)\cdot k_4k_1\cdot k_3k_1\cdot k_5k_2\cdot k_3-2\epsilon(k_1)\cdot k_1k_1\cdot k_4k_1\cdot k_5k_2\cdot k_3+2\epsilon(k_1)\cdot k_1k_1\cdot k_3k_1\cdot k_5k_2\cdot k_4$$
$$+2\epsilon(k_1)\cdot k_1k_1\cdot k_5k_2\cdot k_3k_2\cdot k_4+2\epsilon(k_1)\cdot k_1k_1\cdot k_3k_1\cdot k_5k_3\cdot k_4+2\epsilon(k_1)\cdot k_1k_1\cdot k_5k_2\cdot k_3k_3\cdot k_4$$
$$-2(\epsilon(k_1)\cdot k_1(k_1\cdot k_5k_2\cdot k_3+k_1\cdot k_3(k_2\cdot k_3+k_2\cdot k_4+k_3\cdot k_4))$$
$$-2\epsilon(k_1)\cdot k_4k_1\cdot k_3(k_2\cdot k_3+k_2\cdot k_4+k_3\cdot k_4))k_4\cdot k_5+2\epsilon(k_1)\cdot k_2k_1\cdot k_3\left(k_1\cdot k_5\left(m^2\right.\right.$$
$$\left.\left.+k_1\cdot k_4+k_2\cdot k_4+3k_3\cdot k_4+k_4\cdot k_5\right)-2(k_2\cdot k_3+k_2\cdot k_4+k_3\cdot k_4)\left(m^2+k_4\cdot k_5\right)\right)$$
$$+2\epsilon(k_1)\cdot k_3\left(2k_1\cdot k_5k_2\cdot k_3\left(m^2+k_1\cdot k_4-k_2\cdot k_4-k_3\cdot k_4+k_4\cdot k_5\right)\right.$$
$$+k_1\cdot k_3\left(2(k_2\cdot k_3+k_2\cdot k_4+k_3\cdot k_4)\left(m^2+k_4\cdot k_5\right)+k_1\cdot k_5\left(m^2+k_1\cdot k_4\right.\right.$$
$$\left.\left.\left.-3k_2\cdot k_4-k_3\cdot k_4+k_4\cdot k_5\right)\right)\right)\Big)+(t^{a_3}t^{a_2}t^{a_1})_{o_1o_2}k_1\cdot k_3\left(2\epsilon(k_1)\cdot k_1k_1\cdot k_2k_2\cdot k_3m^2\right.$$
$$-4\epsilon(k_1)\cdot k_4k_1\cdot k_2k_2\cdot k_3m^2-2\epsilon(k_1)\cdot k_1k_1\cdot k_5k_2\cdot k_3m^2-2\epsilon(k_1)\cdot k_1k_1\cdot k_2k_2\cdot k_4m^2$$
$$+4\epsilon(k_1)\cdot k_4k_1\cdot k_2k_2\cdot k_4m^2-2\epsilon(k_1)\cdot k_1k_1\cdot k_2k_3\cdot k_4m^2+4\epsilon(k_1)\cdot k_4k_1\cdot k_2k_3\cdot k_4m^2$$
$$+\epsilon(k_1)\cdot k_1(k_1\cdot k_2)^2k_1\cdot k_5-4\epsilon(k_1)\cdot k_4(k_1\cdot k_2)^2k_1\cdot k_5+\epsilon(k_1)\cdot k_1k_1\cdot k_2k_1\cdot k_3k_1\cdot k_5$$
$$-4\epsilon(k_1)\cdot k_4k_1\cdot k_2k_1\cdot k_3k_1\cdot k_5+4\epsilon(k_1)\cdot k_4k_1\cdot k_2k_1\cdot k_5k_2\cdot k_3-2\epsilon(k_1)\cdot k_1k_1\cdot k_4k_1\cdot k_5k_2\cdot k_3$$
$$+2\epsilon(k_1)\cdot k_1k_1\cdot k_2k_1\cdot k_5k_2\cdot k_4-2\epsilon(k_1)\cdot k_1k_1\cdot k_5k_2\cdot k_3k_2\cdot k_4+2\epsilon(k_1)\cdot k_1k_1\cdot k_2k_1\cdot k_5k_3\cdot k_4$$
$$-2\epsilon(k_1)\cdot k_1k_1\cdot k_5k_2\cdot k_3k_3\cdot k_4+2(\epsilon(k_1)\cdot k_1(k_1\cdot k_2(k_2\cdot k_3-k_2\cdot k_4-k_3\cdot k_4)-k_1\cdot k_5k_2\cdot k_3)$$
$$+2\epsilon(k_1)\cdot k_4k_1\cdot k_2(-k_2\cdot k_3+k_2\cdot k_4+k_3\cdot k_4))k_4\cdot k_5$$
$$+2\epsilon(k_1)\cdot k_3k_1\cdot k_2\left(k_1\cdot k_5\left(m^2+k_1\cdot k_4-3k_2\cdot k_4-k_3\cdot k_4+k_4\cdot k_5\right)\right.$$
$$\left.-2(k_2\cdot k_3-k_2\cdot k_4-k_3\cdot k_4)\left(m^2+k_4\cdot k_5\right)\right)$$
$$+2\epsilon(k_1)\cdot k_2\left(k_1\cdot k_2\left(2(k_2\cdot k_3-k_2\cdot k_4-k_3\cdot k_4)\left(m^2+k_4\cdot k_5\right)\right.\right.$$
$$\left.+k_1\cdot k_5\left(m^2+k_1\cdot k_4+k_2\cdot k_4+3k_3\cdot k_4+k_4\cdot k_5\right)\right)$$
$$\left.\left.-2k_1\cdot k_5k_2\cdot k_3\left(m^2+k_1\cdot k_4+k_2\cdot k_4+k_3\cdot k_4+k_4\cdot k_5\right)\right)\right)$$
$$+2k_1\cdot k_5k_2\cdot k_3\left((t^{a_2}t^{a_1}t^{a_3})_{o_1o_2}\left(\epsilon(k_1)\cdot k_1\left(k_1\cdot k_3\left(-m^2+k_1\cdot k_4+k_2\cdot k_4+k_3\cdot k_4-k_4\cdot k_5\right)\right.\right.\right.$$
$$\left.-k_1\cdot k_2\left(m^2+k_1\cdot k_4-k_2\cdot k_4-k_3\cdot k_4+k_4\cdot k_5\right)\right)+2\left(k_1\cdot k_3\left(\epsilon(k_1)\cdot k_2\left(-m^2+k_1\cdot k_4\right.\right.\right.$$
$$\left.\left.+k_2\cdot k_4+k_3\cdot k_4-k_4\cdot k_5\right)-2\epsilon(k_1)\cdot k_4k_1\cdot k_2\right)+\epsilon(k_1)\cdot k_3k_1\cdot k_2\left(m^2+k_1\cdot k_4\right.$$
$$\left.\left.\left.-k_2\cdot k_4-k_3\cdot k_4+k_4\cdot k_5\right)\right)\right)+(t^{a_3}t^{a_1}t^{a_2})_{o_1o_2}\left(\epsilon(k_1)\cdot k_1\left(k_1\cdot k_2\left(m^2-k_1\cdot k_4+k_2\cdot k_4\right.\right.\right.$$
$$\left.\left.+k_3\cdot k_4+k_4\cdot k_5\right)+k_1\cdot k_3\left(m^2+k_1\cdot k_4+k_2\cdot k_4+k_3\cdot k_4+k_4\cdot k_5\right)\right)$$
$$+2\left(k_1\cdot k_3\left(\epsilon(k_1)\cdot k_2\left(m^2+k_1\cdot k_4+k_2\cdot k_4+k_3\cdot k_4+k_4\cdot k_5\right)-2\epsilon(k_1)\cdot k_4k_1\cdot k_2\right)\right.$$
$$\left.\left.\left.-\epsilon(k_1)\cdot k_3k_1\cdot k_2\left(m^2-k_1\cdot k_4+k_2\cdot k_4+k_3\cdot k_4+k_4\cdot k_5\right)\right)\right)\right)\Big)\Big)$$

$$\mathcal{M}_{\phi g\to\phi\tilde{t}\tilde{\tilde{t}}} = \frac{\alpha_s g_s \pi \epsilon(k_1)\cdot\epsilon^*(k_3)}{k_1\cdot k_2 k_1\cdot k_3 k_2\cdot k_3 k_2\cdot k_4 k_2\cdot k_5\left(m^2+k_4\cdot k_5\right)} \times$$

$$\Big((t^{a_3}t^{a_1}t^{a_2})_{o_1o_2} k_2\cdot k_3 k_2\cdot k_4 \left(\epsilon(k_2)\cdot k_2\left(k_2\cdot k_5(k_1\cdot k_2)^2 + \left(-2k_3\cdot k_4 m^2 + k_2\cdot k_5(k_2\cdot k_3 + 2k_3\cdot k_4)\right.\right.\right.$$
$$-2k_3\cdot k_4 k_4\cdot k_5 + 2k_1\cdot k_3\left(m^2+k_4\cdot k_5\right) - 2k_1\cdot k_4\left(m^2 - k_2\cdot k_5 + k_4\cdot k_5\right)\big) k_1\cdot k_2$$
$$-2k_1\cdot k_3 k_2\cdot k_5\left(m^2 + k_1\cdot k_4 + k_2\cdot k_4 + k_3\cdot k_4 + k_4\cdot k_5\right)\big)$$
$$+2\epsilon(k_2)\cdot k_1\left(k_1\cdot k_2\left(k_2\cdot k_5 m^2 - 2k_3\cdot k_4 m^2 + k_2\cdot k_4 k_2\cdot k_5 + 3k_2\cdot k_5 k_3\cdot k_4\right.\right.$$
$$+(k_2\cdot k_5 - 2k_3\cdot k_4)k_4\cdot k_5 + 2k_1\cdot k_3\left(m^2+k_4\cdot k_5\right) + k_1\cdot k_4\left(k_2\cdot k_5 - 2\left(m^2+k_4\cdot k_5\right)\right)\big)\big)$$
$$-2k_1\cdot k_3 k_2\cdot k_5\left(m^2 + k_1\cdot k_4 + k_2\cdot k_4 + k_3\cdot k_4 + k_4\cdot k_5\right)\big)$$
$$+2k_1\cdot k_2\left(2\epsilon(k_2)\cdot k_4\left(-(k_1\cdot k_2 + k_2\cdot k_3)k_2\cdot k_5 + (k_1\cdot k_4 + k_3\cdot k_4)\left(m^2+k_4\cdot k_5\right)\right.\right.$$
$$-k_1\cdot k_3\left(m^2 - k_2\cdot k_5 + k_4\cdot k_5\right)\big) + \epsilon(k_2)\cdot k_3\left(k_2\cdot k_5 m^2 + 2k_3\cdot k_4 m^2 + k_2\cdot k_4 k_2\cdot k_5\right.$$
$$-k_2\cdot k_5 k_3\cdot k_4 + (k_2\cdot k_5 + 2k_3\cdot k_4)k_4\cdot k_5 - 2k_1\cdot k_3\left(m^2+k_4\cdot k_5\right)$$
$$+k_1\cdot k_4\left(2\left(m^2+k_4\cdot k_5\right) - 3k_2\cdot k_5\right)\big)\big)\big)\big)$$
$$-(t^{a_1}t^{a_3}t^{a_2})_{o_1o_2} k_1\cdot k_2 k_2\cdot k_4\left(2\epsilon(k_2)\cdot k_1 k_2\cdot k_3\left(k_2\cdot k_5 m^2 - 2k_3\cdot k_4 m^2\right.\right.$$
$$+k_2\cdot k_4 k_2\cdot k_5 + 3k_2\cdot k_5 k_3\cdot k_4 + (k_2\cdot k_5 - 2k_3\cdot k_4)k_4\cdot k_5 - 2k_1\cdot k_3\left(m^2+k_4\cdot k_5\right)$$
$$+k_1\cdot k_4\left(k_2\cdot k_5 - 2\left(m^2+k_4\cdot k_5\right)\right)\big) + \epsilon(k_2)\cdot k_2\left(k_2\cdot k_3\left(k_1\cdot k_2 k_2\cdot k_5\right.\right.$$
$$+k_2\cdot k_3 k_2\cdot k_5 - 2k_1\cdot k_4\left(m^2 - k_2\cdot k_5 + k_4\cdot k_5\right) - 2k_3\cdot k_4\left(m^2 - k_2\cdot k_5 + k_4\cdot k_5\right)\big)$$
$$-2k_1\cdot k_3\left(k_2\cdot k_3\left(m^2+k_4\cdot k_5\right) + k_2\cdot k_5\left(m^2 - k_1\cdot k_4 + k_2\cdot k_4 - k_3\cdot k_4 + k_4\cdot k_5\right)\right)\big)\big)$$
$$+2\left(2\epsilon(k_2)\cdot k_4 k_2\cdot k_3\left(-(k_1\cdot k_2 + k_2\cdot k_3)k_2\cdot k_5 + (k_1\cdot k_4 + k_3\cdot k_4)\left(m^2+k_4\cdot k_5\right)\right.\right.$$
$$+k_1\cdot k_3\left(m^2 - k_2\cdot k_5 + k_4\cdot k_5\right)\big) + \epsilon(k_2)\cdot k_3\left(2k_1\cdot k_3\left(k_2\cdot k_3\left(m^2+k_4\cdot k_5\right)\right.\right.$$
$$+k_2\cdot k_5\left(m^2 - k_1\cdot k_4 + k_2\cdot k_4 - k_3\cdot k_4 + k_4\cdot k_5\right)\big) + k_2\cdot k_3\left(2k_3\cdot k_4\left(m^2+k_4\cdot k_5\right)\right.$$
$$+k_2\cdot k_5\left(m^2 + k_2\cdot k_4 - k_3\cdot k_4 + k_4\cdot k_5\right) + k_1\cdot k_4\left(2\left(m^2+k_4\cdot k_5\right) - 3k_2\cdot k_5\right)\big)\big)\big)\big)\big)$$
$$+k_2\cdot k_5\left((t^{a_2}t^{a_1}t^{a_3})_{o_1o_2} k_2\cdot k_3\left(\epsilon(k_2)\cdot k_2\left(\left(k_2\cdot k_4 - 2\left(m^2+k_4\cdot k_5\right)\right)(k_1\cdot k_2)^2\right.\right.\right.$$
$$+\left(-2k_2\cdot k_3 m^2 + 2k_3\cdot k_4 m^2 + k_2\cdot k_3 k_2\cdot k_4 + 2k_2\cdot k_4 k_3\cdot k_4 - 2(k_2\cdot k_3 - k_3\cdot k_4)k_4\cdot k_5\right.$$
$$+2k_1\cdot k_3\left(m^2+k_4\cdot k_5\right) + 2k_1\cdot k_4\left(m^2 + k_2\cdot k_4 + k_4\cdot k_5\right)\big) k_1\cdot k_2$$
$$-2k_1\cdot k_3 k_2\cdot k_4\left(-m^2 + k_1\cdot k_4 + k_2\cdot k_4 + k_3\cdot k_4 - k_4\cdot k_5\right)\big)$$
$$-2\left(2\epsilon(k_2)\cdot k_4 k_1\cdot k_2\left(k_1\cdot k_4 m^2 f - k_2\cdot k_3 m^2 + k_3\cdot k_4 m^2 + k_2\cdot k_3 k_2\cdot k_4\right.\right.$$
$$+(k_1\cdot k_4 - k_2\cdot k_3 + k_3\cdot k_4)k_4\cdot k_5 - k_1\cdot k_2\left(m^2 - k_2\cdot k_4 + k_4\cdot k_5\right)$$
$$+k_1\cdot k_3\left(m^2 - k_2\cdot k_4 + k_4\cdot k_5\right)\big) + k_2\cdot k_4\left(\epsilon(k_2)\cdot k_3 k_1\cdot k_2\left(m^2 + 3k_1\cdot k_4 - k_2\cdot k_4\right.\right.$$
$$+k_3\cdot k_4 + k_4\cdot k_5) + \epsilon(k_2)\cdot k_1\left(2k_1\cdot k_3\left(-m^2 + k_1\cdot k_4 + k_2\cdot k_4 + k_3\cdot k_4 - k_4\cdot k_5\right)\right.$$
$$+k_1\cdot k_2\left(m^2 - k_1\cdot k_4 - k_2\cdot k_4 - 3k_3\cdot k_4 + k_4\cdot k_5\ \right)\big)\big)\big)\big)$$
$$+(t^{a_2}t^{a_3}t^{a_1})_{o_1o_2} k_1\cdot k_2\left(\epsilon(k_2)\cdot k_2\left(k_1\cdot k_2 k_2\cdot k_3\left(2\left(m^2+k_4\cdot k_5\right) - k_2\cdot k_4\right)\right.\right.$$
$$+2k_1\cdot k_3\left(k_2\cdot k_4\left(-m^2 - k_1\cdot k_4 + k_2\cdot k_4 - k_3\cdot k_4 - k_4\cdot k_5\right) + k_2\cdot k_3\left(m^2+k_4\cdot k_5\right)\right)$$
$$+k_2\cdot k_3\left(-2k_1\cdot k_4\left(m^2 + k_2\cdot k_4 + k_4\cdot k_5\right) - 2k_3\cdot k_4\left(m^2 + k_2\cdot k_4 + k_4\cdot k_5\right)\right.$$
$$+k_2\cdot k_3\left(2\left(m^2+k_4\cdot k_5\right) - k_2\cdot k_4\right)\big)\big) + 2\left(2\epsilon(k_2)\cdot k_4 k_2\cdot k_3\left(k_1\cdot k_4 m^2 - k_2\cdot k_3 m^2\right.\right.$$
$$+k_3\cdot k_4 m^2 + k_2\cdot k_3 k_2\cdot k_4 + (k_1\cdot k_4 - k_2\cdot k_3 + k_3\cdot k_4)k_4\cdot k_5$$
$$-k_1\cdot k_2\left(m^2 - k_2\cdot k_4 + k_4\cdot k_5\right) - k_1\cdot k_3\left(m^2 - k_2\cdot k_4 + k_4\cdot k_5\right)\big)$$
$$+k_2\cdot k_4\left(\epsilon(k_2)\cdot k_1 k_2\cdot k_3\left(m^2 - k_1\cdot k_4 - k_2\cdot k_4 - 3k_3\cdot k_4 + k_4\cdot k_5\right) +\right.$$

$$
\begin{aligned}
&+\epsilon(k_2)\cdot k_3\left(2k_1\cdot k_3\left(m^2+k_1\cdot k_4-k_2\cdot k_4+k_3\cdot k_4+k_4\cdot k_5\right)\right.\\
&\left.\left.\left.\left.\left.+k_2\cdot k_3\left(m^2+3k_1\cdot k_4-k_2\cdot k_4+k_3\cdot k_4+k_4\cdot k_5\right)\right)\right)\right)\right)\\
&+2k_1\cdot k_3k_2\cdot k_4\left(\left(t^{a_1}t^{a_2}t^{a_3}\right)_{o_1o_2}\left(\epsilon(k_2)\cdot k_2\left(k_2\cdot k_3\left(-m^2+k_1\cdot k_4+k_2\cdot k_4+k_3\cdot k_4-k_4\cdot k_5\right)\right.\right.\right.\\
&\left.\left.-k_1\cdot k_2\left(m^2-k_1\cdot k_4+k_2\cdot k_4-k_3\cdot k_4+k_4\cdot k_5\right)\right)+2\left(k_2\cdot k_3\left(\epsilon(k_2)\cdot k_1\left(-m^2+k_1\cdot k_4\right.\right.\right.\\
&\left.+k_2\cdot k_4+k_3\cdot k_4-k_4\cdot k_5\right)-2\epsilon(k_2)\cdot k_4k_1\cdot k_2)+\epsilon(k_2)\cdot k_3k_1\cdot k_2\left(m^2-k_1\cdot k_4+k_2\cdot k_4\right.\\
&\left.-k_3\cdot k_4+k_4\cdot k_5)))\right)\\
&+\left(t^{a_3}t^{a_2}t^{a_1}\right)_{o_1o_2}\left(\epsilon(k_2)\cdot k_2\left(k_1\cdot k_2\left(m^2+k_1\cdot k_4\right.\right.\right.\\
&\left.\left.-k_2\cdot k_4+k_3\cdot k_4+k_4\cdot k_5\right)+k_2\cdot k_3\left(m^2+k_1\cdot k_4+k_2\cdot k_4+k_3\cdot k_4+k_4\cdot k_5\right)\right)\\
&+2\left(k_2\cdot k_3\left(\epsilon(k_2)\cdot k_1\left(m^2+k_1\cdot k_4+k_2\cdot k_4+k_3\cdot k_4+k_4\cdot k_5\right)-2\epsilon(k_2)\cdot k_4k_1\cdot k_2\right)\right.\\
&\left.\left.\left.\left.\left.\left.-\epsilon(k_2)\cdot k_3k_1\cdot k_2\left(m^2+k_1\cdot k_4-k_2\cdot k_4+k_3\cdot k_4+k_4\cdot k_5\right)\right)\right)\right)\right)\right)
\end{aligned}
$$

D.3 Squared Amplitudes

Squared Amplitudes for $GG \to G\tilde{t}\tilde{\bar{t}}$, ϵ Scalars

$$
\begin{aligned}
|\mathcal{M}|^2_{\phi g\to\phi\tilde{t}\tilde{\bar{t}}} = \frac{1024\alpha_s\alpha_s^2(D-4)\pi^3}{9st_1u_1^2(s+t_1+u_1)^2u_6^2(s+t_1+u_6)^2}\big(&144m^2st_1^7+576m^2s^2t_1^6+81u_1u_6^2t_1^6\\
&+288m^2ss_4t_1^6+432m^2su_1t_1^6-9u_1^2u_6t_1^6+612m^2su_6t_1^6-9su_1u_6t_1^6+81s_4u_1u_6t_1^6\\
&+864m^2s^3t_1^5+243u_1u_6^3t_1^5+144m^2ss_4^2t_1^5+496m^2su_1^2t_1^5+126u_1^2u_6^2t_1^5+1116m^2su_6^2t_1^5\\
&+225su_1u_6^2t_1^5+567s_4u_1u_6^2t_1^5+1152m^2s^2s_4t_1^5+1440m^2s^2u_1t_1^5+720m^2ss_4u_1t_1^5\\
&-27u_1^3u_6t_1^5+1836m^2s^2u_6t_1^5-71su_1^2u_6t_1^5+153s_4u_1^2u_6t_1^5+936m^2ss_4u_6t_1^5-36s^2u_1u_6t_1^5\\
&+162s_4^2u_1u_6t_1^5+1530m^2su_1u_6t_1^5+216ss_4u_1u_6t_1^5+576m^2s^4t_1^4+243u_1u_6^4t_1^4\\
&+272m^2su_1^3t_1^4+360u_1^2u_6^3t_1^4+972m^2su_6^3t_1^4+558su_1u_6^3t_1^4+1053s_4u_1u_6^3t_1^4-167s^2u_1^2u_6t_1^4\\
&+576m^2s^2s_4^2t_1^4+1408m^2s^2u_1^2t_1^4+576m^2ss_4u_1^2t_1^4-90u_1^3u_6^2t_1^4+2232m^2s^2u_6^2t_1^4\\
&+64su_1^2u_6^2t_1^4+864s_4u_1^2u_6^2t_1^4+1296m^2ss_4u_6^2t_1^4+189s^2u_1u_6^2t_1^4+729s_4^2u_1u_6^2t_1^4+288m^2ss_4^2u_1t_1^4\\
&+2232m^2su_1u_6^2t_1^4+1332ss_4u_1u_6^2t_1^4+1728m^2s^3s_4t_1^4+1728m^2s^3u_1t_1^4\\
&+2304m^2s^2s_4u_1t_1^4-27u_1^4u_6t_1^4+1836m^2s^3u_6t_1^4-132su_1^3u_6t_1^4+63s_4u_1^3u_6t_1^4+324m^2ss_4^2u_6t_1^4\\
&+243s_4^2u_1^2u_6t_1^4+1512m^2su_1^2u_6t_1^4+144ss_4u_1^2u_6t_1^4+2808m^2s^2s_4u_6t_1^4-54s^3u_1u_6t_1^4\\
&+3690m^2s^2u_1u_6t_1^4+387ss_4^2u_1u_6t_1^4+72s^2s_4u_1u_6t_1^4+2034m^2ss_4u_1u_6t_1^4+144m^2s^5t_1^3\\
&+81u_1u_6^5t_1^3+64m^2su_1^4t_1^3+306u_1^2u_6^4t_1^3+324m^2su_6^4t_1^3+405su_1u_6^4t_1^3+81s_4^3u_1u_6t_1^4\\
&+729s_4u_1u_6^4t_1^3+672m^2s^2u_1^3t_1^3+144m^2ss_4u_1^3t_1^3-180u_1^3u_6^3t_1^3+972m^2s^2u_6^3t_1^3\\
&+261su_1^2u_6^3t_1^3+1197s_4u_1^2u_6^3t_1^3+648m^2ss_4u_6^3t_1^3+387s^2u_1u_6^3t_1^3+891s_4^2u_1u_6^3t_1^3\\
&+1926ss_4u_1u_6^3t_1^3+864m^2s^3s_4^2t_1^3+1392m^2s^3u_1^2t_1^3+144m^2ss_4^2u_1^2t_1^3+1440m^2s^2s_4u_1^2t_1^3\\
&-234u_1^4u_6^2t_1^3+1116m^2s^3u_6^2t_1^3-681su_1^3u_6^2t_1^3+189s_4u_1^3u_6^2t_1^3+324m^2ss_4^2u_6^2t_1^3-357s^2u_1^2u_6^2t_1^3\\
&+891s_4^2u_1^2u_6^2t_1^3+2052m^2su_1^2u_6^2t_1^3+648ss_4u_1^2u_6^2t_1^3+2592m^2s^2s_4u_6^2t_1^3+27s^3u_1u_6^2t_1^3\\
&+243s_4^3u_1u_6^2t_1^3+3528m^2s^2u_1u_6^2t_1^3+1278ss_4^2u_1u_6^2t_1^3+774s^2s_4u_1u_6^2t_1^3+2430m^2ss_4u_1u_6^2t_1^3\\
&+1152m^2s^4s_4t_1^3+864m^2s^4u_1t_1^3+864m^2s^2s_4^2u_1t_1^3+2592m^2s^3s_4u_1t_1^3-9u_1^5u_6t_1^3\\
&+612m^2s^4u_6t_1^3-87su_1^4u_6t_1^3-9s_4u_1^4u_6t_1^3-207s^2u_1^3u_6t_1^3+81s_4^2u_1^3u_6t_1^3+1458m^2su_1u_6^3t_1^3\\
&+720m^2su_1^3u_6t_1^3-180ss_4u_1^3u_6t_1^3+972m^2s^2s_4^2u_6t_1^3-165s^3u_1^2u_6t_1^3+81s_4^3u_1^2u_6t_1^3\\
&+3024m^2s^2u_1^2u_6t_1^3+288ss_4^2u_1^2u_6t_1^3-522s^2s_4u_1^2u_6t_1^3+1422m^2ss_4u_1^2u_6t_1^3+2808m^2s^3s_4u_6t_1^3\\
&-36s^4u_1u_6t_1^3+2790m^2s^3u_1u_6t_1^3+162ss_4^3u_1u_6t_1^3+108s^2s_4^2u_1u_6t_1^3+648m^2ss_4^2u_1u_6t_1^3\\
&-270s^3s_4u_1u_6t_1^3+4554m^2s^2s_4u_1u_6t_1^3+81u_1^2u_6^5t_1^2+81su_1u_6^5t_1^2+162s_4u_1u_6^5t_1^2\\
&-198u_1^3u_6^4t_1^2+126su_1^2u_6^4t_1^2+486s_4u_1^2u_6^4t_1^2+162s^2u_1u_6^4t_1^2+324s_4^2u_1u_6^4t_1^2+324m^2su_1u_6^4t_1^2\\
&+972ss_4u_1u_6^4t_1^2+528m^2s^3u_1^3t_1^2+288m^2s^2s_4u_1^3t_1^2-468u_1^4u_6^3t_1^2-1071su_1^3u_6^3t_1^2\\
&-297s^2u_1^2u_6^3t_1^2+810s_4^2u_1^2u_6^3t_1^2+1296m^2su_1^2u_6^3t_1^2+27ss_4u_1^2u_6^3t_1^2+648m^2s^2s_4u_6^3t_1^2\\
&+72s^3u_1u_6^3t_1^2-117s_4u_1^3u_6^3t_1^2+128m^2s^2u_1^4t_1^2++162s_4^3u_1u_6^3t_1^2+1134m^2s^2u_1u_6^3t_1^2
\end{aligned}
$$

$+972ss_4^2u_1u_6^3t_1^2 + 855s^2s_4u_1u_6^3t_1^2 + 972m^2ss_4u_1u_6^3t_1^2 + 576m^2s^4s_4^2t_1^2 + 544m^2s^4u_1^2t_1^2$
$+288m^2s^2s_4^2u_1^2t_1^2 + 1152m^2s^3s_4u_1^2t_1^2 - 99u_1^5u_6^2t_1^2 - 690su_1^4u_6^2t_1^2 - 108s_4u_1^4u_6^2t_1^2$
$-1047s^2u_1^3u_6^2t_1^2 + 162s_4^2u_1^3u_6^2t_1^2 + 738m^2su_1^3u_6^2t_1^2 - 945ss_4u_1^3u_6^2t_1^2 + 648m^2s^2s_4^2u_6^2t_1^2$
$-402s^3u_1^2u_6^2t_1^2 + 243s_4^3u_1^2u_6^2t_1^2 + 2682m^2s^2u_1^2u_6^2t_1^2 + 288ss_4^2u_1^2u_6^2t_1^2 - 1395s^2s_4u_1^2u_6^2t_1^2$
$+1458m^2ss_4u_1^2u_6^2t_1^2 + 1296m^2s^3s_4u_6^2t_1^2 - 18s^4u_1u_6^2t_1^2 + 1296m^2s^3u_1u_6^2t_1^2$
$+243ss_4^3u_1u_6^2t_1^2 + 126s^2s_4^2u_1u_6^2t_1^2 + 648m^2ss_4^2u_1u_6^2t_1^2 - 180s^3s_4u_1u_6^2t_1^2$
$+3240m^2s^2s_4u_1u_6^2t_1^2 + 288m^2s^5s_4t_1^2 + 144m^2s^5u_1t_1^2 + 864m^2s^3s_4^2u_1t_1^2$
$+1152m^2s^4s_4u_1t_1^2 - 17su_1^5u_6t_1^2 - 84s^2u_1^4u_6t_1^2 + 126m^2su_1^4u_6t_1^2$
$-108ss_4u_1^4u_6t_1^2 - 126s^3u_1^3u_6t_1^2 + 1134m^2s^2u_1^3u_6t_1^2 - 18ss_4^2u_1^3u_6t_1^2$
$-675s^2s_4u_1^3u_6t_1^2 + 324m^2ss_4u_1^3u_6t_1^2 + 972m^2s^3s_4^2u_6t_1^2 - 68s^4u_1^2u_6t_1^2 + 1800m^2s^3u_1^2u_6t_1^2$
$+81ss_4^3u_1^2u_6t_1^2 - 396s^2s_4^2u_1^2u_6t_1^2 + 324m^2ss_4^2u_1^2u_6t_1^2 - 936s^3s_4u_1^2u_6t_1^2 + 2232m^2s^2s_4u_1^2u_6t_1^2$
$+936m^2s^4s_4u_6t_1^2 - 9s^5u_1u_6t_1^2 + 630m^2s^4u_1u_6t_1^2 - 378s^3s_4^2u_1u_6t_1^2$
$+1296m^2s^2s_4^2u_1u_6t_1^2 - 297s^4s_4u_1u_6t_1^2 + 3006m^2s^3s_4u_1u_6t_1^2 - 81u_1^3u_6^5t_1 + 162ss_4u_1u_6^5t_1$
$+64m^2s^3u_1^4t_1 - 342u_1^4u_6^4t_1 - 603su_1^3u_6^4t_1 - 405s_4u_1^3u_6^4t_1 - 99s^2u_1^2u_6^4t_1$
$+162s_4^2u_1^2u_6^4t_1 + 324m^2su_1^2u_6^4t_1 - 567ss_4u_1^2u_6^4t_1 + 162ss_4^2u_1u_6^4t_1$
$+243s^2s_4u_1u_6^4t_1 + 128m^2s^4u_1^3t_1 + 144m^2s^3s_4u_1^3t_1 - 171u_1^5u_6^3t_1$
$-927su_1^4u_6^3t_1 - 261s_4u_1^4u_6^3t_1 - 1035s^2u_1^3u_6^3t_1 - 81s_4^2u_1^3u_6^3t_1$
$+162m^2su_1^3u_6^3t_1 - 1638ss_4u_1^3u_6^3t_1 - 198s^3u_1^2u_6^3t_1 + 162s_4^3u_1^2u_6^3t_1 + 810m^2s^2u_1^2u_6^3t_1$
$-567ss_4^2u_1^2u_6^3t_1 - 1638s^2s_4u_1^2u_6^3t_1 + 324m^2ss_4u_1^2u_6^3t_1 - 243s^2s_4^2u_1u_6^3t_1 - 18s^3s_4u_1u_6^3t_1$
$+324m^2s^2s_4u_1u_6^3t_1 + 144m^2s^5s_4^2t_1 + 64m^2s^5u_1^2t_1 + 144m^2s^3s_4^2u_1^2t_1$
$+288m^2s^4s_4u_1^2t_1 - 170su_1^5u_6^2t_1 - 591s^2u_1^4u_6^2t_1 + 126m^2su_1^4u_6^2t_1 - 423ss_4u_1^4u_6^2t_1$
$-528s^3u_1^3u_6^2t_1 + 576m^2s^2u_1^3u_6^2t_1 - 423ss_4^2u_1^3u_6^2t_1$
$+324m^2ss_4u_1^3u_6^2t_1 + 324m^2s^3s_4^2u_6^2t_1 - 107s^4u_1^2u_6^2t_1 + 774m^2s^3u_1^2u_6^2t_1 - 1710s^2s_4u_1^3u_6^2t_1$
$-1413s^2s_4^2u_1^2u_6^2t_1 + 324m^2ss_4^2u_1^2u_6^2t_1 - 1476s^3s_4u_1^2u_6^2t_1 + 1134m^2s^2s_4u_1^2u_6^2t_1$
$-243s^2s_4^3u_1u_6^2t_1 - 666s^3s_4^2u_1u_6^2t_1 + 648m^2s^2s_4^2u_1u_6^2t_1 - 189s^4s_4u_1u_6^2t_1 + 810m^2s^3s_4u_1u_6^2t_1$
$+288m^2s^4s_4^2u_1t_1 + 144m^2s^5s_4u_1t_1 - 8s^2u_1^5u_6t_1 - 24s^3u_1^4u_6t_1 + 126m^2s^2u_1^4u_6t_1$
$-24s^4u_1^3u_6t_1 + 414m^2s^3u_1^3u_6t_1 - 261s^2s_4^2u_1^3u_6t_1 - 576s^3s_4u_1^3u_6t_1 - 171s^2s_4u_1^4u_6t_1$
$+324m^2s^2s_4u_1^3u_6t_1 + 324m^2s^4s_4^2u_6t_1 - 8s^5u_1^2u_6t_1 + 288m^2s^4u_1^2u_6t_1 - 81s^2s_4^3u_1^2u_6t_1$
$-684s^3s_4^2u_1^2u_6t_1 + 324m^2s^2s_4^2u_1^2u_6t_1 - 495s^4s_4u_1^2u_6t_1 + 810m^2s^3s_4u_1^2u_6t_1-$

$$-162s^3s_4^3u_1u_6t_1 - 342s^4s_4^2u_1u_6t_1 + 648m^2s^3s_4^2u_1u_6t_1 - 90s^5s_4u_1u_6t_1 + 486m^2s^4s_4u_1u_6t_1$$
$$-81u_1^4u_6^5 - 81su_1^3u_6^5 - 162s_4u_1^3u_6^5 - 162ss_4u_1^2u_6^5 - 81u_1^5u_6^4 - 324su_1^4u_6^4$$
$$-162s_4u_1^4u_6^4 - 243s^2u_1^3u_6^4 - 162s_4^2u_1^3u_6^4 - 729ss_4u_1^3u_6^4 - 324ss_4^2u_1^2u_6^4$$
$$-162s^2s_4^2u_1u_6^4 - 153su_1^5u_6^3 - 387s^2u_1^4u_6^3 - 387ss_4u_1^4u_6^3 - 234s^3u_1^3u_6^3 - 567ss_4^2u_1^3u_6^3$$
$$-1017s^2s_4u_1^3u_6^3 - 162ss_4^3u_1^2u_6^3 - 891s^2s_4^2u_1^2u_6^3 - 630s^3s_4u_1^2u_6^3 - 162s^2s_4^3u_1u_6^3$$
$$-324s^3s_4^2u_1u_6^3 - 72s^2u_1^5u_6^2 - 144s^3u_1^4u_6^2 - 297s^2s_4u_1^4u_6^2 - 72s^4u_1^3u_6^2$$
$$-567s^2s_4^2u_1^3u_6^2 - 594s^3s_4u_1^3u_6^2 - 243s^2s_4^3u_1^2u_6^2 - 810s^3s_4^2u_1^2u_6^2 - 297s^4s_4u_1^2u_6^2 - 243s^3s_4^3u_1u_6^2$$
$$-243s^4s_4^2u_1u_6^2 - 72s^3s_4u_1^4u_6 - 162s^3s_4^2u_1^3u_6$$
$$-144s^4s_4u_1^3u_6 - 81s^3s_4^3u_1^2u_6 - 243s^4s_4^2u_1^2u_6 - 72s^5s_4u_1^2u_6 - 81s^4s_4^3u_1u_6 - 81s^5s_4^2u_1u_6\Big)$$

$$|\mathcal{M}|^2_{g\,\phi\to\phi\tilde{t}\bar{\tilde{t}}} = -\frac{1024\alpha_s\alpha_s^2(D-4)\pi^3}{9st_1^2u_1(s+t_1+u_1)^2(s_4+t_1+u_6)^2(s+s_4+t_1+u_1+u_6)^2}\times$$
$$\Big(9st_1^8 - 81s_4t_1^8 + 9u_1t_1^8 - 81u_6t_1^8 + 36s^2t_1^7 - 324s_4^2t_1^7 + 36u_1^2t_1^7$$
$$-324u_6^2t_1^7 - 198ss_4t_1^7 + 80su_1t_1^7 - 216s_4u_1t_1^7 - 216su_6t_1^7 - 810s_4u_6t_1^7 - 198u_1u_6t_1^7 + 54s^3t_1^6$$
$$-486s_4^3t_1^6 + 54u_1^3t_1^6 - 486u_6^3t_1^6 - 729ss_4^2t_1^6 + 194su_1^2t_1^6 - 153s_4u_1^2t_1^6 - 783su_6^2t_1^6 - 2106s_4u_6^2t_1^6$$
$$-729u_1u_6^2t_1^6 - 9s^2s_4t_1^6 + 194s^2u_1t_1^6 - 783s_4^2u_1t_1^6 - 288m^2su_1t_1^6 - 137ss_4u_1t_1^6 - 153s^2u_6t_1^6$$
$$-2106s_4^2u_6t_1^6 - 9u_1^2u_6t_1^6 - 1917ss_4u_6t_1^6 - 137su_1u_6t_1^6$$
$$-1917s_4u_1u_6t_1^6 + 36s^4t_1^5 - 324s_4^4t_1^5 + 36u_1^4t_1^5 - 324u_6^4t_1^5$$
$$-909ss_4^3t_1^5 + 192su_1^3t_1^5 + 27s_4u_1^3t_1^5 - 963su_6^3t_1^5 - 2268s_4u_6^3t_1^5 - 909u_1u_6^3t_1^5$$
$$-90s^2s_4^2t_1^5 + 312s^2u_1^2t_1^5 - 576s_4^2u_1^2t_1^5 - 702m^2su_1^2t_1^5 + 488ss_4u_1^2t_1^5 - 576s^2u_6^2t_1^5$$
$$-3888s_4^2u_6^2t_1^5 - 90u_1^2u_6^2t_1^5 - 4212ss_4u_6^2t_1^5 - 721su_1u_6^2t_1^5 - 3996s_4u_1u_6^2t_1^5 + 351s^3s_4t_1^5$$
$$+192s^3u_1t_1^5 - 963s_4^3u_1t_1^5 - 702m^2s^2u_1t_1^5 - 721ss_4^2u_1t_1^5 + 884s^2s_4u_1t_1^5$$
$$-1062m^2ss_4u_1t_1^5 + 27s^3u_6t_1^5 - 2268s_4^3u_6t_1^5 + 351u_1^3u_6t_1^5 - 3996ss_4^2u_6t_1^5 + 884su_1^2u_6t_1^5$$
$$-1071s_4u_1^2u_6t_1^5 - 1071s^2s_4u_6t_1^5 + 488s^2u_1u_6t_1^5 - 4212s_4^2u_1u_6t_1^5 - 1062m^2su_1u_6t_1^5$$
$$+9s^5t_1^4 - 81s_4^5t_1^4 + 9u_1^5t_1^4 - 81u_6^5t_1^4 - 468ss_4^4t_1^4 + 77su_1^4t_1^4 + 54s_4u_1^4t_1^4 - 1784ss_4u_1u_6t_1^5$$
$$-486su_6^4t_1^4 - 1053s_4u_6^4t_1^4 - 468u_1u_6^4t_1^4 + 72s^2s_4^3t_1^4 + 186s^2u_1^3t_1^4 - 99s_4^2u_1^3t_1^4 - 604m^2su_1^3t_1^4$$
$$+603ss_4u_1^3t_1^4 - 549s^2u_6^3t_1^4 - 2754s_4^2u_6^3t_1^4 + 72u_1^2u_6^3t_1^4$$
$$-3627ss_4u_6^3t_1^4 - 711su_1u_6^3t_1^4 - 3105s_4u_1u_6^3t_1^4 + 882s^3s_4^2t_1^4$$
$$+186s^3u_1^2t_1^4 - 549s_4^3u_1^2t_1^4 - 1064m^2s^2u_1^2t_1^4 + 510ss_4^2u_1^2t_1^4$$
$$+1602s^2s_4u_1^2t_1^4 - 1764m^2ss_4u_1^2t_1^4 - 99s^3u_6^2t_1^4 - 2754s_4^3u_6^2t_1^4$$
$$+882u_1^3u_6^2t_1^4 - 5778ss_4^2u_6^2t_1^4 + 1716su_1^2u_6^2t_1^4 - 1134s_4u_1^2u_6^2t_1^4 - 2160s^2s_4u_6^2t_1^4+$$

$$
\begin{aligned}
&+510s^2u_1u_6^2t_1^4-5778s_4^2u_1u_6^2t_1^4-1584m^2su_1u_6^2t_1^4-2574ss_4u_1u_6^2t_1^4+342s^4s_4t_1^4\\
&+77s^4u_1t_1^4-486s_4^4u_1t_1^4-604m^2s^3u_1t_1^4-711ss_4^3u_1t_1^4+1716s^2s_4^2u_1t_1^4\\
&-1584m^2ss_4^2u_1t_1^4+1395s^3s_4u_1t_1^4-1926m^2s^2s_4u_1t_1^4+54s^4u_6t_1^4-1053s_4^4u_6t_1^4\\
&+342u_1^4u_6t_1^4-3105ss_4^3u_6t_1^4+1395su_1^3u_6t_1^4+459s_4u_1^3u_6t_1^4-1134s^2s_4^2u_6t_1^4\\
&+1602s^2u_1^2u_6t_1^4-2160s_4^2u_1^2u_6t_1^4-1926m^2su_1^2u_6t_1^4+2496ss_4u_1^2u_6t_1^4\\
&+459s^3s_4u_6t_1^4+603s^3u_1u_6t_1^4-3627s_4^3u_1u_6t_1^4-1764m^2s^2u_1u_6t_1^4\\
&-2574ss_4^2u_1u_6t_1^4+2496s^2s_4u_1u_6t_1^4-2844m^2ss_4u_1u_6t_1^4-81ss_4^5t_1^3+8su_1^5t_1^3+9s_4u_1^5t_1^3\\
&-81su_6^5t_1^3-162s_4u_6^5t_1^3-81u_1u_6^5t_1^3+198s^2s_4^4t_1^3+32s^2u_1^4t_1^3+18s_4^2u_1^4t_1^3-254m^2su_1^4t_1^3\\
&+184ss_4u_1^4t_1^3-162s^2u_6^4t_1^3-648s_4^2u_6^4t_1^3+198u_1^2u_6^4t_1^3-1296ss_4u_6^4t_1^3-207su_1u_6^4t_1^3\\
&-810s_4u_1u_6^4t_1^3+1008s^3s_4^3t_1^3+48s^3u_1^3t_1^3-72s_4^3u_1^3t_1^3-474m^2s^2u_1^3t_1^3+609ss_4^2u_1^3t_1^3\\
&+813s^2s_4u_1^3t_1^3-828m^2ss_4u_1^3t_1^3-72s^3u_6^3t_1^3-972s_4^3u_6^3t_1^3\\
&+1008u_1^3u_6^3t_1^3-2997ss_4^2u_6^3t_1^3+1629su_1^2u_6^3t_1^3+351s_4u_1^2u_6^3t_1^3-1341s^2s_4u_6^3t_1^3\\
&+315s^2u_1u_6^3t_1^3-2511s_4^2u_1u_6^3t_1^3-1134m^2su_1u_6^3t_1^3-522ss_4u_1u_6^3t_1^3\\
&+738s^4s_4^2t_1^3+32s^4u_1^2t_1^3-162s_4^4u_1^2t_1^3-474m^2s^3u_1^2t_1^3+315ss_4^3u_1^2t_1^3+2424s^2s_4^2u_1^2t_1^3\\
&-1548m^2ss_4^2u_1^2t_1^3+1209s^3s_4u_1^2t_1^3-1512m^2s^2s_4u_1^2t_1^3+18s^4u_6^2t_1^3-648s_4^4u_6^2t_1^3+738u_1^4u_6^2t_1^3\\
&-2511ss_4^3u_6^2t_1^3+2571su_1^3u_6^2t_1^3+1539s_4u_1^3u_6^2t_1^3-1026s^2s_4^2u_6^2t_1^3+2424s^2u_1^2u_6^2t_1^3\\
&-1026s_4^2u_1^2u_6^2t_1^3-2196m^2su_1^2u_6^2t_1^3+5022ss_4u_1^2u_6^2t_1^3+135s^3s_4u_6^2t_1^3\\
&+609s^3u_1u_6^2t_1^3-2997s_4^3u_1u_6^2t_1^3-1548m^2s^2u_1u_6^2t_1^3-630ss_4^2u_1u_6^2t_1^3-2772m^2ss_4u_1^2u_6t_1^3\\
&+3384s^2s_4u_1u_6^2t_1^3-2754m^2ss_4u_1u_6^2t_1^3+99s^5s_4t_1^3+8s^5u_1t_1^3-81s_4^5u_1t_1^3\\
&-254m^2s^4u_1t_1^3-207ss_4^4u_1t_1^3+1629s^2s_4^3u_1t_1^3-1134m^2ss_4^3u_1t_1^3+2571s^3s_4^2u_1t_1^3\\
&-2196m^2s^2s_4^2u_1t_1^3+670s^4s_4u_1t_1^3-1296m^2s^3s_4u_1t_1^3+9s^5u_6t_1^3-162s_4^5u_6t_1^3\\
&+99u_1^5u_6t_1^3-810ss_4^4u_6t_1^3+670su_1^4u_6t_1^3+513s_4u_1^4u_6t_1^3+351s^2s_4^3u_6t_1^3+1209s^2u_1^3u_6t_1^3\\
&+135s_4^2u_1^3u_6t_1^3-1296m^2su_1^3u_6t_1^3+3288ss_4u_1^3u_6t_1^3+1539s^3s_4^2u_6t_1^3+813s^3u_1^2u_6t_1^3\\
&-1341s_4^3u_1^2u_6t_1^3-1512m^2s^2u_1^2u_6t_1^3+3384ss_4^2u_1^2u_6t_1^3+5550s^2s_4u_1^2u_6t_1^3\\
&+513s^4s_4u_6t_1^3+184s^4u_1u_6t_1^3-1296s_4^4u_1u_6t_1^3-828m^2s^3u_1u_6t_1^3-522ss_4^3u_1u_6t_1^3\\
&-2754m^2ss_4^2u_1u_6t_1^3+3288s^3s_4u_1u_6t_1^3-2772m^2s^2s_4u_1u_6t_1^3+81s^2s_4^5t_1^2-64m^2su_1^5t_1^2\\
&+8ss_4u_1^5t_1^2+81u_1^2u_6^5t_1^2-162ss_4u_6^5t_1^2+504s^3s_4^4t_1^2-112m^2s^2u_1^4t_1^2+107ss_4^2u_1^4t_1^2\\
&+104s^2s_4u_1^4t_1^2-126m^2ss_4u_1^4t_1^2+504u_1^3u_6^4t_1^2-486ss_4^2u_6^4t_1^2+5022s^2s_4^2u_1u_6t_1^3\\
&+684su_1^2u_6^4t_1^2+729s_4u_1^2u_6^4t_1^2-243s^2s_4u_6^4t_1^2+99s^2u_1u_6^4t_1^2-162s_4^2u_1u_6^4t_1^2\\
&-324m^2su_1u_6^4t_1^2+567ss_4u_1u_6^4t_1^2+594s^4s_4^3t_1^2-96m^2s^3u_1^3t_1^2+198ss_4^3u_1^3t_1^2+861s^2s_4^2u_1^3t_1^2\\
&-288m^2ss_4^2u_1^3t_1^2+264s^3s_4u_1^3t_1^2+72m^2s^2s_4u_1^3t_1^2+594u_1^4u_6^3t_1^2-486ss_4^3u_6^3t_1^2+1773su_1^3u_6^3t_1^2\\
&+1593s_4u_1^3u_6^3t_1^2+81s^2s_4^2u_6^3t_1^2+1377s^2u_1^2u_6^3t_1^2+972s_4^2u_1^2u_6^3t_1^2-1296m^2su_1^2u_6^3t_1^2\\
&+4059ss_4u_1^2u_6^3t_1^2+18s^3s_4u_6^3t_1^2+198s^3u_1u_6^3t_1^2-486s_4^3u_1u_6^3t_1^2-486m^2s^2u_1u_6^3t_1^2\\
&+1701ss_4^2u_1u_6^3t_1^2+2322s^2s_4u_1u_6^3t_1^2-972m^2ss_4u_1u_6^3t_1^2+171s^5s_4^2t_1^2-112m^2s^4u_1^2t_1^2+
\end{aligned}
$$

$$+99ss_4^4u_1^2t_1^2 + 1377s^2s_4^3u_1^2t_1^2 - 486m^2ss_4^3u_1^2t_1^2 + 1572s^3s_4^2u_1^2t_1^2 - 738m^2s^2s_4^2u_1^2t_1^2$$
$$+248s^4s_4u_1^2t_1^2 - 378m^2s^3s_4u_1^2t_1^2 + 171u_1^5u_6^2t_1^2 - 162ss_4^4u_6^2t_1^2 + 989su_1^4u_6^2t_1^2$$
$$+864s_4u_1^4u_6^2t_1^2 + 972s^2s_4^3u_6^2t_1^2 + 1572s^2u_1^3u_6^2t_1^2 + 1107s_4^2u_1^3u_6^2t_1^2 - 1566m^2su_1^3u_6^2t_1^2$$
$$+4239ss_4u_1^3u_6^2t_1^2 + 1107s^3s_4^2u_6^2t_1^2 + 861s^3u_1^2u_6^2t_1^2 + 81s_4^3u_1^2u_6^2t_1^2 - 738m^2s^2u_1^2u_6^2t_1^2$$
$$+5598ss_4^2u_1^2u_6^2t_1^2 + 5688s^2s_4u_1^2u_6^2t_1^2 - 1458m^2ss_4u_1^2u_6^2t_1^2 + 189s^4s_4u_6^2t_1^2 + 107s^4u_1u_6^2t_1^2$$
$$-486s_4^4u_1u_6^2t_1^2 - 288m^2s^3u_1u_6^2t_1^2 + 1701ss_4^3u_1u_6^2t_1^2 + 5598s^2s_4^2u_1u_6^2t_1^2$$
$$-1296m^2ss_4^2u_1u_6^2t_1^2 + 2502s^3s_4u_1u_6^2t_1^2 - 648m^2s^2s_4u_1u_6^2t_1^2 - 64m^2s^5u_1t_1^2 + 684s^2s_4^4u_1t_1^2$$
$$-324m^2ss_4^4u_1t_1^2 + 1773s^3s_4^3u_1t_1^2 - 1296m^2s^2s_4^3u_1t_1^2 + 989s^4s_4^2u_1t_1^2 - 1566m^2s^3s_4^2u_1t_1^2$$
$$-576m^2s^4s_4u_1t_1^2 + 80su_1^5u_6t_1^2 + 90s_4u_1^5u_6t_1^2 + 729s^2s_4^4u_6t_1^2 + 248s^2u_1^4u_6t_1^2 + 80s^5s_4u_1t_1^2$$
$$+189s_4^2u_1^4u_6t_1^2 - 576m^2su_1^4u_6t_1^2 + 997ss_4u_1^4u_6t_1^2 + 1593s^3s_4^3u_6t_1^2 + 264s^3u_1^3u_6t_1^2$$
$$+18s_4^3u_1^3u_6t_1^2 - 378m^2s^2u_1^3u_6t_1^2 + 2502ss_4^2u_1^3u_6t_1^2 + 2541s^2s_4u_1^3u_6t_1^2$$
$$-738m^2ss_4u_1^3u_6t_1^2 + 864s^4s_4^2u_6t_1^2 + 104s^4u_1^2u_6t_1^2 - 243s_4^4u_1^2u_6t_1^2 + 72m^2s^3u_1^2u_6t_1^2$$
$$+2322ss_4^3u_1^2u_6t_1^2 + 5688s^2s_4^2u_1^2u_6t_1^2 - 648m^2ss_4^2u_1^2u_6t_1^2 - 126m^2s^4u_1u_6t_1^2$$
$$+2541s^3s_4u_1^2u_6t_1^2 + 756m^2s^2s_4u_1^2u_6t_1^2 + 90s^5s_4u_6t_1^2 + 8s^5u_1u_6t_1^2 - 162s_4^5u_1u_6t_1^2$$
$$+567ss_4^4u_1u_6t_1^2 + 4059s^2s_4^3u_1u_6t_1^2 - 972m^2ss_4^3u_1u_6t_1^2 + 4239s^3s_4^2u_1u_6t_1^2$$
$$-738m^2s^3s_4u_1u_6t_1^2 + 81s^3s_4^5t_1 + 81u_1^3u_6^5t_1 + 81su_1^2u_6^5t_1 + 162s_4u_1^2u_6^5t_1 + 162ss_4u_1u_6^5t_1$$
$$+162s^4s_4^4t_1 + 72s^2s_4^2u_1^4t_1 + 144m^2s^2s_4u_1^4t_1 + 162u_1^4u_6^4t_1 + 405su_1^3u_6^4t_1 + 997s^4s_4u_1u_6t_1^2$$
$$+486s_4u_1^3u_6^4t_1 + 162s^2s_4^2u_6^4t_1 + 243s^2u_1^2u_6^4t_1 + 486s_4^2u_1^2u_6^4t_1 - 324m^2su_1^2u_6^4t_1$$
$$+1053ss_4u_1^2u_6^4t_1 + 648ss_4^2u_1u_6^4t_1 + 567s^2s_4u_1u_6^4t_1 + 81s^5s_4^3t_1 - 1458m^2s^2s_4^2u_1u_6t_1^2$$
$$+234s^2s_4^3u_1^3t_1 + 216s^3s_4^2u_1^3t_1 + 162m^2s^2s_4^2u_1^3t_1 + 144m^2s^3s_4u_1^3t_1 + 81u_1^5u_6^3t_1$$
$$+396su_1^4u_6^3t_1 + 405s_4u_1^4u_6^3t_1 + 486s^2s_4^3u_6^3t_1 + 549s^2u_1^3u_6^3t_1$$
$$+729s_4^2u_1^3u_6^3t_1 - 1134m^2su_1^3u_6^3t_1 + 1602ss_4u_1^3u_6^3t_1 + 324s^3s_4^2u_6^3t_1$$
$$+234s^3u_1^2u_6^3t_1 + 486s_4^3u_1^2u_6^3t_1 - 162m^2s^2u_1^2u_6^3t_1 + 2430ss_4^2u_1^2u_6^3t_1 + 1827s^2s_4u_1^2u_6^3t_1$$
$$-324m^2ss_4u_1^2u_6^3t_1 + 972ss_4^3u_1u_6^3t_1 + 2025s^2s_4^2u_1u_6^3t_1 + 630s^3s_4u_1u_6^3t_1 + 324m^2s^2s_4u_1u_6^3t_1$$
$$+243s^2s_4^4u_1^2t_1 + 549s^3s_4^3u_1^2t_1 - 162m^2s^2s_4^3u_1^2t_1 + 216s^4s_4^2u_1^2t_1 - 612m^2s^3s_4^2u_1^2t_1$$
$$-144m^2s^4s_4u_1^2t_1 + 72su_1^5u_6^2t_1 + 81s_4u_1^5u_6^2t_1 + 486s^2s_4^4u_6^2t_1 + 216s^2u_1^4u_6^2t_1 + 243s_4^2u_1^4u_6^2t_1$$
$$-774m^2su_1^4u_6^2t_1 + 693ss_4u_1^4u_6^2t_1 + 729s^3s_4^3u_6^2t_1 + 216s^3u_1^3u_6^2t_1 + 324s_4^3u_1^3u_6^2t_1$$
$$-612m^2s^2u_1^3u_6^2t_1 + 1827ss_4^2u_1^3u_6^2t_1 + 1440s^2s_4u_1^3u_6^2t_1 - 972m^2ss_4u_1^3u_6^2t_1$$
$$+243s^4s_4^2u_6^2t_1 + 72s^4u_1^2u_6^2t_1 + 162s_4^4u_1^2u_6^2t_1 + 162m^2s^3u_1^2u_6^2t_1 + 2025ss_4^3u_1^2u_6^2t_1$$
$$+3168s^2s_4^2u_1^2u_6^2t_1 + 324m^2ss_4^2u_1^2u_6^2t_1 + 1125s^3s_4u_1^2u_6^2t_1 + 2106m^2s^2s_4u_1^2u_6^2t_1$$
$$+648ss_4^4u_1u_6^2t_1 + 2430s^2s_4^3u_1u_6^2t_1 + 1827s^3s_4^2u_1u_6^2t_1 + 324m^2s^2s_4^2u_1u_6^2t_1$$
$$+297s^4s_4u_1u_6^2t_1 + 162m^2s^3s_4u_1u_6^2t_1 + 81s^2s_4^5u_1t_1 + 405s^3s_4^4u_1t_1 - 324m^2s^2s_4^4u_1t_1$$
$$+396s^4s_4^3u_1t_1 - 1134m^2s^3s_4^3u_1t_1 + 72s^5s_4^2u_1t_1 - 774m^2s^4s_4^2u_1t_1 - 144m^2s^5s_4u_1t_1$$
$$+162s^2s_4^5u_6t_1 - 144m^2su_1^5u_6t_1 + 72ss_4u_1^5u_6t_1 + 486s^3s_4^4u_6t_1 - 144m^2s^2u_1^4u_6t_1+$$

$$+297ss_4^2u_1^4u_6t_1 + 288s^2s_4u_1^4u_6t_1 - 162m^2ss_4u_1^4u_6t_1 + 405s^4s_4^3u_6t_1 + 144m^2s^3u_1^3u_6t_1$$
$$+630ss_4^3u_1^3u_6t_1 + 1125s^2s_4^2u_1^3u_6t_1 + 162m^2ss_4^2u_1^3u_6t_1 + 432s^3s_4u_1^3u_6t_1 + 1386m^2s^2s_4u_1^3u_6t_1$$
$$+81s^5s_4^2u_6t_1 + 144m^2s^4u_1^2u_6t_1 + 567ss_4^4u_1^2u_6t_1 + 1827s^2s_4^3u_1^2u_6t_1 + 324m^2ss_4^3u_1^2u_6t_1$$
$$+1440s^3s_4^2u_1^2u_6t_1 + 2106m^2s^2s_4^2u_1^2u_6t_1 + 288s^4s_4u_1^2u_6t_1 + 1386m^2s^3s_4u_1^2u_6t_1$$
$$+162ss_4^5u_1u_6t_1 + 1053s^2s_4^4u_1u_6t_1 + 1602s^3s_4^3u_1u_6t_1 - 324m^2s^2s_4^3u_1u_6t_1 + 693s^4s_4^2u_1u_6t_1$$
$$-972m^2s^3s_4^2u_1u_6t_1 + 72s^5s_4u_1u_6t_1 - 162m^2s^4s_4u_1u_6t_1 - 324m^2su_1^3u_6^4 - 144m^2s^3s_4^2u_1^3$$
$$-324m^2s^2u_1^3u_6^3 - 648m^2ss_4u_1^3u_6^3 + 648m^2s^2s_4u_1^2u_6^3 - 324m^2su_1^4u_6^3$$
$$-144m^2su_1^5u_6^2 - 288m^2s^2u_1^4u_6^2 - 324m^2ss_4u_1^4u_6^2 - 144m^2s^3u_1^3u_6^2$$
$$+324m^2s^2s_4u_1^3u_6^2 + 1296m^2s^2s_4^2u_1^2u_6^2 + 648m^2s^3s_4u_1^2u_6^2$$
$$-144m^2s^5s_4^2u_1 + 288m^2s^2s_4u_1^4u_6 + 648m^2s^2s_4^2u_1^3u_6 + 576m^2s^3s_4u_1^3u_6 + 648m^2s^2s_4^3u_1^2u_6$$
$$+324m^2s^3s_4^2u_1^2u_6 + 288m^2s^4s_4u_1^2u_6 - 648m^2s^3s_4^3u_1u_6 - 324m^2s^4s_4^2u_1u_6 - 324m^2ss_4^2u_1^3u_6^2$$
$$-324m^2s^3s_4^3u_1^2 - 288m^2s^4s_4^2u_1^2 - 324m^2s^3s_4^4u_1 - 324m^2s^4s_4^3u_1 - 324m^2s^3s_4^2u_1u_6^2\big)$$

$$|\mathcal{M}|^2_{\phi\,\phi\to g\tilde{t}\bar{\tilde{t}}} = -\frac{1024\alpha_s\alpha_s^2(D-4)\pi^3}{9s^2s_4^2t_1u_1(s+t_1+u_1)^2(s_4+t_1+u_1)^2}\times$$
$$\big(-144m^2u_1t_1^7 - 81ss_4^2t_1^6 - 576m^2u_1^2t_1^6 + 9s^2s_4t_1^6 - 432m^2su_1t_1^6 - 612m^2s_4u_1t_1^6$$
$$+9ss_4u_1t_1^6 - 81ss_4u_6t_1^6 - 288m^2u_1u_6t_1^6 - 243ss_4^3t_1^5 - 864m^2u_1^3t_1^5 - 126s^2s_4^2t_1^5$$
$$-1836m^2s_4u_1^2t_1^5 + 36ss_4u_1^2t_1^5 - 162ss_4u_6^2t_1^5 - 144m^2u_1u_6^2t_1^5 + 27s^3s_4t_1^5 + 54ss_4u_1^3t_1^4$$
$$-496m^2s^2u_1t_1^5 - 1116m^2s_4^2u_1t_1^5 - 225ss_4^2u_1t_1^5 + 71s^2s_4u_1t_1^5 - 1530m^2ss_4u_1t_1^5$$
$$-567ss_4^2u_6t_1^5 - 1152m^2u_1^2u_6t_1^5 - 153s^2s_4u_6t_1^5 - 720m^2su_1u_6t_1^5 - 936m^2s_4u_1u_6t_1^5$$
$$-216ss_4u_1u_6t_1^5 - 243ss_4^4t_1^4 - 576m^2u_1^4t_1^4 - 360s^2s_4^3t_1^4 - 1728m^2su_1^3t_1^4 - 1836m^2s_4u_1^3t_1^4$$
$$-81ss_4u_6^3t_1^4 + 90s^3s_4^2t_1^4 - 1408m^2s^2u_1^2t_1^4 - 2232m^2s_4^2u_1^2t_1^4 - 189ss_4^2u_1^2t_1^4 + 167s^2s_4u_1^2t_1^4$$
$$-3690m^2ss_4u_1^2t_1^4 - 729ss_4^2u_6^2t_1^4 - 576m^2u_1^2u_6^2t_1^4 - 243s^2s_4u_6^2t_1^4 - 288m^2su_1u_6^2t_1^4$$
$$-387ss_4u_1u_6^2t_1^4 + 27s^4s_4t_1^4 - 272m^2s^3u_1t_1^4 - 972m^2s_4^3u_1t_1^4 - 558ss_4^3u_1t_1^4 - 324m^2s_4u_1u_6^2t_1^4$$
$$-64s^2s_4^2u_1t_1^4 - 2232m^2ss_4^2u_1t_1^4 + 132s^3s_4u_1t_1^4 - 1512m^2s^2s_4u_1t_1^4 - 1440m^2su_1^2t_1^5$$
$$-1053ss_4^3u_6t_1^4 - 1728m^2u_1^3u_6t_1^4 - 864s^2s_4^2u_6t_1^4 - 2304m^2su_1^2u_6t_1^4$$
$$-2808m^2s_4u_1^2u_6t_1^4 - 72ss_4u_1^2u_6t_1^4 - 63s^3s_4u_6t_1^4 - 576m^2s^2u_1u_6t_1^4 - 1296m^2s_4^2u_1u_6t_1^4$$
$$-1332ss_4^2u_1u_6t_1^4 - 144s^2s_4u_1u_6t_1^4 - 2034m^2ss_4u_1u_6t_1^4 - 81ss_4^5t_1^3 - 144m^2u_1^5t_1^3 - 306s^2s_4^4t_1^3$$
$$-864m^2su_1^4t_1^3 - 612m^2s_4u_1^4t_1^3 + 36ss_4u_1^4t_1^3 + 180s^3s_4^3t_1^3 - 1392m^2s^2u_1^3t_1^3$$
$$-1116m^2s_4^2u_1^3t_1^3 - 27ss_4^2u_1^3t_1^3 + 165s^2s_4u_1^3t_1^3 - 2790m^2ss_4u_1^3t_1^3 - 243ss_4^2u_6^3t_1^3 - 81s^2s_4u_6^3t_1^3$$
$$-162ss_4u_1u_6^3t_1^3 + 234s^4s_4^2t_1^3 - 672m^2s^3u_1^2t_1^3 - 972m^2s_4^3u_1^2t_1^3 - 387ss_4^3u_1^2t_1^3 + 357s^2s_4^2u_1^2t_1^3$$
$$-3528m^2ss_4^2u_1^2t_1^3 + 207s^3s_4u_1^2t_1^3 - 3024m^2s^2s_4u_1^2t_1^3 - 891ss_4^3u_6^2t_1^3 - 864m^2u_1^3u_6^2t_1^3$$
$$-891s^2s_4^2u_6^2t_1^3 - 864m^2su_1^2u_6^2t_1^3 - 972m^2s_4u_1^2u_6^2t_1^3 - 108ss_4u_1^2u_6^2t_1^3 - 81s^3s_4u_6^2t_1^3$$
$$-324m^2s_4^2u_1u_6^2t_1^3 - 1278ss_4^2u_1u_6^2t_1^3 - 288s^2s_4u_1u_6^2t_1^3 - 648m^2ss_4u_1u_6^2t_1^3 - 144m^2s^2u_1u_6^2t_1^3+$$

$$+9s^5s_4t_1^3 - 64m^2s^4u_1t_1^3 - 324m^2s_4^4u_1t_1^3 - 405ss_4^4u_1t_1^3 - 261s^2s_4^3u_1t_1^3 - 1458m^2ss_4^3u_1t_1^3$$
$$+681s^3s_4^2u_1t_1^3 - 2052m^2s^2s_4^2u_1t_1^3 + 87s^4s_4u_1t_1^3 - 720m^2s^3s_4u_1t_1^3 - 729ss_4^4u_6t_1^3$$
$$-1152m^2u_1^4u_6t_1^3 - 1197s^2s_4^3u_6t_1^3 - 2592m^2su_1^3u_6t_1^3 - 2808m^2s_4u_1^3u_6t_1^3$$
$$+270ss_4u_1^3u_6t_1^3 - 189s^3s_4^2u_6t_1^3 - 1440m^2s^2u_1^2u_6t_1^3 - 2592m^2s_4^2u_1^2u_6t_1^3$$
$$-774ss_4^2u_1^2u_6t_1^3 + 522s^2s_4u_1^2u_6t_1^3 - 4554m^2ss_4u_1^2u_6t_1^3 + 9s^4s_4u_6t_1^3$$
$$-144m^2s^3u_1u_6t_1^3 - 648m^2s_4^3u_1u_6t_1^3 - 1926ss_4^3u_1u_6t_1^3 - 648s^2s_4^2u_1u_6t_1^3$$
$$-2430m^2ss_4^2u_1u_6t_1^3 + 180s^3s_4u_1u_6t_1^3 - 1422m^2s^2s_4u_1u_6t_1^3 - 81s^2s_4^5t_1^2 - 144m^2su_1^5t_1^2$$
$$+198s^3s_4^4t_1^2 - 544m^2s^2u_1^4t_1^2 + 18ss_4^2u_1^4t_1^2 + 68s^2s_4u_1^4t_1^2 - 630m^2ss_4u_1^4t_1^2 + 9ss_4u_1^5t_1^2$$
$$+468s^4s_4^3t_1^2 - 528m^2s^3u_1^3t_1^2 - 72ss_4^3u_1^3t_1^2 + 402s^2s_4^2u_1^3t_1^2 - 1296m^2ss_4^2u_1^3t_1^2 + 126s^3s_4u_1^3t_1^2$$
$$-1800m^2s^2s_4u_1^3t_1^2 - 162ss_4^3u_6^3t_1^2 - 243s^2s_4^2u_6^3t_1^2 - 243ss_4^2u_1u_6^3t_1^2 - 81s^2s_4u_1u_6^3t_1^2$$
$$+99s^5s_4^2t_1^2 - 128m^2s^4u_1^2t_1^2 - 162ss_4^4u_1^2t_1^2 + 297s^2s_4^3u_1^2t_1^2 - 1134m^2ss_4^3u_1^2t_1^2$$
$$+1047s^3s_4^2u_1^2t_1^2 - 2682m^2s^2s_4^2u_1^2t_1^2 + 84s^4s_4u_1^2t_1^2 - 1134m^2s^3s_4u_1^2t_1^2 - 324ss_4^4u_6^2t_1^2$$
$$-576m^2u_1^4u_6^2t_1^2 - 810s^2s_4^3u_6^2t_1^2 - 864m^2su_1^3u_6^2t_1^2 - 972m^2s_4u_1^3u_6^2t_1^2 + 378ss_4u_1^3u_6^2t_1^2$$
$$-162s^3s_4^2u_6^2t_1^2 - 288m^2s^2u_1^2u_6^2t_1^2 - 648m^2s_4^2u_1^2u_6^2t_1^2 - 126ss_4^2u_1^2u_6^2t_1^2 - 936m^2s_4u_1^4u_6t_1^2$$
$$+396s^2s_4u_1^2u_6^2t_1^2 - 1296m^2ss_4u_1^2u_6^2t_1^2 - 972ss_4^3u_1u_6^2t_1^2 - 288s^2s_4^2u_1u_6^2t_1^2 - 324m^2ss_4^4u_1t_1^2$$
$$-648m^2ss_4^2u_1u_6^2t_1^2 + 18s^3s_4u_1u_6^2t_1^2 - 324m^2s^2s_4u_1u_6^2t_1^2 - 81ss_4^5u_1t_1^2 - 126s^2s_4^4u_1t_1^2$$
$$+1071s^3s_4^3u_1t_1^2 - 1296m^2s^2s_4^3u_1t_1^2 + 690s^4s_4^2u_1t_1^2 - 738m^2s^3s_4^2u_1t_1^2 + 17s^5s_4u_1t_1^2$$
$$-126m^2s^4s_4u_1t_1^2 - 162ss_4^5u_6t_1^2 - 288m^2u_1^5u_6t_1^2 - 486s^2s_4^4u_6t_1^2 - 1152m^2su_1^4u_6t_1^2$$
$$+297ss_4u_1^4u_6t_1^2 + 117s^3s_4^3u_6t_1^2 - 1152m^2s^2u_1^3u_6t_1^2 - 1296m^2s_4^2u_1^3u_6t_1^2 + 180ss_4^2u_1^3u_6t_1^2$$
$$+936s^2s_4u_1^3u_6t_1^2 - 3006m^2ss_4u_1^3u_6t_1^2 + 108s^4s_4^2u_6t_1^2 - 288m^2s^3u_1^2u_6t_1^2 - 648m^2s_4^3u_1^2u_6t_1^2$$
$$-855ss_4^3u_1^2u_6t_1^2 + 1395s^2s_4^2u_1^2u_6t_1^2 - 3240m^2ss_4^2u_1^2u_6t_1^2 + 675s^3s_4u_1^2u_6t_1^2 - 2232m^2s^2s_4u_1^2u_6t_1^2$$
$$-972ss_4^4u_1u_6t_1^2 - 27s^2s_4^3u_1u_6t_1^2 - 972m^2ss_4^3u_1u_6t_1^2 + 945s^3s_4^2u_1u_6t_1^2 - 1458m^2s^2s_4^2u_1u_6t_1^2$$
$$+108s^4s_4u_1u_6t_1^2 - 324m^2s^3s_4u_1u_6t_1^2 + 81s^3s_4^5t_1 - 64m^2s^2u_1^5t_1 + 8s^2s_4u_1^5t_1$$
$$+342s^4s_4^4t_1 - 128m^2s^3u_1^4t_1 + 107s^2s_4^2u_1^4t_1 + 24s^3s_4u_1^4t_1 - 288m^2s^2s_4u_1^4t_1$$
$$+171s^5s_4^3t_1 - 64m^2s^4u_1^3t_1 + 198s^2s_4^3u_1^3t_1 + 528s^3s_4^2u_1^3t_1 - 774m^2s^2s_4^2u_1^3t_1 + 24s^4s_4u_1^3t_1$$
$$-414m^2s^3s_4u_1^3t_1 - 162s^2s_4^3u_6^3t_1 + 162ss_4u_1^3u_6^3t_1 + 243ss_4^2u_1^2u_6^3t_1 + 81s^2s_4u_1^2u_6^3t_1$$
$$+1035s^3s_4^3u_1^2t_1 - 810m^2s^2s_4^3u_1^2t_1 + 591s^4s_4^2u_1^2t_1 - 576m^2s^3s_4^2u_1^2t_1 + 8s^5s_4u_1^2t_1 + 99s^2s_4^4u_1^2t_1$$
$$-144m^2u_1^5u_6^2t_1 - 162s^2s_4^4u_6^2t_1 - 288m^2su_1^4u_6^2t_1 - 324m^2s_4u_1^4u_6^2t_1 + 342ss_4u_1^4u_6^2t_1$$
$$+81s^3s_4^3u_6^2t_1 - 144m^2s^2u_1^3u_6^2t_1 - 324m^2s_4^2u_1^3u_6^2t_1 + 666ss_4^2u_1^3u_6^2t_1 - 126m^2s^4s_4u_1^2t_1$$
$$+684s^2s_4u_1^3u_6^2t_1 - 648m^2ss_4u_1^3u_6^2t_1 + 243ss_4^3u_1^2u_6^2t_1 + 1413s^2s_4^2u_1^2u_6^2t_1 - 648m^2ss_4^2u_1^2u_6^2t_1$$
$$+261s^3s_4u_1^2u_6^2t_1 - 324m^2s^2s_4u_1^2u_6^2t_1 - 162ss_4^4u_1u_6^2t_1 + 567s^2s_4^3u_1u_6^2t_1 + 423s^3s_4^2u_1u_6^2t_1$$
$$-324m^2s^2s_4^2u_1u_6^2t_1 + 603s^3s_4^4u_1t_1 - 324m^2s^2s_4^4u_1t_1 + 927s^4s_4^3u_1t_1 - 162m^2s^3s_4^3u_1t_1$$
$$+170s^5s_4^2u_1t_1 - 126m^2s^4s_4^2u_1t_1 - 144m^2su_1^5u_6t_1 + 90ss_4u_1^5u_6t_1 + 405s^3s_4^4u_6t_1$$
$$-288m^2s^2u_1^4u_6t_1 + 189ss_4^2u_1^4u_6t_1 + 495s^2s_4u_1^4u_6t_1 - 486m^2ss_4u_1^4u_6t_1 + 261s^4s_4^3u_6t_1 -$$

$$
\begin{aligned}
&-144m^2s^3u_1^3u_6t_1+18ss_4^3u_1^3u_6t_1+1476s^2s_4^2u_1^3u_6t_1-810m^2ss_4^2u_1^3u_6t_1+576s^3s_4u_1^3u_6t_1\\
&-810m^2s^2s_4u_1^3u_6t_1-243ss_4^4u_1^2u_6t_1+1638s^2s_4^3u_1^2u_6t_1-324m^2ss_4^3u_1^2u_6t_1\\
&+1710s^3s_4^2u_1^2u_6t_1-1134m^2s^2s_4^2u_1^2u_6t_1+171s^4s_4u_1^2u_6t_1\\
&-324m^2s^3s_4u_1^2u_6t_1-162ss_4^5u_1u_6t_1+567s^2s_4^4u_1u_6t_1+1638s^3s_4^3u_1u_6t_1\\
&-324m^2s^2s_4^3u_1u_6t_1+423s^4s_4^2u_1u_6t_1-324m^2s^3s_4^2u_1u_6t_1+81s^4s_4^5\\
&+81s^5s_4^4+72s^3s_4^2u_1^4+234s^3s_4^3u_1^3+144s^4s_4^2u_1^3+81ss_4u_1^4u_6^3+243ss_4^2u_1^3u_6^3\\
&+81s^2s_4u_1^3u_6^3+162ss_4^3u_1^2u_6^3+243s^2s_4^2u_1^2u_6^3+162s^2s_4^3u_1u_6^3\\
&+243s^3s_4^4u_1^2+387s^4s_4^3u_1^2+72s^5s_4^2u_1^2+81ss_4u_1^5u_6^2+162s^3s_4^4u_6^2\\
&+243ss_4^2u_1^4u_6^2+243s^2s_4u_1^4u_6^2+324ss_4^3u_1^3u_6^2+810s^2s_4^2u_1^3u_6^2\\
&+162s^3s_4u_1^3u_6^2+162ss_4^4u_1^2u_6^2+891s^2s_4^3u_1^2u_6^2+567s^3s_4^2u_1^2u_6^2\\
&+324s^2s_4^4u_1u_6^2+567s^3s_4^3u_1u_6^2+81s^3s_4^5u_1+324s^4s_4^4u_1+153s^5s_4^3u_1\\
&+162s^3s_4^5u_6+72s^2s_4u_1^5u_6+162s^4s_4^4u_6+297s^2s_4^2u_1^4u_6+144s^3s_4u_1^4u_6\\
&+630s^2s_4^3u_1^3u_6+594s^3s_4^2u_1^3u_6+72s^4s_4u_1^3u_6+567s^2s_4^4u_1^2u_6\\
&+1017s^3s_4^3u_1^2u_6+297s^4s_4^2u_1^2u_6+162s^2s_4^5u_1u_6+729s^3s_4^4u_1u_6+387s^4s_4^3u_1u_6\Big)
\end{aligned}
$$

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Erklärung

Hiermit erkläre ich, dass ich die vorliegende Arbeit selbstständig verfasst und keine anderen als die angegebenen Hilfsmittel verwendet habe.

Würzburg, den 27. September 2007

Lisa Edelhäuser