12. Problemset “Theoretical Particle Physics”

July 2, 2017

Jets

12.1 $e^+e^- \to q\bar{q}g$

Compute the cross section for the process $e^+e^- \to q\bar{q}g$ differential in the energies of the outgoing quarks

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}x_1\mathrm{d}x_2}(x_1, x_2, s) = N_c \frac{4\pi\alpha^2 Q^2}{3s}\frac{\alpha_s}{2\pi}C_F\frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)} \tag{1}$$

with Q the electric charge of the quark in units of e. You may assume $s \ll M_Z^2$ and ignore the Z^0-exchange.

1. Compute the total cross section for the process $e^+e^- \to q\bar{q}$.

2. Write the matrix element T for the process $e^+e^- \to q\bar{q}g$, using p_1 and p_2 for the incoming e^- and e^+ and q_1, q_2 and k for the outgoing q, $\bar{q}$ and gluon.

3. Show that no unphysical gluons with polarization vector $\epsilon_\mu(k) = k_\mu$ are produced, i. e. verify the Ward identity $T|_{\epsilon_\mu(k)\to k_\mu} = 0$. *Hint:* use the Dirac equation for the external quarks and antiquarks, e. g.

$$\frac{1}{-\not{p} - \not{k} - m + \mathrm{i}\epsilon}\not{k}v(p) = \frac{1}{-\not{p} - \not{k} - m + \mathrm{i}\epsilon}(\not{k} + \not{p} + m)\,v(p) = -v(p) \tag{2}$$

4. Show that the squared matrix element summed over color, spins and polarizations can be written

$$\sum_{\text{colors,spins, pol.}} |T|^2 = \frac{e^4 g_s^2 Q^2}{s^2} L_{\mu\nu}(p_1, p_2, 0) H^{\mu\nu}(q_1, q_2, k) \tag{3}$$

with the leptonic tensor

$$L_{\mu\nu}(p, q, m) = \mathrm{tr}\left[(\not{p} + m)\gamma_\mu(\not{q} - m)\gamma_\nu\right] \tag{4}$$

the hadronic tensor

$$H^{\mu\nu}(q_1, q_2, k) = \sum_{\text{colors,spins},\epsilon} J^\mu(q_1, q_2, k, \epsilon) J^{\nu,*}(q_1, q_2, k, \epsilon) \tag{5}$$

and current matrix elements

$$J^\mu(q_1,q_2,k,\epsilon) = J_1^\mu(q_1,q_2,k,\epsilon) + J_2^\mu(q_1,q_2,k,\epsilon) \tag{6a}$$

$$J_1^\mu(q_1,q_2,k,\epsilon) = \frac{\bar{u}(q_1)T_a\not{\epsilon}^*(k)(\not{q}_1+\not{k}+m)\gamma^\mu v(q_2)}{(q_1+k)^2-m^2+\mathrm{i}\epsilon} \tag{6b}$$

$$J_2^\mu(q_1,q_2,k,\epsilon) = \frac{\bar{u}(q_1)\gamma^\mu(-\not{q}_2-\not{k}+m)T_a\not{\epsilon}^*(k)v(q_2)}{(q_2+k)^2-m^2+\mathrm{i}\epsilon} \tag{6c}$$

5. Use the same argument as for the Ward identity to show that

$$(q_1^\mu + q_2^\mu + k^\mu)\, J_\mu(q_1,q_2,k,\epsilon) = 0 \tag{7}$$

and using the center of mass momentum $p = p_1 + p_2 = q_1 + q_2 + k$

$$p_\mu H^{\mu\nu}(q_1,q_2,k) = p_\nu H^{\mu\nu}(q_1,q_2,k) = 0 \tag{8}$$

6. The angular dependence of $H^{\mu\nu}(q_1,q_2,k)$ is too complicated. Use the variables

$$x_1 = 2q_1p/p^2, \quad x_2 = 2q_2p/p^2, \quad x_3 = 2kp/p^2 \tag{9}$$

and the formula

$$\int \widetilde{\mathrm{d}q_1}\widetilde{\mathrm{d}q_2}\widetilde{\mathrm{d}k}\,(2\pi)^4\delta^4(q_1+q_2+k-p)\,f(x_1,x_2,x_3) = \frac{s}{128\pi^3}\int \mathrm{d}x_1\mathrm{d}x_2\,f(x_1,x_2,2-x_1-x_2) \tag{10}$$

to integrate over the angles.

7. The integral over the angles

$$\int \mathrm{d}\tilde{\Omega}\, H^{\mu\nu}(q_1,q_2,k) = \hat{H}^{\mu\nu}(p,x_1,x_2) \tag{11}$$

can only depend on the center of mass momentum p. Argue that

$$\hat{H}^{\mu\nu}(p,x_1,x_2) = \left(\frac{p^\mu p^\nu}{p^2} - g^{\mu\nu}\right)\tilde{H}(p,x_1,x_2) \tag{12}$$

with

$$\tilde{H}(p,x_1,x_2) = -\frac{1}{3}\int \mathrm{d}\tilde{\Omega}\, H^\mu{}_\mu(q_1,q_2,k) \tag{13}$$

NB: this is useful because $H^\mu{}_\mu(q_1,q_2,k)$ is much easier to compute than $H^{\mu\nu}(q_1,q_2,k)$.

8. Compute $\tilde{H}(p, x_1, x_2)$ and the differential cross section.

Formulae:

- Sums of color matrices

$$\sum_a T_a T_a = C_F \cdot \mathbf{1} = \frac{N_C^2 - 1}{2N_C} \cdot \mathbf{1} \tag{14}$$