11. Problemset "Theoretical Particle Physics"
July 5, 2017

Mixing / Two Higgs Doublets (redux)

11.1 K^0-$\overline{K^0}$ Oscillations (redux)

Consider the twodimensional Hilbert space spanned by the vectors

$$\begin{pmatrix} a(t) \\ b(t) \end{pmatrix} = a(t)\,|K^0\rangle + b(t)\,|\overline{K^0}\rangle = |\Psi(t)\rangle \tag{1}$$

of K^0-$\overline{K^0}$ superpositions in the rest frame.

1. Solve the equation of motion
$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} a(t) \\ b(t) \end{pmatrix} = \begin{pmatrix} M - \mathrm{i}\frac{\Gamma}{2} & M_{12} - \mathrm{i}\frac{\Gamma_{12}}{2} \\ \overline{M_{12} - \mathrm{i}\frac{\Gamma_{12}}{2}} & M - \mathrm{i}\frac{\Gamma}{2} \end{pmatrix} \begin{pmatrix} a(t) \\ b(t) \end{pmatrix} \tag{2}$$
with parameters $M, \Gamma \in \mathbf{R}$ and $M_{12}, \Gamma_{12} \in \mathbf{C}$.

2. Find the eigenstates
$$|K_L\rangle = \frac{1}{\sqrt{1+|\bar\epsilon|^2}}\left(|K^0_{CP=-1}\rangle + \bar\epsilon\,|K^0_{CP=+1}\rangle\right) \tag{3a}$$
$$|K_S\rangle = \frac{1}{\sqrt{1+|\bar\epsilon|^2}}\left(|K^0_{CP=+1}\rangle + \bar\epsilon\,|K^0_{CP=-1}\rangle\right)\,, \tag{3b}$$
i. e. compute $\bar\epsilon$, and show that
$$\bar\epsilon \approx \frac{\mathrm{i}}{2}\frac{\mathrm{Im}M_{12} - \mathrm{i}\mathrm{Im}\Gamma_{12}/2}{\mathrm{Re}M_{12} - \mathrm{i}\mathrm{Re}\Gamma_{12}/2} \tag{4}$$
is a good approximation. *Hint:* To simplify the calculation, write
$$\begin{pmatrix} M - \mathrm{i}\frac{\Gamma}{2} & M_{12} - \mathrm{i}\frac{\Gamma_{12}}{2} \\ \overline{M_{12} - \mathrm{i}\frac{\Gamma_{12}}{2}} & M - \mathrm{i}\frac{\Gamma}{2} \end{pmatrix} = \begin{pmatrix} A & p^2 \\ q^2 & A \end{pmatrix} \tag{5}$$
with complex numbers p, q and A. Then convince yourself that
$$2\sqrt{M_{12} - \mathrm{i}\Gamma_{12}/2}\sqrt{\overline{M_{12}} - \mathrm{i}\overline{\Gamma_{12}}/2} = (2\mathrm{Re}M_{12} - \mathrm{i}\mathrm{Re}\Gamma_{12})\left(1 + \mathcal{O}\left(\frac{\mathrm{Im}M_{12}}{\mathrm{Re}M_{12}}\right) + \mathcal{O}\left(\frac{\mathrm{Im}\Gamma_{12}}{\mathrm{Re}\Gamma_{12}}\right)\right) \tag{6}$$
and use this approximation.

3. Compute mass and width (or lifetime) of K_L and K_S. *Hint:* Since Γ_{12} is not specified, you must assume for self-consistency that $\mathrm{Re}\Gamma_{12}$ has a value that leads to $\Gamma(K_S) \gg \Gamma(K_L)$.

4. Study the time evolution of a pure state
$$|\Psi(0)\rangle = |K^0\rangle \tag{7}$$
assuming $(m_L - m_S)/\Gamma_{L,S} = \mathcal{O}(1)$.

5. Study the time evolution of a mixed state
$$\rho(0) = \frac{1}{2}\left(|K^0\rangle\langle K^0| + |\overline{K^0}\rangle\langle\overline{K^0}|\right) \tag{8}$$
assuming $(m_L - m_S)/\Gamma_{L,S} = \mathcal{O}(1)$.

11.2 Symmetry Breaking (redux)

Consider a symmetry breaking sector with two Higgs doublets ϕ_1 and ϕ_2 in the $(\mathbf{1},\mathbf{2})_{+1}$ representation of $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ with potential

$$\begin{aligned} V(\phi_1,\phi_2) = {}& \frac{\lambda_1}{4}\left(\phi_1^\dagger\phi_1 - \frac{v_1^2}{2}\right)^2 + \frac{\lambda_2}{4}\left(\phi_2^\dagger\phi_2 - \frac{v_2^2}{2}\right)^2 \\ & + \frac{\lambda_3}{4}\left(\left(\phi_1^\dagger\phi_1 - \frac{v_1^2}{2}\right) + \left(\phi_2^\dagger\phi_2 - \frac{v_2^2}{2}\right)\right)^2 \\ & + \frac{\lambda_4}{4}\left((\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) - (\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1)\right) \\ & + \frac{\lambda_5}{4}\left(\mathrm{Re}(\phi_1^\dagger\phi_2) - \frac{v_1v_2\cos\xi}{2}\right)^2 + \frac{\lambda_6}{4}\left(\mathrm{Im}(\phi_1^\dagger\phi_2) - \frac{v_1v_2\sin\xi}{2}\right)^2 \end{aligned} \tag{9}$$

with

$$\forall i \in \{1,2,3,4,5,6\} : \mathbf{R} \ni \lambda_i > 0 \tag{10a}$$
$$\forall i \in \{1,2\} : \mathbf{R} \ni v_i > 0 \tag{10b}$$
$$\mathbf{R} \ni \xi \in [0, 2\pi) \tag{10c}$$

and the notation
$$\tan\beta = \frac{v_2}{v_1}\,. \tag{11}$$

1. Show that (9) is minimized by
$$\langle\phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v_1\end{pmatrix}, \quad \langle\phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v_2 e^{\mathrm{i}\xi}\end{pmatrix}. \tag{12}$$

2. Why do we need a minimum where the upper components of *both* $\langle \phi_1 \rangle$ and $\langle \phi_2 \rangle$ vanish?

!!! In order to avoid CP-violation, we choose $\xi = 0$ from now on.

3. Compute the masses of the gauge bosons from

$$|D_\mu \langle \phi_1 \rangle|^2 + |D_\mu \langle \phi_2 \rangle|^2 \ . \tag{13}$$

4. Find the Goldstone bosons by expanding around (12).

5. Show that there are two charged and one neutral Goldstone bosons.

6. Determine the masses of the remaining five physical scalar fields (there should be two charged and three neutral ones).