8. Problemset "Theoretical Particle Physics"
June 14, 2017

Gauge Boson Interactions

8.1 Feynman Rules

Study the $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ gauge bosons

$$\vec{W}_{\mu\nu} = \partial_\mu \vec{W}_\nu - \partial_\nu \vec{W}_\mu - g\vec{W}_\mu \times \vec{W}_\nu \tag{1a}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \,. \tag{1b}$$

coupled to a Higgs doublet

$$\mathcal{L}_{\text{gauge/Higgs}} = -\frac{1}{4}\vec{W}_{\mu\nu}\vec{W}^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + (D_\mu\phi)^\dagger D^\mu\phi - \frac{\lambda}{2}\left(\phi^\dagger\phi - \frac{v^2}{2}\right)^2 \,. \tag{2}$$

Choose unitarity gauge for the Higgs field

$$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix} \tag{3}$$

and introduce the mass eigenstates for the neutral gauge boson

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_w & -\sin\theta_w \\ \sin\theta_w & \cos\theta_w \end{pmatrix} \begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix} \,. \tag{4}$$

1. Express $\mathcal{L}_{\text{gauge/Higgs}}$ in terms of the fields W^+, W^-, Z, A and h.
2. Derive all Feynman rules and express all gauge boson couplings in terms of e and θ_w.
3. Are there couplings of
   (a) 3 or 4 neutral gauge bosons or
   (b) massless gauge bosons to Higgs bosons?

8.2 $\mathrm{so}(4) \cong \mathrm{su}(2) \times \mathrm{su}(2)$

1. Find a basis for the Lie algebra so(4) generating the group SO(4) of orthogonal 4×4-matrices with unit determinant.
2. Show that generators of so(4) can be expressed by linear combinations of generators of two copies of su(2). In other words, show that $\mathrm{so}(4) \cong \mathrm{su}(2) \times \mathrm{su}(2)$.

8.3 SO(4) vs. SU(2) $\times$ SU(2)

Just as so(3) $\cong$ su(2) does not imply that the groups are isomorphic (just SO(3) $\cong$ SU(2)/$\mathbf{Z}_2$), we can *not* expect SO(4) $\cong$ SU(2) $\times$ SU(2).

1. Find the exact relationship of SO(4) and SU(2) $\times$ SU(2).