7. Problemset "Theoretical Particle Physics"
June 7, 2017

Spontaneous Symmetry Breaking

7.1 Nonlinear Sigma–Model (redux)

Consider a field in the $\mathbf{3} \times \mathbf{3}$ representation of $\mathrm{SU}(3) \times \mathrm{SU}(3)$ represented by a 3×3 matrix Σ. It transforms under $L \times R \in \mathrm{SU}(3) \times \mathrm{SU}(3)$ as

$$\Sigma \to L \Sigma R^\dagger \,. \tag{1}$$

A general Σ can be parametrized by eight fields $\{\pi_a\}_{a=1,2,\ldots,8}$

$$\Sigma = \mathrm{e}^{\mathrm{i}\lambda_a \pi_a / v} \tag{2}$$

with v an energy scale.

1. Construct a $\mathrm{SU}(3) \times \mathrm{SU}(3)$ convariant derivative D_μ with
$$D_\mu \Sigma \to L D_\mu \Sigma R^\dagger \,. \tag{3}$$
2. Expand the Lagrangian
$$\mathcal{L} = v^2 \operatorname{tr}\left((D_\mu \Sigma)^\dagger D^\mu \Sigma \right) - \frac{1}{2} \operatorname{tr}\left(F_{\mu\nu} F^{\mu\nu} \right) - \frac{1}{2\alpha} \operatorname{tr}\left(\partial^\mu A_\mu \partial^\nu A_\nu \right) \tag{4}$$
as a power series in π_a and compute all terms containing at most four fields.
3. Derive the corresponding Feynman rules:
   (a) propagators for π_a and A^a_μ,
   (b) couplings of the π_a with itself and with the gauge field A_μ.
4. What are the masses of π_a and A^a_μ?

7.2 Minima of Potentials

Consider a multiplet ϕ of fields in a real orthogonal or complex unitary representation R of a Lie group G with potential V:

- $G = \mathrm{SO}(2)$, $R = \mathbf{2}$
$$V(\phi) = \frac{1}{4}\left(\phi^T\phi - v^2\right)^2 \tag{5}$$
- $G = \mathrm{SO}(3)$, $R = \mathbf{3}$, V as in (5)
- $G = \mathrm{SO}(4)$, $R = \mathbf{4}$, V as in (5)
- $G = \mathrm{U}(1)$, $R = \mathbf{1}$,
$$V(\phi) = \frac{1}{2}\left(\overline{\phi}\phi - v^2\right)^2 \tag{6}$$
- $G = \mathrm{SU}(2)$, $R = \mathbf{2}$,
$$V(\phi) = \frac{1}{2}\left(\phi^\dagger\phi - v^2\right)^2 \tag{7}$$
- $G = \mathrm{SU}(2)$, $R = \mathbf{3}$, V as in (7)
- $G = \mathrm{SU}(3)$, $R = \mathbf{3}$, V as in (7)
- $G = \mathrm{SO}(2)$, $R = \mathbf{2} \otimes \mathbf{2}$,
$$V(\phi, \psi) = \frac{1}{2}\left(\phi^T\phi + \psi^T\psi + \alpha(\phi_1\psi_2 - \phi_2\psi_1) - v^2\right)^2 \tag{8}$$
($\alpha \in \mathbf{R}$).

For all these examples

1. show that $V(\phi)$ is invariant under G.
2. find the local and global minima ϕ_0 of the potentials.
3. using a suitable basis, determine which generators t_a of G are broken by these minima, i. e.
$$t_a\phi_0 \neq 0 \tag{9}$$
and which remain unbroken, i. e.
$$t_a\phi_0 = 0\,. \tag{10}$$
4. identify the Goldstone modes.