

## 6. Problemset “Theoretical Particle Physics”

May 31, 2017

### Symmetry Breaking (preview)

#### 6.1 Propagator

Consider the so-called *Stueckelberg Lagrangian* for a single free massive spin one boson

$$\mathcal{L} = -\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu) - \frac{1}{2\alpha}\partial^\mu A_\mu\partial^\nu A_\nu + \frac{m^2}{2}A_\mu A^\mu \quad (1)$$

with two free real parameters  $\alpha$  and  $m$ .

1. Derive the Euler-Lagrange equations of motion for  $A_\mu$ .
2. Derive the corresponding propagator.
3. Discuss the limiting cases of the propagator and the equations of motion
  - (a)  $m \rightarrow 0$
  - (b)  $\alpha \rightarrow 0$
  - (c)  $\alpha \rightarrow 1$  (“*Feynman*”)
  - (d)  $|\alpha| \rightarrow \infty$  (“*Proca*”)

and their combinations.

4. Discuss the dispersion relations for the different polarization states in dependence of  $\alpha$  and  $m$ .

## 6.2 Nonlinear Sigma–Model

Consider a field in the  $\mathbf{3} \times \mathbf{3}$  representation of  $SU(3) \times SU(3)$  represented by a  $3 \times 3$  matrix  $\Sigma$ . It transforms under  $L \times R \in SU(3) \times SU(3)$  as

$$\Sigma \rightarrow L\Sigma R^\dagger. \quad (2)$$

A general  $\Sigma$  can be parametrized by eight fields  $\{\pi_a\}_{a=1,2,\dots,8}$

$$\Sigma = e^{i\lambda_a\pi_a/v} \quad (3)$$

with  $v$  an energy scale.

1. Construct a  $SU(3) \times SU(3)$  covariant derivative  $D_\mu$  with

$$D_\mu\Sigma \rightarrow LD_\mu\Sigma R^\dagger. \quad (4)$$

2. Expand the Lagrangian

$$\mathcal{L} = v^2 \operatorname{tr}((D_\mu\Sigma)^\dagger D^\mu\Sigma) - \frac{1}{2} \operatorname{tr}(F_{\mu\nu}F^{\mu\nu}) - \frac{1}{2\alpha} \operatorname{tr}(\partial^\mu A_\mu \partial^\nu A_\nu) \quad (5)$$

as a power series in  $\pi_a$  and compute all terms containing at most four fields.

3. Derive the corresponding Feynman rules:

- (a) propagators for  $\pi_a$  and  $A_\mu^a$ ,
- (b) couplings of the  $\pi_a$  with itself and with the gauge field  $A_\mu$ .

4. What are the masses of  $\pi_a$  and  $A_\mu^a$ ?