3. Problemset "Theoretical Particle Physics"
May 8, 2015

Applications of SU(3), Representations of su(4)

3.1   SU(3) and the Harmonic Oscillator

Show that the isotropic harmonic oscillator in three dimensions

$$H = \frac{1}{2m} \sum_{i=1}^{3} p_i^2 + \frac{\omega^2}{2m} \sum_{i=1}^{3} x_i^2 \tag{1}$$

has a SU(3) ⊃ SO(3) symmetry. Identify the degenerate SU(3) multiplets **1**, **3**, **6** and **10** in the subspace of the Hilbert space corresponding to low occupation number.

3.2   Generators of su(4)

- Show that the su(4) Lie algebra has 15 independent generators.

- Using the Gell-Mann matrices as inspiration, write down a basis for the su(4) Lie algebra with

$$T_a = \frac{1}{2}\lambda_a \tag{2a}$$

$$\mathrm{tr}(T_a T_b) = \frac{1}{2}\delta_{ab} \tag{2b}$$

3.3   Roots and Weights of su(4)

The weight vectors $(m_1, m_2, \ldots, m_{\mathrm{rank}(g)})$ can be visualized as sets of points in $\mathbf{R}^{\mathrm{rank}(g)}$.

- Determine the weights in the fourdimensional defining representation and visualize them.

- Determine the weights in the complex conjugate fourdimensional representation and visualize them.

- Determine the roots (i. e. the weights in the adjoint representation) and visualize them.