10. Problemset "Theoretical Particle Physics"
June 26, 2015

Mixing

10.1 K^0-$\overline{K^0}$ Oscillations

Consider the twodimensional Hilbert space spanned by the vectors

$$\begin{pmatrix} a(t) \\ b(t) \end{pmatrix} = a(t) \left| K^0 \right\rangle + b(t) \left| \overline{K^0} \right\rangle = \left| \Psi(t) \right\rangle \tag{1}$$

of K^0-$\overline{K^0}$ superpositions in the rest frame.

1. Solve the equation of motion
$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t} \begin{pmatrix} a(t) \\ b(t) \end{pmatrix} = \begin{pmatrix} M - \mathrm{i}\frac{\Gamma}{2} & M_{12} - \mathrm{i}\frac{\Gamma_{12}}{2} \\ \overline{M_{12} - \mathrm{i}\frac{\Gamma_{12}}{2}} & M - \mathrm{i}\frac{\Gamma}{2} \end{pmatrix} \begin{pmatrix} a(t) \\ b(t) \end{pmatrix} \tag{2}$$
with parameters $M, \Gamma \in \mathbf{R}$ and $M_{12}, \Gamma_{12} \in \mathbf{C}$.

2. Find the eigenstates
$$\left| K_L \right\rangle = \frac{1}{\sqrt{1 + |\bar{\epsilon}|^2}} \left(\left| K^0_{CP=-1} \right\rangle + \bar{\epsilon} \left| K^0_{CP=+1} \right\rangle \right) \tag{3a}$$
$$\left| K_S \right\rangle = \frac{1}{\sqrt{1 + |\bar{\epsilon}|^2}} \left(\left| K^0_{CP=+1} \right\rangle + \bar{\epsilon} \left| K^0_{CP=-1} \right\rangle \right), \tag{3b}$$
i. e. compute $\bar{\epsilon}$, and show that
$$\bar{\epsilon} \approx \frac{\mathrm{i}}{2} \frac{\mathrm{Im}M_{12} - \mathrm{i}\mathrm{Im}\Gamma_{12}/2}{\mathrm{Re}M_{12} - \mathrm{i}\mathrm{Re}\Gamma_{12}/2} \tag{4}$$
is a good approximation.

3. Compute mass and width (or lifetime) of K_L and K_S.

4. Study the time evolution of a pure state
$$\left| \Psi(0) \right\rangle = \left| K^0 \right\rangle \tag{5}$$
assuming $(m_L - m_S)/\Gamma_{L,S} = \mathcal{O}(1)$.

5. Study the time evolution of a mixed state
$$\rho(0) = \frac{1}{2} \left(\left| K^0 \right\rangle \left\langle K^0 \right| + \left| \overline{K^0} \right\rangle \left\langle \overline{K^0} \right| \right) \tag{6}$$
assuming $(m_L - m_S)/\Gamma_{L,S} = \mathcal{O}(1)$.