

## 9. Problemset “Theoretical Particle Physics”

June 19, 2015

### Two Higgs Doublet Model(s)

#### 9.1 Symmetry Breaking

Consider a symmetry breaking sector with two Higgs doublets  $\phi_1$  and  $\phi_2$  in the  $(\mathbf{1}, \mathbf{2})_{+1}$  representation of  $SU(3)_C \times SU(2)_L \times U(1)_Y$  with potential

$$\begin{aligned}
 V(\phi_1, \phi_2) = & \frac{\lambda_1}{4} \left( \phi_1^\dagger \phi_1 - \frac{v_1^2}{2} \right)^2 + \frac{\lambda_2}{4} \left( \phi_2^\dagger \phi_2 - \frac{v_2^2}{2} \right)^2 \\
 & + \frac{\lambda_3}{4} \left( \left( \phi_1^\dagger \phi_1 - \frac{v_1^2}{2} \right) + \left( \phi_2^\dagger \phi_2 - \frac{v_2^2}{2} \right) \right)^2 \\
 & + \frac{\lambda_4}{4} \left( (\phi_1^\dagger \phi_1)(\phi_2^\dagger \phi_2) - (\phi_1^\dagger \phi_2)(\phi_2^\dagger \phi_1) \right) \\
 & + \frac{\lambda_5}{4} \left( \text{Re}(\phi_1^\dagger \phi_2) - \frac{v_1 v_2 \cos \xi}{2} \right)^2 + \frac{\lambda_6}{4} \left( \text{Im}(\phi_1^\dagger \phi_2) - \frac{v_1 v_2 \sin \xi}{2} \right)^2 \quad (1)
 \end{aligned}$$

with

$$\forall i \in \{1, 2, 3, 4, 5, 6\} : \mathbf{R} \ni \lambda_i > 0 \quad (2a)$$

$$\forall i \in \{1, 2\} : \mathbf{R} \ni v_i > 0 \quad (2b)$$

$$\mathbf{R} \ni \xi \in [0, 2\pi) \quad (2c)$$

and the notation

$$\tan \beta = \frac{v_1}{v_2}. \quad (3)$$

1. Show that (1) is minimized by

$$\langle \phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle \phi_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \exp i\xi \end{pmatrix}. \quad (4)$$

2. Why do we need a minimum where the upper components of *both*  $\langle \phi_1 \rangle$  and  $\langle \phi_2 \rangle$  vanish?

!!! In order to avoid *CP*-violation, we choose  $\xi = 0$  from now on.

3. Compute the masses of the gauge bosons from

$$|D_\mu \langle \phi_1 \rangle|^2 + |D_\mu \langle \phi_2 \rangle|^2. \quad (5)$$

4. Find the Goldstone bosons by expanding around (4).
5. Show that there are two charged and one neutral Goldstone bosons.
6. Determine the masses of the remaining five physical scalar fields (there should be two charged and three neutral ones).