8. Problemset "Theoretical Particle Physics"
June 12, 2015

Gauge Boson Interactions

8.1 Feynman Rules

Study the $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ gauge bosons

$$\vec{W}_{\mu\nu} = \partial_\mu \vec{W}_\nu - \partial_\nu \vec{W}_\mu - g \vec{W}_\mu \times \vec{W}_\nu \tag{1a}$$
$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \,. \tag{1b}$$

coupled to a Higgs doublet

$$\mathcal{L}_{\text{gauge/Higgs}} = -\frac{1}{4}\vec{W}_{\mu\nu}\vec{W}^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + (D_\mu\phi)^\dagger D^\mu\phi - \frac{\lambda}{2}\left(\phi^\dagger\phi - \frac{v^2}{2}\right)^2 \,. \tag{2}$$

Choose unitarity gauge for the Higgs field

$$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h \end{pmatrix} \tag{3}$$

and introduce the mass eigenstates for the neutral gauge boson

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_w & -\sin\theta_w \\ \sin\theta_w & \cos\theta_w \end{pmatrix} \begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix} \,. \tag{4}$$

1. Express $\mathcal{L}_{\text{gauge/Higgs}}$ in terms of the fields W^+, W^-, Z, A and h.
2. Derive all Feynman rules and express all gauge boson couplings in terms of e and θ_w.
3. Are there couplings of
   (a) 3 or 4 neutral gauge bosons or
   (b) massless gauge bosons to Higgs bosons?