6. Problemset "Theoretical Particle Physics"
May 29, 2015

Symmetry Breaking

6.1 Minima of Potentials

Consider a multiplet ϕ of fields in a real orthogonal or complex unitary representation R of a Lie group G with potential V:

- $G = \mathrm{SO}(2)$, $R = \mathbf{2}$
$$V(\phi) = \frac{1}{4}\left(\phi^T\phi - v^2\right)^2 \tag{1}$$
- $G = \mathrm{SO}(3)$, $R = \mathbf{3}$, V as in (1)
- $G = \mathrm{SO}(4)$, $R = \mathbf{4}$, V as in (1)
- $G = \mathrm{U}(1)$, $R = \mathbf{1}$,
$$V(\phi) = \frac{1}{2}\left(\overline{\phi}\phi - v^2\right)^2 \tag{2}$$
- $G = \mathrm{SU}(2)$, $R = \mathbf{2}$,
$$V(\phi) = \frac{1}{2}\left(\phi^\dagger\phi - v^2\right)^2 \tag{3}$$
- $G = \mathrm{SU}(2)$, $R = \mathbf{3}$, V as in (3)
- $G = \mathrm{SU}(3)$, $R = \mathbf{3}$, V as in (3)
- $G = \mathrm{SO}(3)$, $R = \mathbf{3}$,
$$V(\phi) = \frac{1}{4}\left(\phi^T\phi + \alpha\sum_{i,j,k=1}^{3}\epsilon_{ijk}\phi_i\phi_j\phi_k - v^2\right)^2 \tag{4}$$
($\alpha \in \mathbf{R}$).
- $G = \mathrm{SU}(2)$, $R = \mathbf{2}$,
$$V(\phi) = \frac{1}{2}\left(\phi^\dagger\phi + \alpha(\overline{\phi_1}\phi_2 - \overline{\phi_2}\phi_1) - v^2\right)^2 \tag{5}$$
($\alpha \in \mathbf{R}$).

For all these examples

1. show that $V(\phi)$ is invariant under G.
2. find the local and global minima ϕ_0 of the potentials.
3. using a suitable basis, determine which generators t_a of G are broken by these minima, i. e.
$$t_a \phi_0 \neq 0 \tag{6}$$
and which remain unbroken, i. e.
$$t_a \phi_0 = 0\,. \tag{7}$$
4. identify the Goldstone modes.