5. Problemset "Theoretical Particle Physics"
May 22, 2015 (rev)

Gauge Lagrangian

5.1 Propagator

Consider the so-called *Stueckelberg Lagrangian* for a single free massive spin one boson

$$\mathcal{L} = -\frac{1}{4}\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)\left(\partial^\mu A^\nu - \partial^\nu A^\mu\right) - \frac{1}{2\alpha}\partial^\mu A_\mu \partial^\nu A_\nu + \frac{m^2}{2} A_\mu A^\mu \tag{1}$$

with two free real parameters α and m.

1. Derive the Euler-Lagragrange equations of motion for A_μ.
2. Derive the corresponding propagator.
3. Discuss the limiting cases of the propagator and the equations of motion
   (a) $m \to 0$
   (b) $\alpha \to 0$
   (c) $\alpha \to 1$ ("*Feynman*")
   (d) $|\alpha| \to \infty$ ("*Proca*")

   and their combinations.
4. Discuss the dispersion relations for the different polarization states in dependence of α and m.

5.2 Nonlinear Sigma–Model

Consider a field in the $\mathbf{3} \times \mathbf{3}$ representation of $\mathrm{SU}(3) \times \mathrm{SU}(3)$ represented by a 3×3 matrix Σ. It transforms under $L \times R \in \mathrm{SU}(3) \times \mathrm{SU}(3)$ as

$$\Sigma \to L \Sigma R^\dagger \,. \tag{2}$$

A general Σ can be parametrized by eight fields $\{\pi_a\}_{a=1,2,\ldots,8}$

$$\Sigma = \mathrm{e}^{\mathrm{i}\lambda_a \pi_a / v} \tag{3}$$

with v an energy scale.

1. Construct a $\mathrm{SU}(3) \times \mathrm{SU}(3)$ convariant derivative D_μ with

$$D_\mu \Sigma \to L D_\mu \Sigma R^\dagger \,. \tag{4}$$

2. Expand the Lagrangian

$$\mathcal{L} = v^2 \operatorname{tr}\left((D_\mu \Sigma)^\dagger D^\mu \Sigma\right) - \frac{1}{2} \operatorname{tr}\left(F_{\mu\nu} F^{\mu\nu}\right) - \frac{1}{2\alpha} \operatorname{tr}\left(\partial^\mu A_\mu \partial^\nu A_\nu\right) \tag{5}$$

as a power series in π_a and compute all terms containing at most four fields.

3. Derive the corresponding Feynman rules:

   (a) propagators for π_a and A_μ^a,

   (b) couplings of the π_a with itself and with the gauge field A_μ.

4. What are the masses of π_a and A_μ^a?