4. Problemset "Theoretical Particle Physics"
May 19, 2015

Noether's Theorem in Field Theory

4.1 Lorentz Transformations

Consider infinitesimal Lorentz transformations

$$\delta_\omega x^\mu = \omega^\mu{}_\nu x^\nu \,. \tag{1}$$

1. Derive the conditions on the matrix $\omega^\mu{}_\nu$.
2. Derive the transformations of a *scalar* field $\delta_\omega \phi$.
3. Derive the term σ^μ_ω in the derivation of Noether's theorem.
4. Derive the conserved currents following from invariance under Lorentz transformations.

4.2 Energy Momentum Tensor

Compute energy momentum tensors for degenerate multiplets of

1. real scalar field
$$\mathcal{L} = \frac{1}{2}\sum_i \partial_\mu\phi_i\partial^\mu\phi_i - \frac{m^2}{2}\sum_i \phi_i\phi_i \tag{2}$$
2. spin-1/2 fermions
$$\mathcal{L} = \sum_i \bar\psi_i \left(\mathrm{i}\gamma^\mu\partial_\mu - m\right)\psi_i \tag{3}$$
3. gauge fields
$$\mathcal{L} = -\frac{1}{2}\,\mathrm{tr}\left(F_{\mu\nu}F^{\mu\nu}\right) \,. \tag{4}$$

Are they symmetric or can you make them symmetric?

4.3 Equations of Motion

Derive equations of motion for free scalars, spin-1/2 fermions and gauge fields. Use them to verify the conservation of the energy momentum tensor by explicit calculcation.