3. Problemset "Theoretical Particle Physics"
May 8, 2015

Applications of SU(3)

3.1 SU(3) and the Harmonic Oscillator

Show that the isotropic harmonic oscillator in three dimensions

$$H = \frac{1}{2m}\sum_{i=1}^{3} p_i^2 + \frac{\omega^2}{2m}\sum_{i=1}^{3} x_i^2 \tag{1}$$

has a SU(3) $\supset$ SO(3) symmetry. Identify the degenerate SU(3) multiplets **1**, **3**, $\bar{\mathbf{3}}$ **6** $\bar{\mathbf{6}}$ and **8** in the subspace of the Hilbert space corresponding to low occupation number.

3.2 Gell-Mann–Okubo Relation for Pseudoscalars

Repeat the derivation of the Gell-Mann–Okubo relation from the lecture for the π, K and η mesons in the **8** of $\mathrm{SU}(3)_F$.

Assume again that the Hamiltonian consists of an SU(3) invariant part and a part transforming like thr hypercharge

$$Y = \frac{2}{\sqrt{3}} H_2 = \frac{2}{\sqrt{3}} T_8 \,. \tag{2}$$

Note that for bosons, i. e. (pseudo)scalars and vectors, the expectation of the Hamiltonian will be the square of the mass

$$M^2 = \langle\Psi|H|\Psi\rangle = \langle\Psi|H^{\mathbf{1}}|\Psi\rangle + \langle\Psi|H^{\mathbf{8}}|\Psi\rangle \,. \tag{3}$$

Get the masses from the *Particle Data Group* at `http://pdg.lbl.gov/`.

3.3 Baryon Magnetic Moments

Use an argument similar to the derivation of the Gell-Mann–Okubo relation from the lecture to derice relations among the magnetic moments the p, n, Σ, Ξ and Λ baryons in the **8** of $\mathrm{SU}(3)_F$.

Assume that the magnetic moment is proportional to the expectation of the electric charge

$$Q = T_3 + \frac{Y}{2}\,, \tag{4}$$

i. e.

$$\mu = \text{const.} \cdot \langle \Psi | Q | \Psi \rangle\,. \tag{5}$$