2. Problemset "Theoretical Particle Physics"
May 5, 2015

su(3) and su(4) Representations

2.1 Generators

- Show that the su(4) Lie algebra has independent 15 generators.
- Using the Gell-Mann matrices as inspiration, write down a basis for the su(4) Lie algebra with

$$T_a = \frac{1}{2}\lambda_a \tag{1a}$$

$$\mathrm{tr}(T_a T_b) = \frac{1}{2}\delta_{ab} \tag{1b}$$

2.2 Roots and Weights

- Determine the weights in the fourdimensional defining representation and visualize them.
- Determine the weights in the complex conjugate fourdimensional representation and visualize them.
- Determine the roots and visualize them.

2.3 su(2) ⊂ su(3) ⊂ su(4)

Determine the subspaces in the defining, complex conjugate and adjoint representations of su(4) that correspond to irreducible representations of su(3) and su(2) subalgebras.

2.4 SU(3) and the Harmonic Oscillator

Show that the isotropic harmonic oscillator in three dimensions

$$H = \frac{1}{2m}\sum_{i=1}^{3} p_i^2 + \frac{\omega^2}{2m}\sum_{i=1}^{3} x_i^2 \tag{2}$$

has a SU(3) $\supset$ SO(3) symmetry. Identify the degenerate SU(3) multiplets **1**, **3**, $\bar{\mathbf{3}}$ and **8** in the subspace of the hilbert space corresponding to low occupation number.