12. Problemset Relativistic Quantum Field Theory

January 14, 2018

Compton Scattering

12.1 Unpolarized

Consider the amplitude for Compton scattering from the lecture

$$T = -e^2 \bar{u}_{\sigma'}(p') \left(\frac{\not{\epsilon}^*_{\lambda'}(k') \, (\not{p} + \not{k} + m) \, \not{\epsilon}_{\lambda}(k)}{2pk} - \frac{\not{\epsilon}_{\lambda}(k) \, (\not{p} - \not{k}' + m) \, \not{\epsilon}^*_{\lambda'}(k')}{2pk'} \right) u_{\sigma}(p) \,. \quad (1)$$

1. Compute

$$\sum_{\substack{\text{spins} \\ \text{polarizations}}} |T|^2 \quad (2)$$

directly using the simplified polarization sum

$$\sum_{\sigma=-1,1} \epsilon^{\mu}_{\sigma}(k) \epsilon^{*,\nu}_{\sigma}(k) = -g_{\mu\nu} \,. \quad (3)$$

2. Compute the differential cross section for unpolarized photons and electrons in the rest frame of the incoming electron.

3. Derive the nonrelativistic limit $E_\gamma \ll m_e$.

12.2 Polarized

Consider the amplitude for Compton scattering from the lecture

$$T = e^2 \bar{u}_{\sigma'}(p') \left(\frac{\not\epsilon^*_{\lambda'}(k') \not\epsilon_\lambda(k) \not k}{2pk} + \frac{\not\epsilon_\lambda(k) \not\epsilon^*_{\lambda'}(k') \not k'}{2pk'} \right) u_\sigma(p) \tag{4}$$

in the gauge

$$p\epsilon_\lambda(k) = p\epsilon^*_{\lambda'}(k') = 0\,. \tag{5}$$

1. Use the Dirac algebra

$$\not a \not b = -\not b \not a + 2ab \tag{6a}$$
$$\not a \not a = a^2 \tag{6b}$$

   together with

$$k^2 = k'^2 = 0 \tag{7a}$$
$$(\epsilon_\lambda(k))^2 = (\epsilon^*_{\lambda'}(k'))^2 = -1\,, \tag{7b}$$

   transversality, energy-momentum conservation and our choice of gauge (5) to compute

$$\sum_{\text{spins}} |T|^2 \tag{8}$$

   using traces of at most four Dirac matrices.

2. Compute the differential cross section for polarized photons and unpolarized electrons in the rest frame of the incoming electron.

3. Derive the nonrelativistic limit $E_\gamma \ll m_e$.

4. Sum and average over the polarizations to compare with the result of the first exercise.