11. Problemset Relativistic Quantum Field Theory
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QED

11.1 Kinematics

Consider the scattering of two particles with masses m_1 and m_2 into two particles with masses m_3 and m_4. The respective fourmomenta are $\{p_i\}_{i=1,2,3,4}$ with $p_i^2 = m_i^3$. The Mandelstam variables are

$$s = (p_1 + p_2)^2 = (p_3 + p_4)^2 \tag{1a}$$
$$t = (p_3 - p_1)^2 = (p_4 - p_2)^2 \tag{1b}$$
$$u = (p_4 - p_1)^2 = (p_3 - p_2)^2 \,. \tag{1c}$$

Choose the 3-axis in the center of mass system according to

$$\vec{p}_1 = (0, 0, |\vec{p}_1|) \tag{2a}$$
$$\vec{p}_2 = (0, 0, -|\vec{p}_2|) = -\vec{p}_1 \tag{2b}$$

1. Show that
$$s + t + u = \sum_{i=1}^{4} m_i^2 \,. \tag{3}$$

2. Parametrize the Mandelstam variables by masses, total energy and scattering angle in the center of mass system.

3. Determine the allowed region for the Mandelstam variables.

11.2 $e^-\mu^- \to e^-\mu^-$

Consider the special case $m_1 = m_3 = m_e$ and $m_2 = m_4 = m_\mu$ of the previous problem.

1. Compute the differential cross section
$$\frac{\mathrm{d}\sigma}{\mathrm{d}\omega_e}(\cos\theta_e, s) \tag{4}$$

for unpolarized $e^-\mu^- \to e^-\mu^-$ scattering, i. e. averaged over the initial spins and summed over the final spins, using the result from the lecture

$$\sum_{\text{polarizations}} |T|^2 = 8e^4 \frac{s^2 + u^2 - 4(u+s)(m_e^2 + m_\mu^2) + 6(m_e^2 + m_\mu^2)^2}{t^2} . \tag{5}$$

2. Does the integrated cross section

$$\sigma(s) = \int \mathrm{d}\Omega_e \, \frac{\mathrm{d}\sigma}{\mathrm{d}\omega_e}(\cos\theta_e, s) \tag{6}$$

   exist?

   (a) if yes: compute it!

   (b) if no: why not?

11.3 Scalar Particle Exchange

Instead of QED, consider a theory of two spin-1/2 fermions ψ_1 and ψ_2 coupled to a scalar ϕ with stregths λ_1 and λ_2, respectively

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_I \tag{7a}$$

$$\mathcal{L}_0 = \bar\psi_1(\mathrm{i}\partial\!\!\!/ - m_1)\psi_1 + \bar\psi_2(\mathrm{i}\partial\!\!\!/ - m_2)\psi_2 + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{m^2}{2}\phi^2 \tag{7b}$$

$$\mathcal{L}_I = -\lambda_1\phi\bar\psi_1\psi_1 - \lambda_2\phi\bar\psi_2\psi_2 . \tag{7c}$$

1. Compute the scattering amplitude for $\psi_1\psi_2 \to \psi_1\psi_2$.
2. Compute the spin summed/averaged squared scattering amplitude.
3. Compute the unpolarized differential cross section.
4. Compute the unpolarized total cross section.
5. Compare with $e^-\mu^- \to e^-\mu^-$ in QED: is there an observable difference?

11.4 More Traces

1. Prove

$$\mathrm{tr}\,(\not{a}\not{b}\not{c}\not{d}) = 4\,((ab)(cd) - (ac)(bd) + (ad)(cb)) \tag{8}$$

2. Compute

$$\mathrm{tr}\,(\not{a}_1\not{a}_2\not{a}_3\not{a}_4\not{a}_5\not{a}_6) \tag{9}$$

3. Using the Lorentz transformation properties of γ_5, argue that

$$\mathrm{tr}\,(\gamma_5) = \mathrm{tr}\,(\not{a}_1\gamma_5) = \mathrm{tr}\,(\not{a}_1\not{a}_2\gamma_5) = \mathrm{tr}\,(\not{a}_1\not{a}_2\not{a}_3\gamma_5) = 0\,. \tag{10}$$

4. Which result (up to normalization) do you expect for

$$\mathrm{tr}\,(\not{a}_1\not{a}_2\not{a}_3\not{a}_4\gamma_5)? \tag{11}$$

5. Can you compute the normalization factor?