Wick & Feynman

10.1 Combinatorics

1. Show that

\[ T[\phi(x_1)\phi(x_2)\phi(x_3)] = \phi(x_1)\phi(x_2)\phi(x_3): + \phi(x_1)\phi(x_2)\phi(x_3): + \phi(x_1)\phi(x_2)\phi(x_3): \] (1)

by explicit calculation.

2. Show that

\[ T[\phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)] = \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4): + \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4): + \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4): + \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4): \] (2)

follows from

\[ T\left[\exp\left(\int d^4x \phi(x)j(x)\right)\right] = \exp\left(\int d^4x \phi(x)j(x)\right): \times \exp\left(\frac{1}{2} \int d^4xd^4y j(x) \langle 0|T[\phi(x)\phi(y)]|0\rangle j(y)\right) \] (3)

keeping track of combinatorial factors.

10.2 Fermion Propagator

Show, using the free Dirac fields constructed in section 4.4 that their propagator has the form

\[ \langle 0|T[\bar{\psi}_\alpha(x)\psi_\beta(y)]|0\rangle = \int \frac{d^4k}{(2\pi)^4} \frac{i(k + m1)_{\alpha\beta}}{k^2 - m^2 + i\epsilon} e^{-ik(x-y)}. \] (4)
10.3 Counting Diagrams

In $\phi^3$-theory with

$$L_I = -\frac{\lambda}{3!} \phi^3,$$

(5)

there are

$$(2n - 1)!! = (2n - 1) \cdot (2n - 3) \cdots 5 \cdot 3 \cdot 1 = \prod_{i=1}^{n} (2i - 1)$$

(6)

tree diagrams contributing to $2 \to n$ scattering.

1. Verify this by drawing diagrams for $n = 1$, $n = 2$ and $n = 3$.

2. Prove it by induction, constructing all tree diagrams for $2 \to n$ scattering from the tree diagrams for $2 \to (n - 1)$ scattering.

10.4 Derivative Couplings

Consider a neutral scalar field with interaction

$$L_I = -\frac{\lambda}{4} (\phi \partial_{\mu} \phi)(\phi \partial^\mu \phi)$$

(7)

1. Derive the Feynman rules.

2. Compute the differential cross section for $2 \to 2$ scattering.