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10. Problemset Relativistic Quantum Field Theory
January 10, 2018

Wick & Feynman

10.1 Combinatorics

1. Show that

$$\begin{aligned}\mathrm{T}\left[\phi(x_1)\phi(x_2)\phi(x_3)\right] &= {:}\phi(x_1)\phi(x_2)\phi(x_3){:} + {:}\wick{\c1\phi(x_1)\c1\phi(x_2)}\phi(x_3){:}\\ &+ {:}\wick{\c1\phi(x_1)\phi(x_2)\c1\phi(x_3)}{:} + {:}\wick{\phi(x_1)\c1\phi(x_2)\c1\phi(x_3)}{:} \end{aligned} \tag{1}$$

by explicit calculation.

2. Show that

$$\begin{aligned}\mathrm{T}\left[\phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\right] &= {:}\phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4){:}\\ &+ {:}\wick{\c1\phi(x_1)\c1\phi(x_2)}\phi(x_3)\phi(x_4){:} + {:}\wick{\c1\phi(x_1)\phi(x_2)\c1\phi(x_3)}\phi(x_4){:}\\ &+ {:}\wick{\c1\phi(x_1)\phi(x_2)\phi(x_3)\c1\phi(x_4)}{:} + {:}\wick{\phi(x_1)\c1\phi(x_2)\c1\phi(x_3)}\phi(x_4){:}\\ &+ {:}\wick{\phi(x_1)\c1\phi(x_2)\phi(x_3)\c1\phi(x_4)}{:} + {:}\wick{\phi(x_1)\phi(x_2)\c1\phi(x_3)\c1\phi(x_4)}{:}\\ &+ \wick{\c1\phi(x_1)\c1\phi(x_2)\c2\phi(x_3)\c2\phi(x_4)} + \wick{\c1\phi(x_1)\c2\phi(x_2)\c1\phi(x_3)\c2\phi(x_4)}\\ &+ \wick{\c1\phi(x_1)\c2\phi(x_2)\c2\phi(x_3)\c1\phi(x_4)} \end{aligned} \tag{2}$$

follows from

$$\begin{aligned}\mathrm{T}\left[\exp\left(\int \mathrm{d}^4x\,\phi(x)j(x)\right)\right] &= {:}\exp\left(\int \mathrm{d}^4x\,\phi(x)j(x)\right){:}\times\\ &\exp\left(\frac{1}{2}\int \mathrm{d}^4x\mathrm{d}^4y\,j(x)\left\langle 0\right|\mathrm{T}\left[\phi(x)\phi(y)\right]\left|0\right\rangle j(y)\right) \end{aligned} \tag{3}$$

keeping track of combinatorial factors.

10.2 Fermion Propagator

Show, using the free Dirac fields constructed in section 4.4 that their propagator has the form

$$\left\langle 0\right|\mathrm{T}\left[\psi_\alpha(x)\bar{\psi}_\beta(y)\right]\left|0\right\rangle = \int \frac{\mathrm{d}^4k}{(2\pi)^4}\frac{\mathrm{i}(\not k + m\mathbf{1})_{\alpha\beta}}{k^2 - m^2 + \mathrm{i}\epsilon}\mathrm{e}^{-\mathrm{i}k(x-y)}\,. \tag{4}$$

10.3 Counting Diagrams

In ϕ^3-theory with

$$\mathcal{L}_I = -\frac{\lambda}{3!}\phi^3\,, \tag{5}$$

there are

$$(2n-1)!! = (2n-1)\cdot(2n-3)\cdots 5\cdot 3\cdot 1 = \prod_{i=1}^{n}(2i-1) \tag{6}$$

tree diagrams contributing to $2\to n$ scattering.

1. Verify this by drawing diagrams for $n=1$, $n=2$ and $n=3$.
2. Prove it by induction, constructing all tree diagrams for $2\to n$ scattering from the tree diagrams for $2\to(n-1)$ scattering.

10.4 Derivative Couplings

Consider a neutral scalar field with interaction

$$\mathcal{L}_I = -\frac{\lambda}{4}(\phi\partial_\mu\phi)(\phi\partial^\mu\phi) \tag{7}$$

1. Derive the Feynman rules.
2. Compute the differential cross section for $2\to 2$ scattering.