9. Problemset Relativistic Quantum Field Theory

December 20, 2017

Interactions

9.1 $\lambda\phi^4/4!$

Consider the Lagrangian density

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_I \tag{1}$$

with

$$\mathcal{L}_0 = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{m^2}{2}\phi^2 \tag{2a}$$

$$\mathcal{L}_I = -\frac{\lambda}{4!}\phi^4 \tag{2b}$$

for a neutral scalar field with asymptotic representation

$$\phi(x) = \int \widetilde{\mathrm{d}p}\, \left(a(p)\mathrm{e}^{-\mathrm{i}xp} + a^\dagger(p)\mathrm{e}^{\mathrm{i}xp}\right) \tag{3}$$

1. Compute the scattering matrix element for $2 \to 2$ scattering

$$\langle q_1, q_2|S|p_1, p_2\rangle = \left\langle q_1, q_2 \middle| \mathrm{T}\left[\exp\left(\mathrm{i}\lambda\int \mathrm{d}^4x\, \mathcal{L}_I(x)\right)\right] \middle| p_1, p_2 \right\rangle \tag{4}$$

with

$$|p_1, p_2\rangle = a^\dagger(p_1)a^\dagger(p_2)\,|0\rangle \tag{5}$$

to first order in λ.

2. Verify the four-momentum conservation $p_1 + p_2 = q_1 + q_2$.

3. Compute the differential cross section

$$\frac{\mathrm{d}\sigma_{2\to2}}{\mathrm{d}\Omega_1}(\cos\theta_1, \phi_1) \tag{6}$$

and the total cross section $\sigma_{2\to2}$.

9.2 $\lambda\phi^3/3!$

Consider the Lagrangian density

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_I \tag{7}$$

with

$$\mathcal{L}_0 = \sum_{i=1}^{3} \left(\frac{1}{2} \partial_\mu \phi_i \partial^\mu \phi_i - \frac{m_i^2}{2} \phi_i^2 \right) \tag{8a}$$

$$\mathcal{L}_I = -\frac{\lambda}{2} \left(\phi_1^2 \phi_3 + \phi_2^2 \phi_3 \right) \tag{8b}$$

for three neutral scalar fields with asymptotic representations as in the first exercise.

1. Compute the scattering matrix element for $2 \to 2$ scattering

   $$\langle q_1, 2, q_2, 2 | S | p_1, 1, p_2, 1 \rangle \tag{9}$$

   with

   $$|p_1, i, p_2, j\rangle = a_i^\dagger(p_1) a_j^\dagger(p_2) \, |0\rangle \tag{10}$$

   to the first non-vanishing order in λ.

2. Verify the four-momentum conservation $p_1 + p_2 = q_1 + q_2$.

3. Compute the differential and total cross section.

4. Compute the scattering matrix element for $2 \to 2$ scattering

   $$\langle q_1, 1, q_2, 2 | S | p_1, 1, p_2, 2 \rangle \tag{11}$$

   to the first non-vanishing order in λ.

5. Verify the four-momentum conservation $p_1 + p_2 = q_1 + q_2$.

6. Compute the differential and total cross section.