

## 12. Problemset “Quantum Algebra & Dynamics”

January 18, 2019

### Free Bose Gas / Squeezed States

#### 12.1 Fock Representation

Show that

$$\omega_F(f, g) := (\Omega_F, U_F(f)V_F(g)\Omega_F) = \exp\left(-\frac{1}{4}\langle f, f \rangle - \frac{1}{4}\langle g, g \rangle - \frac{i}{2}\langle f, g \rangle\right) \quad (1)$$

with

$$\langle f, g \rangle = \int dx f(x)g(x) \quad (2)$$

in the Fock representation introduced on page 118 of the lecture notes starting with formula (6.123). See also (5.68) on page 79.

#### 12.2 Squeezed States revisited

Revisit the unitary creation operators

$$U_s(\zeta) = e^{\frac{1}{2}(\bar{\zeta}aa - \zeta a^* a^*)} \quad (3)$$

for squeezed states

$$\Psi_s(\zeta) = U_s(\zeta)\Omega. \quad (4)$$

1. Compute the expectation values

$$\langle x \rangle = (\Psi_s(\zeta), x\Psi_s(\zeta)) \quad (5a)$$

$$\langle p \rangle = (\Psi_s(\zeta), p\Psi_s(\zeta)) \quad (5b)$$

$$\langle x^2 \rangle = (\Psi_s(\zeta), x^2\Psi_s(\zeta)) \quad (5c)$$

$$\langle p^2 \rangle = (\Psi_s(\zeta), p^2\Psi_s(\zeta)) \quad (5d)$$

with

$$x = \frac{1}{\sqrt{2m\omega}}(a + a^*) \quad (6a)$$

$$p = \frac{1}{i}\sqrt{\frac{m\omega}{2}}(a - a^*) \quad (6b)$$

2. Compute the uncertainties

$$(\Delta x)^2 = \langle (x - \langle x \rangle)^2 \rangle \quad (7a)$$

$$(\Delta p)^2 = \langle (p - \langle p \rangle)^2 \rangle \quad (7b)$$

and their product.

3. Compute the time dependence of these expectations for the dynamics generated by the free Hamiltonian

$$H = \omega a^* a. \quad (8)$$