The gravitational constant

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1 Introduction

Sir Isaac Newton's gravitational law from his *Philosophiae Naturalis Principiae Mathematica* is in its modern form known as

$$|\vec{F}| = G \cdot \frac{m \cdot M}{r^2} \tag{1}$$

It describes the attractive force between the two masses m and M seperated by distance r. The strength of this force is defined by the constant of proportionality G, known as the gravitational constant. Besides the speed of light c_0, G has the longest history of measurements. In 1798 Henry Cavendish published results on his experiments to obtain the density of the earth. These results are nowadays known as the first precise ones on Newton's gravitational constant.

Most astonishing is that not much has happened to the accuracy of it. The CODATA[1] recommends G with an accuracy of $\frac{\Delta G}{G} = 1500$ ppm, while most other natural constants' accuracy is below 1 ppm. The actual[2] value is

$$G = (6.67259 \pm 0.00085) \times 10^{-11} \, \frac{m^3}{kgs^2} \tag{2}$$

Several experimenters around the world tried to measure G, as you see in table 1 on page (pagereftable1, and the fact that these results differ so widely shows that it is still a great challenge. Theoretically a determination of G is actually easy: one just has to measure the force between two known masses in a known geometry and compare the result with an appropriate estimate of $\frac{m \cdot M}{r^2}$. Determining the geometry and doing the calculation both require care, but the real challenge lies in measuring the force.

Experimenter	Year	Method	G $[10^{-11} \frac{m^3}{kgs^2}]$	$\frac{\Delta G}{G}$ [ppm]
Cavendish	1798	Torsion balance	6.75 ± 0.05	7400 (stat.)
Boys	1895	Torsion balance	6.658 ± 0.007	1000
Luther	1982	Torsion pendulum	6.6726 ± 0.0005	75
Fitzgerald	1995	Torsion balance	6.6656 ± 0.0006	90
Schwarz	1998	Free fall	6.6873 ± 0.0094	1400
Kündig	2002	Beam balance	6.67407 ± 0.00022	200

Table 1: Measurements of the gravitational constant

Some reasons why measuring the gravitational constant is so difficult, are that gravitation is the weakest of all four known fundamental forces. For example: the gravitational force between two ordinary people in a close distance is about

$$F = \frac{G \cdot M_{Jack} \cdot M_{Jill}}{Distance^2} \approx 10^{-7} N \tag{3}$$

and that´s very tiny. Furthermore one can not shield gravity. If one wanted to produce a region that was free of electric fields, one just would have to construct a conducting shell around the region. This won't work with gravitational forces. By the way Newton's law includes only point masses, no real bodies. This all is why laboratory experiments are so damageable by any disturbance. Even the smallest modification at distribution of mass in the environment of the experiment have big effects. All other forces must be known or, better, be eliminated. Due to the fact that for example the electromagnetic force compared to the gravitational force between a proton and an electron is 10^{40} times larger.

For these reasons G experiments are designed so that it is not necessary to have a gravity-free environment for their success. Most experimenters, however, would like to circumvent one major component of the gravity field that exists

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[1] Committee on Data for Science and Technology of the international Council of Scientific Unions

[2] CODATA enlarged the uncertainty by a factor of 12 in 1998

on Earth, namely the attraction due to the planet itself. Experimenters have commonly attempted to do this by using a torsion fibre to support a bar bell shaped bar. This allows horizontal forces to be expressed in twists of the fibre support without strong dependence on the local acceleration. Fibres have been most commonly used in torsion balances. It is used by measuring the shift of the equilibrium position of the beam due to the gravitational attraction of external masses.

2 Measurements

As you see in table 1 on page (pagereftable1 experiments use often the same technique measuring G, this increases the existence of systematic errors. So there were many people, and still there are, to measure it in a different way. Some even tried to measure it in space. Other experiments use mobile water tanks as masses, and some don't suspend the bar bell, but put it on a liquid mercury layer, but generally they can be described the following way. Here are some methods who measure the gravitational constant.

2.1 Torsion balance

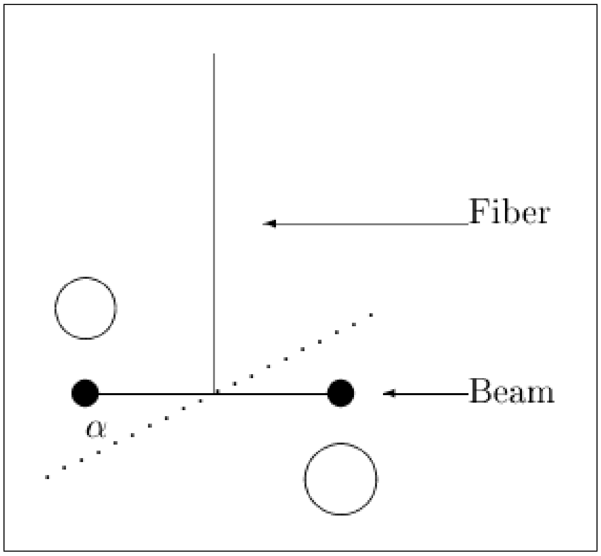

Figure 1: Torsion balance experiment

In general the torsion balance method uses two masses at each end of a bar, that is suspended, e.g. with a fibre in its middle as in figure 1 on page (pagereffigure1. Because fibres twist very easily even small gravitational forces produce a significant shift in the position of equilibrium. Now there is a newer dynamic and a former static way of measuring. The bar bell rotates like a torsion pendulum (see later on) and changes direction with a very small frequency or hangs there in peace. When it just hangs there and other kinds of mass get close to the bar bell one could measure the angle the bar bell needs for an elongation in one direction. It should enlarge, because of the attracting interaction with the additional masses. Optical instruments, like LASERS, are often used to get very precise readings nowadays, therefore a mirror is attached to the bar and the reflected beam is detected to measure the angle.

At the Physikalisch-Technische Bundesanstalt (PTB) in Braunschweig de Boer, Haars and Michaelis [2] have carried out a similar experiment for which planning began in 1976. They used a compensated torsion balance with a mercury bearing, rather than a fibre, to support the beam. This type of bearing is nearly free of static friction. The experimenters made measurements of G at different distances and between different materials.

2.2 Torsion pendulum

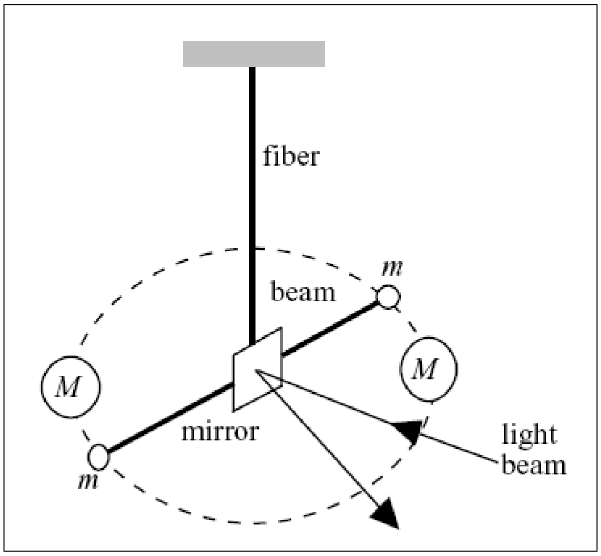

Figure 2: Torsion pendulum experiment

This method requires that the frequency of oscillation of the beam is recorded as seen in figure 2 on page (pagereffigure4. Source masses add a gravitational torque that acts on the beam either to enhance or decrease the effective torsion constant of the fibre. By quantifying the effect of the masses on the period, the value of the gravitational constant can be found. The vast majority of precision G experiments done since Cavendish have used either torsion balances or torsion pendula. Over the ninety years before the 1986 setting of the accepted value[3],

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[3] Only the uncertainty changed in 1998

more than three quarters of all G measurements used one of these two systems.

2.3 Fabry-Pérot pendulum

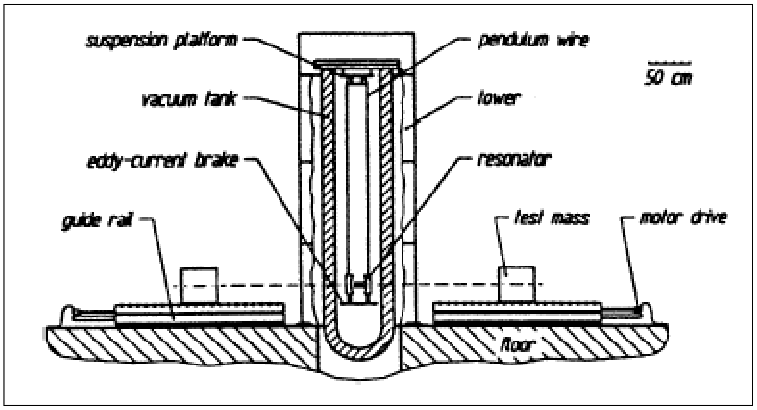

Figure 3: Fabry-Pérot experiment

The first measurement was conducted by Walech, Meyer, Piel and Schurr, at the University of Wuppertal, Germany [6]. The group used two simple pendula to support mirrors defining a microwave Fabry-Pérot resonator. By placing a source mass system on the axis defined by the line joyning the two pendula, the distance between the two pendulum bobs was changed. The resonator measured the separation change, thereby determining G as you see in figure 3 on page (pagereffigure2.

2.4 Free fall

The free-fall method, described in [4], depends on our ability to measure the amount that an external source mass changes the acceleration of a freely falling object. First a source mass is placed above the region in which the "test mass" falls. Here the gravitational pull of the source mass acts in the opposite direction to the attraction of the Earth, decreasing the downward acceleration of the test mass. Second the source mass is placed below the drop region, where it increases the Earth's attraction and the acceleration of the falling mass. If the change in acceleration can be measured accurately, and if the geometry of the source and test masses are well known, then one can determine G as seen in figure 4 on page (pagereffigure5.

2.5 Beam balance

In [9] is described that the weight difference of two test masses is measured. It is changed by the gravitational force of two movable tanks filled with a liquid of known density. G can be calculated from the measured change of the weight

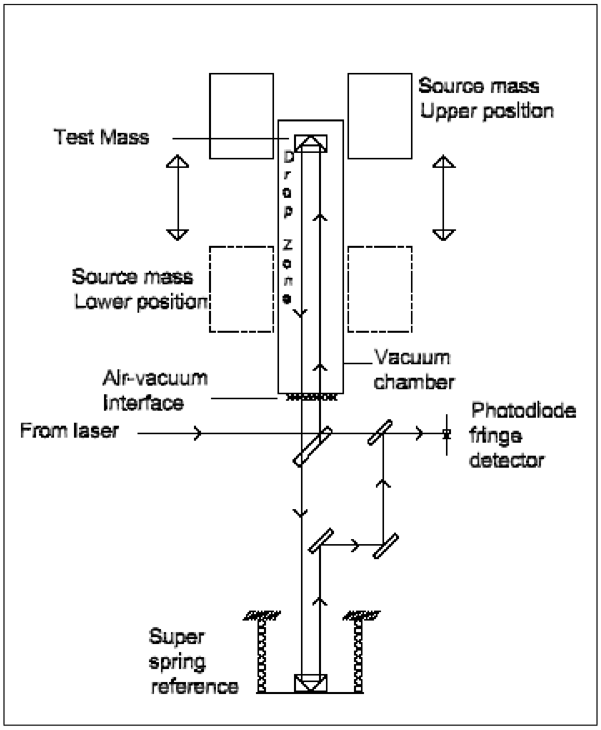

Figure 4: Free fall experiment

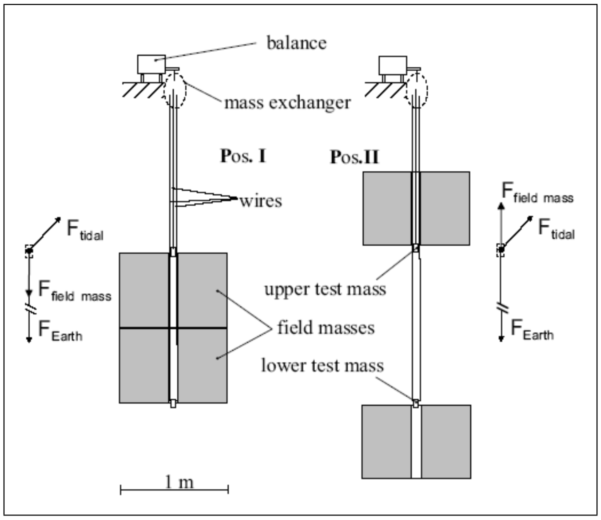

Figure 5: Beam balance experiment

difference. A few attempts to measure G by means of a beam balance have been made, none of which achieved a precision comparable to that of a torsion balance method. Owing to recent developments in beam-balance technology, the beam balance is becoming quite competitive. Further features of the experiment are an optimized mass arrangement and a favorable measuring procedure. This experiment is observable in figure 5 on page (pagereffigure3.

3 Accuracy of measurements

The measurement of the Newtonian gravitational constant, G, the characterization of its properties, and the testing of the inverse square law of gravity are problems that have been under study for many years. Let´s summarize the situation: many experiments have been effected recently, yet at the level of a quarter percent there is no clear value for G. The new experiments use a variety of methods, most moving away from twisted fibre ideology, but their results fail to converge. The large spread in results compared to small error estimates, indicates that there are large systematic errors in various results, such could be:

- inconstancy of the torsional moment of suspension
- disturbances of the ground
- influence of the ambient temperature
- inhomogeneity of the masses
- gradients in the gravitational field
- changes of the surrounding field
- magnetic or electrical influences

There are many efforts now aimed at achieving improved designs of the instrumentation systems employed in the measurement of G. It has to be possible, because there are no confirmed laboratory experiments or astronomical observations that demonstrate definitely, that G varies in either space or time, that its value depends on the temperature or any other physical property of the test masses, that G is modified by the presence of matter placed between gravitationally interacting test masses, or that any other anomalous phenomenon indeed exists, at least at presently achievable levels of measurement sensitivity and stability. G is indeed a mysterious constant, seemingly eternal and unchanging, but veiled from view by our inability to seize it. In the end one has to say that Cavendish's result was hardly corrected after over two hundred years, which was an enormous achievement.

References

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