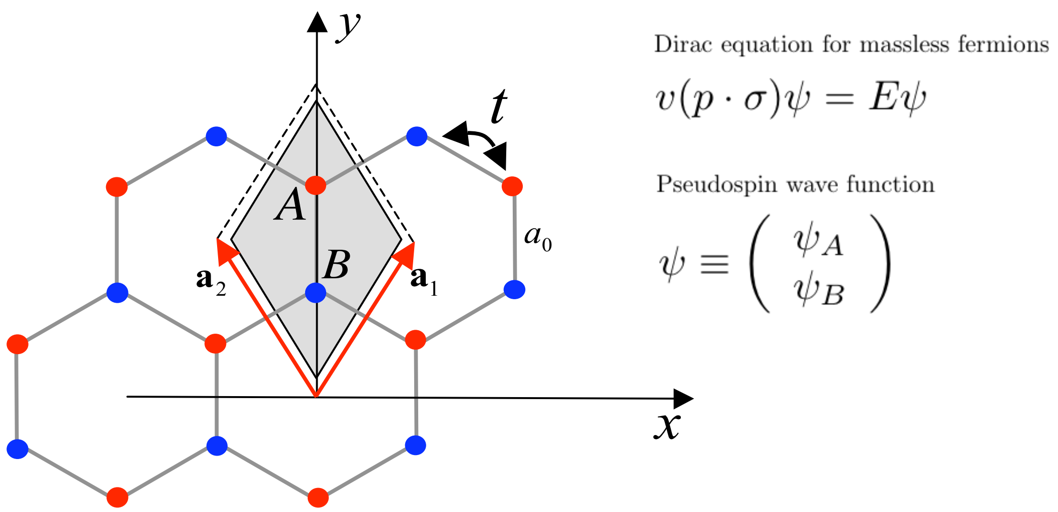
The Emmy Noether project: Correlated spin transport and spin manipulation in graphene and quantum spin Hall insulators

We are interested in mesoscopic aspects of Dirac fermions in graphene and topological insulators, as realized in HgTe-quantum wells, as well as in non-equilibrium electron transport in 1D systems which exhibit Luttinger liquid behavior (carbon nanotubes, edge states of topological insulators). Questions of spin-related phenomena (spin qubits in graphene quantum dots, creation of mobile spin-entangled electrons in superconductor-normal junctions and their mapping to photons) are central to our work.

A. Mesoscopic aspects of graphene:

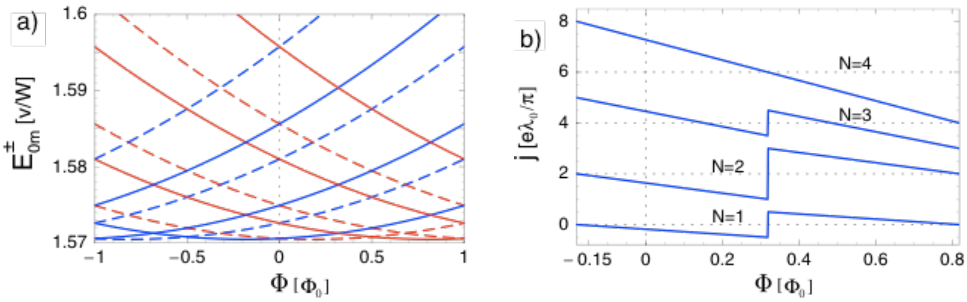
Graphene is a single layer of graphite where carbon atoms are arranged on a honeycomb lattice. The bandstructure of graphene consists of two inequivalent cones or valleys at the corners of the 1st Brillouin zone. It is a gapless semiconductor and due to two sublattices, carriers in graphene behave as if they were ultrarelativistic massless fermions described by the Dirac-Weyl Hamiltonian [1]. Although graphene is the mother-material of all graphitic allotropes (e.g. carbon nanotubes to be discussed later), its advent had to wait till 2004, when A. Geim's group was able to isolate and contact a singe layer of carbon—the first truly two-dimensional condensed matter system was created [2]. Our interest is concerned with mesoscopic phenomena in this material.



Graphene is a single layer of graphite where carbon atoms are arranged in a honeycomb lattice that has a unit cell (derived from Bravais lattice vectors **a1** and **a2**) with a two-atom basis. The band structure in a tight binding model (with nearest neighbor hopping integral t) consists of two inequivalent conical cones (or valleys) where conduction band and valence band touch in the so called Dirac point (or charge neutrality point). For low-energies around the two Dirac points, the dynamics for electrons in both valleys are described by the Dirac equation (the two valleys being connected by time-reversal) for ultrarelativistic massless fermions. The spinor structure is due to the two sublattices A and B of the lattice (and not the real spin). The electrons have a velocity $v \sim ta_0/\hbar \sim 10^6 m/s$ which is about 300 times smaller than the speed of light.

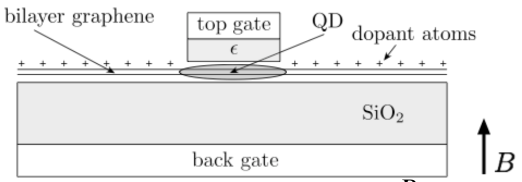
Aharonov Bohm effect in graphene rings:
We have investigated the spectrum of closed graphene rings analytically as well as numerically for the first time [3]. We have found that the valley degeneracy which disguises

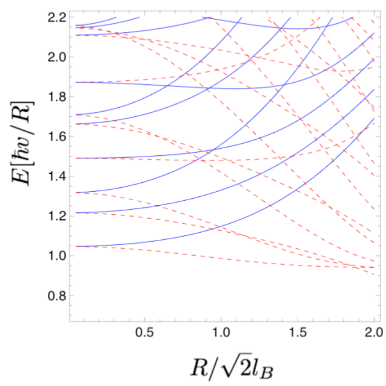
many interesting physical phenomena of massless Dirac fermions (like weak-antilocalization in magnetoresistance [4]) can be lifted controllably by an Aharonov Bohm (AB) flux (even in the absence of intervalley scattering). The breaking of the valley degeneracy is also absolutely crucial when using the electron spin in quantum dots as a quantum bit (qubit) [5,6].



a) Energy levels of a graphene ring with smooth edges as a function of AB-flux ϕ. Levels originating from the two inequivalent valleys are displayed as full and dashed lines, resp. At finite ϕ, the valley degeneracy is lifted. b) Persistent current as a function of ϕ for the first few electrons N on the ring for a single valley. The finite persistent current at ϕ=0 is peculiar to graphene and would allow for a detection of a valley-polarization (taken from [3]).

Gate-tunable quantum dots in single- and bilayer-graphene:
Isolated graphene is a gapless semiconductor (semimetal) and therefore the usual gate-defined quantum dots (QDs), as made e.g. in GaAs 2DEGs [7], cannot be straightforwardly created. However, by breaking the inversion symmetry of the graphene lattice, a band gap will open and would allow for gate-defined QDs in graphene. Inversion can be broken in graphene by applying different potentials on the two sublattices (A and B) of the honeycomb lattice which in practice can be done by interaction with a substrate (like boron-nitride [8]). In bilayer graphene, a voltage between the two layers will open a gap as well and is experimentally easier to control. We have calculated by means of the Dirac equation the bound states of such quantum dots in single- and bilayer graphene as a function of magnetic field [9]

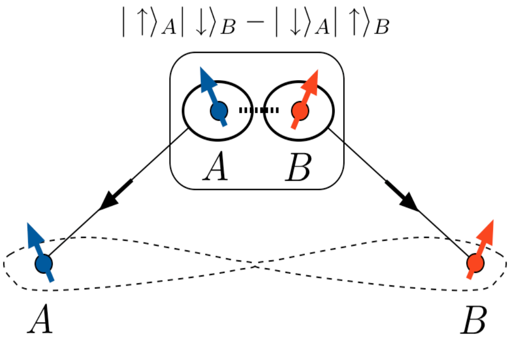




Left: Bilayer graphene QD in magnetic field B: A back gate and dopants on top of the bilayer control the voltage between the layers—leading to a controllable gap opening—-as well as the Fermi energy (band filling). An additional top gate induces a spatially inhomogeneous electrostatic potential which leads to bound states in the conduction band (or valence band) of the bilayer. Another possibility is to use a split top gate (instead of a combination of top gate and dopants) to achieve a similar confinement.
Right: Calculated bound states as function of magnetic length $l_B = \sqrt{\hbar/eB}$ for bilayer QD with radius R=25nm. Full lines and dashed lines distinguish the two valleys of bilayer graphene (similar for single layer) (from [9]).

We found that such QDs would serve as ideal hosts for spin qubits, since the valley degeneracy can be broken controllably which ensures an efficient spin exchange interaction between spins in neighboring QDs [9].

Possible future directions:
In graphene, we want to investigate the potential use of graphene QDs for a host of spin qubits. This topic is currently investigated experimentally in leading groups [10,11]. From the theory side, we need to better understand the single-particle levels of real structures (like bilayer graphene with a voltage) as well as physics of two interacting electrons. This is important for spin-based quantum computing in graphene. Graphene has weak spin-orbit interaction and sparse nuclear spins which makes graphene potentially a good spin conductor with long spin coherence times [12]. We are interested to investigate the creation of spin-entangled electrons at the interface of graphene and superconductors.

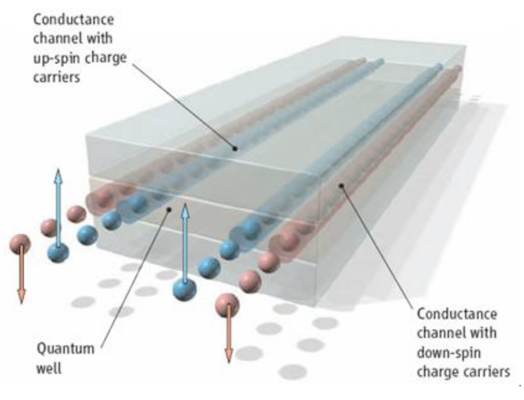


The main idea behind the EPR (Einstein-Podolsky-Rosen [13]) pairs is the spatial separation of an electron pair (A and B) that once interacted (dashed line) so that the spin wave function is entangled (shown is the case of a spin-singlet that could be created e.g. in a superconductor). The main task is the reliable spatial separation of the pair, so that the spin remains in the correlated entangled state.

Such spatially separated spin-entangled electron pairs could be used in quantum computing or quantum communication processes. The creation of entangled electron pairs with the help of Andreev (pair)-processes has been investigated in detail by us for various interacting normal electron systems [14,15,16].

B. Topological insulators:

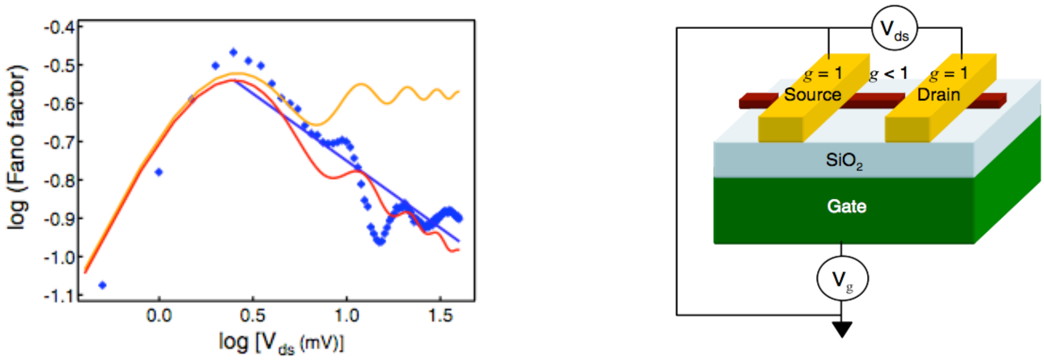
Topological insulators are materials that have insulating bulk states and edge states at their surfaces that can carry current and/or spin and are, due to the strong spin-orbit interaction, protected against disorder-induced scattering which leads to phenomena like the quantum spin Hall effect [17]. In Würzburg, the experimental group of Prof. Laurens Molenkamp intensively investigates such topological insulators in quantum wells made from HgTe [18,19]. There is an interesting connection between graphene and these quantum wells, since the low-energy physics is governed by the same effective Dirac fermions, however, this time with a tunable bulk-mass. Even more interesting, the valley-degree of freedom is replaced by the real-spin degree of freedom, which motivates us to play with the mathematical similarities between these two seemingly very different systems and to predict new interesting (spin)effects.



The HgTe-quantum wells are yet another system where carriers are described by a Dirac equation. The valley degree of freedom of graphene is played here by the real spin and the Dirac fermions are massive. If the sign of the bulk mass is negative (inverted), topologically protected edge states arise that have a definite helicity, i.e. the direction of motion is coupled to the spin of the electron. Since the backscattering between the two opposite spin direction at the same edge is not allowed by virtue of the time-reversal symmetry, the transport is quantized with a conductance of $G = 2e^2/h$. This phenomena known as the quantum spin Hall effect has been predicted in these structures [17] and measured here in Würzburg [18].

C. Non-equilibrium transport in 1D electron channels:

In one-dimensional systems, electron-electron (e-e) interaction—even if weak—shows dramatic effects on transport properties. The Fermi liquid description which beautifully works in higher dimensions breaks down in 1D and a successful alternative is the so called Luttinger liquid (LL) theory [20]. One key feature of this theory is non-linear power-law like behavior of the conductance and shot noise as a function of the bias voltage due to backscattering off impurities, which naturally appear e.g. at the interface of the 1D system with the contacts. However, spin properties in non-linear transport have not been investigated in detail so far and it is our aim to do so. We have established a theory of shot noise in carbon nanotubes taking into account scattering off weak impurities as well as the effects of the non-interacting leads [21].



Fano factor (on a log-log scale) of low-frequency shot noise in a single-walled carbon nanotube measured at 4K [22] as function of gate voltage. A clear non-linear behavior of Fano factor is observed (blue diamonds) and compared to the non-equilibrium transport theory in the framework of the inhomogeneous Luttinger liquid model (depicted on the right) in the weak backscattering regime (Fabry-Perot (FP) regime) [21]. The red curve assumes a LL interaction parameter of g=0.25 whereas the power-law fit (blue straight line) to the data relates to g=0.18. By averaging over many values of gate voltages, the fit gives g=0.21 which is close to the expected value (g=0.2-0.3) for a single-walled carbon nanotube [23,24]. The yellow curve shows the non-interacting result (g=1) of the theory which only describes FP-oscillations but no power-law behavior and clearly fails to describe the measured data.

Possible future directions:

Recently, spin-orbit (S.O.) interaction in carbon nanotubes has been found to be much more relevant than the small intrinsic S.O. coupling of carbon would suggest which is due to the curvature of the tube [25,26]. The interplay of S.O. with possible ferromagnetic contacts is one of possible open questions. Another direction leads us towards the newly discovered edge channels at the surface of a 2D topological insulator. They behave as a pair of spinless LLs with the direction of motion at one edge tightly connected to the spin of the electron. These edge states have been named a helical liquid [27]. They provide a new system where an interplay between e-e interaction and spin can be investigated. We plan to study non-equilibrium transport properties (shot noise) in these systems.

International collaborations:

Recently, we also started to work on problems related to light-matter interaction in quantum confined structures. In this direction, we have active collaborations with TU-Delft (Netherlands) and Stanford University (USA).

At TU-Delft we investigate in collaboration with Prof. Yuli Nazarov (theory) and Prof. Leo Kouwenhoven (experiment) coherent emission from an optical quantum dot attached to superconducting leads (of n- and p-type). The proposed device acts as a source of Josephson radiation in the visible range as well as a correlated two-photon source [28].

At Stanford University and NII Tokyo we have collaborated with the groups of Prof. Yoshihisa Yamamoto on several projects involving quantum simulation with solid state systems: these are the simulation of a Fermi Hubbard model with the help of surface acoustic waves launched on a 2DEG [29] and the investigation of exciton-polaritons subjected to a weak periodically modulated trapping-potential [30]. We recently investigated theoretically the necessary parameter regime where exciton-polaritons and indirect excitons in a periodic potential are expected to undergo a superfluid-to-Mott insulator phase transition [31].

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